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Damage severity estimation from the global stiffness decrease

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Abstract. In actual damage detection methods, localization and severity estimation can be treated separately. The severity is commonly estimated using fracture mechanics approach, with the main disadvantage of involving empirically deduced relations. In this paper, a damage severity estimator based on the global stiffness reduction is proposed. This feature is computed from the deflections of the intact and damaged beam, respectively. The damage is always located where the bending moment achieves maxima. If the damage is positioned elsewhere on the beam, its effect becomes lower, because the stress is produced by a diminished bending moment. It is shown that the global stiffness reduction produced by a crack is the same for all beams with a similar cross-section, regardless of the boundary conditions. One mathematical relation indicating the severity and another indicating the effect of removing damage from the beam. Measurements on damaged beams with different boundary conditions and cross-sections are carried out, and the location and severity are found using the proposed relations. These comparisons prove that the proposed approach can be used to accurately compute the severity estimator.

1. Introduction

Integrity assessment of structures involving vibration-based techniques has got an increased attention in last decades. The core of these techniques is the vibration measurement, which provides information in real time regarding the structure's parameter changes, in this way making possible the detection of damage occurrence [1]. In the last few years, many research reports on vibration-based damage detection techniques have been published [2-9]. In respect to the geometry and orientation, damages can be differently classified. This study is limited to analyze the dynamic behavior of Euler-Bernoulli beams with transverse cracks. The rigidity of such beams is clearly diminished by the cracks, and also the mass is reduced. Among many past studies, two main crack ranges raised up: open cracks assuming the loss of rigidity and mass, and breathing cracks acting just on the rigidity [10].

In the case of open cracks, no contact between the crack edges is present, thus the vibration evolution is considered as linear. In all applications, single degree of freedom (SDOF) models are used for these systems. In the case of breathing cracks, the open and the closed stages alternate. The inertial forces compress the crack and close it. In the next stage, the forces act in the opposite direction and open the crack. This mechanism is known as breathing [11]. Studied papers, see for instance [12] and



[13], presented the SDOF models as always useful for the beams with breathing cracks and proposed bilinear model for nonlinearity. As a consequence, the vibration is considered nonlinear.

Severity estimation for both damage types is usually performed by means of the fracture mechanics approach [14-17]. For each damage geometry, cross-section shape combinations are necessary an individual analysis. This paper deals with the limitations of the above-mentioned approaches and proposes a new behavioral model that globally characterizes the damaged beam's dynamic behavior. In this model, the modal stored energy is introduced as a measure of the natural frequency and the damage severity is found from the decrease of the stored energy due to crack. The energy is evaluated from the beam deflection, which indicates an energy loss when it increases.

2. Actual models of cracked beams

It is largely recognized that, for open cracks, the beam rigidity equally decreases in both senses in transverse directions, e.g; from k in the healthy state to αk in the damaged one. Therefore, the vibration signal is a sinusoid, having decreased frequency in comparison with that of the original intact beam. If the intact and damaged beams are excited with the same amount of energy, a larger deflection is expected for the damaged ones. The behavior of a beam with an open crack close to the beam center is presented in figure 1. The blue line indicates the deflection achieved by the damaged beam, while the dashed line represents the case of the intact beam excited with the same amount of energy.

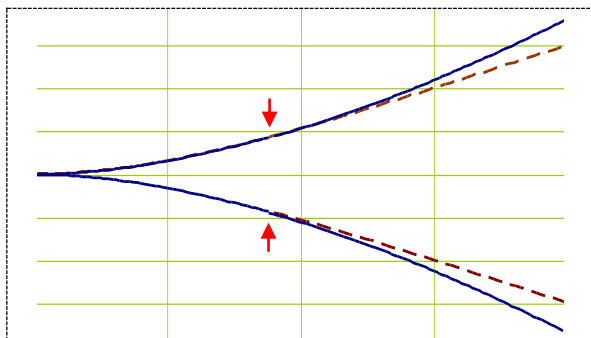


Figure 1. Maximum elongation of a beam with open crack near the center.



Figure 2. Maximum elongation of a beam with breathing crack near the center.

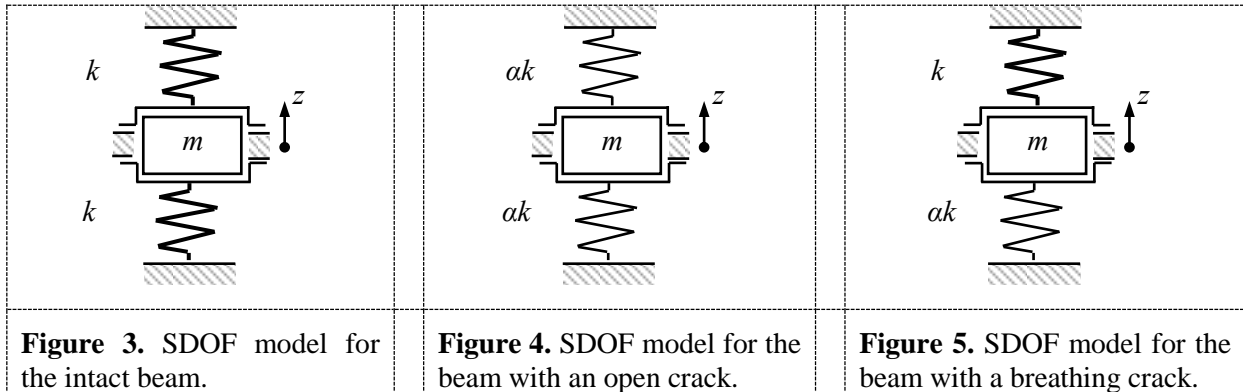
The behavior of a beam with a breathing crack is described in figure 2. One observes that, for the open stage the beam with a breathing crack behaves similar to the beam with an open crack, while in the closed stage it behaves like the healthy beam. For both stages, the amplitude is marked with blue color, the dashed line again representing the case of the intact beam. As a consequence, for a beam with a breathing crack, the stiffness depends on whether or not the crack is closed. In fact, it takes two different values, depending on the bending moment's sign [11]. Usually, the stiffness function $\tilde{k}(z)$ is defined as:

$$\tilde{k}(z) = \begin{cases} k_c = k & \text{if } z \geq 0 \\ k_o = \alpha k & \text{if } z < 0 \end{cases} \quad (1)$$

where z is the transversal direction.

The parameter α is called the stiffness ratio and takes values in the range of 0 to 1. For the stiffness ratio $\alpha = 1$, the system is linear, else it is stepwise linear. The two states correspond to the stages of the breathing mechanism: a) in the closed stage, indicated by index C, the stiffness ratio is equal to the unit and b) in the open stage, marked with O, the stiffness ratio is less than 1. Figures 3, 4 and 5 present the SDOF models of the intact beam, the beam with open crack and closed crack, respectively.

If involving the stiffness from equation (1) to describe the two distinct stages of breathing, the signal displays a half-period T_C similar to that of the intact beam, followed by an increased half-period T_O , for which the decreased rigidity is responsible. Larger amplitude for the open stage is observed.



In [12], the solutions for the angular frequencies are: $\omega_C = \sqrt{k_C / m}$ for the closed crack, and $\omega_O = \sqrt{k_O / m}$ for the open crack. The frequency ω_D of the bilinear oscillator in free vibration, the so-called bilinear frequency, satisfies the condition $\omega_O < \omega_D < \omega_C$, and is derived as:

$$\omega_D = \frac{2\omega_C\omega_O}{\omega_C + \omega_O}. \quad (2)$$

Apparently, the phenomenon is well-described, but some inadvertencies are observed by a deeper analysis [18]. These arise if the analysis is made by means of energetic methods and will be demonstrated in the next section.

3. Derivation of damage severity

Given that the law of conservation of energy is valid, the maximum strain energy in the open and closed stage is the same. In addition, the kinetic energy at the rest position should also be equal to these values. This can be formalized as:

$$TE = \max EP_O = \max EK = \max EP_C = \text{const.} \quad (3)$$

Where TE is the total energy, EP_O and EP_C are the strain energy in open and closed stages, respectively, and EK is the kinetic energy.

3.1. The frequencies of the intact cantilever beam

For a uniform cantilever beam, the kinetic energy of a given mode i is derived as:

$$EK_i = \frac{\bar{m}}{2} \int_0^L [X'_i(x,t)]^2 dx \quad (4)$$

where $X'_i(x,t)$ is the velocity of a point located at distance x from the origin (left beam end), $\bar{m} = \rho A$ is the distributed mass, E is the Young's modulus, I is the moment of inertia and t stays for time. If $X_i(x,t)$ is the beam deflection in transversal direction, and assuming a sinusoidal displacement law, $X_i(x,t)$ can be written as:

$$X_i(x,t) = \phi_i(x) \cdot \sin(\omega_i t) \quad (5)$$

Here $\phi_i(x)$ denotes the mode shape at maximum elongation and ω_i the angular frequency of vibration mode i . Thus, the highest kinetic energy at the rest position is given as:

$$\max EK_i = \frac{\bar{m}}{2} \int_0^L [\phi_i'(x)]^2 dx = \frac{\omega_i^2 \cdot \bar{m}}{2} \int_0^L [\phi_i(x)]^2 dx \quad (6)$$

The potential energy is:

$$EP_i = \frac{EI}{2} \int_0^L [X_i''(x,t)]^2 dx \quad (7)$$

Thus, the pick value is obtained for the extreme positions of the beam as:

$$\max EP_i = \frac{EI}{2} \int_0^L [\phi_i''(x)]^2 dx \quad (8)$$

From equations (6) and (8), the angular frequency is calculated as:

$$\omega_i^2 = \frac{EI \int_0^L [\phi_i''(x)]^2 dx}{\bar{m} \int_0^L [\phi_i(x)]^2 dx} \quad (9)$$

It is known that the mode shape for the cantilever beam is:

$$\phi_i(x) = \cosh \alpha_i x - \cos \alpha_i x - \frac{\cos \alpha_i L + \cosh \alpha_i L}{\sin \alpha_i L + \sinh \alpha_i L} (\sinh \alpha_i x - \sin \alpha_i x) \quad (10)$$

and the modal curvature is:

$$\phi_i''(x) = \alpha_i^2 \left[\cosh \alpha_i x + \cos \alpha_i x - \frac{\cos \alpha_i L + \cosh \alpha_i L}{\sin \alpha_i L + \sinh \alpha_i L} (\sinh \alpha_i x + \sin \alpha_i x) \right] \quad (11)$$

where α is a dimensionless parameter reflecting the boundary conditions. Substituting these expressions in equation (9), one obtains the angular frequency of the i -th mode, as:

$$\omega_i^2 = \frac{\alpha_i^4 EI \int_0^L \left[\cosh \alpha_i x + \cos \alpha_i x - \frac{\cos \alpha_i L + \cosh \alpha_i L}{\sin \alpha_i L + \sinh \alpha_i L} (\sinh \alpha_i x + \sin \alpha_i x) \right]^2 dx}{\bar{m} \int_0^L \left[\cosh \alpha_i x - \cos \alpha_i x - \frac{\cos \alpha_i L + \cosh \alpha_i L}{\sin \alpha_i L + \sinh \alpha_i L} (\sinh \alpha_i x - \sin \alpha_i x) \right]^2 dx} \quad (12)$$

In [19], it is proved that the functions under the two integrals provide the same result $0.25L\delta^2$, where δ is the free end deflection. After substitution and reduction, the angular squared frequency becomes:

$$\omega_i^2 = \frac{\alpha_i^4 EI \cdot 0.25L}{\bar{m} \cdot 0.25L} = \alpha_i^4 \frac{EI}{\bar{m}} \quad (13)$$

Hence, the well-known expression of the natural frequency of a beam for the bending modes is obtained as:

$$f_i = \frac{\omega_i}{2\pi} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{EI}{\bar{m}L^4}} \quad (14)$$

with $\lambda = \alpha / L$ derived from the beam's characteristic equation. For the cantilever, this equation is:

$$\cos \lambda \cosh \lambda + 1 = 0 \quad (15)$$

One can easily observe in Eq. (14) that the term under the square is inversely proportional to the beam deflection at the free end if only dead weight is acting, i.e.:

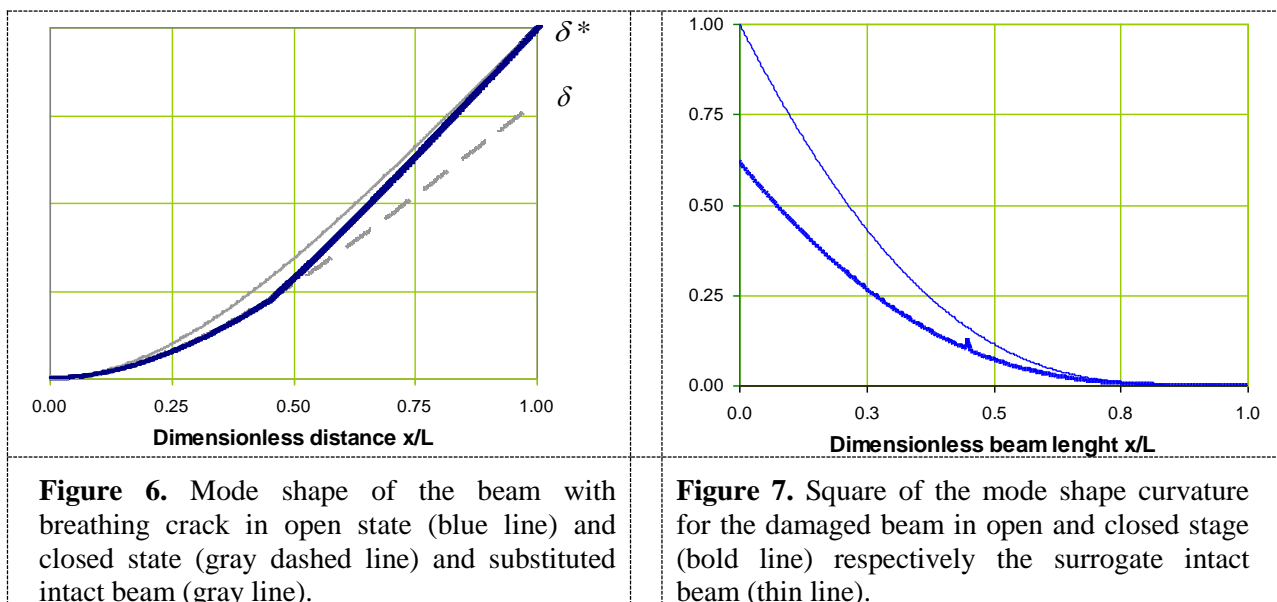
$$\delta = \frac{\bar{m}gL^4}{8EI} \quad (16)$$

where g denotes the gravitational acceleration. From equations (14) and (16), the natural frequencies in respect to the free end deflection are achieved under dead weight as:

$$f_i = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{g}{8\delta(L)}} \quad (17)$$

3.2. The frequencies of a cantilever beam with a breathing crack

Let us now consider the cracked beam vibrates, and in open stage, the free end achieves amplitude δ^* similar to the deflection of the intact beam under dead weight. Because of energetic equilibrium, a smaller amplitude δ is achieved in the closed stage. In figure 6, both open and closed states are depicted on the same side, in order to permit a facile comparison. The shape in the open state is marked with a blue line and that in the closed state with a gray dashed line. Note that the same quantity of strain energy is stored in both states since the larger deflection occurs just due to a supplementary rotation in a narrow slice subjected to damage. In all other damaged beam slices, the same energy is stored as in the intact beam. This is reflected in figure 7 by the squared modal curvature. The strain energy for the open and closed stages are both represented with bold blue line, the single difference between the two distribution consisting in the small hill present in the slice where supplementary rotation is possible.



The above presented example shows that stored energy and deflection ratio must be considered in the analysis. Now, we can replace the damaged beam with a healthy one with decreased rigidity and achieving similar deflection as the damaged in open stage. This will store, according to Castigliano's second theorem, which is applicable to the stepwise linear system under analysis, the same energy as the damaged beam. Let this beam, nominated herein as surrogate beam, have the pick deflection δ^* and the subsequent rigidity $EI_D(c)$. It is worth to mention that the distributed mass \bar{m} is maintained

similar with that of the intact original beam. Also, the surrogate rigidity $EI_D(c)$ is influenced by the damage position c ; the closer the damage to the fixed end, the lower the surrogate rigidity is.

The squared modal curvature for the surrogate beam, represented in figure 7 with thin blue line, achieves larger amplitude as the damaged beam in closed stage, due to the decreased rigidity. The ratio is defined by deflections δ^* and δ , their values being deductible from equation (19).

The rigidity $EI_D(c)$ is derived from the deflections of the original and surrogate beams at the free end, which are:

$$\delta^* = \frac{\bar{m}gL^4}{8EI_D(c)} \quad \text{and} \quad \delta = \frac{\bar{m}gL^4}{8EI} \quad (18)$$

thus

$$EI_D(c) = EI \frac{\delta}{\delta^*} \quad (19)$$

For the intact and surrogate beam, respectively, the angular frequencies ω_i and ω_{i-D} are deductible from:

$$\omega_i^2 = \frac{EI \int_0^L [\phi_i''(x)]^2 dx}{\bar{m} \int_0^L [\phi_i(x)]^2 dx} = \frac{EI}{\bar{m}} \quad \text{and} \quad \omega_{i-D}^2 = \frac{EI_D(c) \int_0^L [\phi_{i-D}''(x)]^2 dx}{\bar{m} \int_0^L [\phi_{i-D}(x)]^2 dz} = \frac{EI_D(c)}{\bar{m}} \quad (20)$$

Hence, the frequency of a cracked beam in closed state is

$$f_{i-D} = f_i \sqrt{\frac{EI_D(c)}{EI}} = f_i \sqrt{\frac{\delta}{\delta^*}} \quad (21)$$

and the dimensionless frequency is

$$\bar{f}_{i-D} = \frac{f_{i-D}}{f_i} = \sqrt{\frac{\delta^*}{\delta}} \quad (22)$$

The deflection ratio, $r = \delta^* \cdot \delta^{-1}$, presented in equations (21) and (22) merits a special remark. Since just the inertial forces act on the beams and no large deflections occur (introducing nonlinearities), this ratio depends only on the damage position c and its depth a .

Now, we change the initial condition. The analysis is made for the free end of the cracked beam removed with a distance δ in such a way that the crack closes. By releasing the free end, the beam vibrates, in the opposite position achieving a larger deflection δ^* . The case becomes similar with that of the closed crack presented in subsection 3.2. The same surrogate beam results, and the same frequency as indicated in equations (21) and (22) are obtained.

3.3. Some considerations regarding the vibration of cracked beams

A conclusion from this study is that the two half-periods T_O and T_C are identical for the free vibration of beams subjected to breathing cracks. Also, it results that the frequency of the damaged beam can be found involving equation (21), basing on the intact beam frequency and the beam deflection in intact respectively damaged state.

If a given damage is located in a slice where the curvature achieves maxima, i.e. the fixed end for the cantilever beam, the lowest value for ratio r is achieved. Here, the lowest deflection ratio and

surrogate rigidity are achieved. This beam is nominated herein as equivalent beam and the rigidity as equivalent rigidity EI_{eq} .

For a beam with an open crack, the damage expansion on the longitudinal axis increases the deflection due to the weak segment between the crack edges. This has as a consequence an additional frequency decrease in comparison with that produced by a breathing crack with a similar depth. The frequency is still deductible from equation (21).

Because the mode shapes and modal curvatures are reduced in equation (20), the equation (21) can be used for deriving the frequencies for damaged beams with other boundary conditions. Obviously, the intact beam's frequencies for the given boundary conditions must be involved.

4. A simple damage severity estimator

As shown in previous section, the damage depth is one of the parameters affecting the natural frequencies. The second parameter is the damage position. Let us firstly consider the damage at the fixed end, and take as variable the damage depth a . In open stage, the deflection at the free end is $\delta_o(0, \bar{a})$, where 0 indicates the damage location, i.e. $c=0$, and $\bar{a}=a/h$ is the dimensionless damage depth. If the deflection of the intact beam is δ , the frequency shift for a beam with rigidity decrease can be calculated as:

$$\Delta f_{i-D} = f_i - f_{i-D} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{8}{g\delta}} - \frac{\lambda_i^2}{2\pi} \sqrt{\frac{8}{g\delta_o(0, \bar{a})}} = \frac{\lambda_i^2}{2\pi} \left(\sqrt{\frac{8}{g\delta}} - \sqrt{\frac{8}{g\delta_o(0, \bar{a})}} \right) \quad (23)$$

Hence, the relative frequency shift, obtained by normalizing the frequency of the intact beam f_i , becomes:

$$\Delta \bar{f}_{i-D} = \frac{f_i - f_{i-D}}{f_i} = \frac{\frac{\lambda_i^2}{2\pi} \left(\sqrt{\frac{8}{g\delta}} - \sqrt{\frac{8}{g\delta_o(0, \bar{a})}} \right)}{\frac{\lambda_i^2}{2\pi} \sqrt{\frac{8}{g\delta}}} = \frac{\sqrt{\delta_o(0, \bar{a})} - \sqrt{\delta}}{\sqrt{\delta_o(0, \bar{a})}} = \gamma(0, \bar{a}) \quad (24)$$

The absolute damage severity $\gamma(0, \bar{a})$ indicates the biggest effect, which a given damage can produce on the cantilever beam, being valid for all bending vibration modes. It can be simply calculated from the deflection of the beam with a breathing crack in intact and damaged state. Based on this damage evaluator, the frequency of a damaged beam with a breathing crack at the fixed end can be found as:

$$f_{i-D} = f_i - f_i \cdot \Delta \bar{f}_{i-D} = f_i \left[1 - \frac{\sqrt{\delta_o(0, \bar{a})} - \sqrt{\delta(L)}}{\sqrt{\delta_o(0, \bar{a})}} \right] = f_i [1 - \gamma(\bar{a})] \quad (25)$$

Let us now consider the crack located elsewhere on the beam, for instance at normalized distance $\bar{c} = c/L$ from the fixed end. Obviously, the deflection of the damaged beam will be smaller. We take now advantage of introducing the surrogate beam. This is concretized in distributing the real rigidity decrease of one beam element onto all elements, as depicted in figure 8 for the second vibration mode. In figure 8.a, the element near the fixed end has a reduced rigidity due to the crack. If this is located on another element, the effect will be decreased in function of the energy stored in that element, thus in respect to $[\bar{\phi}_i''(\bar{c})]^2$. In this way, the effect of damage location upon the severity is introduced. Now, we can write the damage severity as:

$$\gamma(\bar{c}, \bar{a}) = \gamma(0, \bar{a}) \cdot [\bar{\phi}_i''(\bar{c})]^2 \quad (26)$$

$$\gamma(\bar{c}, \bar{a}) = \frac{\sqrt{\delta_o(\bar{c}, \bar{a})} - \sqrt{\delta}}{\sqrt{\delta_o(\bar{c}, \bar{a})}} = \frac{\sqrt{\delta_o(0, \bar{a})} - \sqrt{\delta}}{\sqrt{\delta_o(0, \bar{a})}} \cdot [\phi_i''(\bar{c})]^2 \quad (27)$$

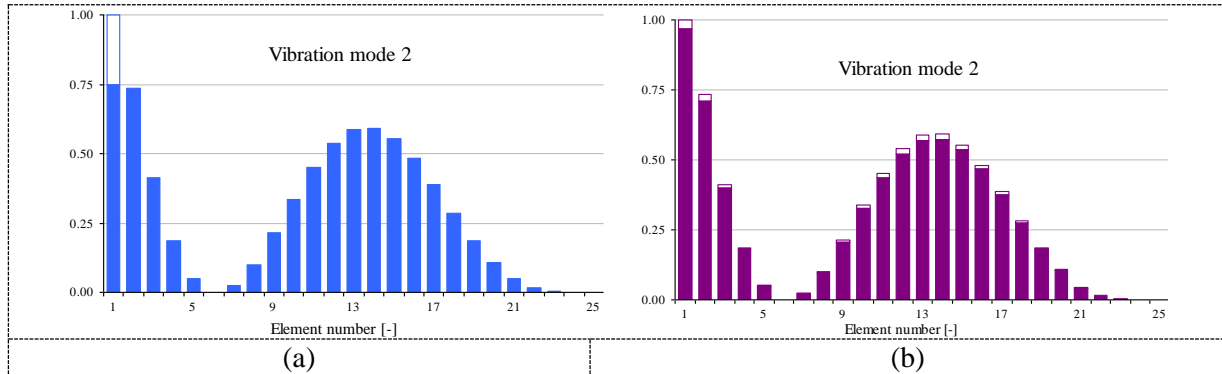


Figure 8. Normalized square of the modal curvature corresponding to bending vibration mode two: (a) damaged beam; (b) equivalent beam.

The natural frequency of a beam with a breathing crack with depth \bar{a} and position \bar{c} is

$$f_{i-D} = f_i \left\{ 1 - \gamma(\bar{a}) \cdot [\phi_i''(\bar{c})]^2 \right\} \quad (28)$$

This simple mathematical relation can be used to develop damage scenarios in order to create databases usable in damage detection procedures. For open cracks, in case of a fine tuning, the mass loss should also be taken into consideration [20].

5. Numerical examples

The robustness of the proposed damage severity indicator was proved by numerical experiments on steel beams with length $L = 600\text{mm}$ and three different cross-sections shapes: square, hexagonal and circular. The cross-section dimensions were chosen as following: the square edges $h_s = 12\text{mm}$, the circle diameter $d = 12\text{mm}$ and the distance between the parallel sides of the hexagon $h_H = 12\text{mm}$. All simulations were made by means of the ANSYS package, the mechanical characteristics being that of the structural steel from the dedicated library. The medium size of the tetrahedral elements, used for meshing, was fixed to 0.5mm .

The damage, in form of a breathing crack, was positioned near the fixed end. Several crack depths were used in the simulations. First a static analysis in order to find the deflections was performed. Afterward, the modal analysis was used to find out the natural frequencies. The achieved results are presented separately for each cross-section shape in tables 1 to 3. Percentile error was contrived, to evaluate the damage severity precision.

Table 1. Results achieved for the hexagonal beam. The absolute damage severity and the relative frequency shift are indicated together with the contrived error.

a [mm]	δ_o [mm]	γ [%]	f_1 [Hz]	RFS [%]	Error [%]
0	0.62271	0	24.82598	0	0
1	0.62572	0.240813	24.76592	0.241942	0.466644
2	0.63387	0.884216	24.60571	0.887279	0.345255
3	0.64781	1.956435	24.33850	1.963586	0.364182
4	0.66976	3.576406	23.93435	3.591538	0.42134
5	0.70426	5.967841	23.33672	5.998789	0.515904
6	0.76214	9.608927	22.42499	9.671274	0.644665

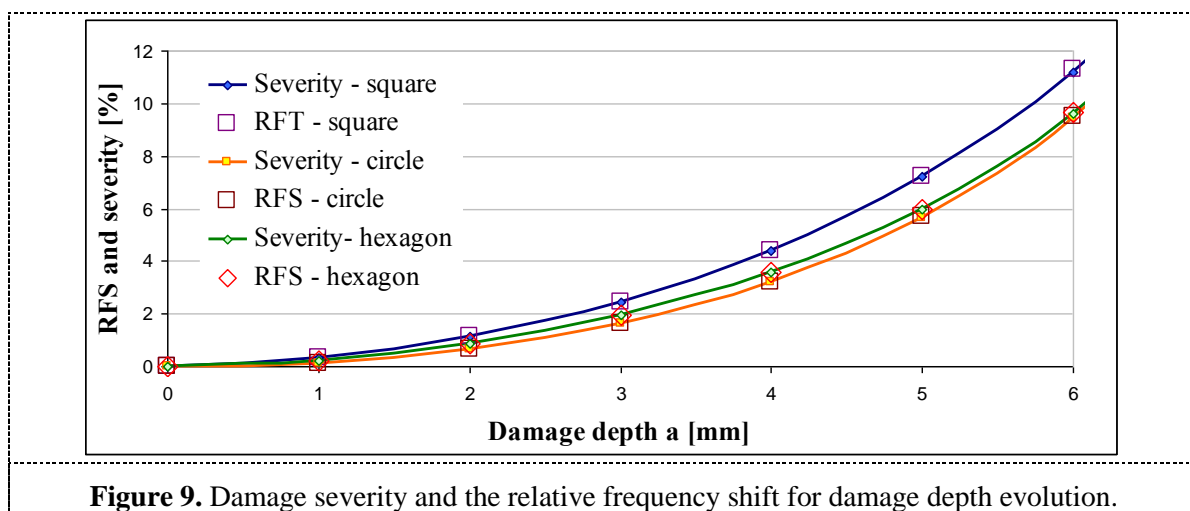
Table 2. Results achieved for the circular beam. The absolute damage severity and the relative frequency shift are indicated together with the contrived error.

a [mm]	δ_o [mm]	γ [%]	f_1 [Hz]	RFS [%]	Error [%]
0	0.69271	0	23.53858	0	0
1	0.69431	0.115289	23.51134	0.115736	0.386
2	0.70192	0.658224	23.38283	0.661669	0.520
3	0.71569	1.618542	23.15626	1.624223	0.349
4	0.73936	3.206153	22.78038	3.221107	0.464
5	0.77817	5.650743	22.20134	5.681056	0.533
6	0.84435	9.423721	21.30601	9.484736	0.643

Table 3. Results achieved for the prismatic beam. The absolute damage severity and the relative frequency shift are indicated together with the contrived error.

a [mm]	δ_o [mm]	γ [%]	f_1 [Hz]	RFS [%]	Error [%]
0	0.51890	0	27.19549	0	0
1	0.52218	0.314563	27.10972	0.315372	0.256585
2	0.53077	1.124509	26.88894	1.127184	0.237287
3	0.54527	2.448032	26.52740	2.456623	0.349687
4	0.56779	4.402185	25.99316	4.421045	0.426585
5	0.60257	7.202114	25.22600	7.241957	0.550169
6	0.65810	11.20349	24.12754	11.28109	0.687906

To calculate the damage severity, the equation (24) was used, where the input terms are the beam deflections. Also equation (24) is used to find the relative frequency shifts, herein the input data consists of the first natural frequencies of the beams.

**Figure 9.** Damage severity and the relative frequency shift for damage depth evolution.

The curves plotted in figure 9 present good similarity with that presented in the literature, see [14-17]. Even more significant is the extremely low error level found between the damage severity and relative frequency shifts. As it was calculated in percentage, the values are indicated in the last column of tables 1 to 3. One can observe that the errors are less than 1%, meaning that the proposed relation to contrive the damage severity is reliable. This can be also concluded from figure 9, where the perfect fit between the compared values is observable.

6. Conclusion

This paper introduces a robust damage severity evaluator that predicts the frequency decrease occurred in a cantilever beam due to a breathing crack. It can be simply calculated using the deflections produced by the dead mass to the intact beam and the beam with a crack in the open state respectively. This introduces the important advantage that evaluator can be used for any cross-section shape. A particular case of this feature is the absolute damage severity, which is derived for the crack located in a slice where the bending moment achieves maxima. Thus, it indicates the biggest effect, which a given damage can produce on the frequencies, and it is unique for all bending vibration modes. In contrary, the damage severity takes, for a given crack, different values in respect to the position on the beam and the vibration mode number. The ratio between the damage severity and the absolute damage severity is actually the normalized squared modal curvature of the intact beam at the crack location. An important conclusion resulted from this study, which permits a correct interpretation of the vibration of beams with breathing cracks, is that the two half-periods T_O and T_C are identical. This happens because the frequencies are proportional to the stored energy which is the same in the open and closed stage. Therefore, the estimator can be involved for a beam with an open crack as well. The single difference here is the damage expansion on the longitudinal axis, producing an increased deflection due to the weak segment between the crack edges. This has as result an additional frequency decrease in comparison with that produced by a breathing crack with similar depth; it is proportional to the deflection increase. In the mathematical relations used to calculate the deflection for different boundary conditions just the numerical value is different. Thus, the deflection ratio r is identical for beams with any support type, because this value is reduced in the ratio. As a consequence, the deflection ratio r for the cantilever beam can be used for any beam configuration. It is the most convenient case, since here the largest deflections are expected and so reduced errors are presumed. Remarkable is the fact that the absolute damage severity estimator permits predicting the frequencies of the damaged beam by involving only the intact beam frequency and the damage depth and its location (in order to calculate the normalized square of the modal curvature for the intact beam). This means that the frequency for damaged beams with any boundary conditions is derivable knowing the crack location and depth.

Acknowledgements

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