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INTRODUCTION

In general, surrounding or underlying the concept of equality there is an idea of sameness. But same in what sense? In mathematics, an equivalence relation is defined as one which is reflexive, symmetric, and transitive,[1] but there are many such relations and so, depending on the case at hand, this is specified further. In geometry, for example, there is a distinction between numerical equality of, say, areas of triangles, and equality (or congruence) of both sizes and shapes. When discussing functions, for example, one distinguishes between identities and equations, and, for different subcontexts, such as matrix algebra, complex variables or vector analysis, particularized definitions are needed. The distinctions, and yet underlying unity, of these various equivalence relations are often only vaguely realized by learners and so could use more thoroughgoing discussion.

In the political realm, where equality is so central to our EuroAmerican views of democracy, justice, and fairness, *equality* is used mostly in regard to the rights or treatment of people vis-a-vis government, institutions, or businesses. While there is a long and ongoing history of philosophical and legal discussions of equality, when used in common catchphrases, it often means quite different things to different people.[2] Further, in common American -English usage, the connotations of *equality* and *equivalence* differ: *equality* "implies the absence of any difference," that is being *exactly* the same, while *equivalence* implies that, although there inay be differences, "they amount to the same thing."[3] These different realms of usage—the mathematical, the political, and the everyday—are, however, not strictly distinct. That they interact is too often ignored; the differing usages, no doubt, influence and support, or, at times, confuse each other.

To enlarge our thinking and stimulate discussion about what equality means, we add a quite different view. Among the Basque of Sainte-Engrâce, France, there is a concept *bardin-bardina* translated as "equal-equal." Consideration of the Basque concept makes us realize that there is cultural variation in even as basic a concept as equality. In addition, elaboration of their concept, within the Basque context, can provide an opportunity to display mathematical ideas used in the promotion of cooperation. All too often, in an attempt to embed mathematical ideas in realistic-sounding contexts, we overlook that we are implicitly transmitting values as we present numerous examples of competition, winning, and financial gain. Here, instead, the focus is on how people organize their interactions to provide mutual assistance and receive mutual benefit.

The Basque concept of equality is underpinned by two operational principles that structure relationships so that everyone both gives and receives. The principles are referred to as *üngürü* and *aldikatzia*. The former is translated into English as "rotation," in the sense of "moving around a centre," and the latter as "'serial replacement' as well as 'alternation."' How these mathematical ideas apply in this context and how they relate to equality is best described in terms of their operation.[4] Where we use some algebraic symbols in the description, the notation is ours and not that of the Basque. The symbols are introduced to succinctly capture and express, in terms familiar to us, the system involved and some of its logical implications. More important, the fact that this translation is possible highlights the mathematical nature of the ideas involved.

CONTEXT

The community of Sainte-Engrâce is in the Basque province of Soule, one of the nine Basque provinces in the Pyrénées-Atlantique which straddle the French-Spanish border. Although the exact origins of the Basque are unknown, it is generally agreed that they predate the French and Spanish-speaking peoples in the region around them by perhaps thousands of years. In the 1970's, at the time of a study of Sainte-Engrâce, there were about two million Basque, with about three-quarters of them living in the Basque provinces in Spain, one-eighth in the Basque provinces in France, and the rest living in other areas of the world. Having their own language, a rich history, their own political and social organization, and long-held traditions, the recent history of the Basque has been marked by conflict with the nation states which encompass them. Nevertheless, the Basque way of life continues, particularly in a place like Sainte-Engrâce which, situated in the high mountains, is one of the most geographically and socially isolated communities in the region. Although the population declined from about 1000 people in the late 1900's to about 375 in the 1970's, the community remains self-reliant, centering on small farms and shepherding.

The mountains which surround the Sainte-Engrâce region range from about 1000 m to 2500 m. The Basque conceive of the region in which they live as enclosed by a circle of mountains with their households forming another circle within that. Whether or not this is actually the case, this spatial model forms the basis for their idea of circularity which pervades many of their interactions. In this circle everyone has neighbors to the left and neighbors to the right. No one is first and no one is last. Everyone's participation is involved in keeping the circle unbroken.

THE GIVING OF BREAD

Until the 1960's, a fundamental circular exchange was the giving of blessed bread. Each household regards its neighbor to the right as its first neighbor. (The directions right and left are as viewed from the center of the circle so that right is clockwise and left is counterclockwise.) The giving of bread took place weekly and was thought of as being given from first neighbor to first neighbor. That is, each Sunday a woman from one particular household, call it H_i, bought two loaves of bread to the church where it was blessed and partially used in a church ritual. Then, before sunset, a portion of the bread was given by H_i to her first neighbor, namely to H_{i+1}. The following week H_{i+1} was the bread-giver and H_{i+2} the bread-receiver. Thus, the giving (and receiving) of bread moved around the circle serially, taking about two years to complete one cycle of about 100 households. While each household was both a giver and receiver of bread, this mode differs from simple reciprocity; only if there were a total of two households would H_i and H_{i+1} directly reciprocate as each other's first neighbor.

FIRST NEIGHBOR OBLIGATIONS

In a more extensive, ongoing, cooperative arrangement, the exchange among neighbors is again predicated on the circular model, but this exchange involves several first neighbors. The *first* first neighbor of H_i is, as in the breadgiving, H_{i+1}, the neighbor to the right; the *second* first neighbor of H_i is the neighbor on the left (H_{i-1}); and the *third* first neighbor is the next on the left (H_{i-2}). Thus, for example, when there is a death in household H_i, the household calls upon its first neighbors for assistance. As a group H_{i-2}, H_{i-1}, and H_{i+1} help to keep the household going, but H_{i+1} provides particular assistance in specific preparations for the funeral. And, on the occasion of a home birth for H_i, it is a woman of household H_{i-1} who serves as the midwife.

Planting, harvesting, threshing, sheep shearing, and pig slaughtering all require the work of more than one person and so, there too, the first neighbors are called upon. These assistances are directly reciprocated by providing food and drink and by the giving of small gifts, but, primarily, the reciprocation is serial, that is, by assisting, when called upon, as the first neighbors of others.

A particularly interesting result of this mode of interaction in the farming yearly round is that households must schedule their work with the obligations of others and to others in mind. Also, for the same chore, each household gets to work with different groups of households and to play different roles within those groups. H_i, for example, works in groups (H_{i-2}, H_{i-1}, H_i, H_{i+1}), (H_{i-3}, H_{i-2}, H_{i-1}, H_i), (H_{i-1}, H_i, H_{i+1}, H_{i+2}), and (H_i, H_{i+1}, H_{i+2}, H_{i+3}), taking the roles of primary household, and first, second, and third first neighbors respectively. And, to avoid causing conflicting obligations for himself or any of his neighbors, H_i cannot schedule his household's work on the same day as the work of H_{i-3}, H_{i-2}, H_{i-1}, H_{i+1}, H_{i+2}, or H_{i+3} because, for example, H_i's third first neighbor (H_{i-2}) is H_{i-3}'s first first neighbor and his first first neighbor (H_{i+1}) is H_{i+3}'s second first neighbor.

We note that the subscript arithmetic is mod n, where n is the number of households and $n \geq 4$. (For $n = 4$, this cooperative mode reduces to a group of 4 households which always work together but with rotating roles.) It is particularly important to recognize that the equivalence relations in modular arithmetic, usually referred to as congruence rather than equality, is

$H_{i+nk}(\text{mod } n) = H_i$ for k = 0, ±1, ±2 .....

That is, to capture the circular nature of the Basque concept, we must involve the algebra and form of equivalence in modern mathematics that applies to cycles.[5]

We further observe that if a particular job takes a group of four households one day, it would take a minimum of

$$\left\lceil \frac{n}{\left\lfloor \frac{n}{4} \right\rfloor} \right\rceil \quad \text{days}$$

to complete the job for all n households.[6] This minimum completion time has a minimum of 4, taken on when n is a multiple of 4, a maximum of 7 taken on when n = 7, and is equal to 5 for n > 12.

SUMMER PASTURING

By far the most intricate cooperative arrangement involves the shepherding and cheese-making groups that work and live together during the summer months. These groups of households share in the ownership of pasturage sites in the mountains. The origin and practices of these groups are part of a long tradition which was described in writing as early as the 1600's. Prior to the 1900's, the ideal ownership group consisted of 10 households, each contributing 50 to 60 ewes and 2 rams to the summer flock and one man to the working unity. The flock of about 550 sheep had to be driven up into the mountains in late May, watched over until they were driven down to the valley for shearing in July, then driven back up to be watched over until returning to their valley homes at the end of September. Additional important aspects of the May to July work were the twice daily milking of the sheep, and the making of cheese from the milk. Different roles were defined that encompassed the various jobs that needed doing, and a formal system of rotation was used to insure that everyone was equal in terms of work contributed, in terms of cheeses produced, and in terms of status.

The households, first of all, had a specific order in the ownership group that remained unchanged from year to year. For the May-July period, for the working group of 10 men, there were 6 explicit roles which required 6 of the men to be together at the mountain site. Thus, calling the households' representatives H_1, H_2,..., H_{10}, and the work roles ranked in status order R_1, R_2,..., R_6, once the sheep were safely at the mountain site, assuming the household count started with H1, the assignments were: $H_1 \rightarrow R_1$, $H_2 \rightarrow R_2$, ... , $H_6 \rightarrow R_6$, and H_7, H_8, H_9, H_{10} returned home. After 24 hours, the rotation would begin: H_7 would ascend the mountain, keeping to the right, and then H_1 would descend, keeping to the left. Their ascent and descent is conceived of as taking place in a circle. Upon his arrival on the mountain, H_7 would take on role R_6 and each of the others would move up one role: $H_2 \rightarrow R_1$, $H_3 \rightarrow R_2$, ... , $H_7 \rightarrow R_6$. Similarly, every 24 hours, at the end of day i, there would take place the rotation up and down of H_{i+6} and H_i, respectively, and the moving up by one role of the others: $H_{i+1} \rightarrow R_1$, $H_{i+2} \rightarrow R_2$, $H_{i+6} \rightarrow R_6$. With 10 men cycling through this rotation, the subscript arithmetic is, of course, mod 10. Thus on, say the 18th day, those present at the mountain site would be H_8, H_9, H_{10}, H_1, H_2, and H_3 in roles R_1 through R_6 respectively. Out of every 10-day period, each man spent 6 consecutive days at the mountain site and 4 days at home. Generally, from May to mid-July, each of the 10 men carried out each of the 6 roles about 6 times with, for reasons of equity to be explained later, an extra turn at R_1 for H_1 and H_2.

From the time of shearing in July until the end of September, because milking and cheese-making were complete, the number of men needed at the mountain site was reduced to two with just two roles, R_1', and R_2'. For this, two men remained on the mountain for 6 consecutive days, alternating daily between roles R1' and R2', After the 6-day period, the pair descended the mountain and the next pair in the cyclic order ascended. Thus, if the period began with H_1 and H_2 in roles R_1' and R_2', then the next day $H_2 \rightarrow R_1'$ and H_1 - R_2', and so on until, on the 7th day, $H_3 \rightarrow R_1'$, while $H_4 \rightarrow R_2'$, or, in general, on the ith day of this second phase,

$$H\left(2\left\lfloor \frac{i-1}{6} \right\rfloor + k\right)(\text{mod } 10) - R_k' \text{ where } k=1 \text{ if } i \text{ odd}, \quad k=2 \text{ if } i \text{ even}.$$

Hence, during a 30-day period, each man spent 6 consecutive days at the mountain site, 3 of them as R_1' and 3 as R_2', and 24 days at home. Usually, each man had two of these 6-day turns on the mountain.

By these rotations, the men's contributions were the same in terms of time spent at home, time spent at the mountain site, time spent in each of the six roles R_1, ..., R_6, and time spent in the roles R1'and R2'. The procedure also insured receiving an equal number of cheeses made from the milk of the sheep. These cheeses were an important part of a household's annual food supply. One responsibility that went with the highest status role (R_1) was making two cheeses and watching over the cheeses that others had previously made. With the exception of the first cheese made on the first day and the first cheese made on the second day, the cheeses made by a person were for his household's use during the year. (The first cheese was sold outside of the community with all the members of the group sharing equally in the profit, and the other was given to the priest or guard of the forest. The extra turns noted before, of H_1 and H_2 being R_1 and, hence, of making more cheese, were to compensate for these cheeses.) In general, a cheese weighed about 8 or 9 kilos. With six turns at being R_1 and making two cheeses on each of these days, each person took home about 100 kilos of cheese.

In cases where a household had fewer sheep than the ideal of 50 to 60, they could own a half share in the cooperative. In that case, two households together owned a full share and together contributed the standard number of sheep as well as two workers, one from each household. The two workers had to alternate their six-day mountain stay so that each did three of the six stays in the May-June segment and one of the two stays in the July-September segment. In this way, they each did half as much work and got half as much cheese as did the others, but they did not modify the rotations up and down the mountain or through the various roles.

A larger cycle in which the annual cycles are embedded is the multi-year cycle. We noted that the ten households are in a fixed order H_1, H_2, ... , H_{10}. The order remains fixed throughout time, but which household representative starts a year as R_1 rotates by one position each year. That is, in a hypothetical Year 1, H_1, H_2, ... , H_6 are the first subgroup at the mountain site but then, in Year 2, the first subgroup would be H_2, H_3, ... , H_7, and so on, from year to year. (To reflect this in our previous statements involving H_i, i should be modified to i+Y-1 where Y is the year number of the cooperative's operation.)

Finally, we introduce the crucial issue of equality of status which becomes particularly significant for groups smaller than the ideal of ten. The six roles, from highest to lowest status are: R_1 = woman of the house; R_2 = master shepherd; R_3 = servant shepherd; R_4 = guardian of non-lactating ewes; R_5 = guardian of lambs; and R_6 = female servant. R_1 is the cheesemaker and is also in charge of cooking and of cleaning the hut in which the six men live. R_6 serves as his servant in the household chores. R_2, the master shepherd, organizes and directs the work of R_3, R_4, and R_5. Because

	Day	1	2	3	4	5	6	7	...	i	...	i+5
Role	R_1	H_1	H_2	H_3	H_4	H_5	H_6	H_7		H_{i+1}		H_{i+5}
	R_2	H_2	H_2	H_4	H_5	H_6	H_7	H_8		H_{i+2}		H_{i+6}
	R_3	H_2	H_4	H_5	H_6	H_7	H_8	H_9		H_{i+3}		H_{i+7}
	R_4	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}		H_{i+4}		H_{i+8}
	R_5	H_5	H_6	H_7	H_8	H_9	H_{10}	H_1		H_{i+5}		H_{i+9}
	R_6	H_6	H_7	H_8	H_9	H_{10}	H_1	H_2		H_{i+6}		$H_{i+10} = H_i$

Figure 1: The rotation through six roles with ten households. (Subscript arithmetic is mod 10.)

there is a decided hierarchy in the roles, the rotation is of special significance in preserving equality. Having ten men rotate through the six roles insures that no status hierarchy is consistently imposed. In particular, whoever serves as R_6 (house servant) when some H_i is R_1 (woman of the house) will serve as R_1 (woman of the house) when that H_i is R_6 (house servant). And, the Basque further note that this H_i will never be above those whom his R_6 will be above when he serves as R_1. (This is seen in Figure 1 where, for example, on day 1, H_1 and H_6 are in roles R_1 and R_6 respectively, but on day 6 their roles are reversed. And, since H_6 is above some or all of H_7, H_8, H_9, H_{10} on days 2-6, H_1 is never above any of them.) In general, using mod 10 subscript arithmetic, on day i, $H_i = R_1$ and $H_{i+5} = R_6$, but on day i+5, their roles are reversed: $H_{i+5} = R_1$, $H_i = R_6$. Also, since H_{i+5} is above H_{i+6}, H_{i+7}, H_{i+8}, and H_{i+9}, H_i is never above them. Similarly, H_{i+5} is never above H_{i+1}, H_{i+2}, H_{i+3}, and H_{i+4}.

After 1900 the number of households in the cooperatives decreased as a result of the overall decrease in the number of community households. With fewer households in each, cycling through the various roles would still insure equality of time and work contributions, but the criteria for the equality of status would not be met without adjusting the number of roles. To view this generally, let n = number of households in the cooperative and, hence, use subscript arithmetic mod n, and let r number of roles. To insure the role reversal of woman of the house/house servant, that is, to insure that

$H_i = R_1$ and $H_{i+r-1} = R_r$ on day i and
$H_{i+r-1} = R_1$ and $H_{i+2r-2} = R_r$ on day i+r-1,

the following relationship between roles and households would have to hold:

$i+2r-2 = i (\mathrm{mod}\ n) = i+n$, or
$2r-2 = n$.

This relationship would also insure that there is no overlap between those whose roles are below those of H_i and those whose roles are below those of H_{i+r-1}, since this criterion is satisfied whenever $n > 2r-2$.

Clearly, the relationship $2r-2 = n$ is satisfied for $r = 6$ and $n = 10$. And, while we do not know how the Basque arrived at the requirements, the Basques knew that there could be at most 5 roles when there were 8 households, 4 roles when there were 6 households, and 3 roles when there were 4 households. To accomodate odd numbers of households and the situations where there were more than the necessary minimum of households, meeting the requirement $n > 2r-2$ became sufficient. In these cases, the stipulation of H_i and H_{i+r-1} being over non-overlapptng groups is maintained, but the requirement of the complete role reversal of H_i and H_{i+r-1} is loosened. The number of roles were reduced, in about 1900, from six roles to five roles by deleting R_4, and then, in about 1940, they were further reduced to four roles by deleting R_5. In the 1960's and 1970, they were still further reduced by either reassorting the functions into three newly titled, but still hierarchically ranked roles, or by creating only two roles by combining into one the master ranks R_1 and R_2, and into another the servant ranks R_3 and R_6.

CONCLUSION

The Basque concept of equal-equal is evidenced by a variety of different circles and cycles. There is, first of all, the giving and receiving of bread in which H_i simply gives to H_{i+1} and the giving moves around the circle made up of all households in the community. There are also the ongoing first neighbor obligations for which the circle of all is divided into fixed, adjacent, overlapping sets of size 4. For the summer pasturing, the community separates into subunits of households, and each subunit is a circle which rotates within itself. There is the annual rotation in which shareholder H_{i+1} replaces shareholder H_i in the starting position of the season's cycle. Beginning with the designated starting household, the season's cycle is made up of two consecutive subcycles: in the first subcycle, one man (H_{i+6}) goes up and one (H_i) comes down the mountain daily, and, in the next subcyele, two (H_{i+2}, H_{i+3}) go up and two (H_i, H_{i+1}) come down every six days. Further, within these subcycles, there can be alternation within a single H_i of a pair of joint owners of the share. And, while on the mountain, the earlier occupants cycle once through $r \leq 6$ roles, and the later ones cycle three times through two roles.

The variety and interrelatedness of the cycles, as well as the cycles themselves, testify to the deep embedding of these ideas in the culture. We not only see algorithms of interaction involving cycles, sequences, and alternation, but a spatial concept of circle underpinning them, as well as an overarching concept of equality uniting them.

The overarching concept, "equal- equal", is not a static relationship as is our conventional mathematical or everyday equality. It is a dynamic process of interaction in which an essential feature is that the participants know what is expected of them and they know what to expect from others. That is, the actors in the process move in synchronization, doing different things, at different times, but together making up a whole. If one were to stop the process at an arbitrary point in time, there would be inequities in what has been contributed, what has been received, and who is superior to whom. But, just as a circle is enclosed by a never-ending line, the process of creating an equal-equal relationship continues throughout the season and throughout the years.

In a previous discussion of the spatial ideas of several cultures,[7] we noted that for many outside of our Euro-American stream, time and space are intimately connected and, what is more, the circle is as fundamental for them as lines and angles are for us. While it is surprising to think that these differences may pervade the concept of equality as well, it may, in fact, be that where equality is conceived of as a static point of balance separating more and less or better and worse, it is often too precarious to be stable or easily attained.[8]

NOTES

1. More specifically, an equivalence relation R on set S is one which satisfies the following for all elements a, b, c of set S:

  Reflexive: aRa
  Symmetric: If aRb, then bRa;
  Transitive: If aRb and bRc, then aRc.

2. For socio-political discussions of equality that were influential in Euro-American culture, see, for example, *Nicomachean Ethics*, Book V, Aristotle, 4th century B.C.E.; Jean Jacques Rousseau's "A Discourse on the Origin of Inequality" (1754) and "The Social Contract" (1762); and John Stuart Mill's "On Liberty" (1859).

3. The phrases quoted appear under the synonyms for same on p. 1289 of *Webster's New World Dictionary of the American Language*, College Edition, World Publishing Co., N.Y., 1966.

4. My discussion of the Basque and their ideas is derived from *A Circle of Mountains: A Basque Shepherding Community*, Sandra Ott, Clarendon Press, Oxford 1981. Of particular relevance are pp. vii-viii, 1-41, 63-81, 103-106, 129-170, and 213-217. The few phrases directly quoted are from p. vii.

5. For a circle of, for example, 5 households, n = 5 and the households are (H_1, H_2, H_3, H_4, H_5). When counting around the circle, the household identified as, say, H_{22} is the same household as H_2, H_7, or H_{-3}. For more about modular arithmetic, see, for example, Chapter 7 on congruences in *Invitation to Number Theory*, Oystein Ore, New Mathematical Library, MAA, Washington, D.C., 1975.

6. The symbol $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. For example, $\lfloor 4.0 \rfloor = 4$, $\lfloor 4.1 \rfloor = 4$, and $\lfloor 4.99 \rfloor = 4$. The symbol $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. For example, $\lceil 4.11 \rceil = 5$, $\lceil 4.99 \rceil = 5$, and $\lceil 5.0 \rceil = 5$.

7. See Chapter 5, "The organization and modeling of space" in *Ethnomathematics: A Multicultural View of Mathematical Ideas*, Marcia Ascher, Brooks/Cole, Belmont, CA, 1991 (paper edition, Chapman & Hall/CRC, New York, 1994.)

8. Other mathematical ideas of the Basque are being studied extensively by Rosyln M. Frank. See, for example, "The Geometry of Pastoral Stone Octagons: The Basque *Sarobe*," R. M. Frank and J. D. Patrick, pp. 77-91 in *Archeoastronomy in the 1990's*, Clive L. N. Ruggles, ed., Loughborough Group D Publications, London, 1993, or "An essay on European ethnomathematics: the coordinates of the septuagesimal cognitive framework in the Atlantic facade," R. M. Frank, 78 pp., ms., 1995. Also, a special counting technique among the Basque living in California is described in "Counting sheep in Basque," Frank P. Arawjo, *Anthropological Linguistics*, 17 (1975) 139-145.