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Buckling analysis of nanoplates using IGA

P Phung-Van¹, M Abdel-Wahab¹, H Nguyen-Xuan²

1 Department of Electrical Energy, Metals, Mechanical Constructions and Systems,
Faculty of Engineering and Architecture, Ghent University, Belgium

2 Center for Interdisciplinary Research in Technology, Hutech University, Ho Chi
Minh City, Vietnam

E-mail: Phuc.PhungVan@UGent.be

Abstract. Isogeometric analysis (IGA) based on HSST is used to simulate buckling analysis of nanoplates. The material properties of nanoplates based on the Mori–Tanaka schemes and the rule of mixture are used. The differential nonlocal equations with size effect are utilized. The nonlocal governing equations are approximated according to IGA, that satisfies naturally the higher-order derivatives continuity requirement in weak form of nanoplates. Several numerical results are presented to demonstrate the reliability of the proposed method.

1. Introduction

There has recently been a fast growth in applications of nanoscale structures, which are primarily concerned with fabrication of FGMs. This is considered a new revolutionary of materials with enhanced functionality. FGM nanoplates, as specific nanostructures, have been applied in the engineering and technology sectors such as mechanical engineering, aeronautic manufacturing industry, nuclear engineering, etc. Therefore, the understanding of the behaviors of the FGM nanoplates is essential for the development of nanostructures because of their huge potential applications in the real life.

To carry out numerical analysis of nanostructures, three general types of modelling approaches can be listed as follows: (i) hybrid atomistic continuum mechanics, (ii) atomistic and (iii) continuum mechanics. In continuum mechanics, one of the well-known theories is the nonlocal continuum theory of Eringen [1]. Finite Element method (FEM) is a very well know numerical techniques and has been used for a wide range of applications [2-15]. Using FEM, Free vibration analysis of FG size-dependent nanobeams was studied by Alshorbagy et al. [16]. The free vibration and buckling analyses of FG nanoplate using Navier solution subjected to thermal load were reported in Ref. [17]. Size-dependent thermal stability analysis of embedded FG annular nanoplates resting on an elastic foundation under various types of thermal using an exact analytical solution was introduced in Ref. [18]. The mechanical behaviour for homogenous nonlinear microbeams [19], FG nonlinear microbeams [20]. A nonlinear microbeams model based on the strain gradient elasticity is introduced in Refs. [21, 22]. Farokhi et al. [23] studied nonlinear dynamics of a geometrically imperfect microbeams with using Hamilton's principle for the nonlinear differential equation of motion for an initially curved beam. free vibration of FGM circle plates using the analytical solution based on the first-order shear deformation theory (FSDT) and the nonlocal theory was derived in Ref [24]. Size-dependent free analysis of FGM square plate based on FSDT was reported by Natarajan et al. [25]. Size-dependent analysis of FG nanoplates using IGA based on quasi-3D theory is recently examined in Ref [26]. Recently, Phung-Van et al. studied static and free vibration analyses of FG-CNTRC nanoplates [27] and nonlinear



transient analysis of FGM nanoplates [28]. Furthermore, mechanical behaviours of FGM composite plates based on the local continuum theory have recently been published in Refs. [29-46].

As one may note, almost cited references deal with the modelling of micro/nano-beams. A very limited literature is available for nano-scale FGM structures. This paper hence aims to fill in this gap by developing a size-dependent geometrically nonlinear transient analysis of FGM nanoplates by a combination of IGA and the nonlocal continuum theory based on HSDT. In particular, we show that IGA based on HSDT fulfilling C2-continuity requirements can easily achieve the higher-order derivatives in the framework of the nonlocal continuum theory, which is of interest in this study. Size effects based on Eringen [1] in the differential nonlocal equations are performed. The effect of nonlocal approach on buckling analysis of the FGM nanoplates with various volume fraction exponents are discussed in details.

2. Functional graded materials

A functionally graded material nanoplate made of ceramic and metal is considered in this research. The properties materials based on the rule of mixture can be given by:

$$P(z) = (P_c - P_m)V_c + P_m ; V_c = \left(\frac{1}{2} + \frac{z}{t}\right)^n \quad (n \geq 0) ; V_m = 1 - V_c \quad (1)$$

where m and c represent the metal and ceramic constituents, respectively, P refers to the effective material properties including the thermal conductivity k , ν Poisson's ratio, ρ density, E Young's modulus and α thermal expansion. V_m and V_c are the volume fraction of the metal and ceramic, respectively, z is the thickness coordinate of plate and varies from $-t/2$ to $t/2$ and n is the volume fraction exponent.

The effective bulk and shear moduli based on the Mori-Tanaka scheme [47] can be defined as:

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m \frac{K_c - K_m}{K_m + \frac{1}{2}\mu_m}} ; \frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m \frac{\mu_c - \mu_m}{\mu_m + f_1}} \quad (2)$$

where $f_1 = \frac{\mu_m(9K_m + 8\mu_m)}{6(K_m + 2\mu_m)}$. The Young's modulus and Poisson's ratio are now expressed by:

$$E_e = \frac{9K_e\mu_e}{3K_e + \mu_e} ; \nu_e = \frac{3K_e - 2\mu_e}{2(3K_e + \mu_e)} \quad (3)$$

3. Theoretical formulation

3.1 Nonlocal elasticity theory

In the nonlocal elasticity theory proposed by Eringen [1], the stress field depends on all points of the considered body, and an equivalent form of differential equations of nonlocal stress at any points \mathbf{x} can be expressed as follows:

$$(1 - \mu\nabla^2)\sigma_{ij} = t_{ij} \quad (4)$$

where $\mu = e_0l$, $0 \leq \mu \leq 4$ is the small-scale effect; l is an internal characteristic length; e_0 is material constant and $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the Laplace operator.

3.2 Displacement field

The displacement field can be expressed as follows [48]:

$$\begin{aligned} u(x, y, z) &= u_0 + z\beta_x + cz^3(\beta_x + w_{,x}) \\ v(x, y, z) &= v_0 + z\beta_y + cz^3(\beta_y + w_{,y}), \quad (-h/2 \leq z \leq h/2) \\ w(x, y, z) &= w_0 \end{aligned} \quad (5)$$

where $c = 4/3h^2$.

The Green strain-displacement relations are now given as:

$$\boldsymbol{\varepsilon} = [\varepsilon_{xx} \ \varepsilon_{yy} \ \gamma_{xy}]^T = \boldsymbol{\varepsilon}_m + z\boldsymbol{\kappa}_1 + z^3\boldsymbol{\kappa}_2 \quad ; \quad \boldsymbol{\gamma} = [\gamma_{xz} \ \gamma_{yz}]^T = \boldsymbol{\varepsilon}_s + z^2\boldsymbol{\kappa}_s \quad (6)$$

where

$$\boldsymbol{\varepsilon}_m = \begin{bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{bmatrix} ; \quad \boldsymbol{\kappa}_1 = \begin{bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{bmatrix} ; \quad \boldsymbol{\kappa}_2 = c \begin{bmatrix} \beta_{x,x} + w_{0,xx} \\ \beta_{y,y} + w_{0,yy} \\ \beta_{x,y} + \beta_{y,x} + 2w_{0,xy} \end{bmatrix} ; \quad \boldsymbol{\varepsilon}_s = \begin{bmatrix} \beta_x + w_{0,x} \\ \beta_y + w_{0,y} \end{bmatrix} ; \quad \boldsymbol{\kappa}_s = 3c \begin{bmatrix} \beta_x + w_{0,x} \\ \beta_y + w_{0,y} \end{bmatrix} \quad (7)$$

The nonlocal equations in Eq. (4) can be rewritten as:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} - \mu \nabla^2 \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \quad (8)$$

where the material constants are given by:

$$Q_{11} = Q_{22} = \frac{E_e}{1-\nu_e^2}, \quad Q_{33} = Q_{44} = Q_{55} = Q_{66} = \frac{E_e}{2(1+\nu_e)}, \quad (9)$$

$$C_{12} = C_{21} = \frac{E_e \nu_e}{1-\nu_e^2}, \quad Q_{13} = Q_{31} = Q_{23} = Q_{32} = 0$$

3.3 Isogeometric analysis

The displacement field of the plate can be expressed as:

$$\mathbf{u}^h(\xi, \eta) = \sum_{I=1}^{m \times n} R_I(\xi, \eta) \mathbf{d}_I \quad (10)$$

where $\mathbf{d}_I = [u_{0I} \ v_{0I} \ \beta_{xI} \ \beta_{yI} \ w_I]^T$ is the vector of degrees of freedom associated with the control point I , and R_I is the shape function as defined in Ref [49].

The in-plane and shear strains can be expressed as:

$$[\boldsymbol{\varepsilon}_0 \ \boldsymbol{\kappa}_1 \ \boldsymbol{\kappa}_2 \ \boldsymbol{\kappa}_s]^T = \sum_{I=1}^{m \times n} [\mathbf{B}_I^m \ \mathbf{B}_I^{b1} \ \mathbf{B}_I^{b2} \ \mathbf{B}_I^s]^T \mathbf{d}_I = \sum_{I=1}^{m \times n} [\mathbf{B}_I^{mb} \ \mathbf{B}_I^s]^T \mathbf{d}_I \quad (11)$$

where

$$\mathbf{B}_I^m = \begin{bmatrix} R_{I,x} & 0 & 0 & 0 & 0 \\ 0 & R_{I,y} & 0 & 0 & 0 \\ R_{I,y} & R_{I,x} & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B}_I^{b1} = \begin{bmatrix} 0 & 0 & R_{I,xx} & 0 & 0 \\ 0 & 0 & R_{I,yy} & 0 & 0 \\ 0 & 0 & 2R_{I,xy} & 0 & 0 \end{bmatrix}, \quad (12)$$

$$\mathbf{B}_I^{b2} = \begin{bmatrix} 0 & 0 & 0 & R_{I,x} & 0 \\ 0 & 0 & 0 & 0 & R_{I,y} \\ 0 & 0 & 0 & R_{I,y} & R_{I,x} \end{bmatrix}, \quad \mathbf{B}_I^s = \begin{bmatrix} 0 & 0 & 0 & R_I & 0 \\ 0 & 0 & 0 & 0 & R_I \end{bmatrix}$$

The governing algebraic equations for buckling analysis can be obtained as follows

$$(\mathbf{K} - \lambda \mathbf{K}_g) \mathbf{d} = \mathbf{0} \quad (13)$$

where

$$\mathbf{K} = \int_{\Omega} [(\mathbf{B}^{mb})^T \mathbf{D}_{mb} \mathbf{B}_I^{mb} + (\mathbf{B}^s)^T \mathbf{D}_s \mathbf{B}_I^s] d\Omega \quad (14)$$

$$\mathbf{K}_g = \int_{\Omega} [(\mathbf{B}^g)^T - \lambda \nabla^2 (\mathbf{B}^g)^T] \mathbf{N}_0 \mathbf{B}^g d\Omega$$

in which

$$\mathbf{D}_{mb} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{E} \\ \mathbf{B} & \mathbf{D} & \mathbf{F} \\ \mathbf{E} & \mathbf{F} & \mathbf{H} \end{bmatrix}; \quad \mathbf{B}_I^s = \begin{bmatrix} 0 & 0 & R_{I,x} & 0 & 0 \\ 0 & 0 & R_{I,y} & 0 & 0 \end{bmatrix}$$

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \int_{-h/2}^{h/2} (1, z, z^2, f(z), zf(z), f^2(z)) C_{ij} dz, \quad i, j = 1, 2, 6$$

$$D_{sij} = \int_{-h/2}^{h/2} [f'(z)]^2 C_{ij} dz, \quad i, j = 4, 5$$

4. Numerical results

In this study, the critical buckling loads $\bar{P}_{cr} = P_{cr} R^2 / D_m$ of Al/Al₂O₃ circular plate (radius $R = 10$, $h = 0.34$) are performed. Table 1 shows Non-dimensional buckling load of simply supported and clamped circle nanoplate. It can be observed that the present results match well with the reference solution [26].

Table 1. Material properties of FGM plates.

| BCs | μ | Model | n | | | | |
|------|-------|---------------|---------|---------|---------|---------|---------|
| | | | 0 | 1 | 2 | 5 | 10 |
| SSSS | 0 | RPT-1 [26] | 22.7572 | 9.6391 | 8.1921 | 6.9000 | 6.0401 |
| | | RPT [26] | 22.7567 | 9.6389 | 8.1920 | 6.9002 | 6.0399 |
| | | Quasi 3D [26] | 23.1059 | 10.0048 | 8.5094 | 7.0907 | 6.1583 |
| | | IGA | 22.7565 | 9.6263 | 8.1811 | 6.8955 | 6.0381 |
| | 1 | RPT-1 [26] | 21.8371 | 9.2241 | 7.8382 | 6.6039 | 5.7855 |
| | | RPT [26] | 21.8367 | 9.2257 | 7.8396 | 6.6053 | 5.7863 |
| | | Quasi 3D [26] | 22.1473 | 9.5755 | 8.1448 | 6.7891 | 5.8991 |
| | | IGA | 21.8371 | 9.2172 | 7.8328 | 6.6045 | 5.7871 |
| | 4 | RPT-1 [26] | 19.4782 | 8.1913 | 6.9602 | 5.8774 | 5.1565 |
| | | RPT [26] | 19.4779 | 8.1920 | 6.9608 | 5.8784 | 5.1572 |
| | | Quasi 3D [26] | 19.7025 | 8.4927 | 7.2273 | 6.0387 | 5.2522 |
| | | IGA | 19.4795 | 8.1788 | 6.9501 | 5.8754 | 5.1572 |
| CCCC | 0 | RPT-1 [26] | 79.3355 | 31.5217 | 26.7612 | 23.1849 | 20.7162 |
| | | RPT [26] | 79.3304 | 31.5202 | 26.7602 | 23.1835 | 20.7141 |
| | | Quasi 3D [26] | 82.3402 | 33.5902 | 28.5574 | 24.4165 | 21.6023 |
| | | IGA | 79.3204 | 31.5156 | 26.7567 | 23.1807 | 20.7108 |
| | 1 | RPT-1 [26] | 69.1774 | 27.4857 | 23.3347 | 20.2163 | 18.0637 |
| | | RPT [26] | 69.1730 | 27.4844 | 23.3338 | 20.2151 | 18.0619 |
| | | Quasi 3D [26] | 71.7287 | 29.2705 | 24.8837 | 21.2721 | 18.8188 |
| | | IGA | 69.1652 | 27.4808 | 23.3311 | 20.2129 | 18.0593 |
| | 4 | RPT-1 [26] | 49.9794 | 19.8579 | 16.8589 | 14.6059 | 13.0507 |
| | | RPT [26] | 49.9762 | 19.8569 | 16.8582 | 14.6050 | 13.0494 |
| | | Quasi 3D [26] | 51.6664 | 21.1039 | 17.9361 | 15.3215 | 13.5514 |
| | | IGA | 49.9809 | 19.8586 | 16.8593 | 14.1510 | 12.7227 |

5. Conclusions

In this paper, size-dependent buckling analysis of nanoplates using IGA based on HSDT is investigated. The material properties of FGM based on the Mori–Tanaka schemes and the rule of mixture are considered. The differential nonlocal equations are utilized to take into account effect of

the size-dependent. The buckling nonlocal governing equations of motion is approximated according to IGA based on HSDT, which satisfies naturally the 3rd derivatives of displacement field. The effects of volume fraction exponent and small scale parameter on buckling analysis of nanoplates are also performed. Numerical results proved high accuracy and reliability of the present method in comparison with other available numerical approaches.

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