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What Has Mathematics Got to Do with Values?

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The popular view of mathematics, and mathematics education in particular, can be described as follows:

(a) Whilst mathematics education can be used to bring politics into the classroom, or to teach children particular values, and in fact mathematics itself can be used for moral or political ends in society, this is all really about uses or perhaps abuses of mathematics.

(b) In essence, mathematical knowledge is about pure concepts, relationships, pattern and structure. It is concerned with proof, and its truths are timeless, certain and absolute.

(c) We may argue about the causes of the Second World War, or whether Lowry was a great artist, we may discuss the role of religion today, but "5 + 6 = 11" is a truth now and forever.

(d) Any other mathematics is inconceivable, and so it is quite independent of time, of place, of culture and even of the people who invented or discovered it.

It is generally recognized that education is not just about the passing on of certain bodies of knowledge, but is also a preparation for life in society. This is enough justification in itself for anti-racist and anti-sexist mathematics. Look at the kinds of examples we draw on in the teaching of mathematics at the moment: percentage increases in pay; simple and compound interest; profit and loss; hire purchase; exchange rates and angles of missile projection to hit a target. Why shouldn't we use examples to reveal prejudice and injustice, and raise children's awareness of social issues? Reactions to the occurrence of these kinds of questions in mathematics lessons, or examinations, as seen in the Daily Telegraph, the Sunday Times and other newspapers, simply reveal how strong are the hidden messages of British values. We have come to accept questions on those topics above, and it is only when we see something about an unusual issue for mathematics, such as SMILE 'O' Level question about expenditure on arms, and the cost of feeding the starving peoples of the world, that people begin to worry.

So, we can point out to the critics of this kind of work in mathematics, that questions we have been using for years convey certain messages, and it rather hypocritical, to say the least, to single out some messages from others, particularly when they are about charity to the Third World, or opposing Apartheid, both issues that British Governments claim to support.

However, the reaction could be that we should make mathematics completely free of all messages. Return it to its pure state, where it is about numbers, patterns, skills, and procedures. This reaction, I believe, is one that most mathematicians and in particular mathematics teachers would give, and consequently it is essential to examine the very heart of the issue, the nature of mathematics itself. A strong case can be made, I am proposing, from within mathematics, for confronting social issues in the mathematics classroom, and in relation to this volume in particular, the issue of racism.

To summarize, the following is the claim that I will be examining here:

Mathematics seems to have a character all of its own, and a position of unique significance in discussions about knowledge, because of its hold on certainty and truth, and because of its purely abstract nature. The mathematical knowledge that we have is fixed, timeless and absolute, as are the logical methods that are used to deduce or calculate. If this is essentially the nature of mathematics, then it has nothing to do with social content, or values. Mathematics teaching should be kept free of all such material.

The evidence against this claim is beginning to mount up. First, I will give some examples from the mathematics classroom that appear to show some fundamental changes taking place. Then some examples will be drawn from philosophy of mathematics.

<u>Examples from mathematics education:</u>

1) Child methods of working, e.g.

$$\frac{1}{2} \div \frac{3}{5} = \frac{5}{10} \div \frac{6}{10} \text{ (common denominators?)}$$

$$= \frac{5 \div 6}{10 \div 10} \quad \text{(this only works for x!)}$$

$$= \frac{5 \div 6}{1}$$

$$= \frac{5}{6} \quad \text{correct!}$$

Now it is not a new idea to suggest that pupils often solve problems in ways that are completely different from the method we taught. But what does this do for our understanding of pupils' replies to our questions? One's first reaction to choosing common denominators for a division of fractions would be that we only do that for addition and subtraction. We would have destroyed that pupil's confidence, and identification with her/his own mathematical thinking. Or else the pupil would have gone on ignoring the teacher! It suggests, perhaps, that the teacher has to listen to every answer given by pupils, and treat them as potentially correct. They require testing and discussion by the class, before rejection or adoption as a good method. This is a very different function for the mathematics teacher from the traditional one of the conveyor of knowledge and algorithms, and arbiter of right and wrong answers

2) The view of the CSMS group [Hart et al 1981], supported by Cockcroft, is that <u>mathematics is a very difficult subject</u>. This is a very worrying statement. After all, what is mathematics about, if not certain kinds of interaction, that we all experience, with the world around us? If this is the case, why should it be so difficult? That is, does our view of mathematics perhaps act as a kind of self-fulling prophecy?

If our notion of mathematics is that it is hierarchical, and that one must learn it in order, from basic concepts to more abstract and difficult ones, and that the mathematical progression is mirrored by our psychological development, and indeed depends on it, then our curriculum, teaching styles, expectations, testing and much else are structured in a most rigid fashion. Yet there is growing evidence to show that young un-taught children (in the traditional sense of the word) often show understanding of concepts that in our hierarchy should only be accessible at 'formal operational' stage. The point is, that I am suggesting that our work is structured by our theories, and so is our testing. Is it then any surprise that our tests confirm our theories?

3) An investigation from a well-known source [SMILE 1981]:

"Consider triangles with integer sides.

There are 3 triangles with perimeter 12 units. Investigate."

No methods, skills or procedures. Not even a question! I have had PGCE and also BEd students ask me what they are supposed to do. Others have chosen to work on rectangles, areas, angles, triangles with other perimeters, etc. What is more, as the teacher, I don't have the answers. Even if I worked on the question for several hours the night before, I would still only have answered the questions that I asked.

With this kind of mathematical work, which is becoming more common in schools now, since GCSE criteria include course work, extended pieces of work, and investigations, the teacher/pupil relationship is changing. We are no longer the possessors of all the knowledge, passing it on in snippets as and when we feel the pupils are ready. All the people in the classroom are participating together in doing mathematics, whether we are aware of the change or not!

4) Here are three quotes concerning the excitement of mathematical creativity, in an adult and then in two children, aged 10 and about 5:

"This fascinated and excited him, spurring him on to feverish activity... He relaxed, satisfied... After coffee I wanted to work but the tension was unbearable... He felt a strange mixture of disappointment at his failure and elation because he felt he knew <u>why</u> he had failed." [Tall 1980 p. 25-34]

"About ten minutes later it happened. Sandra jumped up, knocked over her chair and ālmost shouted "you can!". It was clearly time for a get-together. Sandra described her discovery... She was really thrilled and I believe the others were pleased for her as well." [Atkins 1984 p.3]

"Consider Kevin, who was presented with ten drinking straws of differing lengths. Before I said a word about the straws, he picked them up and said to me,

'I know what I'm going to do,' and proceeded, on his own, to seriate them by length... It wasn't easy for him. He needed a good deal of trial and error as he set about developing his system." [Duckworth 1972 p. 219]

Are these not descriptions of the same kind of activity, despite the differing levels of "sophistication" of the mathematics? One's first reaction to the latter two extracts, perhaps, is how wonderful to have that excitement of discovery, and creativity in the mathematics classroom. Further reflection perhaps leads to the idea that what characterizes mathematics is not sets, quadratic equations, or calculus, but the doing of the business of mathematics, at all levels.

<u>Examples from Philosophy of Mathematics</u>

It is vital but not quite enough, for mathematics teachers and educationalists to believe that our notions of mathematics and of mathematics education have changed. The problem is that we ourselves remain convinced that we must ultimately look to the 'real' mathematicians, in the universities, at the forefront of knowledge, and see what they have to say. All of us are products of the mould, either directly from university or polytechnic teaching, or indirectly at colleges of education, from tutors who were themselves from that tradition.

Here are some quotes from recent, and not so recent, writing of mathematicians reflecting on the nature of their activity, or from philosophers of mathematics who spend al the time reflecting!:

"...all mathematical pedagogy, even if scarcely coherent, rests on a Philosophy of Mathematics" [Thom 1973 p. 204]

"...when he (the professional mathematician) is doing mathematics, he is convinced that he is dealing with an objective reality whose properties he is attempting to determine. But when challenged to give a philosophical account of this reality, he finds it easier to pretend that he does not believe in it after all." [Hersh 1979 p. 32]

"Mathematics is able to deal successfully only with the simplest of situations, more precisely, with a complex situation only to the extent that rare good fortune makes this complex situation hinge upon a few dominant simple factors." [Kac et al 1986 p. 21-22]

"Logic may explain mathematics but cannot prove it. It leads to sophisticated speculation which is anything but trivially true. The domain of triviality is limited to the uninteresting decidable kernel of arithmetic and logic - but even this trivial kernel might sometime be overthrown..." [Lakatos 1978 p. 19]

"Insofar as the propositions of mathematics give an account of reality they are not certain; and insofar as they are certain they do not describe reality." [Einstein 1921]

Obviously these are only extracts, chosen to illustrate a particular point of view. Others can be selected that would demonstrate support for the more traditional view of mathematical knowledge.

Which is correct? The traditional absolutist view, that mathematics is about truth, proof, certainty, structure, and that we the teachers possess some of that, and must convey it to pupils? Or what is sometimes called the fallibilist, or relativist view that all knowledge is relative to time and place, and hence to culture and values? By this latter view, all the knowledge that we have is a library, a body of experience, much of which is well-corroborated and supported, and successful. After all, buildings stay up, most of the time, and space research has taken people to the moon and back. But that knowledge is always vulnerable to new ideas and discoveries, and revolutionary change, as history shows. And it is not the only way that mathematics could have developed.

These are rival perceptions of mathematics, and they have, as I have hinted at, consequent major and significant effects on the teaching of mathematics in schools.

The difficulty here is that we have no certain way of making a universally acceptable choice. The criteria we use for deciding which of two rival theories is better, are themselves open to choice! One way of preferring seems to be which theory is the richer in the sense of the ideas for investigation and research (and possible refutation) generated by it. From this criterion, the fallibilist view is very rich in consequences for study and action, not least of all in the area of concern of this volume.

Mathematics and Values

If one holds the view that mathematical knowledge is social in nature, then one cannot get away from involvement in values. The mathematical ideas that one is teaching would have originated in a certain time and place, as a response to some social needs, whether of the individual mathematician, or of the wider community, scientific or otherwise. History of mathematics becomes a primary source of information for finding out what is mathematics, how it develops, and how it functions in society. It is not simply finding out who discovered binary numbers, or when logarithm tables were first put together. Instead, just as we can examine why the Greeks preferred geometry to number, or why British mathematicians did not develop non-Euclidean geometries, we can also look at how the Chinese were using 'Pascal's Triangle' centuries before he was born, or why Frege developed the supremely abstract mathematical logic. We can be impartial with respect to 'correct' mathematics or 'wrong' mathematics. We can also visualize how mathematics could have developed quite differently.

The South American educationalist Paulo Freire [e.g. Freire and Shor 1987] describes what I have called the traditional or absolutist view as the 'banking concept'. The image he describes is of the teacher baking ideas in the minds of pupils, whose only activities are storing, filing, classifying and retrieving. The alternative view he calls the 'problem-posing concept', whereby people see themselves as having the power to engage in problems that dominate their lives, pose questions for themselves, and develop solutions. He sees the former as associated with oppression, and the latter with freedom.

This may seem somewhat extreme, especially for such a cold climate as Britain! But if one considers what happens when children examine, in the mathematics classroom, racist headlines in the Sun, unemployment patterns in Britain by region, gender and race, the economics of Apartheid, distributions of wealth in Britain and the world etc., the word 'freedom' is not at all inappropriate. By these kinds of study our students do gain freedom, the freedom to examine the assumptions of the society in which they live. These assumptions draw on mathematical ideas and techniques, be they decisions about what constitutes an 'unemployed' person, with a view to keeping the total down, or up, for party-political ends, or what percentage figure is a real increase in funding for the National Health Service.

These mathematical techniques are often developed just in order to solve these kinds of problems, set in the terms of reference of the required solution. This is yet a further illustration of the social nature of mathematical knowledge. Certainly that knowledge gains some objectivity, in that another person or group somewhere else can read about those techniques, in the learned journals, and use them or adapt them to their own particular situation or problem. It cannot be said to have gained objectivity in the sense of corresponding to the 'real world', however, since that reality has been established in certain socially determined ideas. In developing an equation to decide whether to close a coal mine, one has the choice to introduce an element to take account of the social impact on the community, or not. There are no absolute rules to be applied in such a situation.

Conclusion

Mathematics is treated as a special case, by parent, governors, industry, commerce, the DES, pupils and teachers. It can be said that teachers are perhaps suspicious of the reason for this, and pupils single out mathematics both for the importance of qualification that can result, and for the most negative reactions! But there is no doubting the importance with which it is endowed by society. Mathematical knowledge is similarly treated as a special case in the field of scientific, philosophical, sociological and historical thought.

I have attempted to show that one way of looking at mathematical knowledge is to see it to be as much socially determine as any other area of knowledge, and that it has as little claim to timeless and absolute truth as any other. Mathematics teachers cannot claim that issues of justice, morality, freedom, values, are for the discussions in English lessons, or History, or Personal and Social Education, or Geography, but not Mathematics. It could even be claimed that we have a special responsibility, since mathematical techniques and methods are often developed for, and used in, social decisions. They are always developed by people in particular places at particular times, they reflect the current ideas of the mathematical community, for instance in the policies to fund certain research and PhD students or not, in editorial decisions to publish papers in journals or not, and finally they cannot be said to correspond to reality in any definite and absolute sense.

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