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Loopy

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gebra in grade 8 and most of their grade 8 teachers were not ready to teach algebra. This is a prescription for trouble, and we have already seen plenty of it.

As you can see, the political circus they call education in California goes well beyond any question of the merits of reform in the math curriculum. They say California often sets the trends for the rest of the nation to follow. Decades ago that was the case when bumper stickers were all the rage. Maybe it's time to revive the one which said, "Don't follow me, I'm lost."

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As a mathematician and sculptor, I try to create artworks that are infused with an underlying sense of pattern and structure. I hope that viewers of my sculpture will consciously or unconsciously develop an appreciation for the power of mathematical ideas to enrich art. Ideally, they will be inspired to try their own explorations in the area of mathematical art. This paper considers one sculpture in some detail and provides instructions for a paper model that lets one explore its structure.

Figure 1 shows a sculpture I call *Loopy*, made of painted aluminum. It is five feet in diameter, and stands eight feet tall on its steel base. I created it for an outdoor setting at a local hospital; it is intentionally colorful and joyous. There are thirty identically shaped loops, six in each of five bright colors: red, orange, yellow, green, and blue. My vision of the form is abstract; some viewers tell me they see it as a giant brain, but that was not a conscious intention of mine. ("No, it isn't a self-portrait," I tell people.)

On examination, most viewers can discover and articulate essential aspects of the structure and color pattern: Each loop crosses four loops of the four other colors. The ends of the loops always meet in groups of three of the same color. Around each five-sided opening, there is a spiral of five loops, one in each of the five colors. No loop crosses another of the same color. Each bolt is both the vertex of a regular pentagon and the midpoint of the side of an equilateral triangle.

Each loop began as a ten-foot long strip of aluminum that was drilled for bolt holes, rolled into a loop, primed, painted, and bolted into the assembly. Thus, the thirty strips together sum to 300 feet, the length of a football field. While this arithmetic may be impressive in its own way, there is a different sort of mathematical analysis that I wish to emphasize.
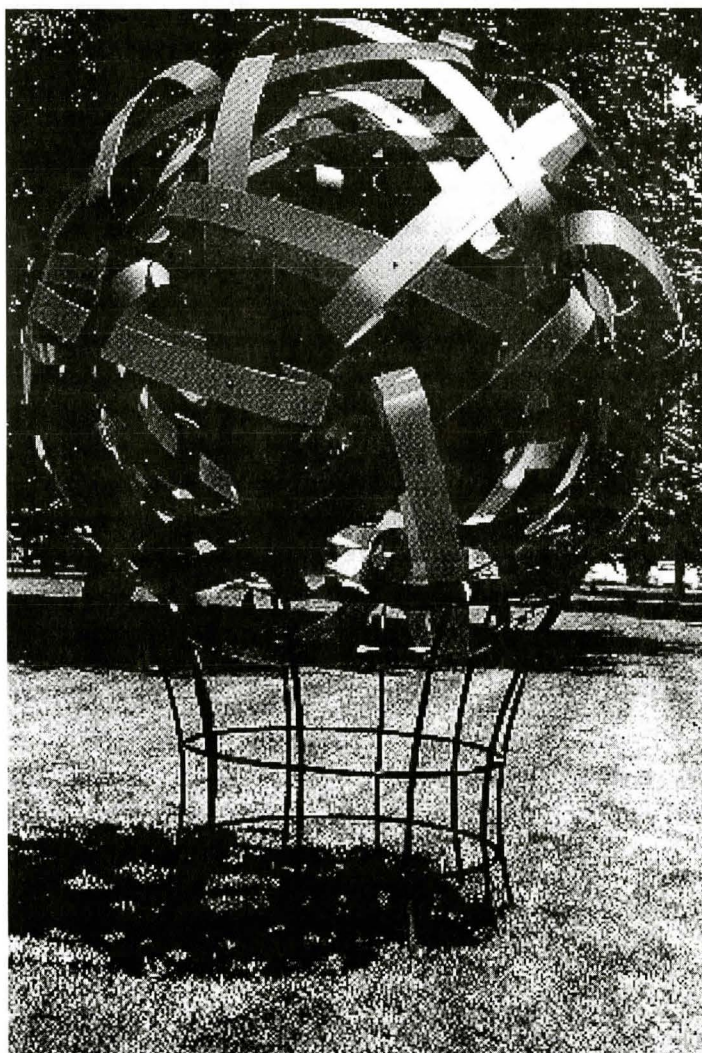

Figure 1
Loopy

Another sort of calculation one could make concerning the sculpture is to determine the angle at which the loops cross. Knowing only that the bolt holes are equally spaced and each loop is part of a great circle, with a bit of spherical trigonometry it is possible to calculate the angle at which loops meet. Pleasantly, it is not necessary to solve this, as the components automatically position themselves at the proper angle. This is analogous to cutting out twenty equilateral triangles and taping them together into an icosahedron. One does not need to know the dihedral angle of an icosahedron beforehand; the model automatically rigidifies to the necessary dihedral angles as the last edges are joined.

The mathematical properties that I consider most worthy of attention involve the structure and coloring. I do not know that I can articulate the intuitive reasons why I feel they are worth study. Perhaps the sculpture itself addresses that ineffable issue. But I do predict that if the reader or a student makes the following paper model, they will come to share some of this understanding. *Loopy* has a two-layer structure, which for architectural simplicity, this paper model simplifies to just one layer.

Cut out thirty strips of heavy paper, e.g., poster stock. Eleven inches by one-half inch is a convenient size if one starts with U.S. standard 8.5x11 inch paper. A paper cutter makes quick work of this. There should be six strips in each of five different colors. Use a hole puncher to make four equally spaced holes in each, as shown in Figure 2. Paper fasteners will be used to join the strips. They allow free rotation, and the proper angle will form on its own.

Begin by making a fivefold spiral as in Figure 3, using five different colors. The placement of the next five strips is indicated by the dotted lines. Note that the pentagon edges are one unit long but the triangle edges are two units long. Each strip has two "end holes" and two "inner holes." Each paper fastener connects one end hole and one inner hole. For a maximally attractive model, consistently place the end hole

inside (underneath) the inner hole it joins. The assembly and coloring of the remainder of the model is then determined by logically keeping in mind three rules:

(1) Every opening is either a pentagon with edge length one or a triangle with edge length two;

(2) Each pentagon is surrounded by five triangles;

(3) The three strips that touch any triangle's edge midpoints are of a single color. See Figure 4.

In building this model, one will appreciate that there is a fundamental structure inherent in it. Why does it work? For example, why is one never forced into a situation where a strip touches another of the same color? Or why doesn't one come to a point around back where a triangle's midpoints contact endpoints of more than one color? A full explanation would involve a discussion of how the group of tetrahedral rotations is a subgroup of the icosahedral group. If
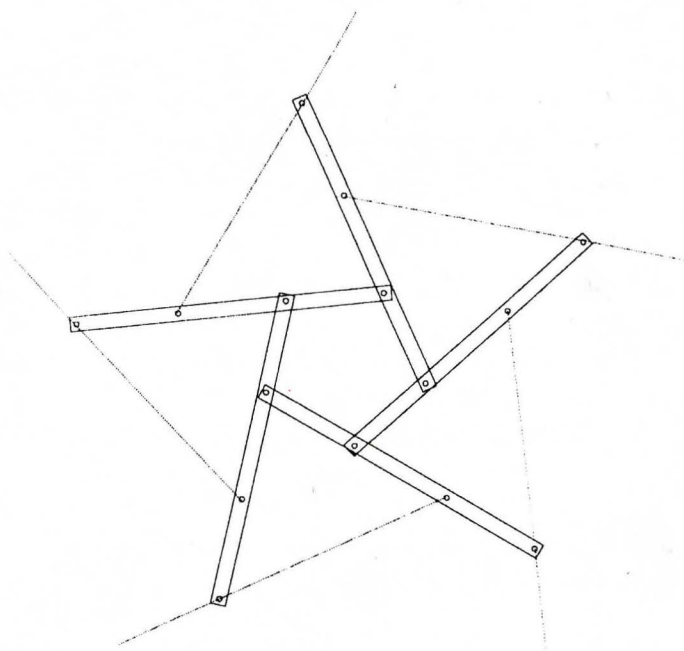

Figure 3
Start with five strips.
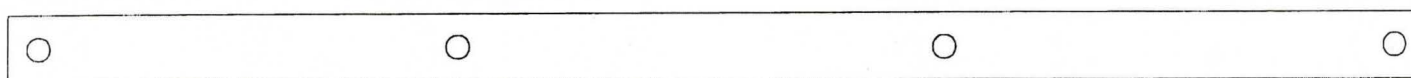Dotted lines indicate position of next five strips.

Figure 2
One strip. Space holes equally.

the geometric model known as "the compound of five tetrahedra" were better known, a study of its thirty edges could serve as a basis for understanding *Loopy*; each presents five groups of six. Similarly, one might wonder at how the paper strips work together as an analog computer, to solve the equation of the angle crossing. As I came to the form intuitively before doing any mathematical analysis, my feeling is that thinking about these questions goes hand in hand with just seeing that the form is beautiful.

For a well-rounded education, it is important that students develop the ability to discover new truths on their own. Let me suggest a few ideas for those who wish to start with this model and then explore further: *Loopy* has small pentagons and large triangles; try making the pentagons twice the edge length of the triangles. Notice you can start with a left-handed or right-handed spiral of five strips; what happens if you turn your model inside out? Or, try making a four-color structure with squares instead of pentagons. Or, design an analogous structure in which all the openings are triangles. Or, explore other color patterns. Can you generalize any of these models to a structure with hundreds of components? These questions might engage curious students.

For me, one key aspect of the math-art connection is that both can concern space, and patterns or structures in space. Both also involve aesthetics in a hard to define manner, which involves much subjectivity. But a theorem or proof of Euclid's can be simply beautiful, and a mathematician sees this as clearly as an artist sees that a line by Leonardo can be simply beautiful. Both fields also involve creative individuals who find satisfaction in creating and communicating new forms that are the outward expression of their inner visions. I feel *Loopy* and my other constructive geometric sculptures are somewhere in this common tradition, and I hope viewers can appreciate the synthesis. Mathematics is about patterns, structures, and relationships of all kinds. Art as much as anything can bring this out.

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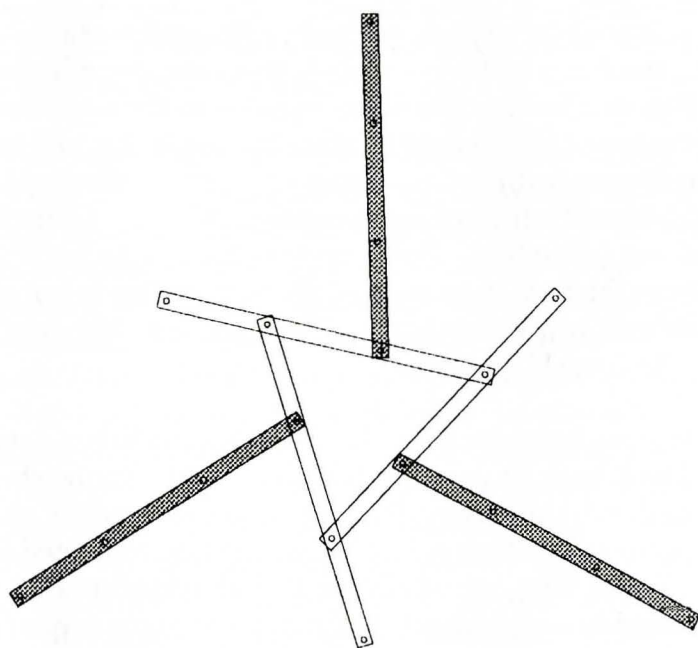
Figure 4
Three strips of same color end at midpoints of edges of triangles.