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The Mathematical Quest For the Perfect Letter

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ABSTRACT
In fifteenth century Europe, geometric and mathematical principles were harnessed in the construction and formulation of "perfect letters." These letters, based on the script of imperial Rome, occupied the attention of both mathematicians and artists of the time. This article discusses this search for the perfect letter.

PERSPECTIVE
Among the legendary accomplishments credited to Pythagoras of Samos (ca. 585-ca. 500 B.C.) was an interest in the geometric design and construction of the letters for the Greek alphabet. According to Apollonius of Messene, a second century grammarian and teacher in Alexandria, Pythagoras sought to achieve visual harmony within each letter through a systemic use of angles, line segments and circular arcs [1]. Whether this story is true remains unresolved; however, its existence does testify to a purposeful application of geometric methods for the construction of letters in the ancient world. Careful analysis of civic inscriptions from the Augustan period (27 B.C. - 14 A.D.) of Imperial Rome shows that their aesthetic appeal was based on the use of rather exact geometric designs and fixed proportions [2]. The thickness of the principal strokes of a letter as compared to the letter's height was in the ratio of 1 to 10. Apparently architects of this period who possessed a mathematical training imposed a canon of inscription design on their lapidary workers. This canon was adhered to for several generations. With the subsequent decline of the Roman Empire, the classical forms of the *litterae lapidariae* were temporarily forgotten and replaced by the Gothic styles of the Middle Ages.

With the intellectual and artistic blossoming of southern Europe in the fifteenth century, the prevailing atmosphere which had separated mathematics from aesthetics during the previous millennium began to change. Applications of mathematics to space and form became more widely recognized and appreciated. One episode of this movement was the renewed quest for the "perfect alphabet." This quest involved mathematics, aesthetics, philosophical, and mystical considerations and resulted in a fascinating series of connections between peoples and ideas. As the humanistic spirit grew in Italy, it manifested itself in many forms, not least of which was a renewed reverence for classical studies, artifacts and institutions. Latin, as a written language and conveyor of ideas, achieved a new prominence. Architectural forms and principles of Imperial Rome were resurrected. In particular, the theories of the architect Marcus Vitruvius Pollio (1st century B.C.) once again became popular.

Figure 1: Vitruvian man.

Existing Roman ruins and buildings were sought out and examined. Their geometrical features were graphically and physically duplicated by the artisans and artists of the time. Special attention was focused on the style and form of the classical inscriptions, the *scriptura monumentalis*, found on many such buildings. The resurrected classical Roman alphabet became an object of admiration and speculation and a new genre of theoretical and didactical writing appeared, *trattati delle lettre antiche*, manuals devoted to the graphical design of the Roman alphabet. By 1459, Roman let-

ters began to appear in panel paintings and frescoes. An example was in the work of Andrea Mantegna, who depicted Roman buildings and monuments and emulated Roman letters in his signatures. Ferrarese and Paduan miniaturists also incorporated Roman letter forms in their initials, imparting, so they believed, a classical charm to their products. These early letters were noticeably slender and possessed a dominating verticality. But gradually lettering, particularly that employed in lapidary work and medallion inscription, drew closer to its historical Roman exemplars. This transition was guided by a pronounced scholarly debate as to the form and shape of the "perfect letter", a debate in which the geometry of the ruler and compass and theories of proportion played prominent roles.

THE QUEST

The first noted commentaries on classical Roman lettering can be found in the writings of Veronese calligrapher and antiquarian Felice Feliciano. Feliciano dedicated a collection of such inscriptions copied from existing Roman structures to his friend, the artist Andrea Mantegna, in 1463 noting "I, Felice Feliciano, have revived this in the antique manner after ancient marble tablets such as are to be found in Rome and elsewhere" [3]. As a disciple of the theories of Vitruvius, he used the Roman architect's system of structure and proportion for the design of his ideal alphabet. Feliciano explained:

> According to ancient practice the letter is shaped from the circle and the square, the sum of whose forms rise to 52 [50 units], from which is drawn the perfect number 10. And thus the width [of the stroke] of your letter should be one-tenth of its height. In this way the letter has as much of the circle as the square [4].

Vitruvius, in turn, was influenced by neo-Pythagorean number theory of his time. For the ancient Greeks 10 was considered a very significant number.

It, of course, numbered the digits of the hand, establishing a special numerical relationship to the human body. For the Pythagoreans 10 was the sum of the tetractys, the numbers that represented the four elements i.e. 1 (fire) + 2 (water) + 3 (air) + 4 (earth) = 10, which, in itself, represented the universe. Further, Vitruvius viewed the human form geometrically as symmetrically contained within a configuration of a circle inscribed within a square: *homo ad quadratum et ad circulum*. In this geometrical complex, the human form was then longitudinally divided into 10 units each of which approximated the facial distance from the chin to the top of the forehead. This geometrizing and proportionalizing of the human form captured the imagination of the renaissance artistic community, particularly members such as Lorenzo Ghiberti and Leonardo da Vinci. Da Vinci's famous "Vitruvian Man" drawing depicting an individual, arms and legs extended, reaching for the circumscribing bounds of a confining circle and square, could well serve as a visual metaphor for the spiritual and intellectual climate of the times [5]. The concept of human centeredness was stressed by Vitruvius in his *De architectura* where he noted:

> Then again, in the human body the central point is naturally the navel. For if a man be placed on his back, with his hands and feet extended, and a pair of compasses centered at his navel, the fingers and toes of his two feet will touch the circumference of a circle described therefrom [6].

This physiographic theme was incorporated into the construction of Renaissance alphabets. For example, the writing manual *Luminario* (1526) of Giovam Baptisa Verini, featured its own representation of a Vitruvian man [7] [Fig. 1].

It is a fascinating realization that the humanistic trends in the graphic arts of this period purposefully associated the proportions and symmetries of the human form with the design of letters, the eventual vehicles for the expression of human thought.

Feliciano's suggestions for the design of an alphabet reflected the Pythagorean mystique now resurrected in the early Renaissance that man, music, architecture, and ultimately the world possess beauty based on a harmony of inherent numerical proportionality. This concept of harmony and beauty was now extended to the letters of the alphabet. While earlier European scribes drew their curved lines by visual reckoning, the calligraphers of the Renaissance, seeking higher

quality and standardization and spiritual significance for their letters, now employed a ruler and compass constructions and a system of fixed proportions.

Felice Feliciano's designs for the Roman alphabet are contained in a slender codex now found in the Vatican Library (Vat. Lat. 6852) [8]. The collection of twenty-five drawings depicting various letters are rendered in watercolor. Each letter contains shading providing a sense of depth for stonecutters who would consult the model. The instruction provided is purely visual; a calligrapher or craftsman would study the letters together with their indicated constructions and then duplicate the image as required. Explanatory text is also nonexistent. This treatise is believed to have been compiled in about 1460, making it the oldest extant reference on the classical calligraphic reforms of the Renaissance. Feliciano crafted each letter within the confines of a circle and a square; however, time has faded his original construction lines and they have been reproduced in later copies [Fig. 2].

While the Feliciano treatise was influential in establishing new directions for lettering, its format diminished possible impact. The mathematical quest for the perfect alphabet would be taken up and carried forward by others.

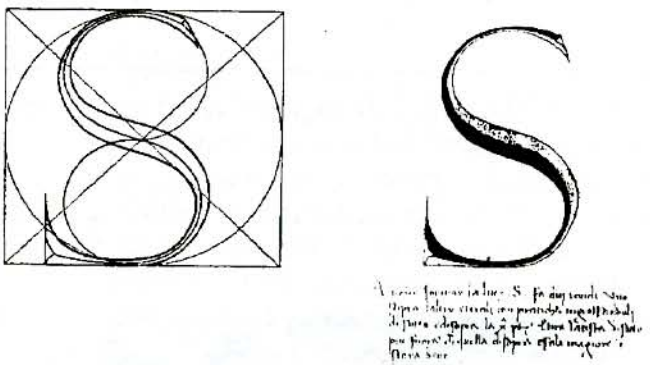

Figure 2: Letters designed by Felice Feliciano.

The first Italian to devise a printed manual on the design of letters was Damiano da Moile of Parma who in 1480 published his *Alphabetum*. This small tome, written for craftsmen, provided specific geometrical instructions for the design of each letter [Fig. 3]. Da

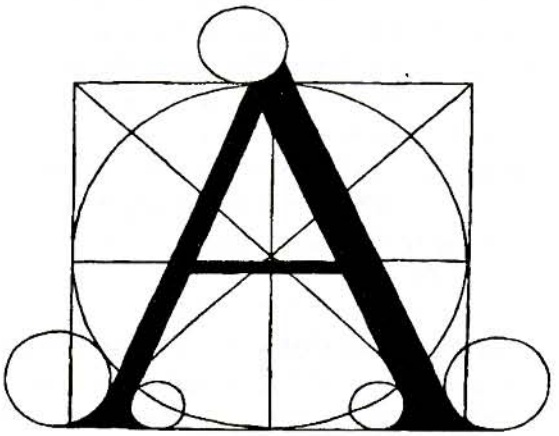

Figure 3: Letter designed by Damiano da Moile.

Moile also employed the guiding circle and the square constructs, but his letters were not constrained within their bounds. In his designs, geometry provided guidance without exerting a strict control. In the *Alphabetum*, the letter prints were made from wood block impressions. The wood block constructions and carving imposed certain restrictions on letter design features. Da Moile's system of proportions was based on a ratio of 1:12; that is, the thickness of the major stroke of a letter was to be 1/12 its height. Twelve was possibly chosen as a multiple of the perfect number 6.

At the height of the Renaissance in 1509, the last of the early theoretical works on Roman alphabet design appeared. This work was an appendix to a treatise on human proportion contained in the *De Divina Proportione* by Luca Pacioli. Pacioli, a theologian and mathematician, was the geometry teacher of Leonardo da Vinci and a friend and companion to Piero della Francesca and Andrea Mantegna. He was known for his work on geometric perspective and was well respected in the artistic and scientific communities. Pacioli's instructions on lettering were directed at "worthy stone cutters and zealous followers of the craft of sculpture and architecture." All requirements were rationalized in a humanistic light.

> . . . from the human body derives all measures and their demonstrations and in it is to be found every ratio and proportion by which God reveals the innermost secrets of nature.

> The ancients, after having considered the right arrangement of the human body, proportioned all their work, particularly the temples, in accordance with it. For in the human body they found two main figures without which it is impossible to achieve anything, namely the perfect circle . . . and the square [9].

He stressed the theory that only the classical geometric instruments, a straight edge and a compass, were necessary for the construction of perfect letters. Pacioli further noted that he had written his text on lettering to demonstrate that "everything comes from the discipline of mathematics." The friar-scholar differs from his predecessors in requiring a proportion within letters of 1:9. In a practical sense, such a ratio produces a bulkier letter, one more recognizable from afar. Theoretical considerations may have also reflected Platonic thinking where it was believed that 9 was a mediator between the dynamic numbers 6 and 12. Pacioli's letters are beautifully illustrated within the *Divina Proportione* [10] [Fig. 4]. There is some speculation that Leonardo himself supplied the illustrations for these letters—he is known to have drawn the regular polyhedra contained in the work; however, there is no firm evidence to support this claim [11]. Da Vinci's involvement with the calligraphic reforms of this period is controversial. While he was known to comment on the geometry and proportions of Roman letters, no specimens of his own constructions are available. An anonymous fifteenth century manuscript devoted to the geometric design of letters resides in the Newberry Library of Chicago [Fig. 5]. At times, the Newberry manuscript has been attributed to Leonardo Da Vinci; however, its authorship has never been fully confirmed. This manuscript bears many features, for example, the use of a 1:9 ratio, that may have influenced Pacioli's work.

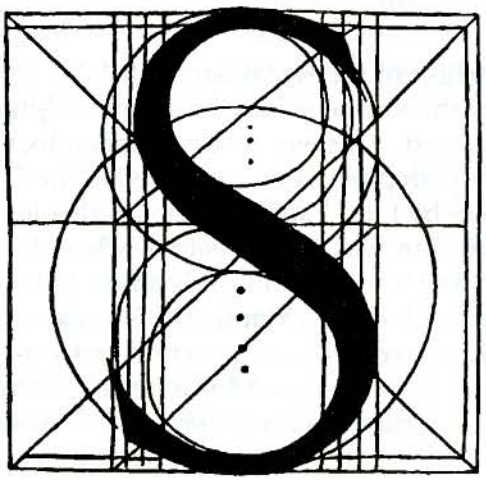

Figure 4: Letter designed by Luca Pacioli.

The first comprehensive manual devoted solely to the art of classical lettering was *Theorica et Practica de Modo Scribendi* by Sigismondo de Fante which appeared in Venice in 1514. This manual considered the construction of both Gothic and classical Roman letters. After its appearance, a proliferation of writing manuals by various authors appeared in Italy in rapid succession: Francesco Torniello (1517), Vicentino (1522), Giovanni Tagliente (1524), Verini (1527), Giambattista (1540), Giovanbattista Palatino (1550), Fernando Ruano (1554) and Giovan Francesco Cresci (1570) [12] [Fig. 6].

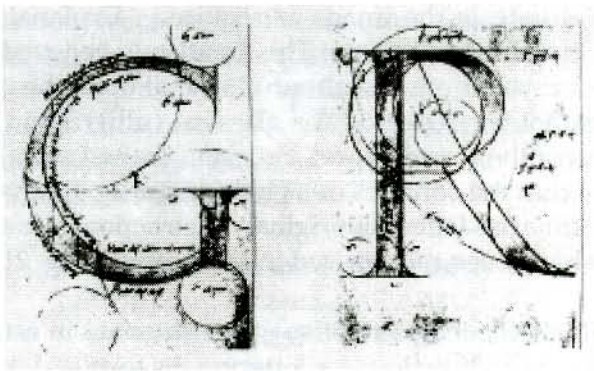

Figure 5: Letters of the anonymous Newberry Library manuscript.

Cresci deviated from his contemporary calligraphic theorists in that he avoided ruler and compass constructions. His admiration for classical Roman letter models was equally as fervent as that of Feliciano or de Fante; however, it was based on the letters of an inscription on the Trajan column in Rome [13]. This column was erected by the Senate and the Roman people in 113 A.D. to commemorate Emperor Trajan's victories on the frontiers of the Danube. Trajan letter models employed a ratio of 1:8 in stem width to height [14]. Cresci's letters are strong, free flowing and devoid of obvious geometric constraints [Fig. 7].

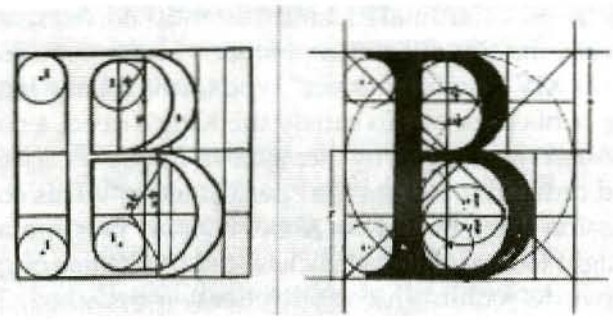

Figure 6: Letters designed by Sigismondo de Fante.

In the preface of his first published writing manual, *Essemplare di piu sorti lettere* (1560), he noted his disdain for the geometric methods of his fellow calligraphers:

> And in drawing every curve of each letter they make more circles than a sphere for the most part contains.... I have come to the conclusion that if Euclid, the prince of Geometry, returned to this world of ours, he would never find that the curves of the letters could, by means of circles made with compasses, be constructed according to the proportion and style of the ancient letters [15].

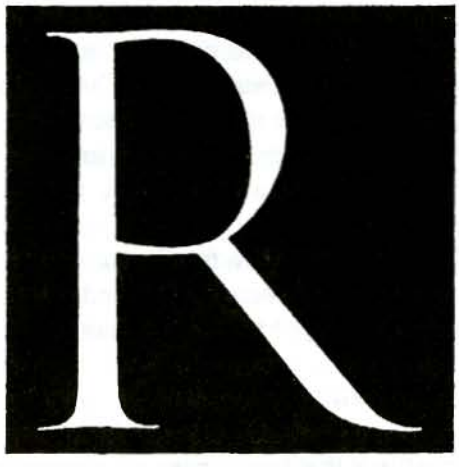

Figure 7: Letter designed by Giovan Francesco Cresci.

At the time, Cresci's letter forms attracted little following, and geometric-based letter rendering prevailed among Italian calligraphers.

Classical lettering models were imported north of the Alps by Albrecht Dürer. In 1525, Dürer published his *Underweysuna der Messung mit dem Zirckel uñ Richtachezt [Course in the Art of Measurement with Compass and Ruler]*. This book was written for *tagliche Arbeiter*, ordinary workmen, to demonstrate applications of geometry to the concrete tasks of architecture and engineering. Dürer felt that "Beauty is the harmony of the parts in relation to each other and the whole" [16] and attempted to insure this harmony in his instructions; however, he also exhibited a spirit of artistic independence. The artist employed two ratios in his lettering, 1:9 and 1:10, while advising his reader to use *"weliche dir am besten gefelt,"* opening the door to multiformity and individual taste. In this respect, Dürer shared the feelings of Leonardo Da Vinci. In his writing Albrecht Dürer considered two letter forms, German Gothic and Roman. In Gothic constructions he dispensed with circular arches and built up each letter from a number of geometric units, squares or trapezoids; the resulting forms were reminiscent of existing Arabic calligraphic styles [17] [Fig. 8].

In surveying the major works of the classic lettering movement of the Renaissance, one final contribution warrants special attention. In 1529, the French royal calligrapher, Geofroy Tory published his theories on the design of *litterae antiquae*, classical letters. Published in Paris, Tory's *Champ Fleury ou L'Art et Science*

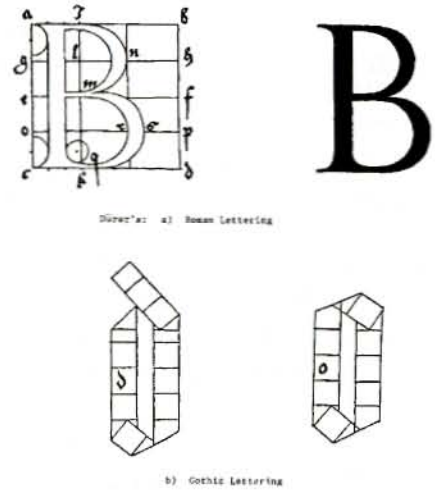

Figure 8: Letters designed by Albrecht Dürer.

de la Proportion des Lettres criticized the popular theories of Fanti, Pacioli and Dürer and offered the author's own version for the construction of perfect letters [18]. A ratio of 1:10 was employed not justified on mathematical or artistic grounds but rather derived from the nine Muses with Apollo added. This calligrapher not only attempted to relate his letters with Vitruvius but also associated them with classical mythology. The design of each individual letter was justified within the premise of *homo ad quadratum et ad circulum.* Almost mystic rendering of letters were constructed upon a square reference grid comprised of 100 smaller squares [Fig. 9].

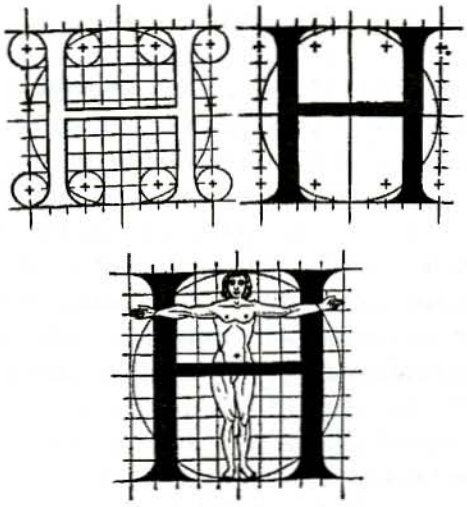

Figure 9: Letters designed by Geofroy Tory.

While it remains questionable if the search for the perfect letter achieved its goal, its quest brought together some of the greatest minds and artistic talent of the Renaissance. An aesthetic dimension was added to the printed and sculpted word and geometry lent itself to producing something living and personal.

SOME LATER EFFORTS

The concept of the "perfect letter" haunted the 16th-17th century architectural and artistic scene. In a sense, its issues involved a conflict between art as an aesthetic expression and mathematics as an aesthetic formulation. The theories of Pacioli, the rationalist, Dürer, the practical artist and Tory, the mystagogue, all reflected a humanistic concern to seek out fundamental laws, even those governing the shape of letters. As the Age of the Enlightment ushered in a sharper scientific spirit, one based on precision and exactitude, it exerted an influence on the search for the perfect letters. Now, design emphasis was on the style of printing fonts rather than the handwritten letter. In 1640, Cardinal Richelieu established, *Imprimerie Royale*, the Royal Printing House. In the year 1692, Louis XIV sought "perfect" type forms for the lettering of his presses. To satisfy the King's need, a commission was formed by the *Academe Royale des Sciences* and ordered to design the "perfect letter." This commission was chaired by Abbe' Nicolas Jaugeon and sought to carry out its task assisted by the use of grid networks within which each letter was designed. The Jaugeon Commission, as it was known, issued its recommendations three years later. Its geometrically perfect letters were each constructed on a square grid composed of 2,304 small squares i.e. 64 large squares each divided into 36 smaller squares. See Figure 10 [19]. The resulting letters were precise in their constructions and were meant to serve as models for the design of type fonts. The royal engraver Rajon freely adapted these designs and cut fonts in a style henceforth to be known as *romain du roi*; however, in general, engravers confronted with what they believed as contrived geometrical patterns simply refused to use them. They felt that ultimately the human eye was the sole judge of proper form and proportion. Eventually the enthusiasm for *litterae antiquae* waned and the use of Gothic letters returned to favor, especially in the printing of religious texts.

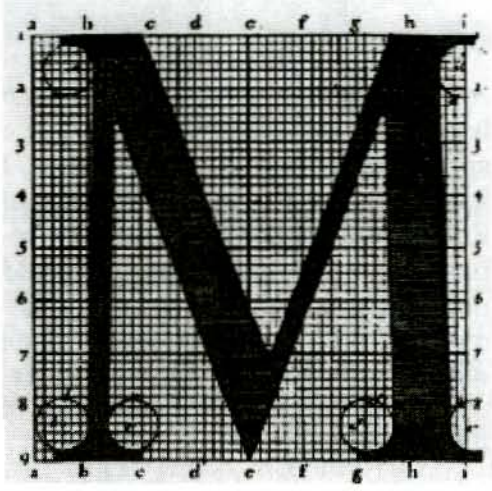

Figure 10: Letter designed by the Jaugeon Commision.

THE ISSUE REVISITED

Rapid technological advances within the last thirty years, including the perfection of photocomposing and the digitalization and computer storage and re-

production of images, have radically altered the graphical potentials of the printing industry. These new potentials have forced a rethinking of processes, techniques and formats, not least of which has been type or letter design.

> The type designer - or better, let us start calling him the alphabet designer - will have to see his task and his responsibility more than before in the coordination of the tradition in the development of letterforms with the practical purpose and the needs of the advanced equipment of today . . . [20].

In essence, the new challenges have resurrected the quest for the perfect alphabet with an included dimension of technological accommodation.

Intrigued by this challenge and recognizing "a good mathematical problem still waiting to be resolved", in 1977, Donald Knuth of Stanford University decided to tackle the problem of mathematically designing the "perfect letter" [21]. Systematically approaching his task, Knuth defined the problems as one of finding the "most pleasing" closed curve, MPC, to fit a set of $n \geq 4$ given points: $Z_1, Z_2, Z_3 \ldots Z_n$ in the plane. He then postulated a set of axioms to clarify the concept of "most pleasing."

The closed curve had to satisfy the following properties:

1. Invariance. If the defining set of points $Z_1, Z_2, Z_3 \ldots Z_n$, are subjected to a Euclidean or projective transformation, the MPC will experience the same transformation.
2. Symmetry. A cyclic permutation of the set of points will not change the form of the MPC.
3. Extensionality. Points from the MPC, if added to the defining set of points, will not alter the solution.
4. Locality. Each segment of MPC between two given points depends only on those points and their immediately adjacent points.
5. Smoothness. MPC is everywhere differentiable.
6. Roundness. If Z_1, Z_2, Z_3, Z_4 are consecutive points of a circle, their MPC will be a circle.

Guided by these postulates, Don Knuth derived his MPC as a piecewise continuous curve where each segment is determined by an appropriate cubic spline. Appropriate MPC's have been devised for each letter of the alphabet and loaded into a computer program. The resulting system, called METAFONT, is flexible and allows its user, through a variation of parameters, to devise an infinite variety of letter forms and fonts. Thus the concept of the "perfect letter" now truly lies within the eye and control of the METAFONT user. Knuth has completed the task attributed so long ago to Pythagoras, labored upon by the artists and calligraphers of the Renaissance and pondered by the Jaugeon Commission. A visual comparison between a METAFONT composition and those proposed by some Renaissance masters is shown in Figure 11.

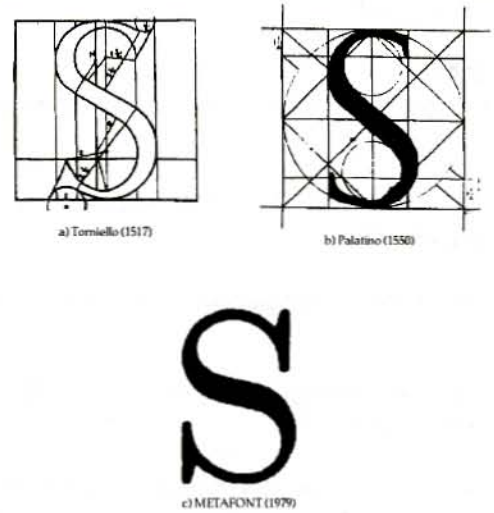

Figure 11: Comparison of classical and modern lettering.

CONCLUSION

This historical discussion has briefly surveyed the conceptual development and evolution of a problem - the mathematical design of the letters of the Western Roman alphabet. While the problem is not a pressing one in terms of utilitarian applications, it possesses a certain psychological and intellectual, as well as mathematical appeal - the combining of mathematics with aesthetic concerns to serve the needs of communication. The various solution attempts have reflected on their times. It can be assumed that the Pythagoreans sought a visual balance and symmetry in the image of written letters to conform with their mystical gestalt of cosmological harmony. Whereas Renaissance calligraphers strove to replicate classical exemplars, they still indulged in experimentation and rationalization. The failure of the Jaugeon's Commission's recommendations may, in part, reflect

the rise of a spirit of individualism and rebellion among French artistians that would later express itself so strongly in the French Revolution. Don Knuth's resolution of the situation while ultimately prompted by a mathematical challenge, was also motivated and facilitated by the existence of new technologies. The scope and power of METAFONT and its companion TEX typesetting system has revolutionized the field of typography and demonstrated how mathematics under the rethinking of an old problem can be applied to new fields. Interestingly, these new resulting techniques of typography have not narrowed choices as to the "perfect letter", rather they have broadened options and personalized the decision of perfection. What a nice result!

REFERENCES

[1] Hilgard, A. "*Scholia in Dionysii Thracis artem grammaticam*", *Lipsiae* (1901) 183-184.

[2] Gordon, A. and J. Gordon. *Album of Dated Latin Inscriptions*, 4 vol., Berkeley, CA. 1958-65.

[3] Bertieri, R. "*Italienische Kalligraphen im 16 Jahrhundert*", *Philobiblon* (1929).

[4] Meiss, M. *Andrea Mantegnas as Illuminator.* New York: Columbia University Press (1957) p. 69.

[5] Payne, R. *Leonardo.* New York: Doubleday and Co., 1978, p.. 432.

[6] Vitruvius. *De architecturu.* Book III, Chapter 1 (see F. Granger translation, *Vitruvius on Architecture*, Cambridge, MA: Harvard University Press, 1962 1:161).

[7] Verini, G. *Luminario.* Tusculano (on Lake Garda), 1526.

[8] Mardersteig, G. (ed). *Felice Feliciano: Alphabetum Romanum.* Verona: Officina Bodoni, 1960.

[9] Wittkower, R. *Architectural Principles in the Age of Humanism.* London (1949), pp. 14, 70.

[10] De Pacioli, Fra Luca. *De Divini Proportione.* Stanley Morison (ed). New York: Kraus Printing Co. (1969).

[11] Goudy, F. W. "The Roman Alphabet: Its Origin and Esthetic Development", *Ars Typographica* (1926) 2:185-208. Goudy gives illustrations of letters he credits to Leonardo p. 204-205 and mentions that they are from a private collection held by one "C.L. Ricketts of Chicago." Later the manuscript containing these letters was acquired by the Newberry Library, Chicago. It presently resides there and is referred to as "the Anonymous Newberry Manuscript."

[12] Meiss, M. "The First Alphabetical Treatises in the Renaissance", *Journal of Typographic Research*, (January, 1969), 3-30.

[13] Anderson, D. "Cresci and His Capital Alphabets", *Visible Language* (Autumn 1971) 331-352.

[14] Goudy, F. W. *ibid.*

[15] Cresci, Giovanni Francesco. *Essemplare de piv sorti lettere.* Rome, 1560.

[16] Panofsky, E. *The Life and Art of Albrecht Dürer.* Princeton, Princeton University Press, 1955, p. 261.

[17] Dürer, Albrecht. *Of the Just Shaping of Letters.* Translated by R. T. Nichol. New York: Dover Publication, (1965), p. 41.

[18] Tory, Geofroy. *Champ Fleury ou L'Art et Science De La Proportion Des Lettres.* Geneve: Gustave Cohen, 1973.

[19] Morison, S. *Calligraphy* 1535-1885, Milan: La Bibliofla, 1962.

[20] Zapf, H. *About Alphabets: Some Marginal Notes on Type Design.* Cambridge, MA: M.I.T. Press, 1970, p. 71.

[21] Knuth, D. *TEX and METAFONT: New Directions on Typesetting.* Providence, RI: American Mathematical Society, 1979.