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Three years ago Pomona College embarked on a program of Freshman Seminars entitled "Critical Inquiry". In these seminars at most fifteen entering freshmen think, discuss and write extensively. Participating faculty choose the subject of their own seminars. My topic of "Mathematical Modeling and Exposition" provokes special difficulties for the students which may best be understood through a model for their "Cognitive And Ethical Growth" developed by William G. Perry, Jr.

Contrary to the expectations of some, these seminars have been just as popular in the sciences and mathematics as they have in the humanities and social sciences. Mathematically oriented seminars have been offered on the philosophy and art of pure mathematics, on societal uses and abuses of statistics, on symmetry, and on mathematical modeling. Topics in the sciences vary from plate tectonics to nuclear war to biological determinism.

My original hope for my seminar was to emphasize mathematical modeling as part of and as a means of mathematical exposition. It was my underlying thesis that a kind of simple, naive, mathematical modeling underlies the way many of us understand the social and physical world around us. If so, then that modeling constitutes an integral part of the way we communicate our ideas.

Fundamental to our exposition of mathematics is our complete acceptance of working from assumptions. At the pure end of the mathematics spectrum we emphasize axiom systems, while at the applied end we value the conceptual and computational simplifications resulting from well-considered assumptions. Even in ordinary everyday discussions we mathematicians tend to be fairly conscious of the assumptions, or axiomatic base, which we bring to bear in understanding social and physical phenomena. (Of course in everyday discourse we tend not to worry too much about consistency of our axioms.)

Not everyone shares our love for axiomatically connected discourse. In our own classrooms, how often are we accused of having our heads in the clouds, just for wanting to think carefully concerning "all those obvious facts about real numbers?" Even our colleagues in the physical an social sciences frequently blur the distinction between "reality" and consequences obtained from assumptions. They naturally influence the way our students think when they come to the mathematics classroom.

So it shouldn't come as a surprise when our students believe they can understand everything as it "really is". No system is too complex, no detail, too fine to deter them. They are filled to overflowing with the confidence that they can understand anything completely. You can easily imagine them saying, "If things seem a little complicated today, then we'll pull an all-nighter and straighten this out for tomorrow." Their optimism is nothing, if not charming.

But they have little use for simplifying assumptions. They are certain that making an assumption is tantamount to the admission that they cannot know everything to be either fact or false. This they find to be far less acceptable and a far more bitter pill than I do. Moreover, at this point they would consider me to be slightly irrelevant if I suggested there might be a logical problem with the concept of knowing everything to be fact or false.

Probably many of you are less surprised than I am, at the lack of sophistication of my freshmen. After all, they are just beginning to mature mentally. Indeed, they are just beginning that mental maturation process which is central to developing their world views. Therefore it would be helpful for us to understand that maturation process, in order to teach them effectively about the relevance of modeling for their worldviews.

William G. Perry, Jr., has suggested a model for understanding the "Cognitive and Ethical Growth" of our students. The model contains a scheme of development consisting of nine literary ordered "positions" and prototypical transitions between them. The positions begin with the simplistic and dualistic attitudes wherein students categorize everything as to "good vs. bad," "right vs. wrong," "true vs. false," and so forth. It then proceeds in discovering relativistic standards, according to Perry, wherein truth becomes relative to context. For example

for the student in Perry's earliest relativist positions (which are just beyond the dualistic positions), good writing in mathematics can be different than good writing in literature classes, because the authorities, the professors, are different. Further growth, if it occurs, moves in this model toward a commitment to a more mature relativism. This position Perry characterizes by the statement of attitude:

> I must be wholehearted while tentative,
> fight for my values yet respect others, believe
> my deepest values right yet be ready to learn.

It seems that Perry's model has much to suggest about the attitudes of students toward mathematical modeling. The attitudes by which I have characterized my students earlier are consistent with Perry's dualistic position at the beginning of the development ladder. They are saying that a description of the "real" world is either right or wrong. In this viewpoint, simplifying assumptions might be seen to make the description wrong. There is very little room for meaningful approximation. Our expectation that they accept a model which only approximates experiential evidence in only a limited set of scenarios should be understood as an expectation that these students make significant strides in growing through Perry's positions. We therefore recognize the implications of those expectations in terms of fundamental personal growth.

Consider my original hopes for my students: that they understand mathematical modeling as a part of, and as a means of mathematical exposition; that they come to use modeling approach for casual understanding of the social and physical world around them. That is, I was hoping that they would come to be aware of how the conclusions and even values they form about the world around them depend on the assumptions they bring to their analyses. I further hoped they would be self-consciously aware of the tentative nature of their assumptions.

Now compare these hopes for my students' development with Perry's position of highest "Cognitive and Ethical Growth" characterized by the statement of attitude above. For me, the correspondence between my hopes, and Perry's position of highest development was amazing and dismaying. Clearly if Perry is right about the positions through which we must progress in our development, and if that progress is as slow as he indicates, then we are forced to realize that my hopes were wildly unrealistic and desperately need modification.

I believe we can develop a freshman pedagogy for mathematical modeling which is comfortable for students in the earliest of the Perry positions. After all, they are well accustomed to accepting some other kinds of models as correct and useful. Toys and dolls are used by all children to model a more complicated reality. Many high school students are fairly sophisticated users of maps and models. They recognize that topographic maps may not be good indicators of economic activity. They also know that a refined map might include economy with topography. Students value these models as aids to studying the world. However, they would probably disagree with my suggestions that their concept of geographical reality depends to a large extent upon such models.

We cannot expect freshmen to accept models as tentative replacements for their reality. That would be tantamount to the expectation of immediate progression to more sophisticated positions in Perry's model. A model as a separate entity can be useful for displaying information about a separately conceived reality. But in order for the model to retain its legitimacy, it must not be held up as a replacement for that reality. For if it is, it will be discarded as being incorrect in some respects, and therefore false in the dualist perspective.

Just as children perform musically long before they acquire an interpretative maturity, so can our freshmen model proficiently independently of their progress toward cognitive maturity. Fortunately they are already familiar with many powerful mathematical concepts and tools. Even regression models and dynamic systems are viable for some of them. Their powers of deduction are frequently equal to the task of finding a conclusions. Subsequent comparisons with data from the real world fit all too well into their dualist's perspective. Thus modeling as a craft, if not as a world view, can be practiced by students in any of the positions of Perry's model.

Realizing this, we can better introduce our students to mathematical modeling. If they can achieve an intellectual understanding of the modeling process early in their cognitive development, then perhaps they can incorporate a modeling attitude in their later development to a tentative relativism. In fact, I hope they can thereby grow more easily in their cognitive and ethical senses, according to Perry's model, toward a more personalized, relativist stance in their worldviews.

Reference

W. G. Perry, Jr., "Cognitive and Ethical Growth: The Making of Meaning," in A. W. Chickering and associates (eds.), The Modern American College, Jossey-Bass Pub. 1981, pp. 76-116.