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The Questionable Probability Theory Behind the Strange Story of The Bell Curve's Bell Curve

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The mathematical underpinnings of Hernstein and Murray's *The Bell Curve* are to be found in the appendices. In the first of these we see a diagram of a few bellshaped (normal) distribution curves with the (scientifically fuzzy) explanation:

"...a common way in which natural phenomena arrange themselves approximately."

The title of the book and the various statistical techniques used do in fact indicate that the authors' interpretation of the observed data assumes that I.Q. is normally distributed in the population. The applicability of many of their statistical methods necessitates that the bellshaped curve prevail. The discussion below explains why a theoretical model based on the conclusions the authors draw from the observed data *will not bring about a bellshaped distribution.*

The normal distribution, even if very prevalent, does not however fall out of the sky. In fact the mathematical criteria needed to produce a normal distribution are not satisfied in the case of the population the authors of *The Bell Curve* hypothesize—a non-homogeneous group in which there is a significant difference between the mean I.Q. of the two groups. The authors cannot have it two ways: either the two population groups—black and white; poor white and middle and upperclass white—are sufficiently homogeneous to generate a bellshaped curve with a common mean, or we are dealing with two distinct populations and the various statistical tests based on the model of a bellshaped curve simply do not apply.

A large number of small, independent, random effects (say, those that combine to generate I.Q.s) may, under certain circumstances, combine to display a collective (statistical) regularity. In particular the sum of a large number of such small random fluctuations may combine into what we call a "stable" limiting distribution law, to which family the bellshaped curve belongs. A good example of when this does happen is the example discussed in *The Bell Curve* of the distribution of the body heights in a class of schoolboys. Similarly a close to bellshaped frequency curve will be observed for the physical sizes in a homogeneous adult population of one gender. There is a reason for this. (For a more detailed discussion see Miriam Lipschütz-Yevick, "Probability and Determinism," *American Journal of Physics*, 1957; a classical and beautiful exposition can be found in the early work *Théorie des Probabilités*, Gauthier-Villars, Paris, 1925,page 103, by the great French mathematician Paul Lévy.)

It so happens that the physical stature of an individual is determined by the sum of the sizes of some two hundred bones making up the skeleton. In a large population of males, say, the small, accidental differences from the mean size—which are caused by a host of environmental and genetic factors—over the whole population for a particular bone fluctuate randomly from individual to individual and quite independently from bone to bone. Some of the bones will be larger than average, some smaller, so that winners are more or less matched equally by losers. Yet even the *largest deviation* from the mean will contribute a negligible part—i.e., be statistically negligible—to the sum of all the individual differences which together determine how physical sizes are statistically distributed over the whole population.

In view of the sloppy theoretical underpinnings of Murray and Hernstein's book, it is doubtful that the measure of these two scholars' achievements would be located at the extreme upper end of such a nonnormal stable distribution curve.

These exactly are the necessary and sufficient conditions—the individual and uniform (collective) smallness of the variations compared to their sum—for the normal distribution to evolve when a large number of small independent random effects, or "errors" conspire together, i.e., sum up, to produce a statistically regular distribution of some "phenotype."

Clearly these conditions would not be satisfied if our population were composed of, say, American males and Japanese females—for the deviations from the mean would not be uniformly small. The result in this case would, most likely, be twopeaked, a bimodal distribution for physical size. And by the same token, *The Bell Curve*'s conclusion that intelligence quotient is distinctly different for the two subpopulations hypothesized, cannot yield a normal distribution with the one subpopulation squeezed into the lower ten percentile. We are, from a theoretical point of view, not in the domain of the normal distribution.

A bellshaped distribution for a phenotype can then be ascribed to a genetic factor only if this factor operates *randomly* and *independently* on each of a large number of genes which conspire together to produce the particular phenotype. And the measure of the factors must be such that the fluctuations in the values of each component are individually and uniformly (i.e., no component deviation is *overwhelmingly* large) negligible against their sum. Once again *The Bell Curve*'s conclusions preclude that these theoretical (mathematical) conditions be satisfied for the distribution of I.Q.s. For *The Bell Curve* concludes that the subpopulation is such that its genotype will systematically land the measure of its intelligence in the lowest ten percentile. The small individual genetically induced components which are summed in this case are neither independent nor randomly distributed in a uniformly negligible manner over the whole population. A bellshaped curve would hence not be statistically generated and empirically observed.

The normal distribution, even if very prevalent, does not however fall out of the sky. In fact the mathematical criteria needed to produce a normal distribution are not satisfied in the case of the population the authors of The Bell Curve hypothesize.

Yet we *do* empirically observe a normal distribution for I.Q.s as well as many other test results. This is compatible with the hypothesis that the normal distribution evolved from a large number of random, independent environmental and genetic fluctuations, whose differences from the mean were individually and uniformly negligible against their total in a single population. Fluctuations whose values lie mainly to the left of the mean (reflecting negative environmental factors) will so sum statistically and similarly for positive variations—collectively producing a bellshaped curve.

The Bell Curve's asssumptions (or conclusions as the case may be) could more easily be fitted into another model, that of a non-normal stable distribution. The graphs in the book showing the high values for measurements of achievement for a small group of elite college graduates, etc., are compatible with this model. To wit, when a few of the measures of the component terms contribute a sizable fraction of the sum (so that the components are statistically not uniformly negligible) a highly skewed distribution will evolve. The distribution of the sum will reflect the distribution of its largest term(s) and a sizable part of the total distribution will be concentrated in the upper tail end of the curve . Such, for instance, is the distribution of wealth and income in most present-day societies. Such too is the distribution of scientific, intellectual, or artistic achievements, where a minute fraction of practitioners makes most of the major contributions.

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In view of the sloppy theoretical underpinnings of Murray and Hernstein's book, it is doubtful that the measure of these two scholars' achievements would be located at the extreme upper end of such a nonnormal stable distribution curve. Let us remember that it has been the hallmark of contemporary authoritarian and racist theory-inspired governments to eliminate the true intellectual elite (those at the upper end of the distribution) and their creations in short order (vide Nazi Germany, Stalinist Russia, Cambodia, Bosnia, Rwanda...).

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