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Puzzles, Patterns, Drums: the Dawn of Mathematics in Rwanda and Burundi.

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1. INTRODUCTION.

Douglas R. Hofstadter received the 1980 Pulitzer Prize for his book *Gödel, Escher, Bach: An Eternal Golden Braid*. It wedded the mathematical results of Kurt Gödel, the graphical art of Escher and the music composed by Bach. Hofstadter showed how a common idea seemed to emerge in three different modes of expression, easing access to the more arduous mathematical part by suggesting the reader to solve a Gödel problem (see [Hof] and [Swa]). Drawings by Escher were alternated by excerpts from Bach's score and dialogues between the imaginary actors Achilles and Tortoise served as intermezzos.

Hofstadter elucidated one of the apogees of modern mathematics, Gödel's theorem, but maybe at another era in history analogous similarities can be discovered between mathematical, graphical and musical expressions. Inventing names for numbers, adding them, making geometric and numeral combinations with pawns, with lines or through music, might have been a comparable summit for humanity, in the times of the dawn of science.

In the middle of Africa, two small countries lived until recently in such an epoch. On the tops and flanks of the almost round hills of Rwanda and Burundi, lived one of the most dense populations of Africa, from agriculture and cattle breeding. There are many similarities between their populations of Hutus, the vast majority of peasants, Tutsis, the former aristocratic cattle-breeders and Twas, the more marginal potters. Their Kinyarwanda and Kirundi languages, their tradition and social history are closely related. Some pretend that, if the word *Rwanda* could be interpreted as *the vast territory*, *(B)urundi* would simply mean *the other [country]*.

Historically, small kingdoms existed in this region within living memory, and each had its own sacred symbols, like for example a *little drum*. There were a multitude of these little principalities dispersed in the mountains, until the legendary *Gihanga* descended from the heavens, along with the thunder, as did other emperors in Sumeria, Mesopotamia or Creta. Besides cattle and seed, he brought fire to the mortals; the memory of this Promethean event would only extin-

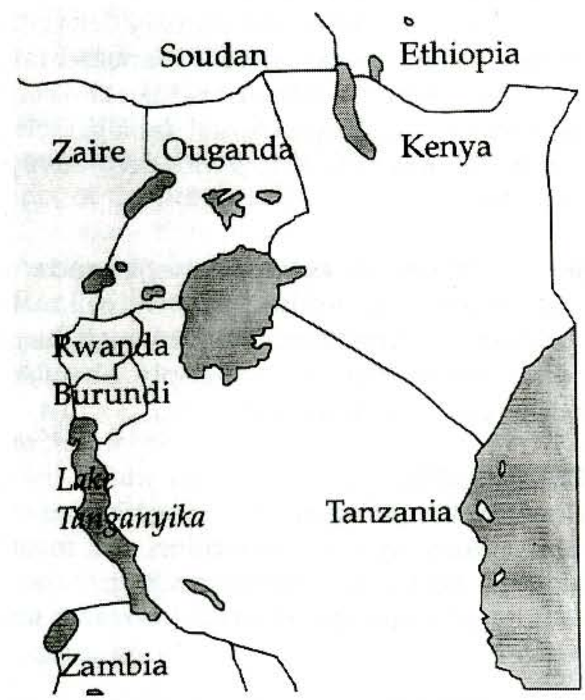


Figure 1

guish in 1933, when the king was converted to Catholicism. *Gihanga* also created the cult of the *large drums*, a very visible sign of his monarchy. These wooden emblems had a symbolic value comparable to the respect a scepter and a national flag have in other nations. The royal drums at the Rwanda court were not beaten but only touched when an important decision had to be to justified through the resonance of their deep bass sound.

There were 4 sacred drums, of about six feet high.

These wooden cylinders were covered by a brown cow-hide and each contained a crystal of quartz, their soul. The most magnificent had been called *Ruoga*, but it was lost in the 15th century. This was believed to be the cause of eleven years of distress, until a determined king could restore its shape *by his knowledge of numbers*. The new sacred drum *Karinga* or *warrant for hope* was placed in the *hut* of worship close to two others and next the oldest drum of all, called *the king is the owner of science*[1].

The sacred drums could not be regarded in absence of the king. Partition-walls protected them from the eyes of the mortals and as a security measure, other non-sacred drums were used when the king had to travel. Still, on other batteries music was indeed performed, for pleasure. It was not the only delectation, since playing on the *igisoro*-board was another favorite diversion, as were the Homeric riddles and *puzzles* that were told during the nocturnal *drum* gatherings where milk was drunk from jars with decorative *patterns* of all kinds.

Important differences between the Rwanda and Burundi culture exist. In this paper we will mainly focus on examples from the former, although the general principles of most topics apply likewise to Burundi, as can be seen from Figures 2, 3 and 4.

2. AN *IGISORO*-PUZZLE.

Two elements, **the traditional *igisoro*-game** and some facts about **counting** without writing, will together provide the setup for the formulation of a puzzle. The idea to propose a problem, to make the reader familiar with some characteristic difficulties, was inspired by Hofstadter's book. An answer to the enigma will be given in §6.

Figure 2
The *igisoro*-board around the hearth in Burundi (see [Acq]).

Figure 4
A drum player in Burundi (see [Acq]).

The traditional *igisoro*-game (see [Cou-Ben] and [Mer]) is played by two opponents on a rectangular wooden board. It is about 2 feet by 1, and has 32 circular cavities, arranged in 4 rows and 8 columns divided in 2 parts (see Figure 5). The players move around with 64 pawns (little stones, seeds or beans) to get enough pawns of the adversary to prevent him from taking pawns on his turn.

The players move their pawns on their own half of the board, following the indicated direction (opposite, as we would say, to the direction of the hands of a watch). A move can start at any cavity containing at least two pawns by collecting all the pawns in it and consists in dropping the pawns one by one in every cavity, after the cavity where the pawns were taken. If the last pawn is dropped in an empty cavity, the move stops. Otherwise, the player may go on by collecting these pawns and doing another similar move; this is called a *bridge*.

Pawns of the adversary may be captured if a move ends in a cavity on the lower row, containing at least one pawn, as should the opponent's two cavities in the same column. Taking pawns is obligatory, if it is possible. If a player has indeed captured some pawns

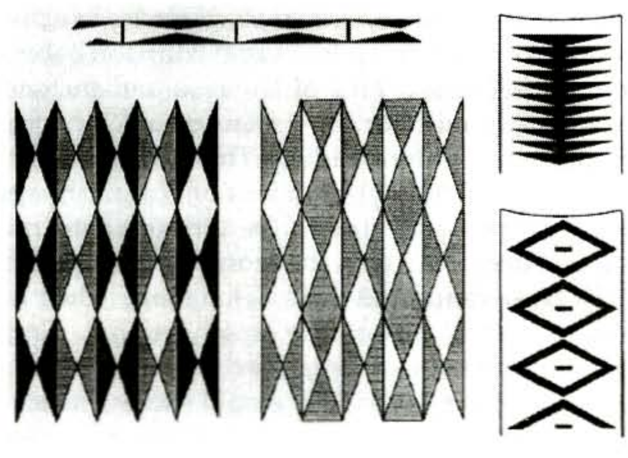


Figure 3
A decorative pattern called *abashi* or *wooden support* (see [Cel] and [Pau]) from the border region of Rwanda, Tanzania and Burundi.

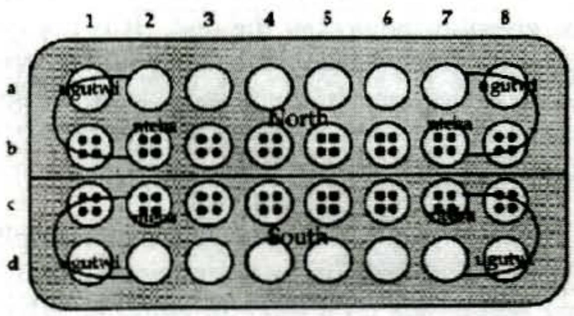


Figure 5
The *igisoro*-board with a few denominations.

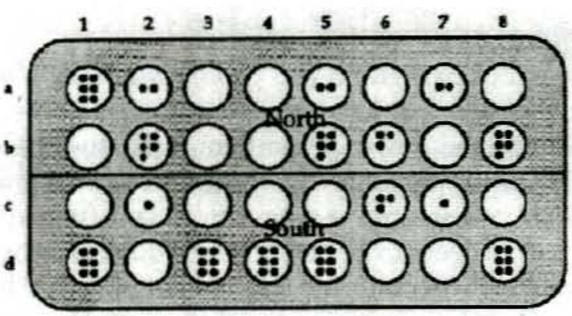


Figure 6
Taking pawns: only if player South could manage to end a move by dropping the last pawn in c2, he could take his oppenent's pawns. For instance, starting at d5, a bridge at c6 reaches to c2. The player goes on with the pawns from a2 and b2, starting at c5.

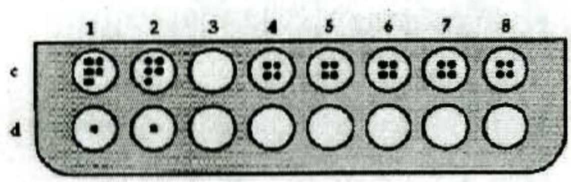
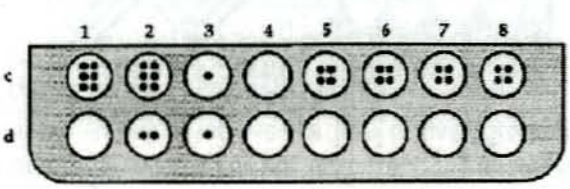


Figure 7
An easy opening in *igisoro* is called *madondi*, meaning *to deal dry and repetitive whips*. The move starts at c3 (above). It is followed by another move starting from d3, with a bridge in d1 (below).



of his adversary, he goes on playing with the pawns he took. He drops, one by one, the pawns he captured in the next cavity after the one where he started the last simple move (or after the last bridge if there has been one) before taking his opponent's pawns (see Figure 6).

There is a particular rule about the direction of movement: if one starts a move or if one makes a bridge at cavities called *nteba* (b2, b7, c2, c7) or *ugutwi* (a1, a8, d1, d8), and if one can, by doing so, get into a situation of capturing pawns by a simple move, without bridging, then the player may reverse the direction.

When a game starts, there are 4 pawns in each of the middle rows (as in Figure 5), and both players begin their opening moves simultaneously. There are different kinds of openings, sometimes with amusing names (see Figure 7), and like in chess they each have their reasons for being used. If one of the players has finished the second movement of his opening, the opponent has the right to take his pawns, and the winner of the previous game starts an attack (if it is the first game they play together, it is a matter of tactics to choose who starts). Each player makes a move, until one of them does not have enough pawns to continue. The game has to be played fast and sanctions are foreseen for a player who hesitates or cheats.

Note that these rules define the *igisoro*-game as it is known in one particular region, and that different versions exist, even within the region of Rwanda and Burundi. Traveling farther, larger variations are encountered. In East Africa, in Tanzania, a similar *soro* or *boa* game is played, while farther North, the *Kabaka* of Uganda play the *okweso*, and going to the West, the Nigerian *Yoruba* call their version *Ayo*. North of the Equator, the game is often performed on a board with only two rows instead of four, while three rows seem to be the tradition in Ethiopia. There may be from six up to fifty holes in a row.

Counting in Burundi and Rwanda was done using a base 10 system, and even for numbers as large as 1,999,999,999 words existed (see [Huy]). It must be pointed out that no consensus about these facts exists among historians[2], but this does not, of course, prevent mathematicians to admire the feat of inventing words for large numbers. There were no written expressions, and one can wonder how the slightest arithmetical operations could ever be executed.

An example of a procedure for executing complicated multiplications without any notation, can be found with the Yoruba (see [Jos]). They have a number system with base 20 and often use substractions to describe numbers. For example, *nineteen* (*nine plus teen*) is expressed as *ookandinlogun*, meaning *one less than twenty* (*ookan = one, dinl = minus, ogun = twenty*). Similarly, the appellation of *525* corresponds to *80 less then 600 plus 5*, or $(20\times3)-(20\times4)+5$. A story from 1887 tells about a counter who used cowry shells. To multiply 19 and 17, he started forming twenty piles of twenty shells each. Next, he took one shell from each pile, and then put three piles aside. These three heaps were rearranged by taking two shells from one of them, and adding it to the two others, the objective being to reduce the involved numbers to twenty:

$$400-20-(20\times2)-(20-3)=(400-80)+3=323$$

The Yoruba example shows some arithmetic operations were indeed done even in civilizations where no form of notation existed: representations with cowries replaced the written symbols.

Before turning again to the *igisoro*-board, we need a more convenient multiplication method, called the *Russian peasant* method. It was already known in ancient Egypt and in Greece, and is said to have found its way during the Middle Ages to Russia, the Middle East and finally back to Africa, in Ethiopia (see [Nel]). To multiply two numbers, like for example 241 and 17, one proceeds in this method as follows: divide 241 by 2, until 1 is reached; if an odd number is encountered, first subtract 1:

$$241\to120\to60\to30\to15\to7\to3\to1$$

The other number, 17, is multiplied as many times by 2:

$$17\to34\to68\to136\to272\to544\to1088\to2176$$

The numbers in this last row, corresponding to odd numbers in the previous row, are added:

$$17+272+544+1088+2176=4097$$

This is the desired result: $4097=241\times17$!

The puzzle: could the reader find out how to perform such a multiplication without writing down any auxiliary calculations, and use but an *igisoro*-board? In other words, a description is asked, of numbers with pawns placed in cavities, and a way for translating the Russian peasant multiplication into this representation. The mathematical justification of the method will be given at the end of the text. It is not quite necessary for solving the puzzle, but could be useful to find an indication. In the next paragraphs, some additional information is given first, to render the proposed answer more plausible.

Note that we do not mean to suggest that multiplications were traditionally done on an *igisoro*-board, but playing with seeds on a piece of wood to solve an arithmetic question may be a diverting and instructive exercise to get an idea about the necessary intellectual efforts needed to realize a mathematical achievement in a given cultural environment.

3. PATTERNS.

The previous paragraph was probably not very helpful to discover a primary explanation on the how and why about the dawn of mathematics. Indeed, the *igisoro*-game is played in different countries, and so it might be conjectured it was introduced from other cultures. However, the genesis of the idea of decorating walls with geometric patterns, can be traced back to its very origin. Indeed, an oral account relates why suddenly someone preferred to decorate his hut by geometric patterns instead of figurative images.

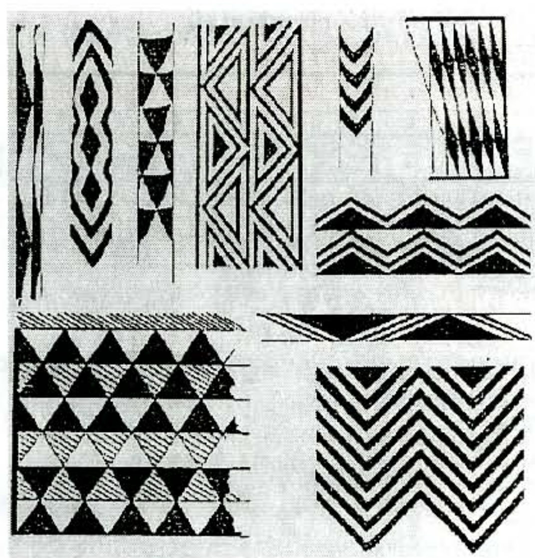


Figure 8

Paintings givin in [Cel]; all have descriptive names in Kinyarwanda. For example, the first is called *umuheha*, or *tube*.

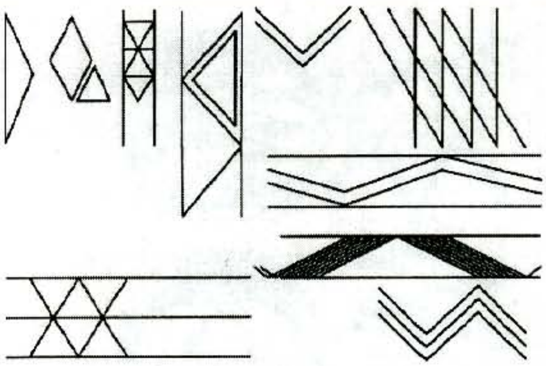


Figure 9
Explanations of the patterns by Celis.

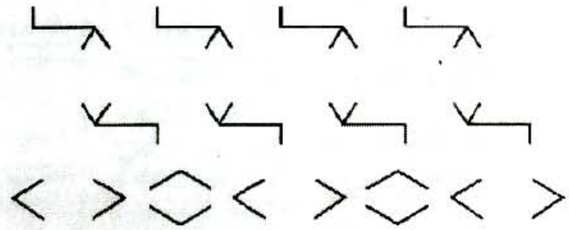


Figure 10
Zaslavasky's symmetry example.

In [Cel] the authors published their discovery of original paintings on enclosures of huts, in an isolated region in Rwanda (see Figure 8). It is difficult to access and hence, most paintings are believed to be traditional concepts, and not the result of an exchange with other cultures, nor the consequence of the ever progressing phenomenon of acculturation. The phenomenon of decorating a hut by the so-called *imigongo* seems to go back in the past for about three centuries, and the oral narration still relates how the legendary notable *Kakira ka Kimenyi* came to install the tradition of embellishing walls:

> Numerous acts in his life proved *Kakira ka Kimenyi* was possessed by neatness; his cattle were held in huts and were slaughtered there, so that no fly would ever touch it. [...] He hated mud and sat on a rock during heavy rainfall. His neatness was so legendary it became a locution to say *isuku ni ya Kakira* (neat like *Kakira*). Plenty of initiatives, *Kakira* would have made these paintings for pleasure, and by solicitude of neatness; first, he made them for his father [...], and then in his own hut. [...] Having made these paintings, he encouraged young girls -- of the aristocracy -- to imitate him. In this way, these paintings spread.

In other regions of Rwanda and Burundi, drawings and patterns were, of course, made too, but then it was on enclosures or walls in the huts, on small baskets, covers of milk jars and decorated drums (see Figure 11). On one of the previous pages (Figure 3), three drawings of the pattern *abashi* are *imigongo* paintings (reported in [Cel]) while the two on the right were found elsewhere in Rwanda on enclosures in huts (reproduced from [Pau]).

G. and T. Celis noted that the patterns they found, are combinations based on just a few elementary constructions. Only vertical, horizontal and three skew directions together with their symmetric directions along the vertical, are enough to form all the motives (see Figure 9). The *imigongo* can be classified by these 8 directions into just a few cases since only parallel lines, isosceles or equilateral triangles and kites are involved. Incidentally, these geometrical observations also led them to reject some other paintings as non-traditional.

A discussion about the use of some Chinese, Arabic and African drawings in the curriculum of pupils from 6 to 16 age was given by J. Williams (see [Nel]). Her comments apply to the present drawings from Rwanda:

> The classification of patterns by their symmetry groups is studied by crystallographers, and can be pursued through multicultural sources of patterns and design. Zaslavsky (see [Zas]) reproduces a picture of embroidered cloths from Kuba, Zaïre (now in the British Museum) which provides a complete set of seven different one-dimensional strip patterns. These patterns involve transformations in one dimension, such as 180° rotations and

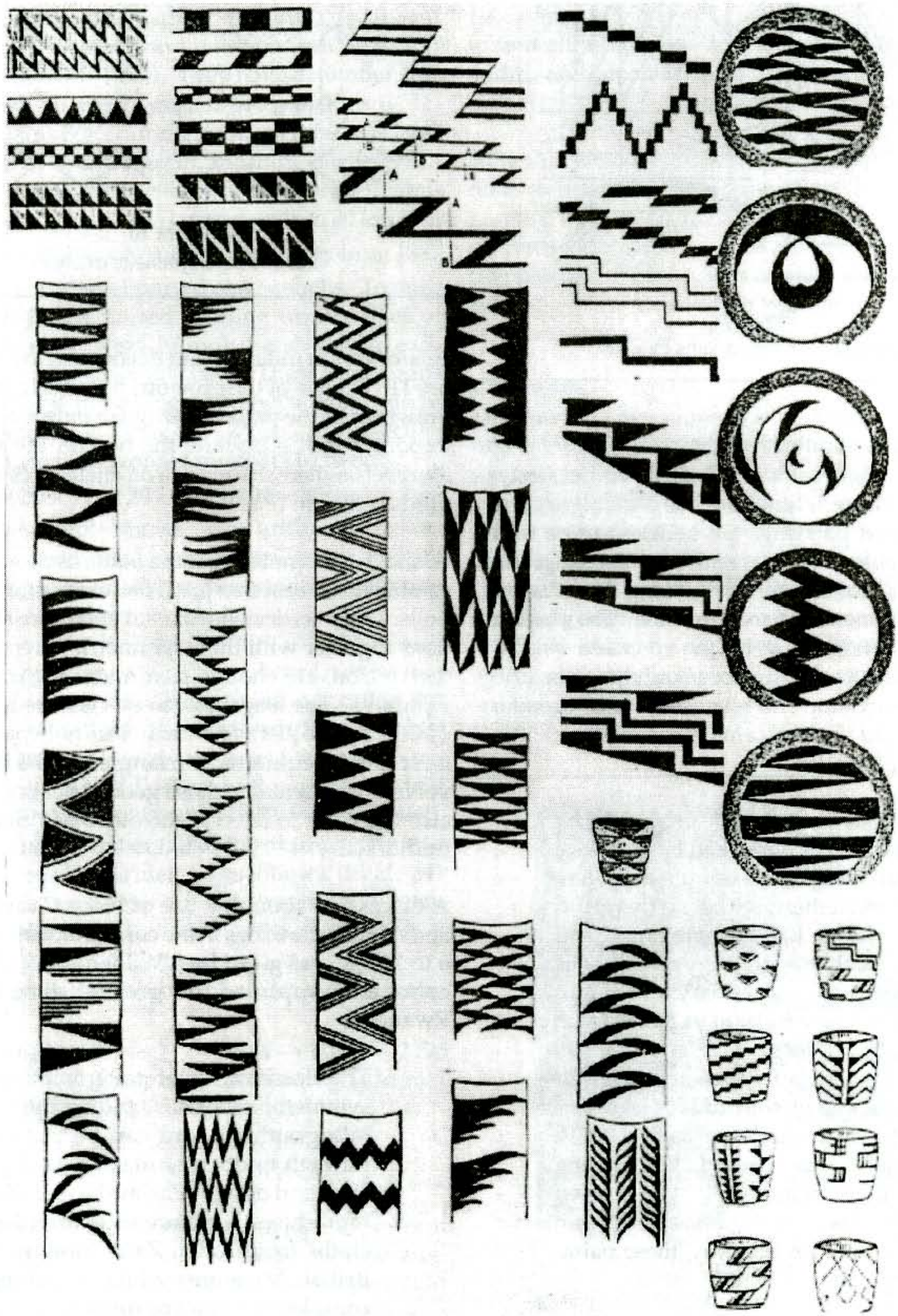


Figure 11
Decorations on baskets, jars and enclosures (from [Pau]).

Figure 12
Percussion staffs; cf. [Nke].

> horizontal and vertical reflections. Group theory can be used to prove that only seven such patterns can exist.

Williams also gave the above drawing (Figure 10), showing two strip patterns with rotational symmetry of order 2, but only one of these has horizontal and vertical lines of symmetry. It is a more mathematical way of appreciating geometric figures: it illustrates that any pattern with two perpendicular reflectional symmetries must have a rotational symmetry of order 2.

4. DRUMS

Some structure was apparent is the *igisoro*-game because of the presence of counters obeying well-defined rules, and in the previous paragraph the reader was invited to cerebrate an *igisoro*-framework for the Ethiopian multiplication method. However, when hearing African music, recognition of some logical basis seems even more difficult. Günther relates that, at the end of the 50's, the royal drums of Rwanda came to the world fair in Brussels. The Belgian audience was not prepared: the 24 drummers made the impression, said one listener, of *insistent, horrendous banging*. Others confessed more politely that *after a while the din overcame one's power of concentration.*

However, ignorance of the underlying structure may have been the main reason for the latter conclusion. Of course, if someone is familiarized to some kind of art, the knowledge about how some craft was accomplished is not necessary to appreciate it, though someone who went to an academy is more likely to appreciate Bach's music. Three rules seem to govern the percussion music in Rwanda and Burundi: the ***hemiola* effect**, the ***additive* rhythm**, and the ***Gestalt* phenomenon**.

Figure 13
Hemiola; cf. [Nke].

Hemiola is about the proportions of rhythmic models in their organization of the rigorous measures of time. Fixed intervals of time are subdivided in an equal number of subintervals by consecutive beats. Possible subdivisions of the intervals of time are two, four, eight or sixteen impulses or else three, six, twelve or twenty-four impulses, even if one starts with the same base interval. This, of course, implies that the proportion of the period between the pulses in both cases is 2:3. Usually, an intermediate rhythm of 4 or 6 beats follows, accentuated by slapping the hand, and by beating wooden sticks: this is called *simple idiophony*. Sometimes, the slow rhythm of 2 or respectively 3 drum-beats is used to reinforce this base rhythm. The faster cadences, of 8 or 16 and 12 or 24 pulses, are the bases for more melodic or percussional rhythms. They form the basic elements of the structure.

Yet, there are often subdivisions that cannot be placed in either this basis 2 or the basis 3-form. One can imagine these as proportions of an alternating basis 2 or basis 3 -form, and so as a successive realization of the proportion 2:3. This drum-beat structure is called

Figure 14
A piece of Rwanda drum music with *Hemiola*; cf. [Bra].

hemiola. It is the serial combination of a 2 or 3 subdivision, each of the same length, possibly with still more subdivisions. R. Brandel (see [Bra]) points out that reversed sectional change, that is, from 2-grouping to 3-grouping, is also encountered. His main example is precisely a piece of music from the royal drums in Rwanda:

> Here the 2/8 groups are organized in 3/4 measures (17 measures in this section), and the 3/8 groups are organized in 3/8 measures (22 measures in this section). Again the true *hemiola* is evident, provided two 3/8 measures are combined.

The consulted references ([Bra], [Nke], [Mic], [Gün]) agree that this *hemiola*-rhythm makes African music so different from Occidental patterns, although alliance with Middle Eastern and Hindu rhythms certainly does not make it unique. The five-unit *hemiola* of Ancient Greece also contained this 2:3 leader-beat contrast, but the rapid succession of unequal leader-beats in a 2:3 length-ratio is the typical African *hemiola* change: the music is distinguished by immediate exchanges of leader-beat: many changes occur within a short space, usually within a measure.

Additive **rhythms** differ from the more Occidental division rhythms, although they both are ways of subdividing an interval of time. The use of unequal groupings is preferred in African music. This attitude of asymmetry is the domain of excellence of the percussion.

To describe what additive rhythms are about, consider an interval with 12 pulses. It can be grouped in two groups of 6+6, but also into 7+5 or 5+7. Also, a measure of 8 beats can be decomposed as 5+3 or 3+5, and as 3+2+3, 2+3+3, or 3+3+2. Inside an interval of time, an equal duration can thus be lengthened or shortened, but of course all pieces should add up to the given number (here, 12 or 8). On a staff, this is written as follows:

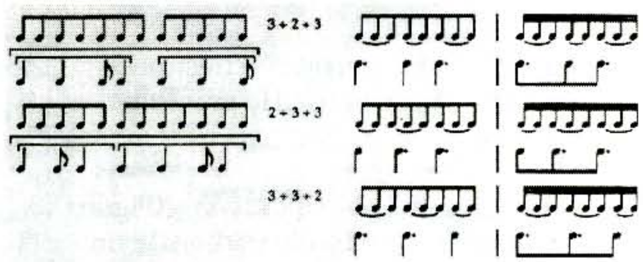


Figure 15
Additive rhythm; cf. [Nke].

Gestaltvariation is the third remarkable feature in the drum-batteries of Rwanda (see [Bra]):

> The coincidence of hemiolic lines inevitably carries with it some kind of *Gestalt* effect, almost as if a new rhythmic pattern, resulting from the composite interplay of all the lines, emerged. Very often the preponderance through timbre, pitch, etc. of one line over the others makes it suitable for *single-line listening* no matter how complex the entire work.

The indications of this *Gestaltvariation* again point towards a similarity with Mediterranean and Asian music, notes Günther (see [Gün]), and others again

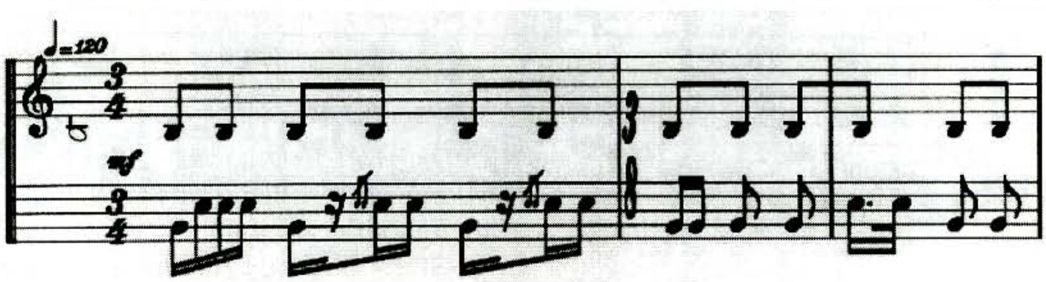


Figure 16
A piece of Rwanda drum music with *Hemiola*; cf. [Bra].

Figure 17
A more involved example of Rwanda drum music; cf. [Bra].

see a link with Ancient Greece. In the example of Figure 14, the deeper toned drums in the ensemble changed from 2-grouping, 3/4, to 3-grouping, 3/8. The leader-drum continued its 2/8 figure (see Figure 16) but is overshadowed by the basses.

> Yet, [says Brandel] because of its lesser obtrusiveness, the listener does not really hear the total counter-rhythm -- he merely feels it. The dynamic accent in the leader drum is almost lacking and the 2/8 grouping is achieved by means of very subtle timbre contrast.

Finally, all these constructions can be put together as in the following piece of percussion from the royal drums in Rwanda. It is more complicated to understand for the non-initiated:

> Despite the galloping strength of the lowest line, the 3-grouping of the top line somehow makes itself quite apparent, and the eventual result is complex pull in two directions.

In contrast to the remarks made at the Belgian '50 world exposition, given in the beginning of this section, it is therefore not the lack of structure and logical constructions that make this music difficult to access for Western listeners, but rather its abundance.

5. INTERMEZZO.

In Hofstadter's masterpiece *Gödel, Esher, Bach*, conversations between Achilles, the Tortoise, and the Tapir alternated the tougher mathematical reasonings that explained the Gödelian concepts. The well-known paradox of Zeno was the inspiration for the creation of these imaginary personages. In the context of Rwanda, actors exchanging Aristotelian sophisms were created by Kagame (see [Kag1]). He called them *Gama* and *Kama*, and some of the exchanges of ideas of the players Kagame invented, suit well to provide us a Hofstadter-like intermezzo. The following excerpt contains riddles related to the present topic about the dawn of mathematical reflection:

Gama: It would be convenient to examine if the bantu-rwandean philosophy has elements related to the notion of "time". I think, at this very moment, about that woman of the Court, who lived under the reign of *Mibambwe III 'Sentabyo*, in the XVIIIth century. One attributes the following reflection to her. It passed afterward onto the common language like a profound adage: *Ko bucya bukira, amaherezo azaba ayahe?*, that is *Since there is day and night, and the end of times, what will there be?*

One can apply this sentence, as you know, upon the events that go on and on, without interruption, even when one expects it to end. That woman certainly had thought profoundly about the progress of "time"! Don' t you think, like myself that her reflection merits the qualification of "philosophical"?

Kama: Up to a certain degree, yes. However, there is much better in this domain. Did you ever hear about the riddles that were solved by *Ngoma*, the son of *Sacyega*? I do not want to confirm that these two personalities have really existed. The solved riddles have been grouped under the name of *Ngoma*, as some lies were gathered under the name of *Semuhanuka*; in the same way the gourmet anecdotes were attributed to *Rugarukirampfizi* and the sly puns to *Semikizi*.

Gama: That is the way it goes with our traditions presenting a certain literary value, characterizing a numinous turn, and of which the various authors are forgotten. Our narrators grouped them in series, and each series got a single, but maybe faithless, name.

Kama: Correct! So one day, our *Ngoma* had to solve another riddle. His father was in debt with the Death about a head of cattle. Thus, it is clearly an invented story, the work of someone regarded as a thinker. The terrible creditor went one day to see *Sacyega* and declared:

> "The debt has to be paid without hesitancy! However, I demand that you pay me a head of cattle, that is nor a bull nor a cow! Failing that head of cattle, I will sacrifice yourself!"
>
> "You ask me something completely impossible!" *Sacyega* begged; "A head of cattle always is a bull or a cow, because one never sees one that is nor the first nor the latter!"
>
> "Your problem!" answered the Death; "or you find me that head of cattle, or you can within eight days from today arrange your affairs."

Informed about this terrible dilemma, *Ngoma* answered his father as follows:

> "It is not so difficult! It suffices to put the Death in the impossibility to claim his incompatible head of cattle. When he will show up at the agreed day, answer him by the words: 'I finally have found what my arrearages are. Yet, to seize it, you cannot come during the day nor at night. At daytime one can see the stars, and at night they are visible. So come between both events and you will get your cattle.'"

The narrators do not tell the continuation and they did not need to. The inventor of the problem only had in mind to formulate two impossibilities and to oppose them one another.

Gama: In fact, the solution attributed to *Ngoma* has an obvious philosophical significance. It points very precisely to the moment whose duration is as impossible to evaluate as it is impossible to meet cattle without genus.

Yet it is clear that the narrators did not recall the entire depth of this point of view. They do not conceive this limit between the day and the night, explicitly based on the passage of non-being to being, as we envisage:

> "non-being of light of
> the stars" to "being of
> light of the stars".

This solution corresponds exactly to the well-known principle of the great meta-physicists: between the being and the non-being there is no third way.

The above quote from Kagame obeys the tradition of the *smiths of intelligence*, the narrators at the royal court who memorized thousands of verses relating a poetic version of the history of Rwanda from about the year 1100 up to the beginning of our century. To ease memorization, a rigorous formal structure based on rhyme, rhythm and tone, was imposed on the text, as in the Homeric verses.

In other poems too, a unit of vocalic quantity could be discovered: the *mora* (see [Cou-Kam]). It consists of one short vowel or half a long vowel, and 9, 10 or 12 *moras* form the basis for the main type of verses. Studies in Kirundi poetry (see [Cou]), confirm these findings about the rigorous formal structure of the poetry in the culture of this region.

Parenthetically, the illustration below comes from a book of modern Kinyarwanda poetry (see [Kag2]). The poem glorifies the creation of the almighty *Imana*, but here the bard does not ask if *God* plays dice: the divine hand covers an *igisoro*-game.

6. COMMENTS.

Returning to the §2 *igisoro*-puzzle, here is first an explanation for numerical example of the Russian peasant method given in that section. The number 241 is written as a sum of powers of 2:

$$241 = 1\times 2^0 + 0\times 2^1 + 0\times 2^2 + 0\times 2^3 + 1\times 2^4 + 1\times 2^5 + 1\times 2^6 + 1\times 2^7$$

The product of 241 and 17 follows from a term by term multiplication:

$$\begin{aligned}241\times 17 &= \left(1\times 2^0 + 0\times 2^1 + 0\times 2^2 + 0\times 2^3 + 1\times 2^4 + 1\times 2^5 + 1\times 2^6 + 1\times 2^7\right)\times 17\\ &= 17\times 1\times 2^0 + 17\times 0\times 2^1 + 17\times 0\times 2^2 + 17\times 0\times 2^3 + 17\times 1\times 2^4 + 17\times 1\times 2^5 + 17\times 1\times 2^6 + 17\times 1\times 2^7\\ &= 17\times 1\times 2^0 + 17\times 1\times 2^4 + 17\times 1\times 2^5 + 17\times 1\times 2^6 + 17\times 1\times 2^7\\ &= 17 + 272 + 544 + 1088 + 2176\\ &= 4097\end{aligned}$$

Figure 18
An illustrationfrom a tome ([Kag2]) of modern Kinyarwanda poems.

In §2, 241 was first divided in halves, with the condition to subtract 1 in case of an odd quotient. This first row of operations allowed to get the non-zero coefficients in the decomposition of 241 as a sum of powers of 2, while the second row, where 17 was doubled, served to obtain the corresponding numbers that had to be added: 17, 272, 544, 1088 and 2176.

The proposed *igisoro*-puzzle asked for a representation of this multiplication diagram on an *igisoro*-board, without using any notation to re-

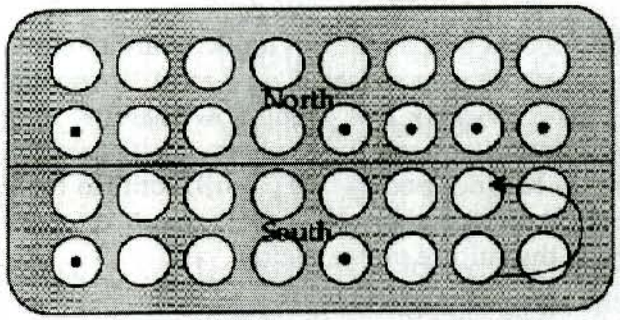


Figure 19
Imaginary representations of 241, *North*, and 17, *South*.

member the operations. A possible solution goes as follows: imagine a pawn in the first cavity (b1 or d1) would represent 2^0, one in the second (b2 or d2) would stand for 2^1, and so on, until the last one (a1 or c1), 2^{15}, is reached. Then, in Figure 19, North would represent the first number, 241, while South would be the second, 17. This is merely a recreating idea by the author, inspired by the Yoruba cowry calculations and the principles of the *igisoro*-game; it does not correspond to any historical data.

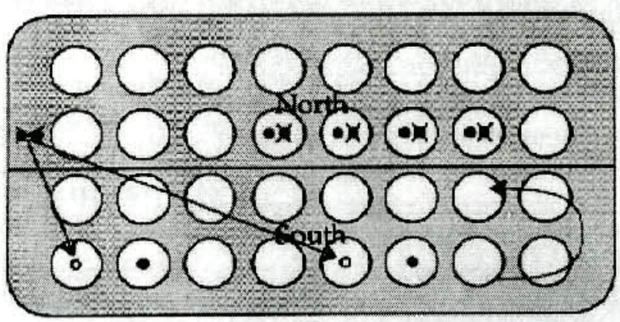


Figure 20
Halving 241 in *North* and doubling 17 in *South*; the pawns of 17 are withheld (white) , the result being 34 (black).

Following the *Russian peasant* method, 241 should now be divided 2. Each pawn in the representation of 241 is replaced by two pawns in the previous cavity, and only half of them are withheld (see North, in Figure 20). 17 is doubled simultaneously by moving its pawns 1 step to the right. There was a problem with the remainder 1 of the division of 241 by 2, since it could not be represented adequately. This fact reminds us we should keep track of the initial value 17, before it was doubled (cf. Figure 20, South, white pawns).

The next consecutive divisions by 2 yield no problem, since the remainder is 0, and thus the results of those multiplications by 2 are not withheld. Note that the operations of halving and doubling are easily executed: it is enough to move the pawns one cavity to the left or the right, respectively. Yet, when 4 pawns on a row are obtained, in b1, b2, b3 and b4, representing the number 15, one has to keep in mind that for the next doubling in South, the initial pawns should again be withheld (see Figure 21).

Finally, when there is only 1 pawn left in North, the procedure stops. In South, the withheld pawns in d1, d5 (2 pawns), d6, d7, d8, c8, c7, c6 and d5 correspond to the numbers 17, 272, 544, 1088 and 2176 and these should be added (see Figure 22).

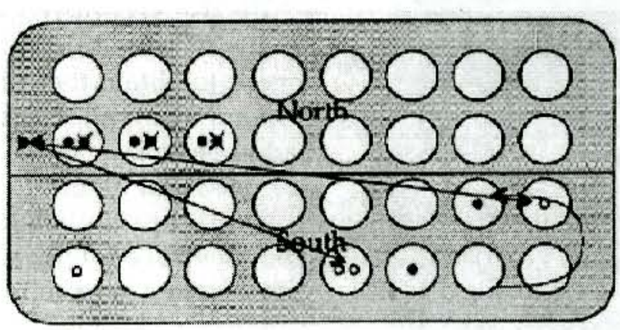


Figure 21
Halving 15 in *North* and doubling 272 in *South*; the pawns of 272 are withheld (white), the result being 544 (black).

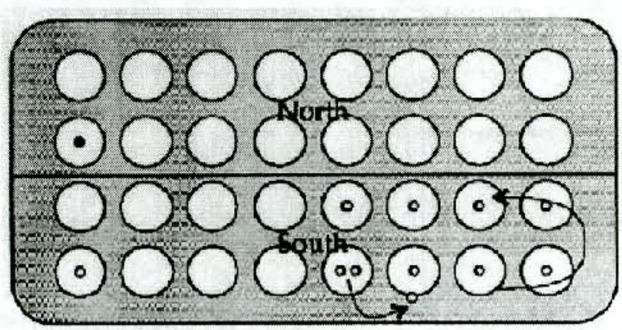


Figure 22
Finally, only 1 pawn remains in *North* and 10 in *South*; the latter should be added.

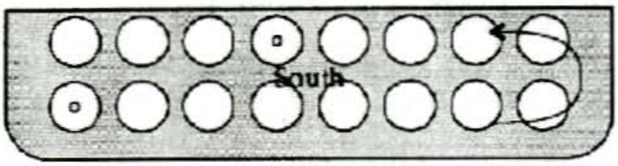


Figure 23
The final result: 1 + 4096 = 4097.

The addition of the 2 pawns in d5 is straightforward: they are replaced by a single one in d6. Now there are 2 pawns in d6, and the procedure continues until every cavity contains but a single pawn. The demanded product can be read off: 1 (the pawn in d1) plus 4096 (1 pawn in c4) yield the required 4097.

An objection to this apparently very easy method could be that the example works so smoothly because of the choice of the numbers 17 and 241. This is indeed partially true: if there are many pawns left to be added, a harder mental computation is necessary in the last step (Figure 23) to convert the answer in base 2 to the final result in base 10.

A final wink to *Gödel, Escher, Bach* is the observation that Hofstadter liked to refer to computer problems, although the subject of his book was a topic out of the domain of the purest mathematics of all. His favorite computer savant was Babbage, but in the present case it might have been entertaining to say a few more words about N. Wirth, the creator of PASCAL. Indeed, instead of **puzzling** about the multiplication procedure on an *igisoro*-board, one could imagine that the cavities corresponded to computer switches. A pawn in a cavity means the switch is closed. Thus, doubling a number by transferring pawns one cavity to the right, corresponds exactly to a computer shift. Of course, the reality is not that simple, but even N. Wirth explained the importance of converting a multiplication to an operation of doubling in his successful book on programming fundamentals (see [Wir]). Note that the prestigious Massachusetts Institute of Technology expressed its appreciation for the *igisoro*-concept by programming it on a computer. They restricted their study to one of the most simple *igisoro*-versions with only 2 rows of 6 holes and 36 counters. Nevertheless, there are still about 1024 possibilities in this very simple situation. Thus, it is a good test case for trying out heuristic methods, applying only advantageous moves. R. C. Bell's classification of *igisoro* among the world's nine best games seems amply justified (see [Zas]).

The design by computer of geometric patterns, as those found in Africa, was the subject of Williams' text entitled *Geometry and Art* (see [Nel]). This author proposed the following key lines of a computer program to form patterns of TRIANGLEs separated by GAPs:

```
FOR N=1 TO ENDX;
        NEWY=0;
        FOR M=1 TO ENDY;
                NEWY=OLDY+GAPY(M);
PROCTRIANGLE(NEWX,NEWY);
        NEXT M;
        NEWX=OLDX+GAPX(N);
NEXT N.
```

Musicians like computer toggling too: Frank Michiels, a researcher at the prestigious Belgian Museum for Central Africa in Tervuren and a recognized percussionist, plays on African **drums** for his computer. The electronics transform the recorded music into notes of any kind, from organ to violin. And still, the African musical structure remains irrefutable!

The **summary** given in Table 1 is easily completed from the present paper. The words in italics refer to some striking terms or names used in the text.

Expression → ↓ Representation	Puzzles	Patterns	Drums
Without writing	*Igisoro* -board	*Kakira*-drawing	*Additive hemiola*
Written	241×17=4097	*Symmetry-groups*	*Staffs-structures*
Computer-screen	*Shift switches*	*Williams' computer patterns*	*F. Michiels' violin-percussion*

Table 1

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[1] This is a quotation from [Per], but one has to be careful about how to translate statements like *by his knowledge of numbers* or *the owner of science*. Some linguists may provide different translations, but of course, being a mathematician, this is a discussion that the author willingly omits. Another problem that was not regarded, is the particular transcription with special punctuation linguists use for the Kinyarwanda or Kirundi words.

[2] from [Cou2] and [Rod] one could conclude 10,000 or *ibihuumbi cumi*, was the highest number that was conceivable, while [Pau] mentions 100,000 or *akahumbi*, but [Kag3] goes indeed as far as one less than 2 billion.