Origin of the multipole pairing interactions

Hideo Sakamoto
Faculty of Engineering, Gifu University, Yanagido, Gifu 501-11, Japan

and

Teruo Kishimoto
Institute of Physics, University of Tsukuba, Ibaraki 305, Japan

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The self-consistent effective interactions in nuclei under the influence of the monopole pairing correlation are investigated. It is shown that the multipole pairing interactions are induced so as to restore the local galilean invariance of the system.

Recently, we have developed a theory of self-consistent effective interactions in nuclei [1]. The framework can be applied for deriving effective interactions to a quite general potential $V(\boldsymbol{r})$ so long as it depends only on coordinates. In the process of performing realistic numerical applications, however, we have realized that there are still some undesirable features to be resolved. They seemed to be associated with certain properties of a single particle field and a pairing field. Such examples, in the case of an isovector giant dipole resonance (GDR), are the following [2]:

(i) The centroid energy of GDR in a deformed nucleus does not coincide with that of a spherical nucleus, in apparent disagreement with experimental observations. It turned out that it is mainly due to the $(\boldsymbol{l}^2-\langle \boldsymbol{l}^2\rangle)$ term in the Nilsson hamiltonian, which is actually velocity dependent.

(ii) The effect of the pair correlation to the energy of GDR is rather large and depends on shell fillings, in spite of the fact that the excitation energy is quite high. It seems to be mainly caused by spurious velocity dependence of the pairing field.

In order to overcome such shortcomings, the investigation of the symmetry properties of a realistic hamiltonian has been initiated and indeed the source of such troubles is traced down to the velocity dependence of the single particle field and the pairing field which causes the violation of local galilean invariance of the system. The local galilean invariance of the internucleon interaction was discussed by Belyaev in connection with the gauge invariance [3]. Then, the derivation of self-consistent effective interactions in a velocity dependent field has been attempted. In this paper, along the line of the symmetry restoring method [4-6], we will show that the multipole pairing interactions should be induced so as to restore the local galilean invariance of the total system, which was originally broken by the velocity dependence of the monopole pairing field.

The standard monopole pairing interaction is written as

$$H_{0\,\mathrm{pair}}=-\tfrac{1}{4}G_0P_0^\dagger P_0,$$
$$P_0^\dagger=\sum_\gamma a_\gamma^\dagger a_{\tilde\gamma}^\dagger=2\sum_{\gamma>0} a_\gamma^\dagger a_{\tilde\gamma}^\dagger, \tag{1}$$

where $a_\gamma^\dagger$ is the creation operator to the single particle state γ, and $\tilde\gamma$ is the time-reversed state of γ. The corresponding pairing field is given by

$$V_{0\,\mathrm{pair}}=-\tfrac{1}{2}\Delta(P_0^\dagger+P_0)=\tfrac{1}{2}(v^\dagger+v),$$
$$v^\dagger=-\Delta\cdot P_0^\dagger, \tag{2}$$

with a gap parameter Δ given by

$$\Delta=\tfrac{1}{2}G_0\langle P_0^\dagger\rangle_{\mathrm{BCS}}=\tfrac{1}{2}G_0\sum_\gamma u_\gamma v_\gamma, \tag{3}$$

where $\langle\ \rangle_{\rm BCS}$ means an expectation value for the BCS ground state and quasiparticle operators $d^\dagger$ and d are defined by the Bogoliubov–Valatin transformation as

$$a^\dagger_\gamma = u_\gamma d^\dagger_\gamma + v_\gamma d_{\tilde\gamma}. \tag{4}$$

Let us introduce, without any specification yet, a one-body operator $\mathcal{O}$,

$$\mathcal{O} = \sum_{\alpha\beta} \langle\alpha|\mathcal{O}|\beta\rangle a^\dagger_\alpha a_\beta, \tag{5}$$

except for its time-reversal property which is assumed to be

$$\hat{T}\mathcal{O}\hat{T}^{-1} = (-)^T \mathcal{O}^\dagger \tag{6}$$

or equivalently

$$\langle\beta|\mathcal{O}|\tilde\alpha\rangle = -(-)^T \langle\alpha|\mathcal{O}|\tilde\beta\rangle, \tag{7}$$

where $\hat{T}$ is the time-reversal operator, and $(-)^T$ is a short-hand notation for the time-reversal phase and is either $+1$ or -1. Then, the following commutation relation can be easily verified:

$$[\mathcal{O}, V_{0\,\rm pair}] = \Delta \sum_{\alpha\beta} \langle\alpha|\tfrac{1}{2}[1+(-)^T]\mathcal{O}|\tilde\beta\rangle (a^\dagger_\alpha a^\dagger_\beta - a_{\tilde\beta} a_{\tilde\alpha}). \tag{8}$$

A particular choice of $\mathcal{O}$ yields, for example,

$$[p_x, V_{0\,\rm pair}] = 0, \tag{9}$$

$$[x, V_{0\,\rm pair}] = \Delta \sum_{\alpha\beta} \langle\alpha|x|\tilde\beta\rangle (a^\dagger_\alpha a^\dagger_\beta - a_{\tilde\beta} a_{\tilde\alpha}). \tag{10}$$

These relations indicate that the monopole pairing field satisfies the translational invariance, but the galilean invariance is not satisfied if the system is superconductive, i.e. $\Delta \neq 0$ [4,7]. More generally, if $\mathcal{O}$ is an operator depending on coordinate variables, then eq. (8) means that the monopole pairing field is not invariant under a local galilean transformation [3].

Now we will introduce an additional interaction defined by

$$H_{\rm int} = \tfrac{1}{2}\tilde\kappa \tilde{F}^\dagger \tilde{F}, \tag{11}$$

with

$$\tilde{F}^\dagger = -\frac{1}{\tilde\kappa}\frac{1}{i\hbar}[\mathcal{O}, v^\dagger], \tag{12}$$

$$\tilde\kappa = -\left(\frac{1}{\hbar}\right)^2 \langle[\mathcal{O}^\dagger, [v^\dagger, \mathcal{O}]]\rangle_{\rm BCS} = -\left(\frac{1}{\hbar}\right)^2 \langle[\mathcal{O}, [v, \mathcal{O}^\dagger]]\rangle_{\rm BCS}. \tag{13}$$

Then, the broken symmetry is restored in RPA order once $H_{\rm int}$ is added to the total hamiltonian, since the following equality is guaranteed:

$$[\mathcal{O}, V_{0\,\rm pair} + H_{\rm int}]_{\rm RPA} = 0. \tag{14}$$

After a little manipulation, the interaction becomes

$$H_{\rm int} = -\tfrac{1}{2} G(\mathcal{O}) P^\dagger(\mathcal{O}) P(\mathcal{O}), \tag{15}$$

with

$$G(\mathcal{O}) = \left[\sum_{\alpha\beta} \frac{1}{4}\left(\frac{1}{E_\alpha} + \frac{1}{E_\beta}\right) |\langle\alpha|\mathcal{O}^\dagger|\beta\rangle|^2\right]^{-1}, \tag{16}$$

$$P^\dagger(\mathcal{O}) = \sum_{\alpha\beta} \langle\alpha|\mathcal{O}|\beta\rangle a^\dagger_\alpha a^\dagger_{\tilde\beta}, \tag{17}$$

where E_α is the quasiparticle energy of state α. It should be remarked that the summation in eq. (16) runs only over the states included for the monopole pairing interaction as appearing in eq. (1). We must mention here that eq. (16) has the same structure as that of the Belyaev identity [3,8] though the present derivation is somewhat simple.

The above interaction is a generalized pairing interaction characterized by the operator $\mathcal{O}$ which may be chosen as a generator of the collective motion under consideration. For example, if we require the local galilean invariance under a collective shape oscillation of a 2^λ-pole mode, the generator can be chosen as $\mathcal{O} = r^\lambda Y_{\lambda\mu} \equiv Q_{\lambda\mu}$ and the interaction becomes

$$H_{\rm int} = -\sum_\mu \tfrac{1}{2} G_{\lambda\mu} P^\dagger_{\lambda\mu} P_{\lambda\mu}, \tag{18}$$

with

$$G_{\lambda\mu} \equiv G(Q_{\lambda\mu}) = \left[\sum_{\alpha\beta} \frac{1}{4}\left(\frac{1}{E_\alpha} + \frac{1}{E_\beta}\right) |\langle\alpha|Q^\dagger_{\lambda\mu}|\beta\rangle|^2\right]^{-1}, \tag{19}$$

$$P^\dagger_{\lambda\mu} \equiv P^\dagger(Q_{\lambda\mu}) = \sum_{\alpha\beta} \langle\alpha|Q_{\lambda\mu}|\beta\rangle a^\dagger_\alpha a^\dagger_{\tilde\beta}. \tag{20}$$

It is quite important to realize that the energy weighted sum rule for $Q_{\lambda\mu}$ now keeps the classical value without being affected by the spurious velocity dependence of the monopole pairing field, owing to eq. (14).

For a spherical system, we may put

$$|\alpha\rangle = |jm\rangle \quad \text{with } E_\alpha = E_{jm} = E_j, \tag{21}$$

and eq. (19) becomes

$$G_{\lambda\mu} = \left[\sum_{jj'} \frac{1}{4} \left(\frac{1}{E_j} + \frac{1}{E_{j'}} \right) \left| \frac{1}{\sqrt{2\lambda+1}} \langle j \| Q_\lambda \| j' \rangle \right|^2 \right]^{-1} \equiv G_\lambda^{\text{self}}. \tag{22}$$

Since G_λ^{self} is independent of μ, eq. (18) becomes

$$H_{\text{int}} = -\tfrac{1}{2} G_\lambda^{\text{self}} (P_\lambda^\dagger \cdot P_\lambda) \equiv H_{\lambda\,\text{pair}} \quad (\lambda \geq 1), \tag{23}$$

which is nothing but a 2^λ-pole pairing interaction. Now it is quite clear that the local galilean invariance is guaranteed if the interaction strength is chosen as the value of eq. (22). In this sense G_λ^{self} has the meaning of the self-consistent strength of the 2^λ-pole pairing interaction. For comparison, the critical value of the strength at which the RPA mode for this interaction breaks down is given by

$$G_\lambda^{\text{crit}} = \left[\sum_{jj'} \frac{1}{E_j + E_{j'}} \left| \frac{1}{\sqrt{2\lambda+1}} \langle j \| Q_\lambda \| j' \rangle (u_j u_{j'} + v_j v_{j'}) \right|^2 \right]^{-1}. \tag{24}$$

For the case of the dipole ($\lambda = 1$) modes, Pyatov and Salamov have already derived a $(P_1^\dagger - P_1)^2$ type of interaction and applied it to the analysis of GDR [4]. It has the advantage that the galilean invariance is restored without violating the translational invariance, though the particle number is not conserved. On the other hand, the $P_1^\dagger P_1$ type of dipole pairing interaction recovers the galilean invariance and conserves the particle number but violates the translational invariance. The reconciliation of the translational invariance, the galilean invariance and the particle number conservation is under investigation.

For the case of the quadrupole ($\lambda = 2$) modes, we have evaluated the values of G_2^{self} for several even-even nuclei. As shown in fig. 1, these values are in good agreement with the empirical values we have been using in our microscopic BET calculations to fit the experimental data of low-lying collective states [2,9,10].

The extensive applications of the prescription along this line to a realistic single particle field including velocity dependent terms such as the $(\boldsymbol{l} \cdot \boldsymbol{s})$ term, the $(\boldsymbol{l} \cdot \boldsymbol{l})$ term, etc., in addition to the pairing field, are now in progress [11]. When such a program is completed, we may have hopefully improved the microscopic understanding of collective mass

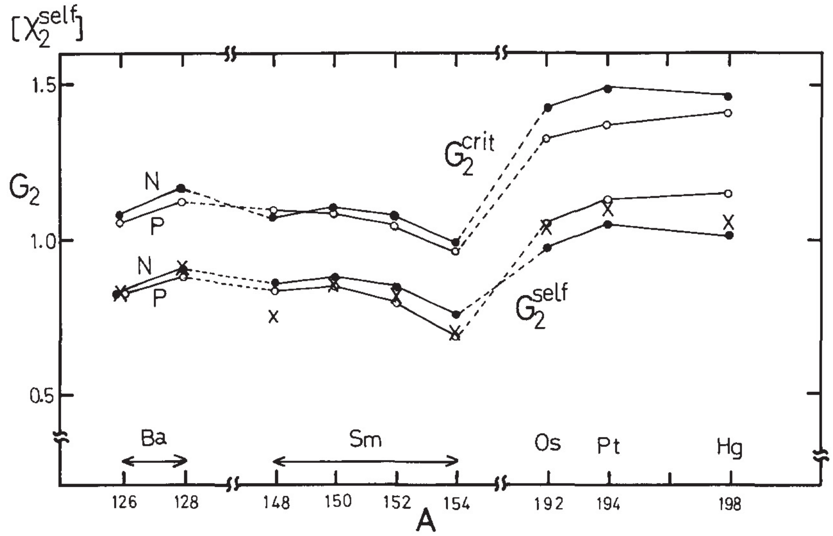

Fig. 1. The strength of the quadrupole pairing interaction. The self-consistent strength G_2^{self} and the RPA critical strength G_2^{crit} are evaluated for the proton side (P) and for the neutron side (N) separately. The empirical value obtained so as to reproduce the experimental data is denoted in the figure by a cross point for each nucleus. Here $\chi_2^{\text{self}} = \frac{8}{5}\pi M\mathring{\omega}^2 / A\langle r^2\rangle$ is a self-consistent strength of the $(Q \cdot Q)$ force when $\Delta N = 2$ excitations are fully renormalized.

parameters in various nuclear collective models. In fact, for a long time such an improvement has been called for as the result of the detailed comparison with experimental data [7].

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References

[1] H. Sakamoto and T. Kishimoto, Nucl. Phys. A 501 (1989) 205, 242.
[2] H. Sakamoto and T. Kishimoto, Annual Report of Tandem Accelerator Center, University of Tsukuba (1983) p. 85; Annual Report of Tandem Accelerator Center, University of Tsukuba (1986) p. 83.
[3] S.T. Belyaev, Sov. J. Nucl. Phys. 4 (1967) 671.
[4] N.I. Pyatov and D.J. Salamov, Nukleonika 22 (1977) 127.
[5] S. Cwiok, J. Kvasil and B. Choriev, J. Phys. G 10 (1984) 903.
[6] O. Civitarese and M.C. Licciardo, Phys. Rev. C 39 (1989) 1550.
[7] A. Bohr and B.R. Mottelson, Nuclear structure, Vol. II (Benjamin, New York, 1975).
[8] I. Hamamoto, Nucl. Phys. A 232 (1974) 445.
[9] T. Kishimoto and T. Tamura, Nucl. Phys. A 192 (1972) 246; A 270 (1976) 317.
[10] H. Sakamoto, Ph.D. Thesis, University of Tsukuba (1987).
[11] S. Kinouchi, H. Sakamoto, T. Kubo and T. Kishimoto, contribution to the Taro Tamura Memorial Symp. (RIKEN, Japan, December 1989).