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Timing and Carrier Synchronization with Channel Estimation in AF Two-Way Relaying Networks

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***Abstract*—Two-way relaying networks (TWRNs) allow for more bandwidth efficient use of the available spectrum since they allow for simultaneous information exchange between two users with the assistance of an intermediate relay node. However, due to superposition of signals at the relay node, the received signal at the user terminals is affected by *multiple impairments*, i.e., channel gains, timing offsets, and carrier frequency offsets, that need to be jointly estimated and compensated. This paper presents the system model for amplify-and-forward (AF) TWRNs in the presence of multiple impairments and proposes least squares and differential evolution based algorithms for joint estimation of these impairments. The Cramér-Rao lower bounds (CRLBs) for the joint estimation of multiple impairments are derived. A minimum mean-square error based receiver is then proposed to compensate the effect of multiple impairments and decode each user's signal. Simulation results show that the performance of the proposed estimators is very close to the derived CRLBs at moderate-to-high signal-to-noise-ratios. It is also shown that the bit-error rate performance of the overall AF TWRN is close to a TWRN that is based on assumption of perfect knowledge of the synchronization parameters.**

I. Introduction

Relaying is a key technology to assist in the communication between two user terminals, especially when there are large transmission distances between them [1]. Unidirectional (one-way) relaying supports communication from a source user to a destination user and has been widely studied in the literature [2]. On the other hand, in two-way relaying networks (TWRNs), the flow of information is bidirectional and the two users exchange information simultaneously with the assistance of an intermediate relay node [3]. Thus, compared with one-way half-duplex relaying, bidirectional relaying is a spectrally more efficient relaying protocol [4]. Both amplify-and-forward (AF) and decode-and-forward (DF) protocols have been developed for TWRNs. In contrast to the DF protocol, the AF protocol is widely adopted, as it requires minimal processing at the relay node [5].

During the two phase communication in AF TWRNs, the two users first transmit their information to the relay node. The relay broadcasts its received signal to both users in the second phase.[1] However, the two users' signals at the relay node undergo different propagation and may not be aligned in time and frequency. Consequently, the superimposed signal broadcasted from the relay node is affected by *multiple impairments*, e.g., channel gains, timing offsets, and carrier frequency offsets. The existing literature does not take all these impairments into account in studying the performance of AF TWRNs [6]. Though, estimation and compensation algorithms have been proposed to counter these impairments in unidirectional relaying networks [7]–[9], the proposed algorithms cannot be directly applied to TWRNs due to differences between the two system models. Particularly, in TWRNs, each user can exploit the knowledge of the self transmitted signal during phase 1 in order to detect the signal from the other user during phase 2. Recently, the effect of channel estimation [5], [10]–[12] or joint channel and carrier frequency offset estimation [13] on the performance of TWRN is analyzed. However, to the best of authors' knowledge, an estimation and decoding scheme for TWRNs in the presence of channel gains, timing offsets, and carrier frequency offsets has not been proposed in the existing literature.

In this paper, a complete synchronization approach, i.e., joint estimation and compensation of channel gains, timing offsets, and carrier frequency offsets for AF TWRNs is proposed. Upon reception of the superimposed signals broadcasted from the relay node, the user nodes first jointly estimate the impairments using *known* training signals and the proposed least squares (LS) or differential evolution (DE) based estimators [14]. Subsequently, the users employ the proposed minimum mean-square error (MMSE) receiver in combination with the estimated impairments to decode the received signal. Each user uses knowledge of its transmitted data to cancel out the self interference and decode the opposing user's signal. The contributions of this paper can be summarized as follows:

- A system model for achieving synchronization and obtaining the channel parameters in AF TWRN is developed.
- New Cramér-Rao lower bounds (CRLBs) for joint estimation of multiple impairments at the user nodes are derived. These bounds can be applied to assess the performance of synchronization and channel estimatiors in AF TWRN networks.
- An LS based estimator for joint estimation of multiple impairments is derived. A DE based algorithm is proposed as an alternative to the LS estimator to significantly

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[1]To ensure spectral efficiency, three or four phase communication protocols for AF TWRN [6] are not considered in this paper.

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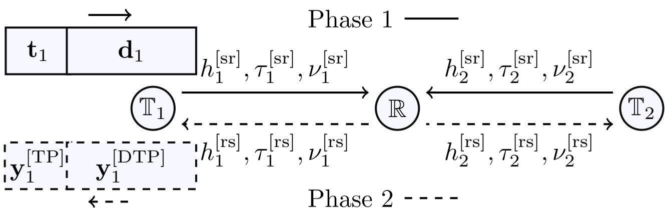

Fig. 1: System Model for AF two-way relay network.

reduce the computational complexity associated with synchronization in AF TWRNs. Simulation results show that the mean square error (MSE) performances of both LS and DE estimators are close to the CRLB at moderate-to-high signal-to-noise-ratios (SNRs).

- An MMSE receiver for compensating the effect of impairments and detecting the signal from the opposing user is derived.
- Extensive simulations are carried out to investigate the estimated MSE and bit-error rate (BER) performances of the proposed transceiver structure. These results show that the BER performance of an AF TWRN can be significantly improved in the presence of practical synchronization errors. In fact, the application of the proposed transceiver results in an overall network performance that is very close to that of an ideal network based on the assumption of perfect knowledge of synchronization and channel parameters.

The remainder of the paper is organized as follows: Section II presents the system model while the new CRLBs for the joint estimation problem are derived in Section III. In Section IV, the LS and DE based estimators for joint estimation of multiple impairments at the user nodes are presented. In Section V, the proposed MMSE receiver is derived, while Section VI presents the simulation results.

Notation: Superscripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, the conjugate, and the conjugate transpose operators, respectively. $\mathbb{E}_x\{\cdot\}$ denotes the expectation operator with respect to the variable x. The operator, $\hat{x}$ represents the estimated value of x. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the real and imaginary parts of a complex quantity. $\mathcal{CN}(\mu, \sigma^2)$ denotes the complex Gaussian distributions with mean μ and variance σ^2. Boldface small letters, $\mathbf{x}$ and boldface capital letters, $\mathbf{X}$ are used for vectors matrices, respectively. $[\mathbf{X}]_{x,y}$ represents the entry in row x and column y of $\mathbf{X}$. $\mathbf{I}_X$ denotes $X \times X$ identity matrix, $\|\mathbf{x}\|$ represents the ℓ_2 norm of a vector $\mathbf{x}$, and $\mathrm{diag}(\mathbf{x})$ is used to denote a diagonal matrix, where its diagonal elements are given by the vector $\mathbf{x}$.

II. System Model

We consider a half-duplex AF TWRN with two user terminals, $\mathbb{T}_1$ and $\mathbb{T}_2$, and a relay node, $\mathbb{R}$, as shown in Fig. 1,. All nodes are equipped with a single omnidirectional antenna. The channel gain, timing offset, and carrier frequency offset between the kth user terminal and the relay node are denoted by h_k, τ_k, and ν_k, respectively, for $k = 1, 2$, where the superscripts, $(\cdot)^{[\mathrm{sr}]}$ and $(\cdot)^{[\mathrm{rs}]}$, are used for the parameters from user terminal to relay node and from relay node to user terminal, respectively. The timing and carrier frequency offsets are modeled as *unknown* deterministic parameters over the frame length, which is similar to the approach adopted in [15] and [16]. Quasi-static and frequency flat fading channels are considered, i.e., the channel gains do not change over the length of a frame but change from frame to frame according to a complex Gaussian distribution, $\mathcal{CN}(0, \sigma_h^2)$. The use of such channels is motivated by the prior research in this field [10], [11].

The transmission frame from each user is comprised of training and data symbols. The exchange of data among the two user terminals is completed in two phases.

1) During the first phase, the transmission frame, $[\mathbf{t}_k, \mathbf{d}_k]$, is transmitted from the kth user, $k = 1, 2$, to an intermediate relay node, where $\mathbf{t}_k$ and $\mathbf{d}_k$ denote the kth user's training and data signal, respectively. This is illustrated in Fig. 1. The signal from the two users is superimposed at the relay node.
2) During the second phase, the relay node amplifies the superimposed signal and broadcasts it back to the users. The users use the training part of the received signal, $\mathbf{y}_k^{[\mathrm{TP}]}$, to jointly estimate the *multiple impairments*, i.e, channel gains, timing offsets, and carrier frequency offsets. The effect of these impairments is compensated and the received signal, $\mathbf{y}_k^{[\mathrm{DTP}]}$, is decoded at the kth user's terminal.

Note that the superscripts $(\cdot)^{[\mathrm{TP}]}$ and $(\cdot)^{[\mathrm{DTP}]}$ denote the signals in training and data transmission periods, respectively and Fig. 1 shows the transmitted frames at the first user terminal, $\mathbb{T}_1$. A similar structure is followed for the second user terminal, $\mathbb{T}_2$.

The received signal at the relay node during the training period, $r^{[\mathrm{TP}]}(t)$, is given by

$$r^{[\mathrm{TP}]}(t) = \sum_{k=1}^{2} h_k^{[\mathrm{sr}]} e^{j2\pi \frac{\nu_k^{[\mathrm{sr}]}}{T} t} \sum_{n=0}^{L-1} t_k(n) g(t - nT - \tau_k^{[\mathrm{sr}]} T) + n(t), \quad (1)$$

where the timing and carrier frequency offsets, $\tau_k^{[\mathrm{sr}]}$ and $\nu_k^{[\mathrm{sr}]}$, are normalized by the symbol duration T, L is the length of training signal t_k, $g(t)$ stands for the root-raised cosine pulse function, and $n(t)$ denotes zero-mean complex additive white Gaussian noise (AWGN) at the relay receiver, i.e., $n(t) \sim \mathcal{CN}(0, \sigma_n^2)$. To avoid amplifier saturation at the relay, the relay node amplifies the received signal, $r^{[\mathrm{TP}]}(t)$, with the power constraint factor, $\zeta = \frac{1}{\sqrt{2\sigma_h^2 + \sigma_n^2}}$, and broadcasts the amplified signal to the users. The received signal at the user terminal, $\mathbb{T}_1$, during the training period, $y^{[\mathrm{TP}]}(t)$ is given by

$$y_1^{[\mathrm{TP}]}(t) = \zeta h_1^{[\mathrm{rs}]} e^{j2\pi \frac{\nu_1^{[\mathrm{rs}]}}{T} t} \, r^{[\mathrm{TP}]}(t - \tau_1^{[\mathrm{rs}]} T) + w_1(t), \quad (2)$$

where $w_1(t)$ denotes the zero-mean complex AWGN at the receiver of $\mathbb{T}_1$, i.e., $w_1(t) \sim \mathcal{CN}(0, \sigma_w^2)$. Substituting (1) into

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(2), $y^{[\text{TP}]}(t)$ is given by

$$y_1^{[\text{TP}]}(t) = \zeta h_1^{[\text{rs}]} e^{j2\pi \frac{\nu_1^{[\text{rs}]}}{T} t} \sum_{k=1}^{2} \Big(h_k^{[\text{sr}]} e^{j2\pi \frac{\nu_k^{[\text{sr}]}}{T}(t-\tau_1^{[\text{rs}]}T))} \times \sum_{n=0}^{L-1} t_k(n) g(t - nT - \tau_k^{[\text{sr}]}T - \tau_1^{[\text{rs}]}T) \Big) + \zeta h_1^{[\text{rs}]} e^{j2\pi \frac{\nu_1^{[\text{rs}]}}{T} t} n(t - \tau_1^{[\text{rs}]}T) + w_1(t) \quad (3)$$

Note that unlike [15], the developed system model in (3) takes into account both the timing errors, from users to the relay node, $\tau_k^{[\text{sr}]}$, $k = 1, 2$, and from relay node back to user terminal $\mathbb{T}_1$, $\tau_1^{[\text{rs}]}$. The received signal in (3), $y_1^{[\text{TP}]}(t)$, is sampled with the sampling time $T_s = T/Q$ and the sampled received signal, $y_1^{[\text{TP}]}(i)$, is given by

$$y_1^{[\text{TP}]}(i) = \sum_{k=1}^{2} \alpha_k e^{j2\pi\nu_k i/Q} \sum_{n=0}^{L-1} t_k(n) g(iT_s - nT - \tau_k T) + \zeta h_1^{[\text{rs}]} e^{j2\pi\nu_1^{[\text{rs}]} i/Q} n(i) + w_1(i) \quad (4)$$

where

- $\alpha_k \triangleq \zeta h_k^{[\text{sr}]} h_1^{[\text{rs}]} e^{-j2\pi\nu_k^{[\text{sr}]}\tau_1^{[\text{sr}]}}$ is the combined channel gain from $\mathbb{T}_1$-$\mathbb{R}$-$\mathbb{T}_1$ and $\mathbb{T}_2$-$\mathbb{R}$-$\mathbb{T}_1$ for $k = 1$ and $k = 2$, respectively,
- $\nu_k \triangleq \nu_k^{[\text{sr}]} + \nu_1^{[\text{rs}]}$ is the sum of carrier frequency offset from $\mathbb{T}_1$-$\mathbb{R}$-$\mathbb{T}_1$ and $\mathbb{T}_2$-$\mathbb{R}$-$\mathbb{T}_1$ for $k = 1$ and $k = 2$, respectively, $\nu_1^{[\text{sr}]} = -\nu_1^{[\text{rs}]}$ because same oscillators are used during transmission from user $\mathbb{T}_1$ to the relay node and from relay node back to user $\mathbb{T}_1$, thus, $\nu_1 = \nu_1^{[\text{sr}]} + \nu_1^{[\text{rs}]} = 0$,
- $\tau_k \triangleq \tau_k^{[\text{sr}]} + \tau_1^{[\text{rs}]}$ is the resultant timing offset from $\mathbb{T}_1$-$\mathbb{R}$-$\mathbb{T}_1$ and $\mathbb{T}_2$-$\mathbb{R}$-$\mathbb{T}_1$ for $k = 1$ and $k = 2$, respectively,

Q is the sampling factor[2], $n = 0, 1, \ldots, L-1$ and $i = 0, 1, \ldots, LQ-1$ are used to denote T-spaced and T_s-spaced samples, respectively, and $n(i)$ has been used in place of $n(iT_s - \tau_1^{[\text{rs}]}T)$, since $n(t)$ denotes the AWGN and its statistics are not affected by time delays. Upon reception of signal broadcasted from the relay, it is assumed that the users first employ coarse frame synchronization to ensure that the superimposed signals are within one symbol duration from each other i.e., $\tau_1 - \tau_2 < 1$, and $\tau_1, \tau_2 \in (-0.5, 0.5)$. This assumption is inline with prior research in this field [15].

Eq. (4) can be written in vector form as

$$\mathbf{y}_1^{[\text{TP}]} = \alpha_1 \mathbf{G}_1 \mathbf{t}_1 + \alpha_2 \mathbf{\Lambda}_2 \mathbf{G}_2 \mathbf{t}_2 + \zeta h_1^{[\text{rs}]} \mathbf{\Lambda}^{[\text{rs}]} \mathbf{n} + \mathbf{w}_1 \quad (5)$$

where

- $\mathbf{G}_k$ is the $LQ \times L$ matrix of the samples of the pulse shaping filter such that $[\mathbf{G}_k]_{i,n} \triangleq g_{\text{rrc}}(iT_s - nT - \tau_k^{[\text{rd}]}T)$,
- $\mathbf{\Lambda}_2 \triangleq \text{diag}\left([e^{j2\pi\nu_2(0)/N}, \ldots, e^{j2\pi\nu_2(LQ-1)/N}]\right)$ is an $LQ \times LQ$ matrix,
- $\mathbf{\Lambda}^{[\text{rs}]} \triangleq \text{diag}\left([e^{j2\pi\nu^{[\text{rs}]}(0)/N}, \ldots, e^{j2\pi\nu^{[\text{rs}]}(LQ-1)/N}]\right)$ is an $LQ \times LQ$ matrix
- $\mathbf{y}_1^{[\text{TP}]} \triangleq [y_1^{[\text{TP}]}(0), \ldots, y_1^{[\text{TP}]}(LQ-1)]^T$
- $\mathbf{t}_k \triangleq [t_k(0), \ldots, t_k(L-1)]^T$,
- $\mathbf{n} \triangleq [n(0), \ldots, n(LQ-1)]^T$, and
- $\mathbf{w}_1 \triangleq [w_1(0), \ldots, w_1(LQ-1)]^T$.

The received signal during the data transmission period, $\mathbf{y}_1^{[\text{DTP}]}$, can be similarly expressed as (5), where training $\mathbf{t}_k$ is replaced by the data $\mathbf{d}_k \triangleq [d_k(0), \ldots, d_k(L-1)]^T$. Note that as anticipated, the data length L is different and larger than the training length L as discussed in Section VI.

Next, without loss in generality, we derive the CRLB and estimators for joint estimation of channel gains, timing offsets, and carrier frequency offsets at the user terminal $\mathbb{T}_1$. Note that the system model in this section and the derived CRLB, estimation, and detection schemes in the following sections can be easily manipulated to detect $\mathbf{d}_1$ at the user terminal $\mathbb{T}_2$. These details are not included to avoid repetition.

III. Cramér-Rao Lower Bound

In this section, the CRLB for joint estimation of multiple impairments at $\mathbb{T}_1$ are derived. The signal model in (5) can be rewritten as

$$\mathbf{y}_1^{[\text{TP}]} = \mathbf{\Omega}\boldsymbol{\alpha} + \mathbf{u}, \quad (6)$$

where $\mathbf{\Omega} \triangleq [\mathbf{G}_1\mathbf{t}_1 \;\; \mathbf{\Lambda}_2\mathbf{G}_2\mathbf{t}_2]$ is an $LQ \times 2$ matrix, $\boldsymbol{\alpha} \triangleq [\alpha_1, \alpha_2]^T$, and $\mathbf{u} \triangleq \zeta h_1^{[\text{rs}]}\mathbf{\Lambda}^{[\text{rs}]}\mathbf{n} + \mathbf{w}_1$. Based on the assumptions and proposed system model in Section II, the received signal vector, $\mathbf{y}_1^{[\text{TP}]}$, is a circularly symmetric complex Gaussian random variable, $\mathbf{y}^{[\text{TP}]} \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{\Sigma})$, with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Sigma}$, given by

$$\boldsymbol{\mu} = \mathbf{\Omega}\boldsymbol{\alpha}, \quad \text{and} \quad (7a)$$

$$\mathbf{\Sigma} = \mathbb{E}\{\mathbf{u}\mathbf{u}^H\} = (\zeta^2\sigma_h^2\sigma_n^2 + \sigma_w^2)\mathbf{I}_{LQ} = \sigma_u^2\mathbf{I}_{LQ}, \quad (7b)$$

respectively. To determine the CRLB, we have to first formulate the parameter vector of interest. The user $\mathbb{T}_1$ has to estimate the channel gains, $\boldsymbol{\alpha}$, timing offsets $\boldsymbol{\tau} \triangleq [\tau_1, \tau_2]^T$, and the carrier frequency offset ν_2. There is no need to estimate ν_1 as this is found to be 0 as explained below (4). As a result, the parameter vector of interest, $\boldsymbol{\lambda}$, is given by

$$\boldsymbol{\lambda} \triangleq [\Re\{\boldsymbol{\alpha}\}^T, \Im\{\boldsymbol{\alpha}\}^T, \nu_2, \boldsymbol{\tau}^T]^T \quad (8)$$

In the following, we derive Fisher's information matrix (FIM) for the estimation of $\boldsymbol{\lambda}$.

Theorem 1: Based on the proposed system model, the FIM, denoted by $\mathbf{F}$, for the estimation of $\boldsymbol{\lambda}$ is given by (9), at the bottom of this page, where

[2]Oversampling is needed to correctly estimate the timing offsets in the presence of pulse shaping.

$$\mathbf{F} = \frac{2}{\sigma_u^2} \begin{bmatrix} \Re\{\mathbf{\Omega}^H\mathbf{\Omega}\} & -\Im\{\mathbf{\Omega}^H\mathbf{\Omega}\} & -\Im\{\mathbf{\Omega}^H\mathbf{D}\mathbf{\Phi}\mathbf{t}_2\} & \Re\{\mathbf{\Omega}^H\mathbf{\Gamma}\mathbf{H}\} \\ \Im\{\mathbf{\Omega}^H\mathbf{\Omega}\} & \Re\{\mathbf{\Omega}^H\mathbf{\Omega}\} & \Re\{\mathbf{\Omega}^H\mathbf{D}\mathbf{\Phi}\mathbf{t}_2\} & \Im\{\mathbf{\Omega}^H\mathbf{\Gamma}\mathbf{H}\} \\ \Im\{\mathbf{t}_2^H\mathbf{\Phi}^H\mathbf{D}\mathbf{\Omega}\} & \Re\{\mathbf{t}_2^H\mathbf{\Phi}^H\mathbf{D}\mathbf{\Omega}\} & \Re\{\mathbf{t}_2^H\mathbf{\Phi}^H\mathbf{D}^2\mathbf{\Phi}\mathbf{t}_2\} & \Im\{\mathbf{t}_2^H\mathbf{\Phi}^H\mathbf{D}\mathbf{\Gamma}\mathbf{H}\} \\ \Re\{\mathbf{H}^H\mathbf{\Gamma}^H\mathbf{\Omega}\} & -\Im\{\mathbf{H}^H\mathbf{\Gamma}^H\mathbf{\Omega}\} & -\Im\{\mathbf{H}^H\mathbf{\Gamma}^H\mathbf{D}\mathbf{\Phi}^H\mathbf{t}_2\} & \Re\{\mathbf{H}\mathbf{\Gamma}^H\mathbf{\Gamma}\mathbf{H}\} \end{bmatrix} \quad (9)$$

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- $\mathbf{\Gamma} \triangleq [\mathbf{R}_1\mathbf{t}_1 \; \mathbf{\Lambda}_2\mathbf{R}_2\mathbf{t}_2]$ is an $LQ \times 2$ matrix, $\mathbf{R}_k \triangleq \frac{\partial \mathbf{G}_k}{\partial \tau_k}$ is an $LQ \times 2$ matrix,
- $\mathbf{D} \triangleq 2\pi \times \text{diag}\{0, \ldots, LQ-1\}$ is an $LQ \times LQ$ matrix,
- $\mathbf{\Phi} \triangleq \alpha_2\mathbf{\Lambda}_2\mathbf{G}_2$ is an $LQ \times L$ matrix, and
- $\mathbf{H} \triangleq \text{diag}\{\alpha_1, \alpha_2\}$ is a 2×2 matrix.

Proof: See Appendix A.

Finally, the CRLB for the estimation of $\boldsymbol{\lambda}$ is given by the diagonal elements of the inverse of $\mathbf{F}$. Note that the CRLB for channel estimation is the sum of the CRLBs for real and imaginary parts of the channel estimation [17].

IV. Joint Parameter Estimation

In this section, the LS estimator for joint estimation of multiple impairments in AF TWRN is derived. Subsequently, the DE based estimator is applied to reduce the computational complexity for obtaining these impairments.

A. LS Estimation

Based on the signal model in (6), the LS estimates of the parameters, $\boldsymbol{\alpha}, \boldsymbol{\tau}$, and ν_2, can be determined by minimizing the cost function, $\mathbf{J}$, according to

$$\mathbf{J}(\boldsymbol{\alpha}, \boldsymbol{\tau}, \nu_2) = \left\| \mathbf{y}_1^{[\text{TP}]} - \mathbf{\Omega}\boldsymbol{\alpha} \right\|^2. \quad (10)$$

Given $\boldsymbol{\tau}$ and ν_2, it is straightforward to show that the LS estimate of $\boldsymbol{\alpha}$, denoted by $\hat{\boldsymbol{\alpha}}$, can be determined as

$$\hat{\boldsymbol{\alpha}} = \left(\mathbf{\Omega}^H\mathbf{\Omega}\right)^{-1}\mathbf{\Omega}^H\mathbf{y}_1^{[\text{TP}]}. \quad (11)$$

Substituting (11) in (10), the estimates of τ_1, τ_2, and ν_2 are obtained via

$$\hat{\tau}_1, \hat{\tau}_2, \hat{\nu}_2 = \arg \min_{\tau_1,\tau_2,\nu_2} \underbrace{-\left(\mathbf{y}_1^{[\text{TP}]}\right)^H \mathbf{\Omega}\left(\mathbf{\Omega}^H\mathbf{\Omega}\right)^{-1}\mathbf{\Omega}^H\mathbf{y}_1^{[\text{TP}]}}_{\triangleq \chi(\tau_1,\tau_2,\nu_2)}, \quad (12)$$

where $\arg\min$ denotes the arguments, τ_1, τ_2, and ν_2, that minimize the expression $\chi(\tau_1, \tau_2, \nu_2)$ and $\mathbf{y}_1^{[\text{TP}]}$ is defined in (6). The channel estimates, $\hat{\alpha}_1$ and $\hat{\alpha}_2$, are obtained by substituting $\hat{\tau}_1, \hat{\tau}_2$, and $\hat{\nu}_2$ back into (11).

The minimization in (12) requires a 3-dimensional exhaustive search over the discretized set of possible timing and frequency offset values, which is inherently very computationally complex. Furthermore, to reach the CRLB (see Fig. 2 in Section VI), the exhaustive search in (12) needs to be carried out with very high resolution[3], which significantly increases the sets of possible values for both timing and frequency offsets and in turn, further increases the complexity of the proposed LS estimator. In the following subsection, DE is employed as a computationally efficient algorithm to carry out the minimization in (12) [14].

[3]Step sizes of 10^{-2} and 10^{-4} for MTOs and MCFOs, respectively.

B. Differential Evolution based Estimation

DE and genetic algorithms are considered as a subclass of *evolutionary algorithms* since they attempt to evolve the solution for a problem through recombination, mutation, and survival of the fittest. More specifically, DE is an optimization algorithm, where a number of parameter vectors are generated and updated at each iteration in order to reach the solution [14]. Following the detailed steps and parameterization of the DE algorithm outlined in [9] and changing the estimation vector length to 3 parameters, $\hat{\tau}_1, \hat{\tau}_2$, and $\hat{\nu}_2$, the minimization in (12) is achieved. Substituting these estimates in (11) also generates the desired channel estimates.

Remark 1: The computational requirements of the LS and the DE algorithms are quantified using CPU execution time [18]. The execution time is observed by setting training length $L = 80$, when an Intel Core i7-2670QM CPU @ 2.20 GHz processor with 8 GB of RAM is used. It has been observed that comparing to the LS estimator, the DE algorithm is capable of estimating the multiple impairments approximately 10^4 times more quickly.

The computational complexity of the LS and the DE algorithms can also be compared by calculating the number of additions plus multiplications. By following the steps outlined in [9, (20)-(21)], we find that an LS algorithm requires 1.84×10^{13} multiplications and additions, however the DE algorithm needs 1.66×10^9 multiplications and additions in order to estimate the multiple impairments. This method of computational complexity also verifies that DE algorithm is capable of estimating the multiple impairments approximately 10^4 times more quickly.

Note that a large number of additions and multiplications are not the point of concern here. This is because the proposed LS and DE estimation methods are applied for initialization only once at system start-up. Afterwards, the estimates of previously transmitted frames may be used to update the new estimates since timing and carrier frequency offsets do not rapidly change from frame to frame. This is due to the fact that oscillator properties are mainly affected by temperature and other physical phenomena that do not rapidly fluctuate with time [19].

V. MMSE Receiver and Data Detection

In this section, an MMSE receiver for compensating the effect of impairments and detecting the signal from user $\mathbb{T}_2$ is derived. Following (5), the received signal at user $\mathbb{T}_1$ during the data transmission period, $\mathbf{y}_1^{[\text{DTP}]}$, is given by

$$\mathbf{y}_1^{[\text{DTP}]} = \alpha_1\mathbf{G}_1\mathbf{d}_1 + \alpha_2\mathbf{\Lambda}_2\mathbf{G}_2\mathbf{d}_2 + \zeta h_1^{[\text{rs}]}\mathbf{\Lambda}^{[\text{rs}]}\mathbf{n} + \mathbf{w}_1 \quad (13)$$

The user $\mathbb{T}_1$ has to decode the signal $\mathbf{d}_2$ using the received signal $\mathbf{y}_1^{[\text{DTP}]}$, the estimated impairments, $\hat{\boldsymbol{\alpha}}$, $\hat{\boldsymbol{\tau}}$, $\hat{\nu_2}$, and its own data $\mathbf{d}_1$. Let us define an $LQ \times 1$ vector $\mathbf{z} \triangleq \mathbf{y}_1^{[\text{DTP}]} - \alpha_1\mathbf{G}_1\mathbf{d}_1$. Using $\mathbf{y}_1^{[\text{DTP}]}$, $\mathbf{d}_1$, $\hat{\tau}_1$, $\hat{\alpha}_1$, user $\mathbb{T}_1$ can estimate the vector $\mathbf{z}$ as

$$\begin{aligned}\hat{\mathbf{z}} &\triangleq \mathbf{y}_1^{[\text{DTP}]} - \hat{\alpha}_1\hat{\mathbf{G}}_1\mathbf{d}_1 \\ &= \alpha_2\mathbf{\Lambda}_2\mathbf{G}_2\mathbf{d}_2 + \mathbf{u} \end{aligned} \quad (14)$$

where $\hat{\mathbf{G}}_1 = \mathbf{G}_1|_{\tau_1=\hat{\tau}_1}$ and $\mathbf{u} \triangleq \zeta h_1^{[\text{rs}]}\mathbf{\Lambda}^{[\text{rs}]}\mathbf{n} + \mathbf{w}_1$ is defined in (6). Applying MMSE based detection, the signal from user

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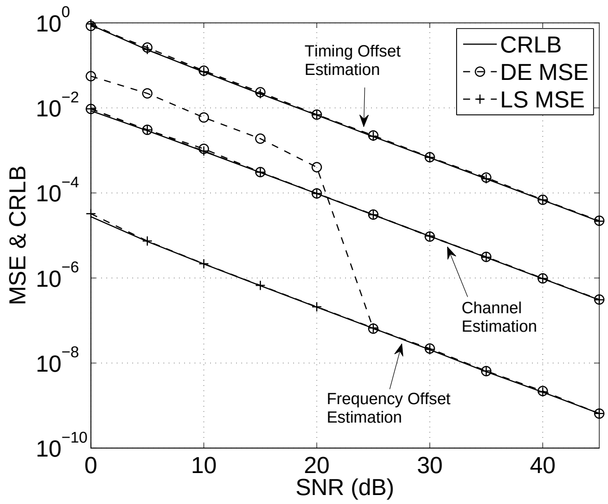

Fig. 2: CRLBs and MSE for LS & DE based estimation of channel gains, timing offsets, and carrier frequency offsets.

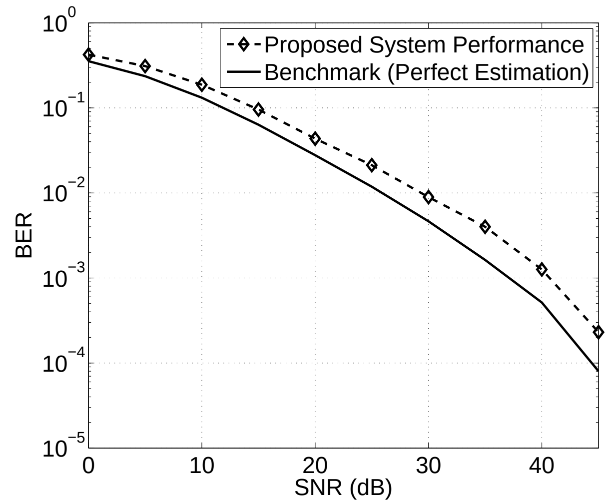

Fig. 3: BER of the proposed AF TWRN with DE based estimation and perfect estimation.

$\mathbb{T}_2$ can be decoded as

$$\hat{\mathbf{d}}_2 = (\hat{\mathbf{\Phi}}^H\hat{\mathbf{\Phi}} + \sigma_u^2\mathbf{I}_L)\hat{\mathbf{\Phi}}^H\hat{\mathbf{z}} \tag{15}$$

where $\hat{\mathbf{\Phi}} \triangleq \hat{\alpha}_2\hat{\mathbf{\Lambda}}_2\hat{\mathbf{G}}_2$, $\hat{\mathbf{\Lambda}}_2 = \mathbf{\Lambda}_2|_{\nu_2=\hat{\nu}_2}$, and $\hat{\mathbf{G}}_2 = \mathbf{G}_2|_{\tau_2=\hat{\tau}_2}$. In order to *benchmark* the decoding performance of the overall AF TWRN, the benchmark data detection $\hat{\mathbf{d}}_2^{[\mathrm{BM}]}$, that is based on the perfect knowledge of multiple impairments, is given by

$$\hat{\mathbf{d}}_2^{[\mathrm{BM}]} = (\mathbf{\Phi}^H\mathbf{\Phi} + \sigma_u^2\mathbf{I}_L)\mathbf{\Phi}^H\hat{\mathbf{z}} \tag{16}$$

where $\mathbf{\Phi} \triangleq \alpha_2\mathbf{\Lambda}_2\mathbf{G}_2$.

VI. Simulation Results

In this section, we present simulation results to evaluate the estimation and BER performance of the AF TWRN. The training length is set to $L = 80$ and data transmission length is set to $L = 400$, which results in the synchronization overhead of 16.6%. Without loss of generality, it is assumed that during the training period, linearly independent, unit-amplitude phase shift keying (PSK) training signals are transmitted from two users. Such TSs have also been considered previously, e.g., in [15].[4]. During the data transmission period, quadrature phase-shift keying (QPSK) modulation is employed for data transmission. The oversampling factor is set to $Q = 2$ and the noise variances, $\sigma_n^2 = \sigma_w^2 = 1/\mathrm{SNR}$. The synchronization parameters, τ_1, τ_2, and ν_2, are assumed to be uniformly distributed over the range $(-0.5, 0.5)$. All the channel gains are modeled as independent and identically distributed complex Gaussian random variables with $\mathcal{CN}(0, 1)$. In the following, the MSE and BER simulation results are averaged out over 600 frames with 400 data symbols per frame, where random realization of Rayleigh fading channel gains is generated every frame.

Fig. 2 plots the CRLBs, derived in Section III, and estimation MSE for joint estimation of multiple impairments. Without loss of generality, the CRLBs and the MSE estimation performance for α_1, τ_1, and ν_2 are presented only, where similar results to that of α_1 and τ_1 are observed for α_2 and τ_2, respectively. Fig. 2 shows that the mean-square estimation error of the proposed LS and DE estimators is very close to the derived CRLBs at moderate-to-high SNRs. Note that the mean square estimation error of LS estimator is close to the derived CRLBs for whole considered range of SNR (0-45 dB). On the other hand, for the DE based estimation, MSE for frequency offset estimation gets close to the CRLB after 25 dB. This is because, unlike LS estimator, the DE estimator is not based on the exhaustive search criteria and is computationally efficient than LS estimator.

Fig. 3 illustrates the BER performance of overall AF TWRN, that employs the DE based estimator and MMSE receiver to decode the received signal. The DE based estimator is employed because LS estimator is computationally very complex (see Remark 1 in Section IV). The BER performance of the proposed estimation and decoding schemes ($\hat{\mathbf{d}}_2$ in (15)) is compared with the the benchmark decoding scheme ($\hat{\mathbf{d}}_2^{[\mathrm{BM}]}$ in (16)), which assumes perfect knowledge of multiple impairments. Fig. 3 shows that the BER performance of the proposed overall AF TWRN is close to the performance of a TWRN that is based on the assumption of perfect knowledge of synchronization parameters, i.e., there is a performance gap of just 2-3 dB only at moderate-to-high SNRs. To the best of our knowledge, our algorithm is the first complete solution for the joint estimation and compensation of channel gains, timing offsets, and carrier frequency offsets. Hence, the performance of the proposed algorithm cannot be compared with any existing algorithm. For example, the algorithm in [11], which estimate and compensate the effect of channel parameters in AF TWRN, fail to decode the received signal and show very poor BER performance in the presence of multiple impairments.

VII. Conclusions

This paper has proposed the system model for AF TWRN in the presence of multiple impairments, i.e., channel gains,

[4]The design of the optimal training sequences is outside the scope of this paper.

timing offsets, and carrier frequency offsets. In order to extract the user's information from the received signal, each user jointly estimates the multiple impairments and compensates their effect from the received signal. CRLBs for joint estimation of multiple impairments are derived and simulation results show that the mean-square estimation error of the applied LS and DE estimators is very close to the derived CRLBs. Next, MMSE based received is derived to compensate the effect of multiple impairments and decode the user's information. The BER performance of overall AF TWRN, employing the proposed estimation and decoding schemes, is close to the lower bound BER (performance gap of 2-3 dB only), that assumes perfect knowledge of multiple impairments.

Appendix A
Derivation of $\mathbf{F}$

The $(\ell, q)^{\text{th}}$ element of the 7×7 FIM is given by [17]

$$[\mathbf{F}(\boldsymbol{\theta})]_{\ell,q} = 2\Re\left\{\frac{\partial \boldsymbol{\mu}^H}{\partial \theta_\ell}\boldsymbol{\Sigma}^{-1}\frac{\partial \boldsymbol{\mu}}{\partial \theta_q}\right\} + \mathrm{Tr}\left(\boldsymbol{\Sigma}^{-1}\frac{\partial \boldsymbol{\Sigma}}{\partial \theta_\ell}\boldsymbol{\Sigma}^{-1}\frac{\partial \boldsymbol{\Sigma}}{\partial \theta_q}\right). \tag{A.1}$$

Accordingly, derivatives in (A.1) can be derived as

$$\frac{\partial \boldsymbol{\mu}}{\partial \Re\{\alpha_k\}} = -j\frac{\partial \boldsymbol{\mu}}{\partial \Im\{\alpha_k\}} = \boldsymbol{\Lambda}_k \mathbf{G}_k \mathbf{t}_k, \tag{A.2}$$

$$\frac{\partial \boldsymbol{\mu}}{\partial \tau_k} = \boldsymbol{\Lambda}_k \mathbf{R}_k \mathbf{t}_k \alpha_k, \quad \frac{\partial \boldsymbol{\mu}}{\partial \nu_2} = j\mathbf{D}\boldsymbol{\Lambda}_2 \mathbf{G}_2 \mathbf{t}_2 \alpha_2, \tag{A.3}$$

where $\mathbf{R}_k \triangleq \partial \mathbf{G}_k / \partial \tau_k$ and $\boldsymbol{\Lambda}_1 = \mathbf{I}_{LQ}$. Since, $\boldsymbol{\Sigma}$ is not a function of $\boldsymbol{\lambda}$, we have

$$\frac{\partial \boldsymbol{\Sigma}}{\partial \nu_2} = \frac{\partial \boldsymbol{\Sigma}}{\partial \tau_k} = \frac{\boldsymbol{\Sigma}}{\partial \Re\{\alpha_k\}} = \frac{\boldsymbol{\Sigma}}{\partial \Im\{\alpha_k\}} = 0. \tag{A.4}$$

After substituting the derivatives in (A.2), (A.3), and (A.4) into (A.1) and carrying out straightforward algebraic manipulations, the FIM, $\mathbf{F}$, can be obtained as shown in (9).

References

[1] J. Boyer, D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1820–1830, Oct. 2004.

[2] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[3] T. J. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, "Broadcast capacity region of two-phase bidirectional relaying," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 454–458, Jan. 2008.

[4] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, p. 379389, Feb. 2007.

[5] P. Cheng, L. Gui, Y. Rui, Y. J. Guo, X. Huang, and W. Zhang, "Compressed sensing based channel estimation for two-way relay networks," *IEEE Wireless Communication Letters*, vol. 1, no. 3, pp. 201–204, Jun. 2012.

[6] S. Tian, Y. Li, and B. Vucetic, "A near optimal amplify and forward relaying in two-way relay networks," in *Proc. IEEE ICC*, 2011.

[7] Y. Tian, X. Lei, Y. Xiao, and S. Li, "SAGE based joint timing-frequency offsets and channel estimation in distributed MIMO systems," *Elsevier Journal Computer Commun.*, vol. 33, no. 17, pp. 2125–2131, Jul. 2010.

[8] A. A. Nasir, H. Mehrpouyan, S. D. Blostein, S. Durrani, and R. A. Kennedy, "Timing and carrier synchronization with channel estimation in multi-relay cooperative networks," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 793–811, Feb. 2012.

[9] A. A. Nasir, H. Mehrpouyan, S. Durrani, S. D. Blostein, R. A. Kennedy, and B. Ottersten, "Transceiver design for distributed STBC based AF cooperative networks in the presence of timing and frequency offsets," *IEEE Trans. Signal Process.*, vol. 61, no. 12, pp. 3143–3158, Jun. 2013.

[10] C. Wang, T. C.-K. Liu, and X. Dong, "Impact of channel estimation error on the performance of amplify-and-forward two-way relaying," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1197–1207, Mar. 2012.

[11] F. Gao, C. Schnurr, I. Bjelakovic, and H. Boche, "Optimal channel estimation and training design for two-way relay networks," *IEEE Trans. Commun.*, vol. 57, no. 10, pp. 3024–3033, Oct. 2009.

[12] T. H. Pham, Y. C. Liang, A. Nallanathan, and G. H. Krishna, "On the design of optimal training sequence for bidirectional relay networks," *IEEE Signal Process. Lett.*, vol. 16, no. 3, p. 200203, Mar. 2009.

[13] G. Wang, F. Gao, Y.-C. Wu, and C. Tellambura, "Joint CFO and channel estimation for CP-OFDM modulated two-way relay networks," in *Proc. IEEE ICC*, 2010.

[14] K. V. Price, R. M. Storn, and J. A. Lampinen, *Differential Evolution: A practical approach to global optimization.*, G. Rozenberg, T. Bck, J. Kok, H. Spaink, and A. E. Eiben, Eds. Springer-Verlag Berlin Heidelberg, 2005.

[15] X. Li, C. Xing, Y.-C. Wu, and S. C. Chan, "Timing estimation and resynchronization for amplify-and-forward communication systems," *IEEE Trans. Signal Process.*, vol. 58, no. 4, pp. 2218–2229, Apr. 2010.

[16] O. Besson and P. Stoica, "On parameter estimation of MIMO flat-fading channels with frequency offsets," *IEEE Trans. Signal Process.*, vol. 51, no. 3, pp. 602–613, Mar. 2003.

[17] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. NJ: Prentice Hall, 1993.

[18] N. Moller, "On Schoonhage's algorithm and subquadratic integer GCD computation," *Mathematics of Computation*, vol. 77, pp. 589–607, Jan. 2008.

[19] B. P. Singh and R. Singh, *Electronic devices and integrated circuits*. Pearson Education India, 2006.