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Robustness in the personnel shift scheduling problem: the modelling and validation of different proactive and reactive strategies

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To my "Silver Lining"

"If you are not willing to learn, no one can help you.
If you are determined to learn, no one can stop you."
Zig Ziglar, 1926 - 2012

Dankwoord

"It always seems impossible, until it's done." (Nelson Mandela, 1918 - 2013)

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Jonas Ingels
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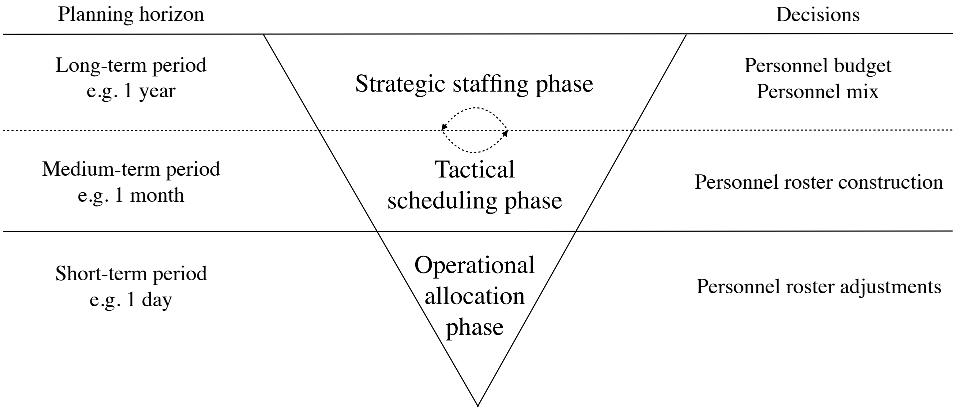
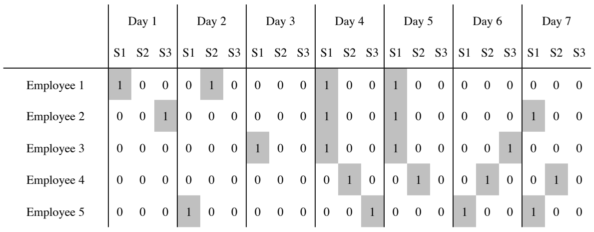
Samenvatting

In dit boek, onderzoeken we de impact van onzekerheid in het personeelsplanningsproces en hoe de personeelsplanner daarop moet anticiperen en reageren om robuuste personeelsroosters te bekomen. Dit is belangrijk omdat het personeelsplanningsproces er in elke organisatie voor moet zorgen dat een wenselijk *service level* kan aangeboden worden aan de klanten aan een minimale personeelskost en een maximale personeelstevredenheid. Dit proces bestaat uit drie hiërarchische fases (Figuur 1), i.e. de *strategic staffing phase*, de *tactical scheduling phase* en de *operational allocation phase.* Elk van deze fases wordt gekenmerkt door een verschillend niveau van beslissingsvrijheid voor de personeelsplanner en een verschillend niveau van onzekerheid omtrent het aantal benodigde werknemers en de beschikbaarheid van deze werknemers in de toekomst.



Figuur 1 Het personeelsplanningsproces

In de strategische fase (*strategic staffing phase*) heeft de personeelsplanner een grote beslissingsvrijheid om beslissingen te nemen voor een periode van één jaar. Deze beslissingen omvatten het bepalen van het personeelsbudget en de gewenste personeelsmix. Het budget bepaalt hoeveel geld beschikbaar moet zijn om het loon van werknemers te betalen en om werknemers aan te nemen, trainen en ontslaan. Dit budget moet er voor zorgen dat de organisatie kan beschikken over de gewenste personeelsmix om de doelstellingen omtrent het *service level* en de personeelskost en tevredenheid te bereiken in de toekomst. Deze personeelsmix wordt gedefinieerd door het aantal beschikbare werknemers en hun vaardigheden. In een omgeving waarin de toekomst perfect kan voorspeld worden, is het relatief gemakkelijk om deze strategische beslissingen te maken. Het wordt echter moeilijk als er onzekerheid is omtrent het aantal werknemers dat nodig zal zijn en welke werknemers op verschillende tijdstippen beschikbaar zullen zijn. In dit geval moet de personeelsplanner een aantal voorspellingen en assumpties maken waarop de beslissingen uiteindelijk gebaseerd worden.



	Day 1			Day 2			Day 3			Day 4			Day 5			Day 6			Day 7		
	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
Employee 1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
Employee 2	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
Employee 3	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0
Employee 4	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0
Employee 5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0

Figuur 2 Een voorbeeld van een personeelsrooster met drie shifts (S1, S2, S3) per dag

De tactische beslissingen (*tactical scheduling phase*) bestaan uit het opstellen van een personeelsrooster voor een middellange termijn van bijvoorbeeld één maand. Zo'n rooster bepaalt voor elke werknemer welke shift hij/zij moet werken op elke dag (Figuur 2). Belangrijk hierbij is dat een aantal beperkingen voldaan moet worden, bijvoorbeeld een minimale rustperiode tussen twee werkshifts en een minimum en maximum aantal werkuren. Bovendien moet ervoor gezorgd worden dat er tijdens elke shift een vooraf bepaald aantal werknemers werken. Verder is de beslissingsvrijheid in deze fase kleiner dan in de strategische fase aangezien rekening gehouden moet worden met de strategische beslissingen, i.e. het budget en de personeelsmix. De personeelsplanner beschikt echter wel over betere informatie omtrent het aantal werknemers dat beschikbaar zal moeten zijn en welke werknemers effectief kunnen werken. Deze informatie is echter nog steeds onderhevig aan onzekerheid gegeven de middellange tijdsperiode. Hierdoor moet de personeelsplanner nieuwe voorspellingen en assumpties maken vooraleer het personeelsrooster opgesteld kan worden.

Het personeelsrooster dat opgesteld werd in de tactische fase, wordt uitgevoerd in de operationele fase (*operational allocation phase*). In deze fase, beschikt de personeelsplanner over de meest up-to-date informatie omtrent personeelsbenodigdheden en beschikbaarheid. Deze informatie kan echter verschillen van de eerder gemaakte voorspellingen en assumpties waardoor er meer werknemers nodig zijn en/of werknemers onverwacht ziek zijn. In dit geval voldoet het tactische rooster niet langer om het gewenste *service level* te kunnen aanbieden aan klanten aan een minimale personeelskost en maximale personeelstevredenheid. Daarom moet de personeelsplanner aanpassingen maken aan het rooster. De aanpassingsmogelijkheden zijn echter sterk beperkt door de gemaakte strategisch en tactische beslissingen. De personeelsplanner moet immers rekening houden met de personeelsmix en het opgestelde rooster. De aanpassingen die de personeelsplanner uitvoert moeten daardoor passen in strategische en tactische context. Het is immers niet mogelijk om extra werknemers aan te nemen of het volledige personeelsrooster te veranderen.

De impact van de strategische en tactische beslissingen mag dus niet onderschat worden. Deze beslissingen bepalen immers in welke mate de personeelsplanner onverwachte verschillen tussen de strategische en tactische assumpties enerzijds en de operationele werkelijkheid anderzijds effectief en efficiënt kan oplossen met een minimale impact op het *service level* en de personeelskost en tevredenheid. Het is daarom belangrijk dat de personeelsplanner anticipeert op onverwachte omstandigheden door het toepassen van proactieve strategieën in de strategische en tactische fase. Deze strategieën moeten ervoor zorgen dat het opgestelde personeelsrooster robuust is bij onverwachte omstandigheden, i.e. stabiel en flexibel. Een rooster is stabiel als het onverwachte gebeurtenissen kan absorberen zonder de interventie van de personeelsplanner. Een rooster is flexibel als de personeelsplanner onverwachte gebeurtenissen met minimale aanpassingen kan oplossen. Het is uiteraard niet mogelijk om perfect op onverwachte gebeurtenissen te anticiperen. Hierdoor zijn reactieve strategieën onmisbaar in de operationele fase. Deze strategieën moeten ervoor zorgen dat de proactief voorziene robuustheid naar behoren gebruikt wordt om onverwachte gebeurtenissen op te lossen.

In hoofdstuk 1, geven we een uitgebreide inleiding over het belang van robuustheid om met onzekerheid om te gaan in het personeelsplanningsproces. Bovendien geven we een overzicht van de verschillende onderzoeken die we ondernomen hebben om een bijdrage te leveren aan de academische literatuur omtrent robuustheid in personeelsplanning.

In hoofdstuk 2, stellen we een methodologie voor die toelaat om de impact van proactieve en reactieve strategieën op de robuustheid te evalueren. Deze methodologie bestaat uit drie stappen. In een eerste stap wordt het personeelsrooster opgesteld met inbegrip van de proactieve strategieën. Dit personeelsrooster wordt in de volgende stap uitgevoerd in de operationele fase. Dit houdt in dat we onverwachte gebeurtenissen simuleren en indien nodig aanpassingen doen aan het personeelsrooster aan de hand van reactieve strategieën. In de laatste stap wordt uiteindelijk een evaluatie gemaakt om de robuustheid van het personeelsrooster en de bijhorende proactieve en reactieve strategieën te bepalen. De twee laatste stappen worden veelvuldig herhaald om een inzicht te krijgen op de impact van de strategieën over verschillende scenario's.
In dit hoofdstuk onderzoeken we ook het nut van *reserve duty scheduling strategies* om capaciteitsbuffers te bekomen. Dit betekent dat we proactief werknemers toewijzen als reserve tijdens een bepaalde shift. Deze toewijzing gebeurt op basis van de definitie van strategieën die verschillen op basis van *reserve duty staffing requirements* en *reserve time-related constraints* die bepalen hoeveel reserve werknemers er moeten zijn en hoeveel keer een werknemer als reserve kan ingeschakeld worden. Bovenop deze proactieve strategieën onderscheiden we twee reactieve strategieën die verschillen in de beslissingsvrijheid die ze voorstellen. De resultaten tonen aan dat werknemers als reserve moeten ingeschakeld worden op basis van een strategie die zowel probleem-specifieke *reserve duty staffing requirements* en *reserve time-related constraints* definieert. Bovendien moet er sterk rekening gehouden worden met het profiel van de vraag naar werknemers over elke shift en dag in het personeelsrooster en bepaalt de reactieve beslissings-

vrijheid in welke mate op onverwachte gebeurtenissen moet geanticipeerd worden.

In hoofdstuk 3, definiëren we de onzekerheid omtrent de toekomst in een *uncertainty set*. We bepalen verschillende proactieve strategieën om de karakteristieken van deze *uncertainty set* te kenmerken. Deze strategieën worden aangevuld met verschillende reactieve strategieën die een verschillende beslissingsvrijheid voorstellen in de operationele fase. Een analyse van de resultaten geeft aan dat het belangrijk is om over zoveel mogelijk shifts een kleine buffer van werknemers te voorzien om onverwachte omstandigheden op te vangen. Deze buffer mag kleiner zijn en voorzien worden voor een lager aantal shifts als de beslissingsvrijheid in de operationele fase toeneemt.

In hoofdstuk 4, bekijken we de mogelijkheid om werknemers met verschillende vaardigheden werk te laten uitwisselen om op onverwachte omstandigheden te reageren. In dit opzicht, stellen we proactieve strategieën voor die de uitwisselbaarheid van werk bevorderen op het niveau van individuele werknemers en op groepsniveau. Uit de resultaten blijkt dat het uiterst belangrijk is dat we ervoor zorgen dat de uitwisseling van werk tijdens de gepaste shifts toegepast kan worden. Bovendien moet er een reactieve strategie zijn die deze uitwisseling ook effectief begeleidt.

In hoofdstuk 5, onderzoeken we de *trade-off* tussen het aannemen van meer mensen en het aannemen van minder mensen in combinatie met het toelaten van *overtime*. Beide proactieve opties hebben gevolgen voor de mogelijkheden om reactief met onverwachte gebeurtenissen om te gaan. Meer aangenomen mensen zorgen tegelijkertijd voor een hogere kost en meer aanpassingsmogelijkheden. Minder mensen aannemen is goedkoper maar moet gecompenseerd worden door proactief meer *overtime* toe te wijzen om de verwachte vraag naar werknemers te voldoen. Bovendien zorgt dit ervoor dat *overtime* reactief minder gebruikt kan worden om onverwachte gebeurtenissen op te lossen. De resultaten tonen aan dat het mogelijk is om met minder werknemers die *overtime* kunnen werken een lagere kost te behalen. Er moet echter wel een strategie gebruikt worden zodat *overtime* zowel proactief als reactief toegepast kan worden. Bovendien kan het proactief creëren van een capaciteitsbuffer in combinatie met *overtime* een positieve impact hebben op de robuustheid van een personeelsrooster. Hiervoor moeten dan wel extra werknemers aangenomen worden.

In een laatste hoofdstuk, worden finaal een aantal algemene conclusies voorgesteld en worden mogelijkheden tot toekomstig onderzoek gegeven.

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An overview of robustness in personnel scheduling

1.1 General introduction

The personnel management and planning process is extensively investigated in the operations research literature (Burke et al., 2004; De Bruecker et al., 2015; Dorne, 2008; Ernst et al., 2004a,b; Van den Bergh et al., 2013). These review papers discuss the personnel shift scheduling problem as one of the most important problems in personnel scheduling because of its relevance in many different application areas such as manufacturing, airline industry, call centres, healthcare and transportation. However, this problem has been mostly investigated assuming a deterministic operating environment in which all inputs to the personnel planning process are known and fixed. In reality however, organisations often operate in a stochastic operating environment and face a degree of uncertainty during their operations. Van den Bergh et al. (2013) distinguish uncertainty of demand, uncertainty of capacity and uncertainty of arrival. Uncertainty of demand may cause the expected personnel demand to differ from the actual demand. As a result of uncertainty of capacity, employee availability cannot be guaranteed due to absenteeism, sick leave, etc. In this book, we explicitly study the personnel shift scheduling problem with fixed start and end times for the different shifts. As a result, only the uncertainty of demand and the uncertainty of capacity are considered, since the uncertainty of arrival impacts the starting times of the specific tasks during the shifts.

Uncertainty may result in the occurrence of unexpected events, which may disrupt or have a negative impact on the quality of the operations of an organisation in terms of the *service level* and the *personnel cost and satisfaction.* A *service level* is postulated by the organisation to guarantee customers are served by the right person at the right time and place. This service level objective requires organisational flexibility in the personnel planning process to deal with unexpected events. *Personnel costs* represent a large portion of the operating costs of an organisation (Ernst et al., 2004b; Van den Bergh et al., 2013). Disruptions can cause important deviations between the planned and actual cost for the personnel resources. The planned cost is the estimated cost when a personnel roster is composed, while the actual cost is the cost incurred during the execution of the roster. Focusing solely on cost minimisation during the roster construction leads to a low planned cost but also to a limited number of options to deal with disruptions. In this case, the organisation needs to resort to expensive corrective actions, which increase the actual cost and may also lead to a lower *personnel satisfaction.* This satisfaction is an important objective in the personnel (re-)scheduling literature and should therefore be taken into account (Bard and Purnomo, 2005b; Pato and Moz, 2008; Topaloglu and Selim, 2010).

As a result, the impact of uncertainty needs to be considered in the personnel planning process to attain the desired service level in such a way that the associated personnel cost and satisfaction is optimised. However, stochasticity and robustness have only received limited attention in personnel scheduling. In this respect, we investigate different strategies to improve the robustness of the personnel planning process by facilitating the anticipation of and reaction to the occurrence of unexpected events such that the impact on the service level and the personnel cost and satisfaction is minimised.

We emphasise the importance of this research by providing a detailed definition of the personnel planning process (Section 1.1.1) and robustness (Section 1.1.2). Based on these definitions, we provide the contribution of our research and an outline of our studies in Section 1.2.

1.1.1 The personnel planning process

The personnel planning process consists of three hierarchical phases, i.e. the strategic staffing phase, the tactical scheduling phase and the operational allocation phase (Abernathy et al., 1973; Burke et al., 2004). These phases are characterised by a certain level of decision freedom corresponding to a specific planning horizon (Figure 1.1) and are usually treated separately. Given the differing lengths of the planning horizon, each phase also represents a different level of uncertainty about the future operating environment, i.e. the future personnel demand and employee availability.

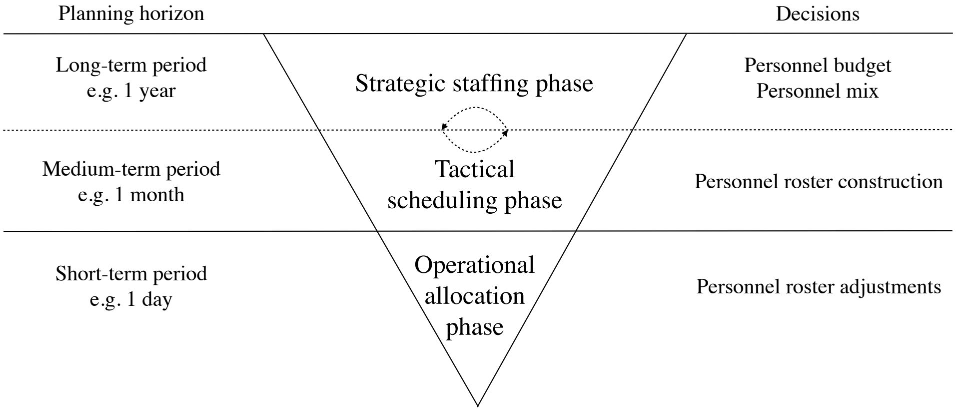


Figure 1.1 The personnel planning process

The strategic staffing phase

In the strategic staffing phase, organisations make decisions concerning a long-term period. Given the long-term nature of the decisions, the personnel planner faces a large uncertainty about the future operating environment. As such, a number of assumptions and predictions need to be made concerning the demand for skilled employees and the availability of these employees. In a deterministic operating environment, these assumptions and predictions are a perfect representation of the future operating environment. This allows the organisation to perfectly analyse the impact of each strategic decision given the future realisations of the personnel demand and employee availability. However, organisations often operate in a stochastic environment and face variability. This variability may cause the actual personnel demand and employee availability to differ from their assumed and predicted values, which increases the complexity of the decision-making process in the strategic staffing phase.

The strategic decisions include the determination of the total *personnel budget* and the desired *personnel mix*. The *personnel budget* defines the amount of monetary resources that the personnel planner can spend on personnel wages and on hiring, firing and training of employees over the course of, for example, a year. This budget should enable the organisation to employ a personnel mix that contributes to the attainment of the organisational objectives in the future operating environment, i.e. the provision of a service level to customers at an optimised personnel cost and satisfaction. The *personnel mix* is characterised by the number of available employees and the skills these employees possess. As the number of skills per employee increases, the level of cross-training rises.

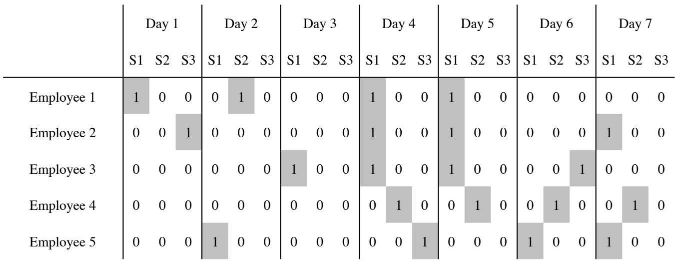
The strategic decisions about the personnel budget and mix are certainly interrelated and should, therefore, be considered concurrently. A specific personnel mix requires the availability of an appropriate budget. However, the organisation aims to minimise its operating costs and wishes to offer a budget that is as small as possible. Hence, the personnel budget impacts the attainable personnel mix while the desirable personnel mix impacts the need for a specific budget. In this respect, there exists a trade-off between overtime and the number of (cross-trained) employees (Koutsopoulos and Wilson, 1987; Tan, 2003). Overtime increases the personnel wage cost but reduces the need for a high number of (cross-trained) employees. Similarly, a high number of (cross-trained) employees increases the hiring and training cost but diminishes the demand for overtime. This trade-off has an important impact on the smooth execution of the organisation's operations and should be thoroughly investigated to enable the provision of an appropriate personnel budget and mix.

The tactical scheduling phase

Given the strategic personnel budget and mix, organisations make decisions concerning a medium-term period in the tactical scheduling phase. In this phase, the personnel planner acquires more up-to-date information on the future operating environment. Given the medium-term nature of the tactical decisions however, a level of uncertainty concerning the personnel demand and employee availability remains. As such, new assumptions and predictions are made for a medium-term period, which can differ from the long-term assumptions and predictions. Given that the tactical decisions are restricted by the strategic decisions, this difference may be problematic in terms of the service level and the corresponding personnel cost and satisfaction.

In a deterministic operating environment, the medium-term assumptions and predictions perfectly reflect the actual personnel demand and employee availability the organisation faces within the medium-term period. In a stochastic operating environment however, variability may cause the actual personnel demand and employee availability to differ from their assumed and predicted values. This uncertainty in demand and capacity distorts the information the personnel planner can employ. As such, it becomes more difficult for the personnel planner to take those tactical decisions that provide the desired service level at an optimised personnel cost and satisfaction.

The tactical decisions entail the construction of a baseline personnel shift roster for a medium-term period. This roster represents the employee schedules, which define the line of work for each employee. A line of work indicates for each day whether the employee has a day off or works a particular duty, i.e. a working assignment characterised by the corresponding (skill, day, shift)-combination. In Figure 1.2, we provide an example of a baseline personnel shift roster for a period of 1 week (Monday-Sunday). This roster consists of 5 single-skilled employees and indicates for each employee whether the employee receives a day off or needs to perform a duty, i.e. an early shift (S1), day shift (S2) or late shift (S3).



	Day 1			Day 2			Day 3			Day 4			Day 5			Day 6			Day 7		
	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
Employee 1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
Employee 2	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
Employee 3	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0
Employee 4	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0
Employee 5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0

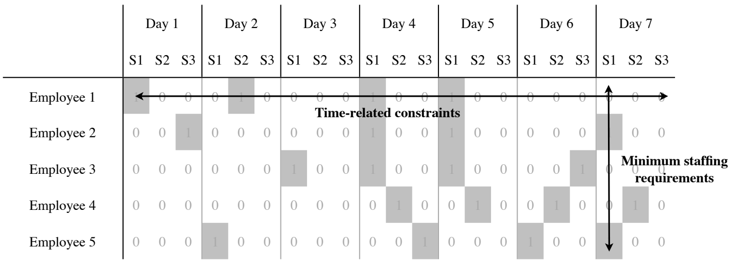
Figure 1.2 An example of a baseline personnel shift roster with three shifts (S1, S2, S3) per day

During the roster construction, the personnel planner needs to consider a number of constraints, which are imposed by the government and labour organisations or are the result of company-specific regulations (Burke et al., 2004; Van den Bergh et al., 2013). Typical examples of constraints imposed by the government or labour organisations are time-related constraints, e.g. the maximum number of hours an employee is allowed to work and the minimum number of hours an employee has to work during a specific period. These types of constraints are imposed within each employee schedule (Figure 1.3) and may, in general, not be violated. Company-specific regulations include the definition of minimum staffing requirements, which determine the number of skilled employees that need to be available during specific shifts within the medium-term period. This constraint is imposed over all employee schedules and ensures a specific service level to customers (Figure 1.3). Note that this type of constraint should not be violated but can be at the expense of a cost. If the minimum staffing requirements exceed the number of scheduled employees, the organisation needs to temporarily hire an external employee or reduce its service level. If the number of scheduled employees exceeds the minimum staffing requirements, the employees experience idle time and the organisation incurs overstaffing and an unnecessary cost.

Naturally, the tactical and strategic decisions can also be made concurrently (cf. Figure 1.1). If this is the case, the roster construction and the determination of the personnel mix and budget are performed simultaneously, which results in a better alignment between the strategic and tactical decisions (Maenhout and Vanhoucke, 2013).

The operational allocation phase

The outcome of the tactical scheduling phase represents the baseline personnel shift roster in the operational allocation phase. In this phase, the personnel planner monitors the execution of the



	Day 1			Day 2			Day 3			Day 4			Day 5			Day 6			Day 7		
	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
Employee 1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
Employee 2	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
Employee 3	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0
Employee 4	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0
Employee 5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0

Figure 1.3 Time-related constraints and minimum staffing requirements

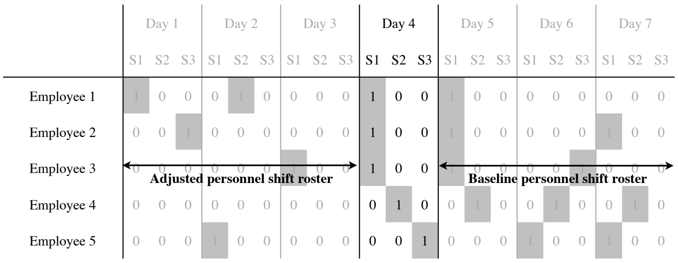
baseline personnel shift roster while accounting for the most recent information, which is subject to a small or no level of uncertainty. In this respect, the organisation receives up-to-date information at the start of day d concerning the personnel demand and employee availability on this day. Based on this information, the minimum staffing requirements and employee availabilities are adjusted. As such, the personnel planner obtains the actual values for the personnel demand and employee availability for day d. In a stochastic operating environment, these values may differ from their assumed and predicted values in the strategic staffing and tactical scheduling phases. These differences may affect the workability and/or feasibility of the baseline personnel shift roster, which provides an incentive to the personnel planner to adjust the personnel shift roster on day d (Bard and Purnomo, 2005a).

The hierarchical nature of the personnel planning process influences the number of available adjustment possibilities in the operational allocation phase. These adjustment possibilities include the allocation of overtime, re-assignments of employees and cancellations (Bard and Purnomo, 2005a; Shebalov and Klabjan, 2006). Nevertheless, the adjustment possibilities are certainly limited by the strategic and tactical decisions.

The strategic decisions determine the number of available (skilled) employees and the degree to which overtime can be allocated. A large number of hired employees with a high degree of cross-training offers more adjustment possibilities than a smaller number of hired employees with a lower degree of cross-training. Naturally, these additional adjustment possibilities require a higher personnel budget.

The tactical baseline personnel shift roster is reconsidered on a day-by-day basis in the operational allocation phase. Hence, we do not adjust the duties on preceding and succeeding days. Naturally, the duties on the previous days have finished and can no longer be adjusted. The duties on the following days could be adjusted in conjunction with the duties on the day under consideration. This could be useful to increase the number of adjustment possibilities by moving a duty, which is superfluous on top of the minimum staffing requirements on a succeeding day, to day d if the organisation experiences a shortage in the supply of employees. However, this also means that the last day in the baseline personnel shift roster could be adjusted each time a disruption occurs on a previous day. Moreover, it is possible that the baseline personnel shift roster needs to be adjusted again once up-to-date information is obtained during the last day.

This creates a high degree of uncertainty concerning the duties the employees are scheduled to perform on the following days and is therefore avoided. Additionally, we only possess up-to-date information for day d and focus on the operational phase and do not consider tactical rerostering. As such, the utilisation of a planning horizon of one day throughout the operational allocation phase ensures that the decision freedom is the same for each day d and does not decline as the number of remaining days in the personnel roster decreases. In this way, the baseline personnel roster gradually changes to reflect the actual execution. In Figure 1.4 for example, the duties on the days preceding day 4 represent the previously-made adjustments in the operational allocation phase. The duties succeeding day 4 reflect the duties that were originally scheduled in the baseline personnel shift roster. Irrespective of the gradual changes, it is important that the personnel roster remains workable in terms of the time-related constraints. Hence, we need to consider the duties that were adjusted in the previous days and the duties that were originally scheduled in the baseline personnel shift roster during the succeeding days. As such, the baseline personnel shift roster limits the adjustment possibilities that can be performed in the operational allocation phase.



	Day 1			Day 2			Day 3			Day 4			Day 5			Day 6			Day 7		
	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
Employee 1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
Employee 2	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
Employee 3	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0
Employee 4	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0
Employee 5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0

Figure 1.4 The assignments the personnel planner can adjust in the operational allocation phase on day 4

1.1.2 A definition of robustness in personnel scheduling

The actual personnel demand and employee availability the organisation faces is only known in the operational allocation phase. However, the organisation cannot postpone the strategic and tactical decisions until perfect information is available. Therefore, the decisions in the strategic staffing phase and the tactical scheduling phase are based on a number of assumptions and predictions. Given the stochastic nature of the organisation's operating environment in the operational allocation phase, these assumptions and predictions are not a perfect reflection of the actual personnel demand and employee availability. Moreover, the decision freedom in this short-term phase is highly dependent on the strategic and tactical decisions (cf. Section 1.1.1). Therefore, inappropriate decisions in the strategic staffing and tactical scheduling phases reduce the quality of the baseline personnel shift roster in the operational allocation phase. In this phase, an unexpected change in the personnel demand and/or employee availability can create understaffing or overstaffing, i.e.

- A larger-than-expected demand and/or the unexpected unavailability of employees leads to understaffing, which reduces the service level. The personnel planner should resolve this understaffing with a minimal impact on the personnel cost and satisfaction but the recovery options are limited by the strategic and tactical decisions, i.e. the personnel mix and budget and the baseline personnel shift roster.
- A smaller-than-expected demand results in overstaffing, which should be avoided because of the impact on the personnel cost and satisfaction. Overstaffing leads to idle employees who need to be paid their wages. In order to avoid this unnecessary cost, overstaffing can be resolved through cancellations. However, a cancellation affects the work-life balance of employees and can only be executed if the time-related constraints in the schedule of the corresponding employee remain satisfied. Hence, the strategic and tactical decisions should help to limit the overstaffing in the operational allocation phase.

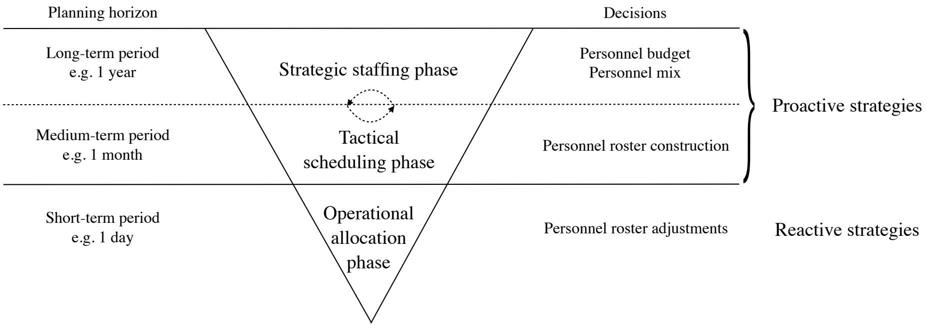


Figure 1.5 Robust strategies in the personnel planning process

It is clear that the organisation should protect the quality of the baseline personnel shift roster by anticipating the variability that causes the actual personnel demand and employee availability to differ from their assumed and predicted values. Hence, the personnel planner should consider robustness as a second objective in the strategic staffing and tactical scheduling phases in order to construct a high-quality baseline personnel shift roster that is a good representation of the actual personnel demand and employee availability. In this respect, a baseline personnel shift roster is robust if it is both *stable* (Dück et al., 2012) and *flexible* (Ionescu and Kliewer, 2011) when unexpected changes occur in the operational allocation phase.

Roster *stability* reflects the absorption capability of a baseline personnel shift roster, i.e. the capability to recover from disruptions without the intervention of the personnel planner. Absorption is possible if an additional number of skilled employees on top of the expected demand is scheduled to work a duty. These overstaffed employees constitute a capacity buffer such that a larger-than-expected demand can be absorbed as long as this additional demand does not exceed

the buffer size. Therefore, capacity buffers aim to ensure the quality of the baseline personnel shift roster in terms of the associated service level and personnel cost and satisfaction.
Roster *flexibility* refers to the degree to which the personnel planner can efficiently and effectively adjust the baseline personnel shift roster to ensure its quality in the operational allocation phase. More specifically, roster flexibility enables the personnel planner to execute those adjustments that have a minimal impact on the baseline personnel shift roster. Hence, a flexible baseline personnel shift roster entails a number of well-positioned adjustment possibilities such that each disruption can be resolved with a minimum number of adjustments that have a minimum impact on the service level and the associated personnel cost and satisfaction.

In order to achieve robustness in the personnel planning process, proactive strategies need to be applied in the strategic staffing phase and/or the tactical scheduling phase (Figure 1.5). However, hedging against all unexpected events in a proactive way would be very expensive. Therefore, reactive strategies in the operational allocation phase (cf. Figure 1.5) are indispensable to provide the opportunity to properly utilise the proactively built-in robustness in response to operational variability.

Proactive strategies
Proactive strategies aim to facilitate the anticipation of the operational variability and the occurrence of unexpected events such that the resulting disruptions can be efficiently and effectively resolved by absorption and/or adjustments. Hence, these strategies improve the *stability* and/or *flexibility* of baseline personnel shift rosters in the operational allocation phase.
Different strategies are mentioned in the literature to increase the robustness by focusing on the *stability* of a personnel roster. In the tactical scheduling phase, the strategic personnel mix and budget determine the extent to which the personnel demand can be covered but also the extent to which proactive strategies can be included. As such, an optimal size of additional staff to respond to uncertainty can be determined (Koutsopoulos and Wilson, 1987; Tan, 2003) in the strategic staffing phase. This additional staff can be utilised to facilitate the inclusion of time buffers or capacity buffers in the tactical scheduling phase. Time buffers can be used to deal with uncertain activity durations in project management (Hazir et al., 2010), unexpected delays of tasks in personnel task scheduling problems (Dück et al., 2012; Ehrgott and Ryan, 2002; Tam et al., 2011) and machine breakdowns in job shop scheduling (Davenport et al., 2001). Capacity buffers occur under different names, among which reserve crew and preferred requirements. In the tactical scheduling phase, the introduction of reserve duties as a means to improve robustness has been studied in the airline industry given that unforeseen demand (Dillon and Kontogiorgis, 1999), employee sickness (Dillon and Kontogiorgis, 1999; Moudani and Mora-Camino, 2010), technical failures (Rosenberger et al., 2002; Sohoni et al., 2006) and unexpected task delays due to adverse weather conditions or unscheduled maintenance (Sohoni et al., 2006) may lead to severe and costly delays. The definition of preferred staffing requirements is another way to introduce capacity buffers. These requirements are higher than the minimum staffing requirements and are typically applied in hospitals (De Causmaecker and Vanden Berghe, 2003; Dowsland and

Thompson, 2000; Topaloglu and Selim, 2010). Time and capacity buffers may also be combined to establish overtime during specific time periods. This overtime increases the capacity during these time periods and the total working time of employees over all time periods. Campbell (2012) distinguishes prescheduled fixed overtime and prescheduled on-call overtime. Moreover, apart from time or capacity buffers, it is also possible to improve the robustness by focusing on the teams that perform a sequence of tasks. Tam et al. (2014) study the airline crew scheduling problem and define the concept of unit crewing as keeping crew with different skills and ranks together for as long as possible in a pairing to minimise delay propagations due to uncertainty of arrival in the operational phase.

According to Li and Li (2000), the strategic personnel budget should consider the personnel mix and employee cross-training as the availability of cross-trained employees offers a certain leeway to respond to uncertainty. In this respect, swaps or the number of substitution possibilities in a tactical roster is a main indicator for the robustness in crew and aircraft scheduling (Dück et al., 2012) because it is an indication of the roster *flexibility.* A swap is the exchange of duties between employees while a substitution possibility entails the potential to change the assignment of one employee. Indeed, a crew and/or aircraft swap is one of the available recovery actions to overcome operational disruptions (Abdelghany et al., 2004, 2008; Eggenberg et al., 2010; Gao et al., 2009; Ionescu and Kliewer, 2011). Shebalov and Klabjan (2006) focus on the maximisation of move-up crews. Move-up crews are crews that can be swapped to overcome operational disruptions such as airport shutdowns. A crew swap is recognised as a cost-efficient option when disruptions occur. Ionescu and Kliewer (2011) also introduce swaps in the scheduling phase to improve the flexibility of the original roster to respond to delays caused by weather and aircraft breakdowns. The authors propose a stochastic optimisation model for which the recourse function represents the benefits of swaps. In contrast to Shebalov and Klabjan (2006), the focus is more on the utility of a swap in the time-space network rather than a pure maximisation of the number of swaps. This utility depends on the likelihood of delay propagations. Substitution possibilities can be obtained through cross-training. Campbell (1999) investigates the impact of the cross-utilisation of employees for different levels of demand variability and employee cross-training. Similarly, Olivella and Nembhard (2016) determine the optimal level of cross-training in work teams to deal with variations in the demand mix and employee availability.

Reactive strategies

Reactive strategies enable the personnel planner to recover the baseline personnel roster when disruptions occur in the operational allocation phase. As a reaction to schedule disruptions, different studies adapt the original roster to minimise the resource shortages (e.g. Bard and Purnomo (2005a); Koutsopoulos and Wilson (1987); Shebalov and Klabjan (2006); Stojkovic et al. (1998); Veelenturf et al. (2016)) and/or the schedule delays of the disrupted tasks (e.g. De Bruecker et al. (2014); Dück et al. (2012); Ehrgott and Ryan (2002); Hazir et al. (2010); Ionescu et al. (2011); Tam et al. (2011)). Additionally, almost all of these studies minimise the schedule changes as a result of the propagation of these disruptions to other tasks. At the same time, the resources must be used in an efficient way as the operational costs are minimised (Bard and Purnomo, 2005a; Dück et al., 2012; Hazir et al., 2010; Ionescu et al., 2011; Koutsopoulos and Wilson, 1987; Shebalov and Klabjan, 2006; Stojkovic et al., 1998; Tam et al., 2011; Veelenturf et al., 2016).

Hence, reactive strategies need to help the personnel planner to exploit the proactively built-in mechanisms. Different studies in the literature propose a reactive decision support model, which is determined by the available options to match supply and demand, e.g. reserve duties, schedule changes and overtime. The conversion of a reserve duty into a working duty is a general recovery action in the airline industry (Abdelghany et al., 2004, 2008; Thiel, 2005). Schedule changes include the execution of swaps or substitution possibilities, which may comprise conversions of a day-off assignment into a working assignment. However, the latter should be utilised with care because of the negative impact on the personal lives of employees (Bard and Purnomo, 2005a; Camden et al., 2011; Schalk and Van Rijckevorsel, 2007). These reactive strategies may be enhanced by allowing overtime to extend the daily working time and the total working time over the complete planning horizon (Easton and Rossin, 1997). An overtime extension of a standard duty prolongs the daily working time, which may simultaneously add overtime to the total working time of the affected employee. Moreover, the total working time can be extended by assigning overtime shift duties on top of the regular-time shift duties.

1.2 Research contribution and outline

In this book, we consider a personnel planning process that is stochastic in terms of the personnel demand and employee availability. We investigate a general personnel shift scheduling problem, which entails the assignment of employees to a duty, i.e. an early, late or night shift, or to a day off for each day in the planning horizon (cf. Figure 1.2). The problem under study is characterised by the following components, i.e.

- Objectives: the objectives consider general objectives in a personnel scheduling context such as wage costs (personnel cost), personnel preferences (personnel satisfaction) and understaffing costs (service level).
- Constraints: the number of constraints is limited to
  - The staffing requirements: This vertical constraint determines that each duty should be staffed by a required number of personnel members.
  - Time related constraints: These horizontal constraints are imposed within an employee schedule. We distinguish sequence, counter and series constraints (Bilgin et al., 2012), i.e.
    - Counter constraints: each employee is allowed to work one duty per day and a minimum and maximum number of hours over the total planning horizon.
    - Sequence constraints: each employee needs to receive a minimum rest period between consecutive duties.

- ∗ Series constraints: each employee can work a maximum number of consecutive duties.

The literature reviews of Ernst et al. (2004b) and Van den Bergh et al. (2013) indicate that a problem with these characteristics is a general personnel shift scheduling problem as these objectives and constraints are valid for different applications in many problem areas.
Note that the personnel shift scheduling problem is different than the more general personnel task scheduling problem (Krishnamoorthy and Ernst, 2001). In the personnel task scheduling problem, different individual tasks are paired to compose one singular duty or shift. Specific applications of the personnel task scheduling problem include the airline and railway crew scheduling problems. For these problems, a multitude of proactive and reactive strategies are already available in the literature (cf. supra).

In order to manage the stochastic personnel demand and employee availability in a personnel shift scheduling context, we aim to model robustness to enable the creation of a decision support system that optimises robustness. As such, we describe four studies that we position based on the distinction between proactive and reactive strategies in Figure 1.6. These studies were designed to contribute to the academic literature by answering the following research questions, i.e.

RQ1 How can we define and test the robustness of a personnel shift roster in the personnel planning process?
In Chapter 2, we establish a three-step methodology to imitate the personnel planning process and to determine the robustness of a baseline personnel shift roster. In the first step, we imitate the tactical scheduling phase and construct a baseline personnel shift roster for a medium-term period. In the second step, we imitate the operational allocation phase and subject the baseline personnel shift roster to a repeated day-by-day simulation and adjustment component. For each day in the baseline personnel shift roster, we simulate the operational variability and, if necessary, perform adjustments to the baseline personnel shift roster. In the third step, we evaluate the planned and actual performance of the baseline personnel shift roster. The planned performance reflects the quality of the baseline personnel shift roster in the tactical scheduling phase, while the actual performance expresses the quality of the baseline personnel shift roster after the the day-by-day operational allocation phase.
We evaluate the planned and actual performance to express the robustness of a personnel shift roster in each chapter. In this respect, we perform the three-step methodology and apply different proactive and/or reactive strategies in the first and second step. The corresponding robustness can be evaluated in the third step trough a comparison of the planned and actual performance of these strategies. Note that the first step can be expanded to represent the integrated strategic staffing and tactical scheduling phase.

RQ2 What is the impact of proactive strategies in the strategic staffing and tactical scheduling phases on the robustness of baseline personnel shift rosters?

In each chapter, we define a different proactive strategy to improve the stability and/or flexibility of a baseline personnel shift roster in the operational allocation phase, i.e.

- We apply different strategies to enhance the stability of the baseline personnel shift roster in Chapters 2 and 3, i.e.
    - In Chapter 2, we aim to increase the robustness of a personnel shift roster by providing capacity buffers in the tactical scheduling phase. In this phase, we apply different strategies that aim to introduce reserve duties on top of the working duties in the baseline personnel shift roster. In this respect, we propose a number of strategies that differ based on the imposed reserve duty staffing requirements and reserve duty time-related constraints.
    - In Chapter 3, we employ robust optimisation principles in the tactical scheduling phase. Robust optimisation is based on the solution of a robust counterpart, i.e. a deterministic but worst-case formulation of a personnel shift scheduling problem that is uncertain in terms of the personnel demand and employee availability. This formulation is based on an uncertainty set that embeds the values these parameters may obtain. For this uncertainty set, we define strategies to determine its underlying characteristics. Given these strategies, we aim to introduce capacity buffers in the baseline personnel shift roster.
- In Chapter 4, we focus on the flexibility of the baseline personnel shift roster in the operational allocation phase as follows, i.e.
    - In the strategic staffing phase, we define a heterogenous workforce with a (fluctuating) number of single- and cross-trained employees. In order to investigate the impact of varying degrees of cross-training of the workforce, we distinguish different skill possession settings. These settings characterise the number and type of skill each employee possesses.
    - In the tactical scheduling phase, we investigate how a proactive maximisation of the value of employee substitutability may improve the adjustment capability of the constructed baseline personnel shift roster in the operational allocation phase. In this respect, we compare two strategies to model employee substitutability, i.e. on the level of the individual employee and on the group level.
- We consider roster stability and flexibility in Chapter 5. In this chapter, we define scheduled overtime strategies to introduce a combination of capacity and time buffers. These strategies involve the assignment of employees to overtime during a shift extension or a complete shift in the tactical scheduling phase. These shift extensions increase the daily working time of employees, which negatively impacts the number of duties these employees can perform over the total planning horizon. In this respect, we investigate the trade-off between the workforce size and scheduling overtime in the tactical scheduling phase. As such, we integrate the strategic staffing and tactical scheduling phases to simultaneously determine the workforce and construct the medium-term baseline personnel shift roster with overtime considerations.

RQ3 What is the impact of reactive strategies in the operational allocation phase on the robustness of baseline personnel shift rosters?

We utilise the following reactive strategies to exploit and facilitate the roster stability and/or flexibility in the operational allocation phase, i.e. conversions, reassignments/substitutions, cancellations and unscheduled overtime strategies.

- In Chapter 2, we convert the tactically scheduled reserve duties into working duties. This conversion can pertain to the same shift or to differing shifts, i.e. the conversion corresponds to the same shift or the conversion occurs from one shift to another. Moreover, we can increase the operational flexibility by allowing reassignments of working duties between shifts (Chapters 2 - 5) and skills (Chapter 4), reassignments of a day off to a working duty (Chapters 3-5) and cancellations of reserve/working duties (Chapters 2 - 5).
- Unscheduled overtime strategies are applied in Chapter 5 to extend the daily and total working time. However, the flexibility to reactively allocate unscheduled overtime in the operational allocation phase depends on the proactively scheduled overtime in the tactical scheduling phase. Hence, we investigate the trade-off between proactively scheduling overtime in the tactical scheduling phase and reactively allocating overtime in the operational allocation phase.

The answers to these research questions provide the following contributions to the academic literature, i.e.

- The contribution of Chapter 2 is fourfold. First, we propose a general three-phase method that combines proactive and reactive scheduling procedures to assess the robustness of a personnel shift roster. Second, we formulate a tactical scheduling model and an operational decision model that only uses internal resources with a specific focus to employ reserve duties to resolve schedule disruptions. Third, we evaluate the actual performance and the planned performance of different proactive strategies to schedule reserve duties. Fourth, insight is acquired in how the strategies introduce some robustness and we formulate managerial guidelines to cope with uncertainty of demand and/or capacity for a general personnel shift scheduling problem.
- The contribution of Chapter 3 is threefold. First, we explicitly utilise the principles of robust optimisation to formulate the deterministic version of a stochastic personnel shift scheduling problem based on the definition of the characteristics of an uncertainty set. Second, we tailor this uncertainty set to our specific problem such that a sufficient degree of uncertainty is introduced during appropriate (shift, day)-combinations in the baseline personnel shift roster constructed in the tactical scheduling phase. Third, we utilise appropriate measures to analyse the robustness of the proposed strategies and compare their performance to the minimum cost scheduling strategy. Based on this analysis, we provide managerial guidelines that are applicable in many problem settings. These guidelines can be utilised by the personnel planner to construct a baseline personnel shift roster in the

tactical scheduling phase that is a good approximation of the uncertainty in the operational allocation phase.

- The contribution of Chapter 4 consists of the definition of a specific terminology to address different types of substitutability. Moreover, we compose a new mathematical formulation to model substitutability of employees on the individual level and on the group level, which we compare to the minimum cost formulation. Finally, we propose and formulate practical guidelines to increase the robustness in relation to the problem characteristics (degree of cross-training and demand profile). In this context, we identify the best proxy included in the model formulation for the construction of a medium-term personnel shift roster to mimic the short-term operating environment.

- The contribution of Chapter 5 consists of the definition of time buffer and hiring strategies. In this respect, we investigate the trade-off between proactively scheduling overtime such that a lower number of employees can be hired and reactively allocating overtime to resolve disruptions. These strategies are mathematically formulated in an integrated staffing and scheduling model and an operational allocation model. Moreover, we compare the planned and actual performance of the formulated strategies to provide managerial guidelines about their impact on the robustness of a personnel shift roster.

Given these contributions, we are able to highlight those strategies that contribute to robustness in the personnel planning process in Chapter 6. In this chapter, conclusions are drawn about the applicability of proactive and reactive strategies and directions for future research are provided.

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NationaleBank
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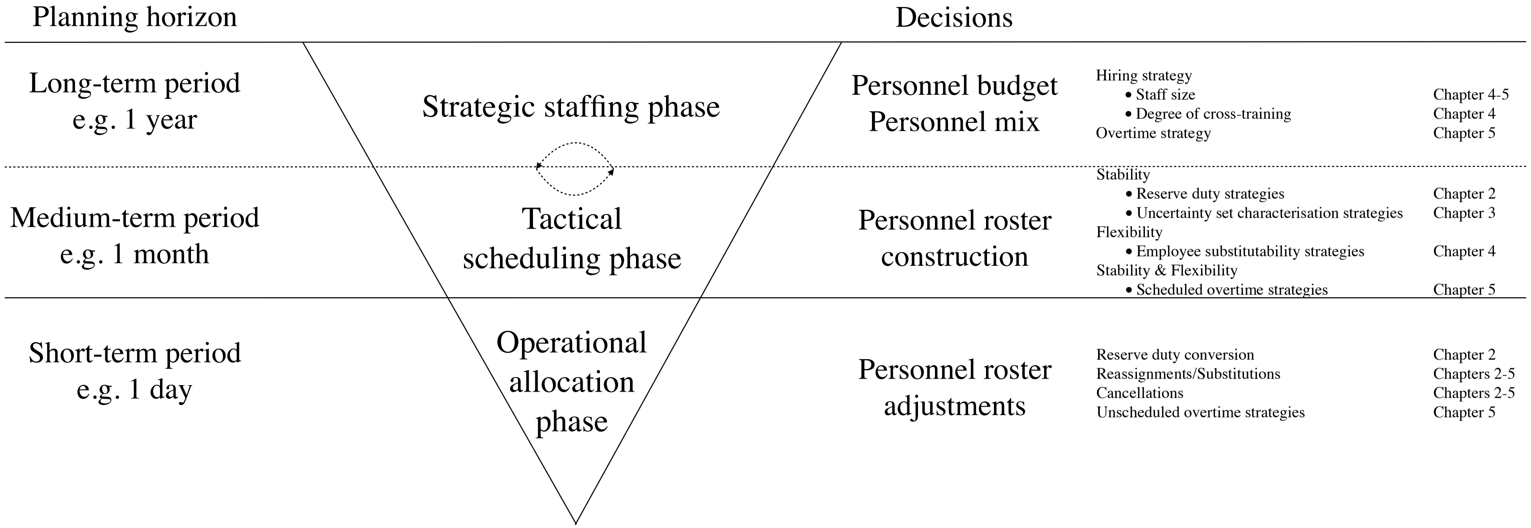


Figure 1.6 Overview figure

Publications in international journals

- Ingels, J. and Maenhout, B. (2015). The impact of reserve duties on the robustness of a personnel shift roster: An empirical investigation. Computers & Operations Research, 61, 153-169.
- Ingels, J. and Maenhout, B. (2017). Employee substitutability as a tool to improve the robustness in personnel scheduling. *Accepted for publication in OR Spectrum. DOI: 10.1007/s00291-017-0476-0.*
- Ingels, J. and Maenhout, B. (2017). The impact of overtime as a time-based proactive scheduling and reactive allocation strategy on the robustness of a personnel shift roster. *Accepted for publication in Journal of Scheduling. DOI: 10.1007/s10951-017-0512-6.*

Unpublished working paper

- Ingels, J. and Maenhout, B. The impact of the defining characteristics of uncertainty sets in robust optimisation: an empirical investigation of a personnel shift scheduling problem. *Under revision in Omega (second round).*

2

The impact of reserve duties on the robustness of a personnel shift roster

In personnel scheduling, a duty roster is typically constructed under the assumption of a deterministic model. However, organisations operate in a stochastic environment and the conjectures made about the personnel demand and the availability of employees may prove to be insufficient to represent reality. In order to anticipate these unexpected events, we investigate the impact of introducing reserve duties in the medium-term personnel shift roster. Reserve duties are scheduled by enforcing specific staffing requirements and/or time-related constraints that consider reserve duties only. We propose a three-step methodology that imitates the personnel planning process to evaluate the delivered robustness in terms of the actual performance. After a personnel roster is constructed, unexpected events are simulated and an optimisation model determines the required adjustments to balance supply and demand. Detailed computational experiments are presented to provide managerial insights into different strategies to schedule reserve duties.

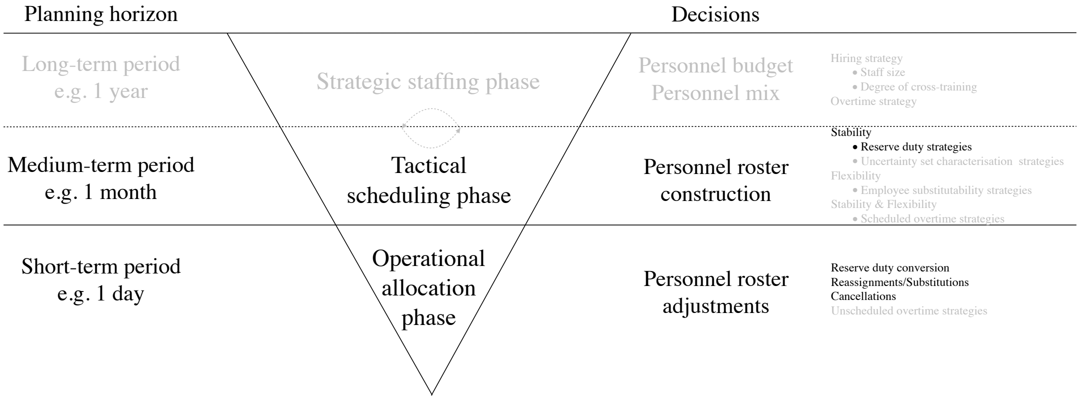


Figure 2.1 The research focus in Chapter 2

2.1 Introduction

In this chapter, we aim to increase the robustness of a personnel shift roster by proactively scheduling reserve duties when this baseline roster is constructed for the medium term. We compare different strategies for scheduling reserve duties and evaluate these strategies on a day-to-day basis by means of a simulation and reactive re-scheduling component. Starting from the baseline roster, we simulate the uncertainty of demand and capacity for every day. If necessary, a reactive optimisation and adjustment mechanism converts these reserve duties into working duties on a daily basis and/or adapts the baseline personnel shift roster in order to restore its workability and/or feasibility. In this way, the roster is revised for every day of the planning horizon and afterwards we evaluate the incorporated robustness based on the planned performance indicated by the tactical shift roster and the actual performance indicated by the operational shift roster.

The remainder of the chapter is organised as follows. In Section 2.2, we give an overview of the relevant literature dealing with the concept of reserve duties in personnel scheduling. In Section 2.3, we define and formulate the problem under study. We discuss the underlying personnel shift scheduling problem and formulate the tactical and operational daily adjustment models mathematically. The research methodology is explained in Section 2.4. The test design and computational experiments are discussed in Section 2.5. In Section 2.6, conclusions are drawn.

2.2 Literature review

The planning of reserve duties in the scheduling phase has mostly been studied in the airline industry, i.e. a task-scheduling context (Dillon and Kontogiorgis, 1999; Moudani and Mora-Camino, 2010; Sohoni et al., 2006). Dillon and Kontogiorgis (1999) developed an automated system to construct optimal schedules for the reserve crew given unforeseen demand and employee sickness in a real-life environment. They indicate that sufficient reserve duties have to be available during days when the foreseen and unforeseen demand is high, during weekends and holidays and by the end of the month. Moudani and Mora-Camino (2010) focus on a dynamic programming approach to efficiently solve the crew reserve assignment problem in response to employee sickness. They indicate that the number of reserve duties should on the one hand be sufficient to prevent disruptions but on the other hand this number should not be too high in order to ensure the availability of crew for other activities. Sohoni et al. (2006) developed a two-stage strategy to optimise the efficient utilisation of the available reserve crew in case of adverse weather conditions, and technical failures and unscheduled maintenance of planes. In the first stage, the demand for reserve duties is estimated using simulation. In the second stage, patterns that cover the reserve duty requirements are generated. Potthoff et al. (2010) study the operational railway crew rescheduling problem when a disruption occurs. They conclude that reserve duties help to cover tasks and limit the number of adjustments but the position of these duties (in time and space) has a major impact.

2.3 Problem definition and formulation

As disruptions compromise the workability of the baseline personnel shift roster, we try to improve the roster robustness by assigning reserve duties to the employees on particular days and shifts, which embodies a proactive mechanism in order to protect the roster against uncertainty. The robustness of the baseline personnel shift roster is then evaluated in the operational phase where different sources of uncertainty arise on the day of operation. As a response to unbalances in supply and demand, a reactive mechanism tries to minimise the number of shortages and changes to the baseline roster and converts reserve duties into working duties. It should be clear that our study comprehends two main phases, i.e. the tactical scheduling phase and the operational allocation phase, which are elaborated in sections 2.3.1 and 2.3.2 respectively.

2.3.1 Scheduling phase

In order to obtain general results, we study a general personnel shift scheduling problem that assigns the employees to working duties or reserve duties. The shift characteristics, the personnel information, the objectives and the constraints are all general and common in the literature of personnel shift scheduling problems. The main difference with other personnel scheduling models (Burke et al., 2004; Van den Bergh et al., 2013) is that specific staffing requirements and time-related constraints are stipulated to schedule the reserve duties. We assume that all input is known and deterministic. The model can be categorised as $AS\,1|RV\,|S||LRG$ according to the classification of De Causmaecker and Vanden Berghe (2011) and is formulated as follows, i.e.

Notation

Sets

N	set of employees (index i)
D	set of days (index d)
S	set of shifts (index j)
$T^{''}_{dj}$	set of shifts that cannot be assigned the day after day d and shift assignment j (index s)

Parameters

R^{w}_{dj}	working duty staffing requirements for shift j and day d
R^{r}_{dj}	reserve duty staffing requirements for shift j and day d
c^{w}_{idj}	wage cost of assigning a working duty during shift j on day d to employee i
c^{r}_{idj}	wage cost of assigning a reserve duty during shift j on day d to employee i
c^{wu}_{dj}	cost of understaffing working duties during shift j on day d
c^{ru}_{dj}	cost of understaffing reserve duties during shift j on day d
p_{idj}	preference penalty cost for a working or reserve duty during shift j on day d for employee i
l_j	duration of shift j
$\eta^{wr,min}_{i}$	minimum number of hours that need to be assigned to employee i
$\eta^{wr,max}_{i}$	maximum number of hours that can be assigned to employee i
$\eta^{r,min}_{i}$	minimum number of reserve duty hours that need to be assigned to employee i
$\eta^{r,max}_{i}$	maximum number of reserve duty hours that can be assigned to employee i
$\theta^{wr,max}_{i}$	maximum number of consecutive working assignments for employee i

Variables

$x^{'w}_{idj}$	1 if employee i receives a working duty during shift j on day d, 0 otherwise
$x^{'r}_{idj}$	1 if employee i receives a reserve duty during shift j on day d, 0 otherwise
$x^{'wu}_{dj}$	the shortage of working duties for shift j and day d
$x^{'ru}_{dj}$	the shortage of reserve duties for shift j and day d

Mathematical formulation

$$\min \quad \sum_{i\in N}\sum_{d\in D}\sum_{j\in S}((c^{w}_{idj}+p_{idj})x^{'w}_{idj}+(c^{r}_{idj}+p_{idj})x^{'r}_{idj})+\sum_{d\in D}\sum_{j\in S}(c^{wu}_{dj}x^{'wu}_{dj}+c^{ru}_{dj}x^{'ru}_{dj}) \tag{2.1}$$

Objective function (2.1) considers multiple objectives, i.e. minimising the wage cost, maximising personnel satisfaction and ensuring the staffing requirements as well as possible. Wage costs are incurred when an employee is assigned to a working or a reserve duty on a specific day and shift. The personnel satisfaction is quantified using day-shift preference penalty costs, regardless of the type of duty. These preference penalty costs express the aversion of an employee to work a specific shift on a specific day. As such, a lower preference penalty cost indicates a higher willingness to be assigned to that shift on that day (Bard and Purnomo, 2005a; Maenhout and Vanhoucke, 2010). The preference penalty cost for a day off is equal to zero. The sum of these costs over all days and shifts is minimised. Furthermore, ensuring the staffing requirements is pursued as the understaffing is minimised for each shift on every day. Note that the overstaffing for a shift is also minimised in the objective function since wage costs are associated with each duty.

$$\sum_{i\in N} x^{'w}_{idj}+x^{'wu}_{dj}\geq R^{w}_{dj} \qquad \forall d\in D,\forall j\in S \tag{2.2}$$

$$\sum_{i\in N} x^{'r}_{idj}+x^{'ru}_{dj}\geq R^{r}_{dj} \qquad \forall d\in D,\forall j\in S \tag{2.3}$$

The staffing requirements define the number of employees that is required for each shift on each day. As personnel members can be assigned to reserve duties, we additionally define staffing requirements for reserve duties (cf. eq. (2.3)) on top of the staffing requirements for regular working duties (cf. eq. (2.2)). Both these constraints are relaxed as they allow shortages by the definition of slack variables.

$$\sum_{j \in S} (x^{'w}_{idj} + x^{'r}_{idj}) \leq 1 \qquad \forall i \in N, \forall d \in D \tag{2.4}$$

$$(x^{'w}_{idj} + x^{'r}_{idj}) + \sum_{s \in T^{''}_{dj}} (x^{'w}_{i(d+1)s} + x^{'r}_{i(d+1)s}) \leq 1 \qquad \forall i \in N, \forall d \in D, \forall j \in S \tag{2.5}$$

$$\sum_{d \in D} \sum_{j \in S} l_j (x^{'w}_{idj} + x^{'r}_{idj}) \leq \eta_i^{wr,max} \qquad \forall i \in N \tag{2.6}$$

$$\sum_{d \in D} \sum_{j \in S} l_j (x^{'w}_{idj} + x^{'r}_{idj}) \geq \eta_i^{wr,min} \qquad \forall i \in N \tag{2.7}$$

$$\sum_{d'=d}^{d+\theta_i^{wr,max}} \sum_{j \in S} (x^{'w}_{id'j} + x^{'r}_{id'j}) \leq \theta_i^{wr,max} \qquad \forall i \in N, \forall d \in D \tag{2.8}$$

$$\sum_{d \in D} \sum_{j \in S} l_j x^{'r}_{idj} \leq \eta_i^{r,max} \qquad \forall i \in N \tag{2.9}$$

$$\sum_{d \in D} \sum_{j \in S} l_j x^{'r}_{idj} \geq \eta_i^{r,min} \qquad \forall i \in N \tag{2.10}$$

The time-related constraints are imposed on the line-of-work of a single employee. In this study, we incorporate different counter, sequence and series constraints. An employee can only be assigned to one duty per day (eq. (2.4)) and a minimum rest period between duties is imposed (eq. (2.5)). Other constraints are considered in the form of the maximum (eq. (2.6)) and minimum (eq. (2.7)) number of hours a personnel member can be assigned to. Moreover, we restrict the maximum number of consecutive duties (eq. (2.8)). All these constraints take all duties into account, i.e. both the working duties and the reserve duties. In order to introduce reserve duties as a proactive policy in the tactical scheduling phase, we explicitly add constraints that restrict the maximum and minimum number of reserve hours an individual employee is assigned to (cf. eqs. (2.9) and (2.10) respectively).

$$\begin{aligned} x^{'w}_{idj} &\in \{0,1\} && \forall i \in N, \forall d \in D, \forall j \in S \\ x^{'r}_{idj} &\in \{0,1\} && \forall i \in N, \forall d \in D, \forall j \in S \\ x^{'wu}_{dj} &\geq 0 && \forall d \in D, \forall j \in S \\ x^{'ru}_{dj} &\geq 0 && \forall d \in D, \forall j \in S \end{aligned} \tag{2.11}$$

The integrality conditions on the decision variables are stated in equations (2.11).

2.3.2 Allocation phase

In the operational phase, the deterministic personnel roster is subject to operational variability with respect to the demand for personnel and unexpected absenteeism of employees planned to perform a particular duty. We assume that a more accurate estimation of the personnel demand and unavailability as a result of the operational variability can be ascertained on the day of operation. We consider a short-term period with a length of one day. As a response to this variability, decisions need to be taken to balance demand and supply. The baseline personnel shift roster needs to be adapted in the short term to take the new demand and availabilities into account and/or to restore feasibility. In this study, the potential adjustments include the conversion of a reserve duty into a working duty or vice versa, reassignments to other shifts and duty cancellations. As the planning period in this allocation phase is much smaller than the planning period in the scheduling phase, we have to optimise the operational decision model multiple times, once for each day. In contrast to previous models in literature, the proposed operational model tries to resolve the disrupted roster exclusively using internal resources by changing the work duty assignments and/or employing reserve duty assignments. The symbols and the underlying mathematical model for one single day are formulated below.

Notation

Sets

N	set of employees (index i)
S	set of shifts (index j)

General parameters

d	day under consideration in the operational planning horizon
c^{w}_{idj}	wage cost of assigning a working duty during shift j on day d to employee i
c^{r}_{idj}	wage cost of assigning a reserve duty during shift j on day d to employee i
c^{wu}_{dj}	cost of understaffing working duties during shift j on day d
p_{idj}	preference penalty cost for a working or reserve duty during shift j on day d for employee i
l_j	duration of shift j
κ^{α}_{idj}	1 if employee i is allowed to receive an assignment during shift j on day d, 0 otherwise
κ^{f}_{id}	the total number of hours employee i has to receive on day d

Simulation parameters

a_{id}	1 if employee i is available on day d, 0 otherwise
R'^{w}_{dj}	simulated working duty staffing requirements for shift j and day d

Roster change parameters

x'^{w}_{idj}	1 if employee i received a working duty during shift j on day d in the baseline roster, 0 otherwise
x'^{r}_{idj}	1 if employee i received a reserve duty during shift j on day d in the baseline roster, 0 otherwise
$c^{w\delta}_{idj}$	roster change cost for assigning a working duty during shift j on day d to employee i with $c^{w\delta}_{idj} > 0$ if $(x'^{w}_{idj} + x'^{r}_{idj}) = 0$ $c^{w\delta}_{idj} = 0$ otherwise
$c^{r\delta}_{idj}$	roster change cost for assigning a reserve duty during shift j on day d to employee i with $c^{r\delta}_{idj} > 0$ if $x'^{r}_{idj} = 0$ $c^{r\delta}_{idj} = 0$ otherwise
c^{v}_{id}	duty cancellation cost for employee i on day d with $c^{v}_{id} > 0$ if $\sum_{j \in S}(x'^{w}_{idj} + x'^{r}_{idj}) = 1$ and $a_{id} = 1$ $c^{v}_{id} = 0$ otherwise

Variables

x^{w}_{idj}	1 if employee i receives a working duty during shift j on day d, 0 otherwise
x^{r}_{idj}	1 if employee i receives a reserve duty during shift j on day d, 0 otherwise
x^{v}_{id}	1 if employee i receives a day off on day d, 0 otherwise
x^{wu}_{dj}	the shortage of working duties for shift j and day d

Mathematical formulation

$$\begin{aligned}\min \quad & \sum_{i \in N} \sum_{j \in S} ((c^{w}_{idj} + c^{w\delta}_{idj} + p_{idj}) x^{w}_{idj} + (c^{r}_{idj} + c^{r\delta}_{idj} + p_{idj}) x^{r}_{idj}) \\ & \qquad + \sum_{i \in N} c^{v}_{id} x^{v}_{id} + \sum_{j \in S} c^{wu}_{dj} x^{wu}_{dj} \end{aligned} \tag{2.12}$$

In the allocation phase, the same objectives are optimised as in the scheduling phase. Objective (2.12) minimises the assignment cost, preference penalty cost and the understaffing of the staffing requirements. However, the assignment cost does not only consist of a wage cost c^{w}_{idj} (c^{r}_{idj}) but also includes a change cost $c^{w\delta}_{idj}$ ($c^{r\delta}_{idj}$) if an employee is assigned to another shift than he was assigned to in the baseline roster (cf. x'^{w}_{idj} and x'^{r}_{idj}). A duty cancellation cost c^{v}_{id} is also incorporated when duties, assigned in the tactical phase, are cancelled in the operational phase and the employee has a day off. As the duty cancellation cost c^{v}_{id} is standard lower than the duty wage cost (c^{w}_{idj} or c^{r}_{idj}), duties are preferably cancelled. Note that the personnel members can still be assigned to a working or reserve duty in order to retain the feasibility of the line-of-work of an individual employee and to have a correct idea of the wage cost.

$$\sum_{i \in N} x^{w}_{idj} + x^{wu}_{dj} \geq R'^{w}_{dj} \qquad \forall j \in S \tag{2.13}$$

Equation (2.13) denotes the operational staffing requirements. These constraints stipulate that the scheduled number of employees is at least equal to the demand R'^{w}_{dj}, which is the modified demand for personnel as a result of the operational variability. When the assigned number of employees is smaller than the staffing requirements, a shortage will occur denoted by the variable x^{wu}_{dj}. Note that in the allocation phase there are no explicit staffing requirements for reserve duties.

$$\sum_{j \in S} (x^{w}_{idj} + x^{r}_{idj}) \leq a_{id} \qquad \forall i \in N \tag{2.14}$$

In the operational phase, employees can become unavailable at the time of their original assignment as a result of uncertainty of capacity. This availability is represented by the parameter a_{id}, which is equal to 1 if employee i is available to work on day d.

$$\sum_{j \in S} (x^{w}_{idj} + x^{r}_{idj}) + x^{v}_{id} = 1 \qquad \forall i \in N \tag{2.15}$$

Equation (2.15) stipulates that an employee is assigned to a duty or to a day off on day d. In case employee i is assigned to a duty in the tactical scheduling phase and he is available to work on day d, his duty is cancelled if $x^v_{id} = 1$.

$$(x^w_{idj} + x^r_{idj}) \leq \kappa^\alpha_{idj} \qquad \forall i \in N, \forall j \in S \tag{2.16}$$

Equation (2.16) imposes limitations in the shifts an employee can be assigned to as a result of the schedule of the employee. The parameter κ^α_{idj} is equal to 1 if employee i is allowed to be reassigned to shift j on day d if this does not violate the time-related constraints given the duties of the employee on the other days.

$$\sum_{j \in S} l_j (x^w_{idj} + x^r_{idj}) \geq \kappa^f_{id} a_{id} \qquad \forall i \in N \tag{2.17}$$

Equation (2.17) tries to assign an employee to a duty in an attempt to avoid cancellations of duties that violate the time-related constraints. This is reflected by the parameter κ^f_{id}. However, as a result of the uncertainty of capacity, it is possible that the employee is not available, which explains that the right-hand side is multiplied with a_{id} to ensure the feasibility of this allocation model. Note that whenever a scheduled employee is not available to work, the minimum number of hours for this employee is modified with this unavailability. Hence, the employee does not need to catch up a missed working assignment in case of an unavailability.

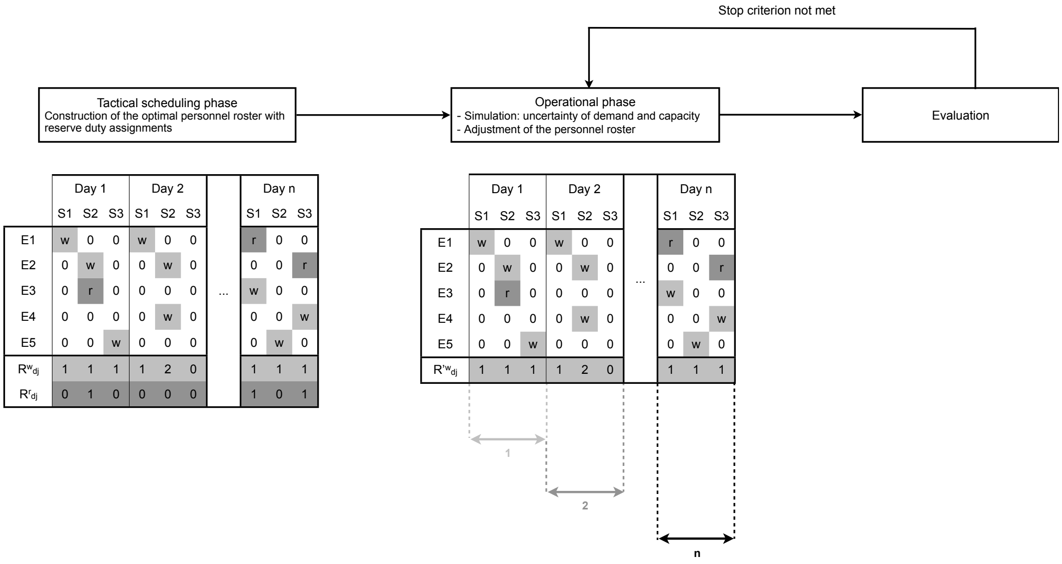
$$\begin{aligned} x^w_{idj} &\in \{0,1\} & \forall i \in N, \forall j \in S \\ x^r_{idj} &\in \{0,1\} & \forall i \in N, \forall j \in S \\ x^v_{id} &\in \{0,1\} & \forall i \in N \\ x^{wu}_{dj} &\geq 0 & \forall j \in S \end{aligned} \tag{2.18}$$

Constraints (2.18) embody the integrality conditions.

2.4 Methodology

As we want to determine the robustness of a personnel roster through simulation, we imitate the personnel planning process in our solution methodology, which encompasses three steps. First, we construct the baseline personnel shift roster in the tactical scheduling phase (Section 2.4.1). In this step, we compose a roster using a branch-and-price algorithm by assigning a working duty, a reserve duty or a day-off on each day to every employee. In this step, the reserve duties are planned according to a pre-defined reserve duty scheduling strategy. Second, we use these baseline rosters as input to the operational phase, which consists of two main components, i.e. the simulation of the occurrence of unexpected events on discrete points in time through a Monte Carlo simulation of uncertainty of capacity and demand, and the adjustment of the personnel roster (Section 2.4.2). The period under consideration in this allocation step is one day, which implies that we repeatedly simulate the uncertainty of demand and capacity on a day-to-day basis until the end of the tactical roster. Based on the simulation output of a particular day, adjustments to the baseline roster may be required. These adjustments entail the conversion of reserve duties into working duties or vice versa, reassignments to other shifts and duty cancellations. Third, we evaluate the adjusted roster by assessing the actual performance in the operational decision phase, which consists out of the actual effectivity and the actual efficiency. This actual performance is compared to the planned performance resulting from the tactical scheduling phase (Section 2.4.3). As we like to have a general idea about the robustness of a personnel roster, we perform multiple simulations and reiterate the operational allocation phase and evaluation step until a stop criterion is met. Our approach has some similarities with the methodologies of Abdelghany et al. (2008) and Bard and Purnomo (2005a) as these papers integrate daily simulations and roster adjustments to determine the roster robustness.

Figure 2.2 gives a conceptual overview of our methodology. The initial roster consists of n days and 5 employees who either receive a working duty (denoted by 'w'), reserve duty (denoted by 'r') or a day-off. Employees receive these assignments to satisfy the staffing requirements for working duties (R^w_{dj}) and reserve duties (R^r_{dj}) and the different time-related constraints. Figure 2.2 also shows the absence of reserve duty staffing requirements during the operational phase. In the remainder of this section, we discuss the individual steps of our methodology in more detail.



	Day 1			Day 2			...	Day n		
	S1	S2	S3	S1	S2	S3		S1	S2	S3
E1	w	0	0	w	0	0		r	0	0
E2	0	w	0	0	w	0		0	0	r
E3	0	r	0	0	0	0		w	0	0
E4	0	0	0	0	w	0		0	0	w
E5	0	0	w	0	0	0		0	w	0
R^w_{dj}	1	1	1	1	2	0		1	1	1
R^r_{dj}	0	1	0	0	0	0		1	0	1

	Day 1			Day 2			...	Day n		
	S1	S2	S3	S1	S2	S3		S1	S2	S3
E1	w	0	0	w	0	0		r	0	0
E2	0	w	0	0	w	0		0	0	r
E3	0	r	0	0	0	0		w	0	0
E4	0	0	0	0	w	0		0	0	w
E5	0	0	w	0	0	0		0	w	0
R'^w_{dj}	1	1	1	1	2	0		1	1	1

Figure 2.2 Methodology to evaluate the robustness of tactical rosters

2.4.1 Tactical scheduling phase: Branch-and-price

The tactical rosters are constructed using a branch-and-price algorithm that optimises the model described in Section 2.3.1. The methodology is conform to the approaches of Barnhart et al.

(1998) and Maenhout and Vanhoucke (2010).

2.4.2 Operational phase: Simulation and integer programming (IP) optimisation

In the operational phase, we consider the days of the tactical planning period one-by-one in chronological order. Operational variability arises on the day of operation and adjustments to the baseline personnel shift roster are required to restore the workability and/or feasibility of the roster. In our methodology, this phase consists of a simulation step and an adjustment step, and is repeated for every single day of the tactical roster.

Simulation step

The operational variability is imitated by simulating two parameters of the allocation decision model (2.12)-(2.18), i.e. the uncertainty of capacity is represented by the parameter a_{id} and the uncertainty of demand by the parameter R'^{w}_{dj}. These parameters are simulated using the GNU Scientific Library (Gough, 2009).

The availability a_{id} of employee i on day d is the result of a binary stochastic distribution, i.e. a Bernouilli distribution, that is determined by the probability of unavailability $P_{id}(X = 0)$. The binary stochastic variable X will take the value of 1 if the employee is available on the day under consideration and 0 otherwise. The probability $P_{id}(X = 0)$ is calculated based on the following formula, i.e.

$$P_{id}(X = 0) = P(X = 0) * f(days\,absent_{id}) \qquad \forall i \in N \qquad (2.19)$$

The probability $P(X = 0)$ represents the general probability that an employee is not available. This probability is multiplied with a decreasing function $f(days\,absent_{id}) = q^{days\,absent_{id}}$ (with q a constant) that has a maximum value of one. The value of the function is dependent on the number of days employee i has already been absent before day d as a result of the operational variability. This function embodies the idea that, when an employee has been unavailable once, the probability that he will be unavailable in the future decreases. The probability $P_{id}(X = 0)$ is therefore a conditional probability depending on past occurrences (Barmby, 2002). Thus, we utilise a probability of absenteeism to represent short-term and unexpected absenteeism. In this respect, we do not explicitly consider employees that may call in sick for a longer-term period of multiple days and assume that the availability of an employee is only known at the start of each day d and cannot be anticipated beyond day d. Moreover, the decreasing function $f(days\,absent_{id})$ ensures that the employee is always available after a number of absent days.

The operational staffing requirements (R'^{w}_{dj}) are also simulated for every shift in the operational planning period. We generate these new staffing requirements by using the tactical staffing requirements as the mean or expected value for a particular distribution. In general, the demand for service is characterised by a Poisson distribution (Ahmed and Alkhamis, 2009; Yeh and Lin, 2007).

Adjustment step

When disruptions occur as a result of the operational variability, adjustments may be required to the baseline personnel shift roster based on the operational allocation model (2.12)-(2.18). The input to the operational decision model is the tactical personnel roster and the output of the simulation step for the operational planning period. Since the operational planning period is limited to one day, the problem size is small. As a result, we utilise IP optimisation by applying the commercial software package Gurobi (Gurobi Optimization, Inc., 2015).

This adjustment step may be conceived as a so-called fixed reactive mechanism or as an adjustable reactive mechanism depending on the degree of flexibility that is allowed to re-schedule a (reserve) duty. Both methods operate as follows, i.e.

- The *fixed reactive mechanism* only allows adjustments to the baseline roster that convert reserve duties into working duties of the same shift and duty cancellations. This mechanism assumes that the current schedule of the employees cannot be changed on the short-term. As this method offers a low flexibility, the fixed reactive mechanism gives a clear idea how accurate reserve duties are scheduled according to a particular strategy.
- The *adjustable reactive mechanism* offers a lot more flexibility by allowing the conversion of a reserve duty to a working duty of the same shift, the change of working and/or reserve duties to another type of duty to any other shift of the planning period and the cancellation of working and/or reserve duties. Only feasible adjustments are allowed, respecting the time-related constraints. This reactive mechanism assumes a more flexible work environment where employees can be re-scheduled to another moment in time at the expense of a change penalty cost.

Thus, the fixed reactive mechanism is completely dependent upon the constructed personnel shift roster and how reserve duties are assigned to specific shifts in the tactical scheduling phase. In the adjustable reactive mechanism additional flexibility is provided in the operational phase as all working or reserve duties scheduled on a particular day may be re-assigned.

Note that we do not allow reassignments of employees with a day-off and ensure the application of the fixed or adjustable reactive mechanism through the definition of the roster change cost parameters.

2.4.3 Evaluation

Incorporating reserve duties leads to a baseline personnel shift roster that represents a higher-than-minimal cost in the tactical scheduling phase but leads to the advantage that the cost increase during the operational phase may be within limits as a result of the increased robustness (Morton and Popova (2004)). However, it is important to make this trade-off between the cost and the inserted robustness and not to tolerate too much of a cost build-up for extra robustness during the roster construction (Shebalov and Klabjan (2006), Tam et al. (2011)). Therefore, the objective is to construct robust rosters at an acceptable cost. The cost of robustness is the premium cost that is required in order to increase the robustness of a personnel roster. Hence, for the evaluation of a personnel roster we consider the *planned performance* and the *actual performance*, which consist of different metrics as shown in Figure 2.3.

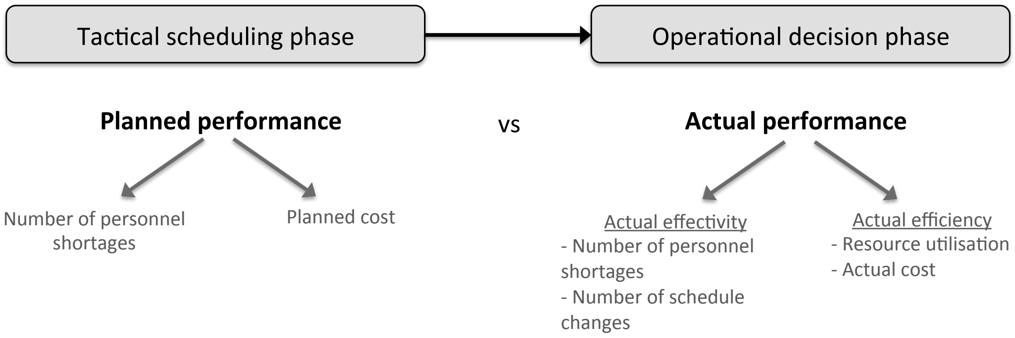


Figure 2.3 Performance measures to evaluate the personnel shift rosters

In our computational results, we calculate the metrics of each scenario in relation to the following two proactive and reactive reference cases, i.e.

- **0% robustness**: This case starts from a tactical personnel roster without any *proactive* reserve duty assignments. As we employ the fixed *reactive* mechanism in the operational phase for this case, every unexpected event has an immediate impact and will lead to an imbalance in supply and demand.
- **100% robustness**: This case maximises the *proactive* robustness by proactively introducing a maximum reserve duty buffer for each day in the tactical scheduling phase. This buffer is determined on top of the working duty staffing requirements. All employees that are originally not scheduled to a working duty but to a day off (i.e. $|N| - \sum_{j \in S} R^{w}_{dj}, \forall d \in D$), comprise a reserve pool and can be employed as a reserve crew during each shift on that particular day. By making use of the adjustable *reactive* mechanism in the operational phase, some flexibility is ensured as these reserve duties may be converted into another type of duty on the same day.

For both cases, we relax the time-related constraints (eqs. (2.6) to (2.8)). Only the time-related constraint that an employee can be assigned to one duty per day (eq. (2.4)) and the minimum rest period between duties (eq. (2.5)) are maintained. In this way, maximum flexibility for scheduling the personnel resources is guaranteed in both these reference cases in order to satisfy the staffing requirements.

Planned performance

This criterion evaluates the baseline personnel shift roster resulting from the tactical scheduling phase and is composed out of two metrics, i.e.

- The planned *total assignment cost* includes the wage cost (i.e. $\sum_{i\in N}\sum_{d\in D}\sum_{j\in S}(c^{w}_{idj}x'^{w}_{idj}+c^{r}_{idj}x'^{r}_{idj})$) and the preference penalty cost (i.e. $\sum_{i\in N}\sum_{d\in D}\sum_{j\in S}p_{idj}(x'^{w}_{idj}+x'^{r}_{idj})$) for scheduling working and reserve duties in the tactical phase. The premium cost of robustness of a roster is then calculated in comparison with the '0% robustness' case, i.e.

$$PP_1 = \frac{total\ assignment\ cost}{total\ assignment\ cost(0\%\ robustness)} - 1 \tag{2.20}$$

- The planned *number of understaffed working duties* x'^{wu}_{dj} summed over all days and shifts, i.e.

$$PP_2 = \sum_{d\in D}\sum_{j\in S} x'^{wu}_{dj} \tag{2.21}$$

Actual performance

This criterion measures the outcome after the operational decision phase and evaluates the resulting robustness of a roster by means of the actual effectivity and the actual efficiency, which should be as high as possible.

Actual effectivity

The actual effectivity or robustness measures the ability of the baseline personnel shift roster to cope with disruptions and considers two metrics, i.e.

- The *number of shortages* are the shortages for working duties that still occur in the operational phase after the adjustment step ($=\sum_{d\in D}\sum_{j\in S}x^{wu}_{dj}$). The related metric AP_1 is calculated as follows, i.e.

$$AP_1 = \frac{\#\ shortages(0\%\ robustness) - \#\ shortages}{\#\ shortages(0\%\ robustness) - \#\ shortages(100\%\ robustness)} \tag{2.22}$$

- The *number of changes* to the baseline personnel shift roster are the adjustments required to restore its workability and/or feasibility. The baseline roster has been adapted when there is a change from a duty in one shift to a duty during another shift (i.e. $x^{w}_{idj}=1$ if $(x'^{w}_{idj}+x'^{r}_{idj})=0$), from a working duty to a reserve duty (i.e. $x^{r}_{idj}=1$ if $\sum_{j\in S}x'^{w}_{idj}=1$) and in case duties are cancelled (i.e. $x^{v}_{id}=1$ if $\sum_{j\in S}(x'^{w}_{idj}+x'^{r}_{idj})=1$ and $a_{id}=1$). The conversion of a reserve duty to a working duty during the same shift is not observed as a change to the baseline roster. The related metric AP_2 is calculated as follows, i.e.

$$AP_2 = \frac{\#\ changes(100\%\ robustness) - \#\ changes}{\#\ changes(100\%\ robustness) - \#\ changes(0\%\ robustness)} \tag{2.23}$$

Note that the reference scenarios assume maximum flexibility to cover the staffing requirements. This may imply that real-life scenarios with more stringent time-related constraints, may result in a higher number of shortages or a higher number of necessary changes compared to the reference scenarios and that these metrics do not fall into the range between 0 and 1.

Actual efficiency
The actual efficiency measures the efficient use of the personnel resources by means of the following metrics, i.e.

- The *resource utilisation* evaluates the number of reserve duties that are converted into a working duty in the operational decision phase, i.e. $x^{w}_{idj} = 1$ if $\sum_{j\in S} x'^{r}_{idj} = 1$. The related metric AP_3 is calculated as follows, i.e.

  $$AP_3 = \frac{\#\, converted\, reserve\, duties}{total\, available\, reserve\, duties} \tag{2.24}$$

  This utilisation ratio has a link with the operational decision phase in the numerator and the tactical scheduling phase in the denominator. Hence, this metric gives a direct indication how well the reserve duties are planned in the tactical phase and should be as high as possible.

- The *actual assignment cost* should be as low as possible and includes the actual wage costs (i.e. $\sum_{i\in N}\sum_{d\in D}\sum_{j\in S}(c^{w}_{idj}x^{w}_{idj} + c^{r}_{idj}x^{r}_{idj})$), the preference penalty cost (i.e. $\sum_{i\in N}\sum_{d\in D}\sum_{j\in S} p_{idj}(x^{w}_{idj} + x^{r}_{idj})$) and the cancellation costs (i.e. $\sum_{i\in N}\sum_{d\in D} c^{v}_{id}x^{v}_{id}$). The related metric AP_4 is calculated as follows, i.e.

  $$AP_4 = \frac{actual\ assignment\ cost}{actual\ assignment\ cost(0\%\ robustness)} - 1 \tag{2.25}$$

In order to make an overall assessment of the robustness of a personnel shift roster, we compare the planned performance (PP) (cf. the objective function of the tactical scheduling phase (eq. (2.1)) and the actual performance (AP) (cf. the objective function of the operational allocation phase (eq. (2.12))).

2.5 Computational experiments

In this section, we provide computational insights into our methodology to improve the robustness of personnel shift rosters. In Section 2.5.1, we describe our test design to provide more information on the settings used in the tactical and operational phase and define the different strategies to schedule reserve duties. In Section 2.5.2, we assess the planned and actual performance of the formulated reserve duty scheduling strategies. Based on these results, we analyse the applicability of the strategies and determine which strategy performs best under different conditions. All tests were carried out on an Intel Core processor 2.5 GHz and 4GB RAM. The required CPU-time to produce the medium-term personnel shift rosters is on average 0.03 (1.3) seconds for test instances with 10 (20) employees and a planning period of 7 (28) days. In the operational phase, the time required to calculate the parameters κ^{α}_{idj} and κ^{f}_{id} is negligible while rescheduling a single day takes on average 0.0037 (0.0047) seconds for instances with 10 (20) employees.

2.5.1 Test design

We provide detailed information on the generated problem instances, the parameters used as input to the tactical roster construction and the reserve duty scheduling strategies in Section 2.5.1.1. In Section 2.5.1.2, we define our simulation procedure and the parameters for the adjustment step.

2.5.1.1 Scheduling phase

We generate test instances in line with the description of a general shift scheduling problem as follows, i.e.

Problem size
The instances count 10 or 20 employees for a planning period of 7 or 28 days. All employees work full-time and have the same skills. In the analysis of our computational experiments, we report on instances with 10 employees and a planning period of 7 days as the conclusions for instances with a larger problem size are very similar.

Staffing requirements
The shift-based staffing requirements for the working duties (eq. (2.2)) are randomly generated in a structured and controlled way based on three indicators proposed by Vanhoucke and Maenhout (2009). These indicators all have a value between 0 and 1 and are defined as follows, i.e.

- The *Total Coverage Constrainedness* (TCC) is the ratio between the total number of required duties and the theoretical maximum number of possible duties for a shift scheduling problem (= the number of employees × the number of days) as employees are allowed to work only one duty per day. We generate the working duty staffing requirements based on three different TCC-values, i.e. 0.30, 0.40 and 0.50.

- The *Day Coverage Distribution* (DCD) measures the variability of the required duties over the days of the planning period. A DCD-value of 0 indicates that the staffing requirements are evenly distributed over the days, a value of 1 indicates maximal variability such that the requirements are maximal for a couple of days (= the number of employees) and zero for the other days. The working duty staffing requirements are generated with DCD-values of 0.00, 0.25 and 0.50.

- The *Shift Coverage Distribution* (SCD) measures the variability of the required duties over the shifts for a single day. The interpretation of the value of the SCD-indicator is similar to the DCD-indicator. The working duty staffing requirements are generated with SCD-values of 0.00, 0.25 and 0.50.

For each parameter setting, we generate 10 instances, which implies that the computational experiments are based on 270 (= 10×3×3×3) instances for each problem size dimension.

Time-related constraints
In order to have an unambiguous idea of the robustness introduced by a particular strategy, the number of imposed time-related constraints is kept to a minimum in our problem formulation. The imposed constraints and their corresponding parameter values for a period of 7 (28) days are the following, i.e.

- An employee is assigned a maximum of 40 (160) hours over 7 (28) days (eq. (2.6)).
- An employee is assigned a minimum of 32 (128) hours over 7 (28) days (eq. (2.7)).
- An employee is assigned to duties on maximum 5 consecutive days (eq. (2.8)).

Note that the constraint that limits the number of consecutive days (eq. (2.8)) is redundant for instances with 7 days, which simplifies the construction of the tactical roster. However, when we construct a medium-term personnel roster for a period of 28 days, this constraint and the increased problem size have a significant impact on the CPU-time. This is evident from the comparison of the CPU-time for instances with 10 employees and a planning period of 7 days (0.03 seconds) and for instances with 20 employees and a planning period of 28 days (1.3 seconds).

Objective function
Table 2.1 gives an overview of the objective function weights (eq. (2.1)). Note that the highest priority is given to preserving the staffing requirements of the regular working duties as well as possible. Experiments with other objective function weights respecting the relative priorities have been performed and confirm the validity of the results that are described in Section 2.5.2.

Table 2.1 Objective function weights for the tactical roster construction

Objective function parameter	**Weight**
Working duty wage cost (c^w_{idj})	10
Reserve duty wage cost (c^r_{idj})	9
Preference penalty cost (p_{idj})	[1,5]
Understaffing working duties (c^{wu}_{dj})	20
Understaffing reserve duties (c^{ru}_{dj})	18

Reserve duty scheduling strategies
The staffing requirements for reserve duties (eq. (2.3)) and the parameter values for the reserve time-related constraints (eqs. (2.9) and (2.10)) are specified by the imposed reserve duty scheduling strategy as follows, i.e.

- **Strategy 1 -** In this strategy, no reserve duties are introduced in the tactical personnel roster. Every employee is assigned to working duties only and all constraints that concern reserve duties (eqs. (2.3), (2.9) and (2.10)) are relaxed. This strategy is similar to the ‘0%

robustness' reference case but is subject to all time-related constraints including equations (2.6)-(2.8)).

- **Strategy 2 -** Reserve duties constitute the surplus assignments for a specific employee and for a specific shift. This means that reserve duties are in fact overstaffed duties on top of the regular working duty staffing requirements (eq. (2.2)). Reserve duties are introduced in the tactical personnel roster but are not the result of reserve duty constraints (eqs. (2.3), (2.9) and (2.10)). They are merely the result of the minimum required hours over all employees (eq. (2.7)) exceeding the total staffing requirements for working duties and are positioned according to the wage and preference penalty cost of the corresponding employees.
- **Strategy 3 -** In the third strategy, we explicitly define reserve duty staffing requirements R^r_{dj} (eq. (2.3)). The reserve duty requirements are determined by a particular positioning scheme and a postulated buffer size ratio b (25%, 50%, 75%, 100%), i.e. $\sum_{d \in D} \sum_{j \in S} R^r_{dj} = b \times \sum_{d \in D} \sum_{j \in S} R^w_{dj}$. We do not impose reserve time-related constraints (eqs. (2.9) and (2.10)).
- **Strategy 4 -** The reserve duties are the result of imposing reserve time-related constraints (eqs. (2.9) and (2.10)) only.
- **Strategy 5 -** This strategy combines strategy 3 and 4 and considers both reserve duty staffing requirements (eq. (2.3)) and reserve time-related constraints (eqs. (2.9) and (2.10)) to schedule reserve duties in the tactical personnel roster. The reserve duty staffing requirements are determined by a particular positioning scheme and the sum of the minimum imposed reserve duty hours over all employees.

The positioning of the reserve duty staffing requirements in strategies 3 and 5 can be determined based on a time-based gauge or a resource-based gauge.
A *time-based gauge* calculates the reserve duty staffing requirements as a function of the days/shifts in the tactical planning period. In this perspective, we define the levelling strategy, i.e.

- *Time levelling* (Strategy 3.1) - The required reserve duties R^r_{dj} are levelled over the days of the planning period.

A *resource-based gauge* calculates the reserve duty staffing requirements based on the working duty staffing requirements. We distinguish three different schemes, i.e.

- *Fixed ratio* (Strategy 3.2) - The reserve duty staffing requirements R^r_{dj} are calculated as a fixed ratio of the working duty staffing requirements.
- *Work-based high* (Strategy 3.3) - The reserve duty staffing requirements R^r_{dj} are distributed over all shifts ($R^w_{dj} > 0$) but priority is given to those shifts with the highest working duty staffing requirements.
- *Work-based low* (Strategy 3.4) - The reserve duty staffing requirements R^r_{dj} are distributed over all shifts ($R^w_{dj} > 0$) but priority is established for shifts with the lowest working duty staffing requirements.

2.5.1.2 Operational phase

In this section, we discuss the parameter settings of the simulation step and the adjustment step for our computational experiments. In order to obtain general insights in the robustness of the different reserve duty scheduling strategies, the simulation and adjustment steps are repeated 1,000 times.

Simulation step
Each day in the planning period, we simulate the uncertainty of demand and the uncertainty of capacity or employee availability. The employee availability is simulated with the Bernoulli distribution. The probability is determined by equation (2.19). The basic probability $P(X = 0)$ is based on a study performed by SD Worx (2013) for 15,864 organisations in Belgium. They report that short-term sick leave has an occurrence probability of 2.44%. Similar absenteeism percentages have been found by the European Foundation for the Improvement of Living and Working Conditions (2010) for Europe (3%-6%) and by the Bureau of Labor Statistics (2013) for the United States (2.9%). We also performed simulations for an absenteeism percentage $P(X = 0)$ of 5% and 10% and observed similar conclusions as for the results discussed in Section 2.5.2. This basic probability $P(X = 0)$ is multiplied with a decreasing function $q^{days\,absent_{id}}$ with $q = 0.8158$. The value of q is based on experimentation and is determined such that $P_{id}(X = 0)$ approximates 0 after 28 days of absence. An absence exceeding one month is indeed considered as a long-term absence.

Adjustment step
In this phase, we optimise the objective (eq. (2.12)) with the same weights as in the tactical scheduling phase (cf. Table 2.1). Additionally, we minimise the number of changes to the baseline roster by accounting a change cost of 5 for changes in shift assignments (i.e. $c^{w\delta}_{idj}$ and $c^{r\delta}_{idj}$) and cancellations (i.e. c^{v}_{id}).

The constraints and corresponding parameter values to calculate the value of κ^{α}_{idj} and κ^{f}_{id} are respectively constraints (2.5), (2.6) and (2.8), and constraint (2.7) with the same parameter values as for composing the tactical roster. More specifically, we consider the assignments in the personnel shift roster before day d (adjusted tactical roster) and after day d (original tactical roster). As such, we establish the feasible assignments on day d. Note that the reserve time-related constraints (eqs. (2.9) and (2.10)) and the reserve duty staffing requirements (eq. (2.3)) are now ignored. Moreover, a reassignment from a day off to a working assignment is not allowed, which is indicated in the parameter value for κ^{α}_{idj}.

2.5.2 Computational results

In order to use the metrics defined in Section 2.4.3, we first evaluate the two reference cases, i.e. '0% robustness' and '100% robustness'. Table 2.2 provides the absolute values of the different components that are evaluated for these two scenarios.

As there are no reserve duties planned in the '0% robustness' case, Table 2.2 shows that the *planned performance* of this scenario has a lower wage cost compared to the ‘100% robustness’ case. Both reference cases cover the staffing requirements almost perfectly. However, for some instances a small level of understaffing was unavoidable given the generated distribution of the working duty staffing requirements and the minimum rest constraint (eq. (2.5)).

When evaluating the *actual performance*, we observe that the robustness introduced in the ‘100% robustness’ case limits the number of shortages significantly by converting several reserve duties and making multiple changes to the baseline personnel shift roster. As the number of reserve duties is high for the ‘100% robustness’ case, the conversion rate of reserve duties is on the low side as a lot of reserve duties are cancelled. When comparing the incurred actual costs, there is a cost increase of the total assignment cost for incorporating maximal robustness as a result of the higher preference penalty cost and the cancellation cost of many duties. As a result of the penalty costs, the actual performance of the ‘0% robustness’ case betters the ‘100% robustness’ case. A higher penalty cost for shortages would result in a worse performance especially of the ‘0% robustness’ case. Note that both reference scenarios violate the time-related constraints and that the associated schedules are not feasible. Both reference cases have far fewer number of working duties included after the operational phase than required by the time-related constraints. This results in significantly smaller wage costs.

In general, the introduced robustness leads to the advantage that the incurred costs are under control as the actual performance is within the limits of the planned performance for the ‘100% robustness’ case. When there are no reserve duties planned, however, the actual performance is not under control any more.

Table 2.2 ‘0% robustness’ case and ‘100% robustness’ case

	0% Robustness	**100% Robustness**
Planned performance	355.17	733.50
Number of shortages	0.04	0.04
Total assignment cost	318.13	696.47
Total wage cost	279.63	657.96
- Number of working duties	27.96	27.96
- Number of reserve duties	0.00	42.04
Preference penalty cost	38.50	38.50
Actual performance	428.35	578.29
Number of shortages	8.38	0.86
Number of changes	0.00	7.56
Number of converted reserve duties	0.00	6.27
Total assignment cost	260.72	523.37
Total wage cost	195.88	270.90
- Number of working duties	19.59	27.09
- Number of reserve duties	0.00	0.00
Number of duties cancelled	7.70	41.22
Preference penalty cost	26.36	46.34

Based on the comparison of these two reference cases, we can conclude that in terms of cost there will be a trade-off between the cost for shortages, the cost for cancelling reserve duties and the cost for changes to the baseline roster. On the one hand, too many reserve duties (proactive mechanism) will lead to high cancellation costs whereas too few reserve duties will lead to high costs for shortages. On the other hand, allowing a larger schedule flexibility on the operational level (reactive mechanism) will lead to a higher cost for changes and a lower number of shortages.

In this section, we determine the impact of the individual reserve duty scheduling strategies (Section 2.5.2.1), compare their impact on the robustness of personnel rosters (Section 2.5.2.2) and consider the impact of re-scheduling flexibility (Section 2.5.2.3). The results are averaged over all simulation runs for all computational experiments unless otherwise stated.

2.5.2.1 The impact of the reserve duty scheduling strategies

In this section, we discuss strategies 3, 4 and 5 in more detail and elaborate on the possible interpretations and parameter settings for these strategies.

Strategy 3: Impact of reserve duty staffing requirements

This strategy introduces reserve duties by explicitly formulating reserve duty staffing requirements R^r_{dj} on top of the working duty staffing requirements R^w_{dj}. The reserve duty requirements are determined by a particular positioning scheme and a postulated buffer size (cf. supra).

Schemes for positioning reserve duty staffing requirements

Figure 2.4 shows the results of the actual performance over the defined positioning schemes. The results are aggregated over different buffer size ratios to express the general trend. The results are displayed for the fixed reactive mechanism only.

Figure 2.4 reveals that, using the fixed reactive mechanism, the actual performance of the *fixed ratio* and the *work-based high*-scheme is significantly better than the *time levelling*-scheme[1], which is a strategy that is commonly used in practice when there is little information on the expected working duty staffing requirements. Moreover, the standard deviation and the minimal and maximal actual performance (σ, lb, ub) over all simulation runs and (TCC, DCD, SCD)-combinations shows that the *fixed ratio* (74.93, 396.00, 978.00) and *work-based high*-scheme (75.00, 386.00, 955.00) provide personnel rosters that are more stable in the operational allocation phase than the *time levelling*-scheme (79.14, 394.00, 1,013.00).

The *work-based high*-scheme reduces the number of shortages drastically compared to the ‘0% robustness’-scenario (AP_1) to only 4.47 shortages on the average while the number of duty cancellations (AP_2) and the total assignment cost are about the same for all positioning schemes (AP_4). Hence, the increased robustness is the result of the better positioning of the reserve duties, which is reflected by the resource utilisation ratio AP_3.

Buffer size of the reserve duty staffing requirements

In Figure 2.5, we show the impact of the buffer size for the best-performing positioning scheme, i.e. *work-based high*. However, we observe the same trends for the other positioning schemes.

Figure 2.5 reveals that an increased size of the reserve duty requirements leads in general to a higher robustness that is accompanied with higher planned costs (cf. PP). The actual performance indicates that the number of shortages decreases (AP_1) and the reserve duty conversion

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[1] p-value<0.01 (Mann-Whitney U test)

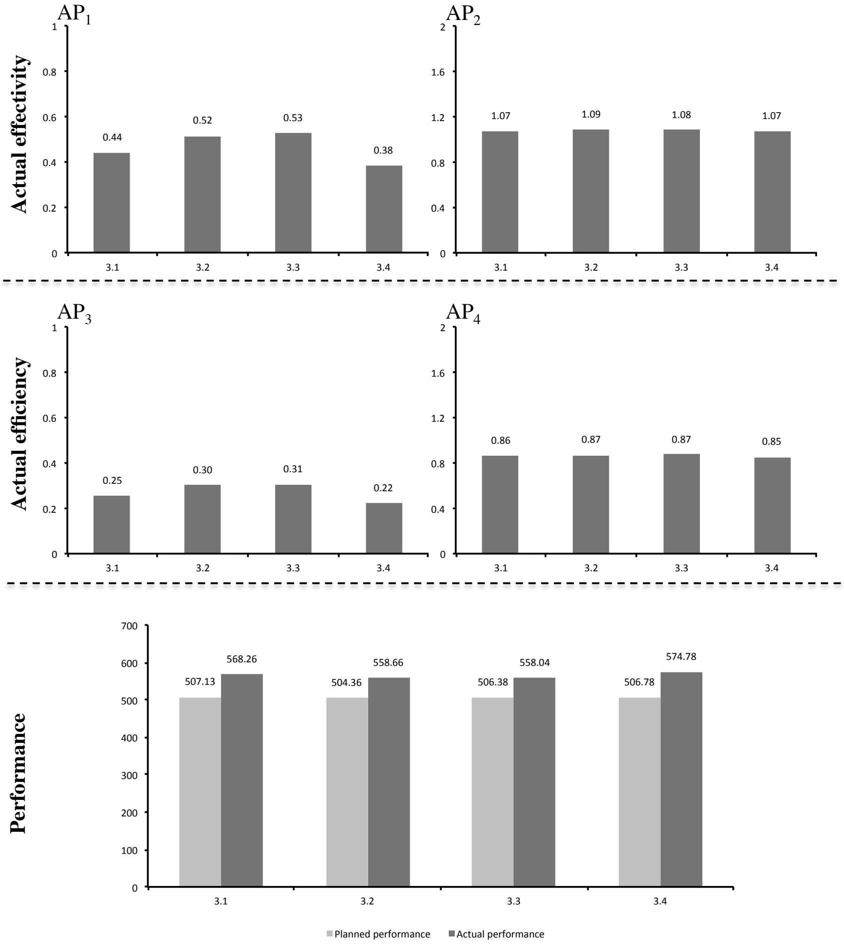


Figure 2.4 Strategy 3 - Impact of the positioning scheme for reserve duties (3.1: levelling, 3.2: fixed ratio, 3.3: work-based high, 3.4: work-based low)

rate increases (AP_3) despite of a higher number of reserve duty cancellations (AP_2). This increased number of cancellations, however, leads to a higher total assignment cost (AP_4) for larger buffer sizes, whereas the wage and preference penalty costs are very similar for the different buffer sizes.

Impact of the structure of the working duty staffing requirements

Further analysis reveals that the robustness behaviour with an increasing buffer size is very dependent upon the height of the working duty staffing requirements. For instances with a lower number of required working duties (i.e. TCC = 0.30), Figure 2.5 shows the general effect as the number of shortages (AP_1) decreases and the reserve duty conversion steadily increases (AP_3). This effect is no longer discernible for a higher number of working duties (i.e. TCC =

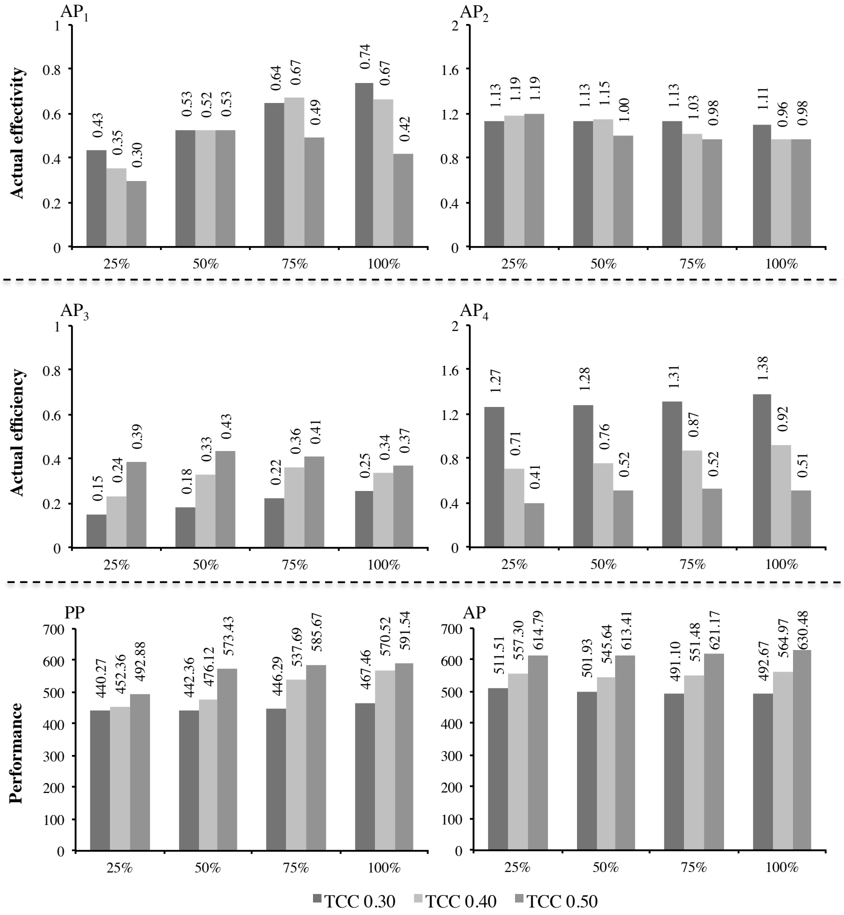


Figure 2.5 Strategy 3 - Impact of the buffer size ratio b corresponding to reserve duty requirements for different TCC-values

0.40 or 0.50), which implies that there is an upper limit to the number of reserve duties planned. The total number of duties that can be assigned to the personnel is limited by the time-related constraints. As the number of working duties increases, the ability to schedule reserve duties decreases. If the number of required reserve duties becomes too high as a result of the buffer size ratio and/or the required number of working duties, it is very difficult to assign the reserve duties to the right shift and day as all shifts are understaffed. At some point, the number of reserve duty conversions start to decrease and the number of duty cancellations and the number of shortages start to increase. This effect is most visible for the fixed reactive mechanism with TCC equal to 0.50.

Examining the variability of the working duty staffing requirements reveals that the higher the variability over the days and shifts the better schemes *work-based high* and *fixed ratio* perform compared to the others. The robustness of these strategies even improves if the variability over the shifts increases. Even more, the optimal buffer size may decrease as reserve duties are better assigned to the right shifts.

The trade-off between the number of shortages, changes and cancellations determines an optimal buffer size ratio revealed by the actual performance (AP), which is dependent upon the number of required working duties. The optimal buffer sizes amount to 75%, 50% and 50% given the TCC-values of 0.30, 0.40 and 0.50 respectively. However, it is interesting to note that these buffers do not necessarily reflect the best results in terms of variability of the actual performance. Organisations that aim to ensure the best possible stability in their actual costs rather than the best average actual cost, should consider larger buffer sizes. These sizes are 100%, 75% and 50% for a TCC-value of 0.30, 0.40 and 0.50 and result in a small extra average actual cost, i.e. 1.57, 5.84 and 0, respectively.

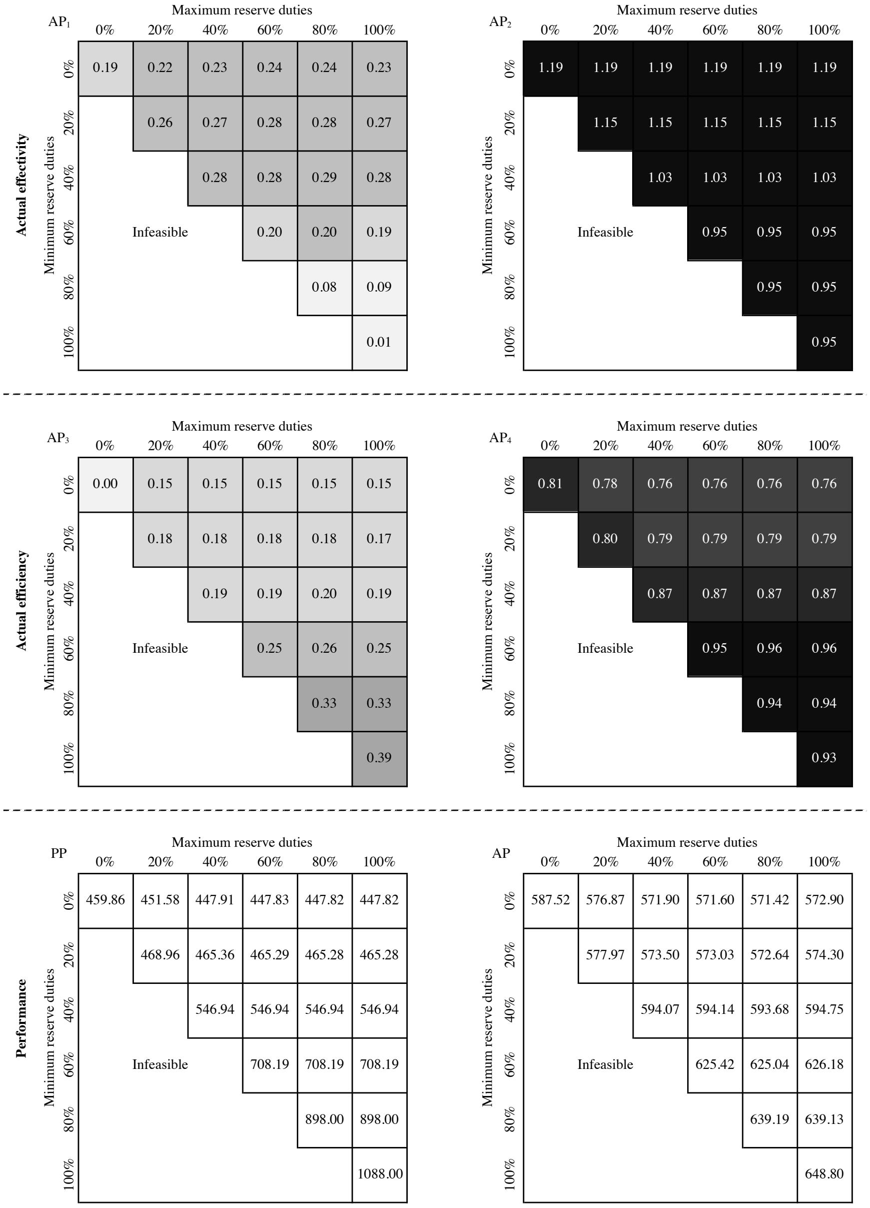
Strategy 4: Impact of reserve time-related constraints

This strategy introduces reserve duties by explicitly formulating reserve time-related constraints, i.e. the minimum and maximum number of reserve hours (eqs. (2.9) and (2.10)) for every employee. Figure 2.6 shows the results for this strategy for several constraint parameter values for the fixed reactive mechanism. The minimum number of reserve hours (vertical axis) and the maximum number of reserve hours (horizontal axis) are both expressed as a percentage of the maximum number of hours for an employee. When the maximum number of hours is 40 for a period of 7 days and the minimum number of reserve hours is 20%, the reserve time-related constraints enforce that an employee has to work at least 8 reserve hours.

Figure 2.6 reveals that the introduced robustness by imposing time-related constraints on the number of reserve hours is relatively low. We observe that varying the maximum reserve hours per employee has no significant effect on the results. All indicators show more or less the same results for different parameter values. Introducing reserve duties in the personnel roster by imposing a minimum number of reserve hours per employee has a larger impact on the results. Increasing this minimum, increases in general the number of cancellations (AP_2), the number of reserve duty conversions (AP_3) and the assignment cost (AP_4), which stagnates when minimum 60% of the duties are reserve duties. However, there is no clear effect on the number of shortages (AP_1). Performance measure AP_1 increases and reaches a maximum around 40% before it decreases.

Impact of the structure of the working duty staffing requirements

The robustness behaviour of strategy 4 is only dependent upon the size of the working duty requirements. Setting the minimum number of reserve hours between 20% and 60% of the total number of hours improves the number of shortages. However, for a higher required number of working duties, a better result is obtained if the minimum number of reserve hours per employee is closer to 20%. This is explained as follows. When personnel is primarily assigned to reserve duties and there is no specific strategy to assign the reserve duties to specific days and shifts, the reserve duties are scheduled based on wage costs and preference penalty costs only, while at the same time the working duties cannot be staffed in a satisfactory way. The higher the



Actual effectivity

AP$_1$	Maximum reserve duties					
Minimum reserve duties	0%	20%	40%	60%	80%	100%
0%	0.19	0.22	0.23	0.24	0.24	0.23
20%		0.26	0.27	0.28	0.28	0.27
40%			0.28	0.28	0.29	0.28
60%		Infeasible		0.20	0.20	0.19
80%					0.08	0.09
100%						0.01

AP$_2$	Maximum reserve duties					
Minimum reserve duties	0%	20%	40%	60%	80%	100%
0%	1.19	1.19	1.19	1.19	1.19	1.19
20%		1.15	1.15	1.15	1.15	1.15
40%			1.03	1.03	1.03	1.03
60%		Infeasible		0.95	0.95	0.95
80%					0.95	0.95
100%						0.95

Actual efficiency

AP$_3$	Maximum reserve duties					
Minimum reserve duties	0%	20%	40%	60%	80%	100%
0%	0.00	0.15	0.15	0.15	0.15	0.15
20%		0.18	0.18	0.18	0.18	0.17
40%			0.19	0.19	0.20	0.19
60%		Infeasible		0.25	0.26	0.25
80%					0.33	0.33
100%						0.39

AP$_4$	Maximum reserve duties					
Minimum reserve duties	0%	20%	40%	60%	80%	100%
0%	0.81	0.78	0.76	0.76	0.76	0.76
20%		0.80	0.79	0.79	0.79	0.79
40%			0.87	0.87	0.87	0.87
60%		Infeasible		0.95	0.96	0.96
80%					0.94	0.94
100%						0.93

Performance

PP	Maximum reserve duties					
Minimum reserve duties	0%	20%	40%	60%	80%	100%
0%	459.86	451.58	447.91	447.83	447.82	447.82
20%		468.96	465.36	465.29	465.28	465.28
40%			546.94	546.94	546.94	546.94
60%		Infeasible		708.19	708.19	708.19
80%					898.00	898.00
100%						1088.00

AP	Maximum reserve duties					
Minimum reserve duties	0%	20%	40%	60%	80%	100%
0%	587.52	576.87	571.90	571.60	571.42	572.90
20%		577.97	573.50	573.03	572.64	574.30
40%			594.07	594.14	593.68	594.75
60%		Infeasible		625.42	625.04	626.18
80%					639.19	639.13
100%						648.80

Figure 2.6 Strategy 4 - Impact of reserve time-related constraints expressed as a percentage of the maximum number of hours that can be assigned to employees ($\eta_i^{wr,max}$)

staffing requirements, the larger the number of shortages after the scheduling phase that cannot be resolved in the operational phase because of the lack of well-positioned reserve duties.

The higher the variability of the working duty staffing requirements, the worse the actual performance. The introduction of time-related constraints without an underlying positioning strategy is only acceptable, but not advisable, when the working duty staffing requirements are levelled over the days and shifts.

Strategy 5

The results for this strategy are similar to observed results for strategy 3 and 4 as the best way to position the reserve duties is based on the *work-based high*-scheme and each personnel member should perform a minimum number of reserve hours.

2.5.2.2 Comparison of reserve duty scheduling strategies

In this section, we compare the performance of the five strategies to schedule reserve duties. In Figure 2.7, we compare the result corresponding to the best parameter setting for each strategy (cf. Section 2.5.2.1) based on its actual performance.

When we compare strategies 1 and 2, we evaluate the value of introducing surplus assignments as reserve duties in the tactical personnel roster. The actual performance of strategy 2 is significantly better than strategy 1[2], which does not include any reserve duties. Both strategies do not bring the robustness to a satisfactory level as the positioning of the reserve duties for strategy 2 is very arbitrary.

The comparison of strategies 2 and 4 shows the impact of explicitly imposing reserve time-related constraints. Strategy 4 has a slightly better ability to handle disruptions (AP_1) but has a similar performance for the other metrics. As such, these strategies do not result in a significantly different actual performance.

As the actual performance (AP) for strategies 1, 2 and 4 is rather bad, the proper scheduling of reserve duties to particular days and shifts of the planning period is required. This effect is demonstrated when we analyse strategies 3 and 5, for which reserve duty staffing requirements are explicitly defined. Both strategies are able to decrease the number of shortages (AP_1) significantly leading to a better actual performance (AP). Hence, in order to obtain a roster with a satisfactory robustness, imposing reserve duty staffing requirements are indispensable.

Overall, the best strategies are strategies 3 and 5 and the actual performance of only these two strategies corresponds to the planned performance within limits.[3] These strategies anticipate the operational variability through appropriately positioned capacity buffers, which are largest for strategy 5. As such, the planned performance actually exceeds the actual performance for this strategy. In this respect, the number of shortages and the reserve duty conversion are noticeably better for strategy 5. This beneficial effect decreases when the variability of the staffing requirements increases. In general, strategy 3 and 5 provide the best average actual performance with the highest level of stability (cf. Figure 2.A.1 in Appendix 2.A).

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[2] p-value<0.01 (Mann-Whitney U test)

[3] p-value<0.01 (Mann-Whitney U test)

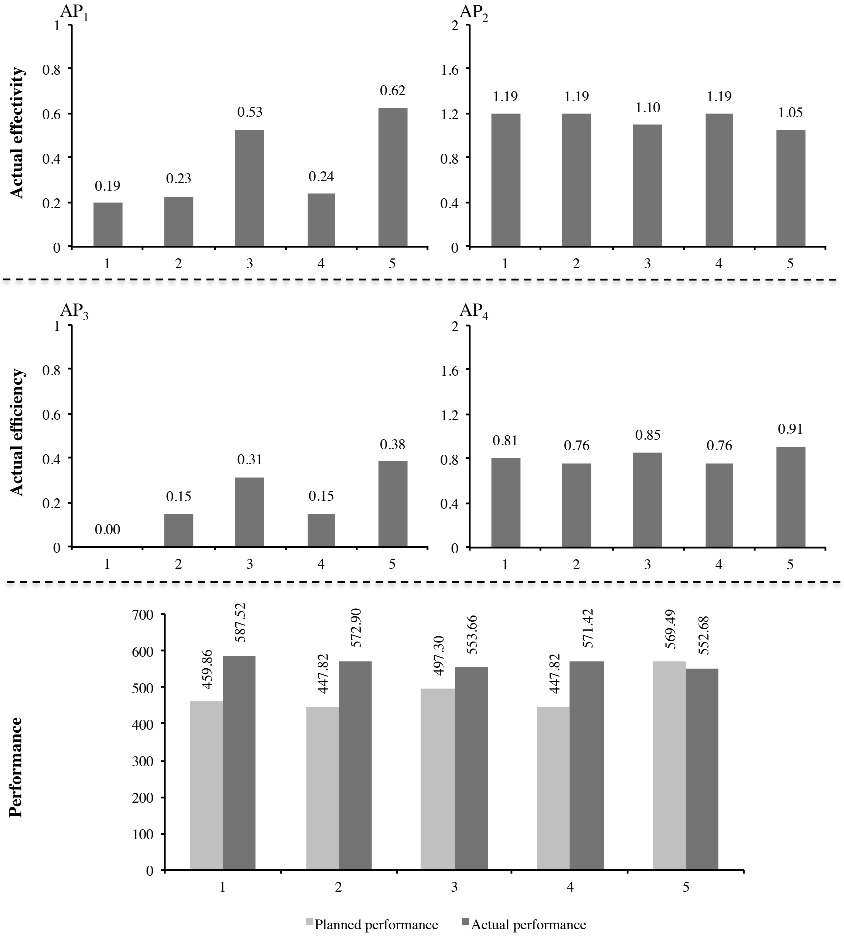

Figure 2.7 Comparison of the 5 reserve duty scheduling strategies using the fixed reactive mechanism

2.5.2.3 The introduction of flexibility in the re-scheduling phase

The comparison of the fixed and adjustable reactive mechanism highlights the influence of introducing re-scheduling flexibility in the operational phase on the robustness of a roster. In comparison with Figure 2.7, Figure 2.8 shows that the adjustable reactive mechanism results in general in a higher actual effectivity compared to the fixed reactive mechanism. The improved actual effectivity is a result of a lower number of shortages (AP_1) despite the higher number of changes to the baseline roster (AP_2). The adjustable reactive mechanism converts a higher number of reserve duties leading to a higher utilisation rate (AP_3) and a higher total assignment cost (AP_4).

Figure 2.8 reveals that strategies 3 and 5 still perform significantly better when the adjustment

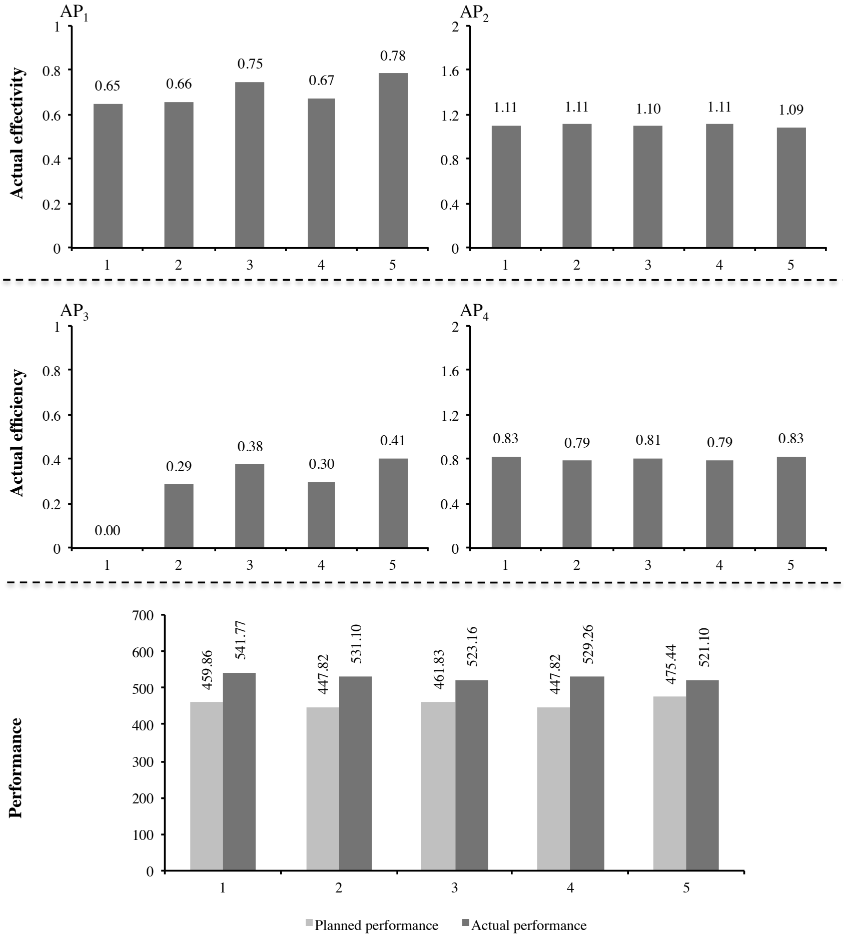

Figure 2.8 Comparison of the 5 reserve duty scheduling strategies using the adjustable reactive mechanism

capability is increased.[4] However, when the reactive flexibility in the operational phase increases, the difference between the different strategies to introduce reserve duties decreases and it becomes less important to explicitly formulate the reserve duty requirements on a shift basis as a function of the working duty requirements. This implies that strategies with a worse planning of reserve duties or that completely lack a clear positioning of reserve duties (i.e. strategies 1, 2 and 4) at the tactical level are compensated by the higher flexibility at the operational level. However, strategy 3 and 5 continue to provide the best average actual performance with the highest level of stability (cf. Figure 2.A.2 in Appendix 2.A) but require a smaller buffer size compared to the fixed mechanism.

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[4] p-value$<$0.01 (Mann-Whitney U test)

2.6 Conclusions

In this chapter, we investigate the impact of reserve duties on the robustness of personnel shift rosters. Our methodology consists of three steps. In the first step, we construct the baseline personnel shift roster using a branch-and-price algorithm. We proactively plan reserve duties according to a reserve duty scheduling strategy. Five strategies are proposed that differ in the presence of reserve time-related constraints and reserve duty staffing requirements. In order to determine the robustness of the baseline personnel shift roster, we use this roster as input to the operational phase. This second step consists of a simulation and reactive re-scheduling component. We perform the simulation of the uncertainty of demand and capacity followed by the required adjustments on a day-to-day basis until we reach the end of the baseline roster. In the third step, the adjusted roster is evaluated based on its actual performance, which consists of the actual effectivity and the actual efficiency. We can conclude that introducing capacity buffers is required to keep the actual performance of a personnel shift roster under control. When a reserve duty strategy is devised, there is a trade-off between the cost for shortages and the cost for scheduling reserve duties. The higher the resource buffer determined proactively in the planning phase by scheduling reserve duties, the lower the number of shortages but the higher the wage costs and the cost for cancelling the redundant reserve duties. Based on this trade-off the buffer size and the positioning of the reserve duties over the planning horizon should be carefully determined by defining reserve duty staffing requirements as a function of the working duty staffing requirements. Defining reserve time-related constraints further improves the robustness of the personnel roster. The ability to deal with disruptions is further increased by allowing a larger reactive re-scheduling flexibility on the operational level, which will give rise to an additional trade-off between the number of changes to the baseline roster and the number of shortages.

2.A Appendix - Boxplot of the strategy comparison

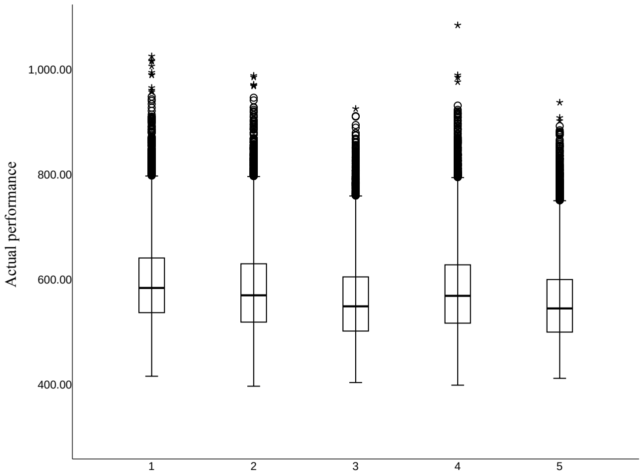


Figure 2.A.1 Comparison of the 5 reserve duty scheduling strategies using the fixed reactive mechanism

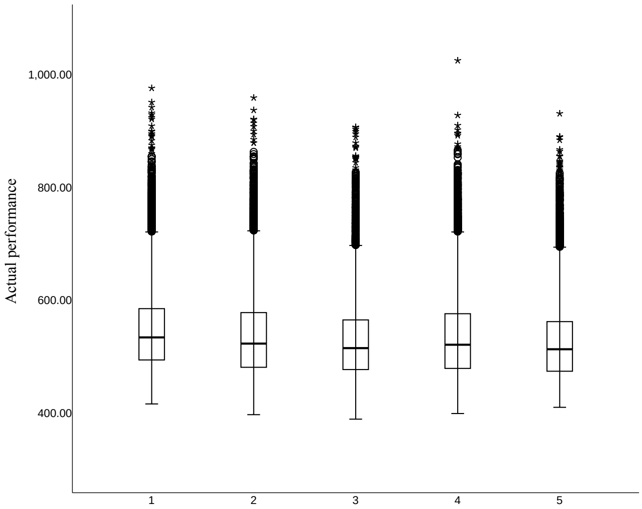


Figure 2.A.2 Comparison of the 5 reserve duty scheduling strategies using the adjustable reactive mechanism

3

The impact of the defining characteristics of uncertainty sets to construct robust personnel shift rosters

Organisations need to construct a baseline personnel shift roster for a medium-term period based on forecasts about the future personnel demand and employee availability. However, variability arises in the short-term and renders these forecasts incorrect. In order to anticipate this variability, we propose a new approach based on the principles of robust optimisation to solve a deterministic formulation of a personnel shift scheduling problem that is uncertain in terms of the personnel demand and employee availability. This formulation is based on an uncertainty set, which determines the actual values the personnel demand and employee availability may obtain. For this uncertainty set, we propose different strategies to define its underlying characteristics. We validate the robustness of these strategies through a performance comparison with a deterministic minimum cost scheduling strategy based on a three-step methodology. This methodology consists of roster construction, day-by-day simulations and roster adjustments, and evaluation.

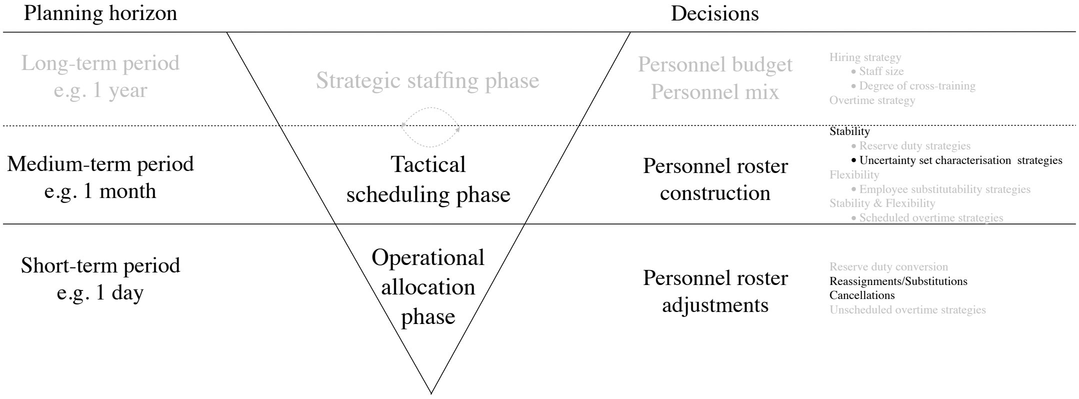


Figure 3.1 The research focus in Chapter 3

3.1 Introduction

The personnel planner can proactively apply robust optimisation in the tactical scheduling phase to anticipate operational variability. Robust optimisation utilises information on uncertain parameter values, such as the uncertain personnel demand and employee availability. This information is embedded in a robust counterpart, which is formulated as the deterministic version of a model with uncertain parameter values (Gregory et al., 2011) and is based on the definition of uncertainty sets. These uncertainty sets include the uncertain values that the parameters may obtain (Bertsimas et al., 2011; Hazir et al., 2010) and are characterised by their structure and scale.

Soyster (1973) proposes an uncertainty set whose structure ensures the feasibility of a solution for all the worst-case realisations of the parameter with uncertain values. However, this approach is very conservative and results in a high cost of robustness, i.e. the cost difference between the robust solution and the minimum cost solution (Gregory et al., 2011). The desirable level of conservatism should be determined based on the trade-off between the degree of robustness and the cost of robustness (Bertsimas and Thiele, 2006). It is therefore important to define more appropriate uncertainty sets that are less conservative. Ben-Tal and Nemirovski (2000) define uncertainty sets, which protect the robust solution against the worst possible joint realisations of the parameter with uncertain values (Fabozzi et al., 2007). Bertsimas and Sim (2004) define an uncertainty set and control the level of conservatism by establishing an uncertainty budget. This budget imposes a maximum on the degree to which parameters may receive a different-than-expected value.

Ultimately, robust optimisation ensures that the obtained solution remains feasible (Bertsimas and Brown, 2009; Gabrel et al., 2014) and workable in terms of costs (Gabrel et al., 2014; Tütüncü and Koenig, 2004) for all (worst-case) realisations within the boundaries of the uncertainty sets. As such, the size or scale of the uncertainty set reflects the risk aversion of the decision maker (Fabozzi et al., 2007). The size depends on the lower and upper bound imposed on the uncertain parameter values. In the personnel scheduling problem, this lower and upper bound determine the minimum and maximum number of employees that are required to cover the uncertain personnel demand while considering the uncertain employee availabilities. The difference between the point forecast, i.e. the expected value, and the lower bound and upper bound of the uncertain parameter values determines the range forecast and respectively entails the backward deviation measure and the forward deviation measure.

In this chapter, we investigate the utility of robust optimisation principles and evaluate different proactive strategies to characterise the uncertainty set underlying the robust counterpart of a general personnel shift scheduling problem that is stochastic in terms of the personnel demand and employee availability. Since smaller-than-expected staffing requirements result in overstaffing and larger-than-expected staffing requirements in shortages, we investigate the ability of these strategies to define staffing requirements that provide a good representation of the uncertain personnel demand and employee availability. We distinguish strategies that determine the deviation measures and the available uncertainty budget. Furthermore, the allocation of the uncertainty budget to different shifts on different days is controlled by two types of allocation strategies. The first type is based on explicit limits imposed on the allowable scaled deviations for each shift and day. The second type defines benefit values for each shift and day to ensure that the uncertainty budget is maximally used during the most appropriate shifts and days.

In correspondence with Abdelghany et al. (2008), Bard and Purnomo (2005a) and Chapter 2, we utilise a problem-specific three-step methodology to validate the robustness of the proposed strategies. In the first step, we utilise column generation to construct a baseline personnel shift roster for a medium-term period. During the roster construction in the tactical scheduling phase, we introduce the proposed uncertainty set characterisation strategies to define a deterministic variant of a stochastic problem. In the second step, we imitate the operational allocation phase and test the robustness of the baseline personnel shift roster. Each day, we simulate the uncertainty of demand and capacity, and adjust the baseline personnel shift roster to restore its feasibility and/or workability. In the third step, the robustness is evaluated by comparing the planned and actual performance of the robust baseline personnel shift rosters and the minimum cost baseline personnel shift rosters, which are constructed based on the assumption of a deterministic operating environment.

The remainder of this chapter is organised as follows. In Section 3.2, we formulate the robust counterpart of a general personnel shift scheduling problem with uncertain personnel demands and employee availabilities. We elaborate on the three-step methodology in Section 3.3. The test design and computational experiments are discussed in Section 3.4. In this section, we evaluate the formulated strategies and test their effectivity in increasing the robustness of a personnel shift roster. In Section 3.5, conclusions are drawn.

3.2 Problem definition and formulation

In this chapter, we aim to improve the stability of baseline personnel shift rosters in the operational allocation phase through the application of robust optimisation principles in the tactical scheduling phase. Robust optimisation is based on the optimisation of a robust counterpart, which is a deterministic representation of a stochastic problem. We formulate this robust counterpart by establishing an uncertainty set that considers the operational variability of the personnel demand and employee availability in the definition of the staffing requirements. These stochastic staffing requirements may be smaller or larger than the expected staffing requirements. Smaller- and larger-than-expected stochastic staffing requirements respectively represent 'requirements diminutions' and 'requirements enlargements'. A 'requirements diminution' is defined to reduce the number of scheduled employees and the associated operational overstaffing and cancellations. In contrast, a 'requirements enlargement' aims to increase the number of scheduled employees by inserting a personnel buffer. These personnel buffers enable the reduction of the operational shortages and reassignments.

In Section 3.2.1, we define and formulate the personnel shift scheduling problem with stochastic staffing requirements. These staffing requirements are embedded in an uncertainty set in Section 3.2.2. In Section 3.2.3, we define and formulate the robust counterpart based on the stochastic personnel shift scheduling problem and the corresponding uncertainty set.

3.2.1 Definition of the stochastic personnel shift scheduling problem

In order to obtain widely applicable results, we study a personnel shift scheduling problem that is general in terms of personnel characteristics, time-related constraints and objectives (Burke et al., 2004; Van den Bergh et al., 2013). This problem can be categorised as $AS\,1|RV\,|S||LRG$ (De Causmaecker and Vanden Berghe, 2011) and entails the assignment of a set of employees with homogeneous skill characteristics to a duty or a day off over a planning period of multiple days. These assignments are subject to a number of time-related constraints and need to contribute to the achievement of the desired service level at an optimal personnel cost and satisfaction. The degree in which this service level is obtained, depends on the uncertain personnel demand and the uncertain availability of the scheduled employees. Both types of uncertainties are considered by a tactical personnel shift scheduling problem that is stochastic in terms of the staffing requirements, i.e.

Notation

Sets

N	set of employees (index i)
D	set of days (index d)
S	set of shifts (index j)
$T_{dj}^{''}$	set of shifts that cannot be assigned the day after day d and shift assignment j (index s)

Parameters

c^w_{idj}	wage cost of assigning an employee i to shift j on day d
c^{wu}_{dj}	shortage cost for shift j on day d
p_{idj}	preference penalty cost if an employee i receives a shift assignment j on day d
l_j	duration of shift j
$\eta^{w,min}_i$	minimum number of hours that need to be assigned to employee i
$\eta^{w,max}_i$	maximum number of hours that can be assigned to employee i
$\theta^{w,max}_i$	maximum number of consecutive working assignments for employee i

Variables

x^w_{idj}	1 if employee i receives a shift assignment j on day d, 0 otherwise
x^v_{id}	1 if employee i receives a day off on day d, 0 otherwise
$\tilde{R}^w_{dj}$	stochastic staffing requirements for shift j on day d
x^{wu}_{dj}	the shortage of employees for shift j and day d

Problem definition

$$\min \quad \sum_{i\in N}\sum_{d\in D}\sum_{j\in S}(c^w_{idj}+p_{idj})x^w_{idj} \tag{3.1a}$$

$$+\sum_{d\in D}\sum_{j\in S}c^{wu}_{dj}x^{wu}_{dj} \tag{3.1b}$$

The objective (3.1a) is to assign employees to shifts such that the wage cost and preference penalty cost is minimised. Moreover, the understaffing of shifts is optimised through objective (3.1b). Since a cost is incurred for each duty, we implicitly minimise overstaffing.

$$\sum_{i\in N}x^w_{idj}+x^{wu}_{dj}\geq \tilde{R}^w_{dj} \qquad \forall d\in D,\forall j\in S \tag{3.2}$$

Constraint (3.2) defines the staffing requirements, which are relaxed by allowing understaffing. The staffing requirements actually represent the required number of employees given the unknown personnel demand and availability of employees. In this way, we need to schedule an uncertain number of employees for each shift and day to account for the variability in the personnel demand and the occurrence that one or more of the scheduled employees are unexpectedly unavailable. Hence, the definition of the staffing requirements ($\tilde{R}^w_{dj}$) accounts for uncertainty of demand and employee availability.

$$\sum_{j\in S}x^w_{idj}+x^v_{id}=1 \qquad \forall i\in N,\forall d\in D \tag{3.3}$$

$$x^w_{idj}+\sum_{s\in T''_{dj}}x^w_{i(d+1)s}\leq 1 \qquad \forall i\in N,\forall d\in D,\forall j\in S \tag{3.4}$$

Each employee receives a duty or a day off (eq. (3.3)) and requires a minimum rest period between two consecutive duties (eq. (3.4)).

$$\sum_{d\in D}\sum_{j\in S} l_j x^{w}_{idj} \leq \eta^{w,max}_{i} \qquad \forall i \in N \tag{3.5}$$

$$\sum_{d\in D}\sum_{j\in S} l_j x^{w}_{idj} \geq \eta^{w,min}_{i} \qquad \forall i \in N \tag{3.6}$$

$$\sum_{d'=d}^{d+\theta^{w,max}_{i}} (1 - x^{v}_{id'}) \leq \theta^{w,max}_{i} \qquad \forall i \in N, \forall d \in D \tag{3.7}$$

The other time-related constraints include a maximum number of hours each employee can receive (eq. (3.5)), a minimum number of hours each employee has to receive (eq. (3.6)) and a maximum number of consecutive working days (eq. (3.7)).

$$\begin{aligned} x^{w}_{idj} &\in \{0,1\} & \forall i \in N, \forall d \in D, \forall j \in S \\ x^{v}_{id} &\in \{0,1\} & \forall i \in N, \forall d \in D \\ \tilde{R}^{w}_{dj} &\geq 0\, and\, integer & \forall d \in D, \forall j \in S \\ x^{wu}_{dj} &\geq 0 & \forall d \in D, \forall j \in S \end{aligned} \tag{3.8}$$

Finally, constraints (3.8) define the integrality conditions.

3.2.2 The characterisation of the uncertainty set

The stochastic staffing requirements in model (3.1)-(3.8) indicate the operational variability of the personnel demand and employee availability. In robust optimisation, uncertainty is embedded in an uncertainty set. In this chapter, we focus on the definition of an uncertainty set by means of the following two characteristics, i.e.

- The *deviation measures* control the degree to which the staffing requirements are assumed to maximally differ from their expected value (R^{w}_{dj}). In this perspective, we distinguish *backward deviation measures* ($\hat{R}^{w-}_{dj}$) and *forward deviation measures* ($\hat{R}^{w+}_{dj}$). These measures indicate the maximum degree to which the stochastic staffing requirements ($\tilde{R}^{w}_{dj}$) may be smaller and larger than the expected staffing requirements, respectively. Hence, these measures aim to ensure that the tactical personnel shift roster is constructed based on staffing requirements that provide a good approximation of the actual staffing requirements in the operational allocation phase. As such, operational costs for overstaffing and cancellations and for reassignments and shortages can be avoided.
- The *uncertainty budget* ($\tilde{g}$), which is available for distribution over all shifts and days in the planning period, imposes a maximum on the degree to which and the number of shifts for which the deviation measures may be utilised to diminish or enlarge the stochastic staffing requirements compared to the expected staffing requirements. This degree is expressed as a percentage for each shift and day in the planning period and the sum of these percentages

provides the total ‘included uncertainty budget’. Hence, the uncertainty budget ensures that different-than-expected staffing requirements are not anticipated as their worst-case counterparts for all shifts and days.

Moreover, the allocation of the uncertainty budget to different shifts and days can be guided by a constraint and/or an objective function component, i.e.

- The constraint imposes explicit limits on the *scaled deviations*, i.e. the percentages with which the stochastic staffing requirements differ from the expected staffing requirements. These limits range between 0% and 100% for each shift and day ($\tilde{e}_{dj}$).
- The objective function component introduces *benefit values ($\tilde{q}_{dj}$)* for the stochastic staffing requirements during each shift and day. These benefit values promote the inclusion of the uncertainty budget to diminish and enlarge the staffing requirements during the most appropriate shifts and days.

We formulate the uncertainty set for the stochastic staffing requirements in the model below, i.e.

Notation

Uncertainty set parameters

R^w_{dj}	expected staffing requirements for shift j on day d
$\hat{R}^{w-}_{dj}$	backward deviation measure for the staffing requirements for shift j on day d
$\hat{R}^{w+}_{dj}$	forward deviation measure for the staffing requirements for shift j on day d
$\tilde{q}_{dj}$	benefit value of the stochastic staffing requirements for shift j on day d
$\tilde{e}_{dj}$	maximum scaled deviation for the staffing requirements for shift j on day d
$\tilde{g}$	uncertainty budget

Uncertainty set variables

$\tilde{R}^w_{dj}$	stochastic staffing requirements for shift j on day d
$\tilde{b}_{dj}$	scaled deviation for the staffing requirements for shift j on day d

Problem definition

$$\max \quad \sum_{d \in D} \sum_{j \in S} \tilde{q}_{dj} \tilde{R}^w_{dj} \tag{3.9}$$

The objective (3.9) is to maximise the benefit value of the staffing requirements.

$$\tilde{R}^w_{dj} \geq R^w_{dj} - \tilde{b}_{dj} \hat{R}^{w-}_{dj} \qquad \forall d \in D, \forall j \in S \tag{3.10a}$$

$$\tilde{R}^w_{dj} \leq R^w_{dj} + \tilde{b}_{dj} \hat{R}^{w+}_{dj} \qquad \forall d \in D, \forall j \in S \tag{3.10b}$$

We define the limits to diminish or enlarge the stochastic staffing requirements compared to the expected staffing requirements in equations (3.10a) and (3.10b). These limits are defined by the backward ($\hat{R}_{dj}^{w-}$) and forward ($\hat{R}_{dj}^{w+}$) deviation measures, respectively.

$$\sum_{d\in D}\sum_{j\in S}\tilde{b}_{dj} \leq \tilde{g} \tag{3.11a}$$

$$\tilde{b}_{dj} \leq \tilde{e}_{dj} \qquad \forall d \in D, \forall j \in S \tag{3.11b}$$

The variable $\tilde{b}_{dj}$ determines the degree to which the deviation measures are utilised to diminish or enlarge the stochastic staffing requirements ($\tilde{R}_{dj}^{w}$). The total deviation over the planning period is restricted by an uncertainty budget (eq. (3.11a)) and the scaled deviation per shift and day cannot exceed a given maximum (eq. (3.11b)).

$$\begin{aligned} \tilde{R}_{dj}^{w} &\geq 0 \, and \, integer \quad \forall d \in D, \forall j \in S \\ \tilde{b}_{dj} &\geq 0 \quad \forall d \in D, \forall j \in S \end{aligned} \tag{3.12}$$

Constraints (3.12) define the domains of the variables.

3.2.3 Definition of the robust counterpart

The robust counterpart is the deterministic representation of a stochastic model. In this section, we formulate the robust counterpart by integrating the uncertainty set formulation (cf. eqs. (3.9)-(3.12) in Section 3.2.2) into the stochastic model (cf. eqs. (3.1)-(3.8) in Section 3.2.1). This results in objective function (3.13), which is subject to constraints (3.2)-(3.8) and (3.10)-(3.12).

$$\min \quad \sum_{i\in N}\sum_{d\in D}\sum_{j\in S}(c_{idj}^{w} + p_{idj})x_{idj}^{w} + \sum_{d\in D}\sum_{j\in S} c_{dj}^{wu} x_{dj}^{wu} - \sum_{d\in D}\sum_{j\in S}\tilde{q}_{dj}\tilde{R}_{dj}^{w} \tag{3.13}$$

We define this robust counterpart based on different strategies that determine the characteristics of the underlying uncertainty set (eqs. (3.9)-(3.12)). The strategies are formulated in Section 3.4.1 and aim to enable the construction of a personnel shift roster based on staffing requirements that are a good approximation of the actual staffing requirements in the operational allocation phase. As such, we ensure that we obtain an appropriate capacity buffer at appropriate positions, which was identified as important in Chapter 2. In addition to Chapter 2, we enable the anticipation of smaller-than-expected staffing requirements to avoid operational cancellations and overstaffing and to enable larger capacity buffers.

3.3 Procedure

We apply a three-step methodology to improve and validate the robustness of personnel shift rosters. This methodology consists of the following steps, i.e.

- In the first step, we construct a baseline personnel shift roster for a medium-term period by applying column generation (Section 3.3.1). In this tactical scheduling phase, we consider robust optimisation principles and employ different proactive uncertainty set characterisation strategies in model (3.9)-(3.12) (Section 3.2.2), which models the basis for the robust counterpart (Section 3.2.3) and aims to improve the robustness of personnel shift rosters.

- We test the robustness of the baseline personnel shift roster in the second step, which consists of an imitation of the operational allocation phase on a day-by-day basis (Section 3.3.2). Each day of the planning horizon, we simulate the operational variability and perform adjustments to the baseline personnel shift roster.
- In the third step, we compare the planned and actual performance of the baseline personnel shift roster (Section 3.3.3). This comparison aims to evaluate the robustness of the proposed strategies that determine the characteristics of the uncertainty set.

The validation of the improved robustness through simulation has been examined by other authors (Abdelghany et al., 2008; Bard and Purnomo, 2005a) and is based on the developed methodology in Chapter 2.

3.3.1 Tactical scheduling phase

In the tactical scheduling phase, we aim to construct a baseline personnel shift roster that optimises the employee assignments (x^w_{idj}) and the stochastic staffing requirements ($\tilde{R}^w_{dj}$). Given the multitude of decision possibilities, the baseline personnel shift roster is generated using column generation. This means that the problem formulation is decomposed into a restricted master problem, which considers only a subset of all possible columns, and a subproblem, which defines the structure of a column (Barnhart et al., 1998; Vanderbeck, 2000). In our column generation procedure, a column is defined for each employee and entails a feasible personnel schedule, i.e. a line of work that satisfies the time-related constraints in equations (3.3)-(3.7).
We iteratively solve the restricted master problem and subproblem until it is no longer possible to improve the objective function value.

Restricted master problem
In the restricted master problem, we need to determine the stochastic staffing requirements that can be optimally combined with personnel schedules. In order to formulate this problem, we define a set of columns for each employee i (K^w_i), a parameter that indicates whether an employee i receives a shift assignment j on a day d in a column k ($\bar{x}^w_{kidj}$=1) or not ($\bar{x}^w_{kidj}$=0), a pattern cost (c^w_{ik}) that indicates the cost of a column k for an employee i and a variable z^w_{ik} that is 1 if employee i is assigned to column k and 0 otherwise. Note that the cost parameter c^w_{ik} includes the wage cost and preference penalty cost corresponding to each shift assignment in the column, i.e. $c^w_{ik} = \sum_{d \in D} \sum_{j \in S} (c^w_{idj} + p_{idj}) \bar{x}^w_{kidj}$.

These definitions result in the following restricted master problem, i.e.

$$\min \quad \sum_{i\in N}\sum_{k\in K_i^w} c_{ik}^w z_{ik}^w + \sum_{d\in D}\sum_{j\in S} c_{dj}^{wu} x_{dj}^{wu} - \sum_{d\in D}\sum_{j\in S} \tilde{q}_{dj}\tilde{R}_{dj}^w \tag{3.14}$$

Objective (3.14) minimises the pattern costs corresponding to the selected employee columns and the cost for understaffing. Simultaneously, the objective ensures that the best benefit value is obtained for the stochastic staffing requirements, which are subject to constraints (3.10) and (3.11).

$$\sum_{i\in N}\sum_{k\in K_i^w} \bar{x}_{kidj}^w z_{ik}^w + x_{dj}^{wu} \geq \tilde{R}_{dj}^w \qquad \forall d\in D, \forall j\in S \tag{3.15a}$$

$$\sum_{k\in K_i^w} z_{ik}^w = 1 \qquad \forall i\in N \tag{3.15b}$$

We redefine the constraint that imposes the staffing requirements (eq. (3.15a)) and ensure that one column is selected for each employee (eq. (3.15b)).

$$\begin{aligned} z_{ik}^w &\geq 0 \quad \forall i\in N, \forall k\in K_i^w \\ x_{dj}^{wu} &\geq 0 \quad \forall d\in D, \forall j\in S \\ \tilde{R}_{dj}^w &\geq 0 \quad \forall d\in D, \forall j\in S \\ \tilde{b}_{dj} &\geq 0 \quad \forall d\in D, \forall j\in S \end{aligned} \tag{3.16}$$

Finally, we relax the restricted master problem in equation (3.16) by defining the variables as continuous.

In every iteration, we solve the restricted master problem for the given set of columns. To be able to solve the restricted master problem in the first iteration, we start by solving model (3.9)-(3.12) and utilise these generated stochastic staffing requirements ($\tilde{R}_{dj}^w$) as a parameter in model (3.1)-(3.8) to obtain an initial schedule for each employee in the set K_i^w.
After the solution of the restricted master problem, we determine the value for the dual variables of constraints (3.15a) and (3.15b), i.e. ρ_{dj} and ω_i respectively.

Pricing problem
The pricing problem solves the subproblem by incorporating dual information from the master problem. We repeatedly solve the pricing problem to construct a schedule for each individual employee. In this problem, we minimise the reduced cost of a column as follows, i.e.

$$\min \quad \sum_{d\in D}\sum_{j\in S}(c_{idj}^w + p_{idj} - \rho_{idj})x_{idj}^w - \omega_i \tag{3.17}$$

For each employee, we minimise this objective function (3.17) subject to the time-related constraints (eqs. (3.3)-(3.7)) to ensure the feasibility of each column. Each column with a negative reduced cost is added to the set K_i^w as it may help to decrease the objective function value of the restricted master problem.

If we can find a new employee column, we resolve the restricted master problem (eqs. (3.14)-(3.16)). If this is not the case, we have found the optimal LP-solution. If this LP-solution is integer, we have simultaneously found the optimal IP-solution. Otherwise, we utilise the given set of employee columns to resolve the restricted master problem as an integer problem.

3.3.2 Operational allocation phase

In the operational allocation phase, we reconsider the baseline personnel shift roster on a day-by-day basis for each day in the planning horizon. We imitate the dynamic operating environment, with which organisations are faced, through the application of a simulation and an adjustment component (cf. Chapter 2). For each day d, we simulate the operational variability and execute adjustments to the baseline personnel shift roster to restore its workability and/or feasibility. The personnel planner may execute the following recovery options, i.e.

- Reassignments: we distinguish two types of reassignments to recover shortages, i.e. *between-shift changes* and *day-off-to-work changes.* The first type reassigns a working employee to a different shift while the second reassigns an employee with a day off to a duty.
- Cancellations: duties, which are redundant on top of the staffing requirements, can be cancelled to recover overstaffing.

These recovery options are embedded in the operational decision model in Appendix 3.A, which is based on model (2.12)-(2.18) in Chapter 2. Different levels of operational flexibility may be examined by considering all or a combination of these options through the definition of the associated cost parameters in this model.

Given the adjustments, we update the baseline personnel shift roster on day d and repeat the operational allocation phase for day $d + 1$. In order to obtain a clear insight into the robustness of the original baseline personnel shift roster, we repeatedly execute this day-by-day process.

3.3.3 Robustness evaluation

We evaluate the robustness of baseline personnel shift rosters by comparing their planned and actual performance (cf. Chapter 2). Table 3.1 provides an overview of the building blocks of the planned and actual performance.

The planned performance consists of three building blocks, i.e.

- The *planned cost* comprises the shortages and total assignment cost, i.e. the wage cost and preference penalty cost.
- Similar to the shortages, the *planned overstaffing* represents the overstaffing of the expected (R_{dj}^w) or stochastic ($\tilde{R}_{dj}^w$) staffing requirements for minimum cost or robust baseline personnel shift rosters, respectively.

Table 3.1 Building blocks of the planned and actual performance

PLANNED PERFORMANCE	**ACTUAL PERFORMANCE**
Planned cost	**Actual cost**
Shortages (in shifts)	Shortages (in shifts)
Total assignment cost	Total assignment cost
Wage cost	Wage cost
Preference penalty cost	Preference penalty cost
	Number of duties cancelled
	Number of duty changes
	Between-shift changes
	Day-off-to-work changes
Planned overstaffing	**Actual overstaffing**
Planned uncertainty budget	**Actual uncertainty budget**
Included uncertainty budget	Included uncertainty budget
Requirements diminution *(NB, AVG, VAR)*	Requirements diminution *(NB, AVG, VAR)*
Requirements enlargement *(NB, AVG, VAR)*	Requirements enlargement *(NB, AVG, VAR)*

- The *planned uncertainty budget* provides the combined information embedded in the 'included uncertainty budget', the 'requirements diminution' and the 'requirements enlargement' as follows, i.e.
  - The 'included uncertainty budget' expresses the sum of the scaled deviations between the expected and stochastic staffing requirements, i.e. $\sum_{d \in D} \sum_{j \in S} \tilde{b}_{dj}$.
  - The 'requirements diminution (enlargement)' is defined by a triplet *(NB, AVG, VAR)*, i.e. the *number* of shifts with diminution (enlargement), the *average* negative (positive) difference between the stochastic staffing requirements ($\tilde{R}^w_{dj}$) and the expected staffing requirements (R^w_{dj}) and the *variance* of this difference over all shifts in the planning horizon.

The actual performance is very similar to the planned performance and we elaborate on the following components, i.e.

- The *actual cost* includes the shortages, the total assignment cost and the number of changes in the duties of the employees. The total assignment cost comprises the wage cost, preference penalty cost and the cost for cancellations of duties. We also account a cost for the number of duty changes, i.e. between-shift changes and day-off-to-work changes.
- The *actual uncertainty budget* indicates the uncertainty budget that ideally should have been included and distributed over 'requirements diminution and enlargement' in the tactical scheduling phase, i.e.
  - Equations (3.18) determine the 'included uncertainty budget' based on the backward and forward budget, i.e. the difference between the actual demand and the expected staffing requirements (R^w_{dj}) as a percentage of the deviation measures. The actual demand depends on the simulated staffing requirements (R'^w_{dj}) and the unavailability of employees who were assigned to the considered shift in the baseline personnel shift roster (x'^w_{idj}). Note that if the backward ($\hat{R}^{w-}_{dj}$) or forward ($\hat{R}^{w+}_{dj}$) deviation measures

are zero, their respective equations (eqs. (3.18b) and (3.18c)) are considered to be zero.

- The 'requirements diminution (enlargement)' provides the *number* of shifts with actual diminution (enlargement), the *average* actual negative (positive) difference between the actual demand and the expected staffing requirements (R^{w}_{dj}) and the actual *variance* of this difference over all shifts in the planning horizon.

$$included\ uncertainty\ budget = \sum_{d \in D} \sum_{j \in S} \max \left(actual\ backward\ budget_{dj}, actual\ forward\ budget_{dj} \right) \tag{3.18a}$$

$$actual\ backward\ budget_{dj} = \frac{R^{w}_{dj} - (R'^{w}_{dj} + \sum_{i \in N}(1 - a_{id}) x'^{w}_{idj})}{\hat{R}^{w-}_{dj}} \qquad \forall d \in D, \forall j \in S \tag{3.18b}$$

$$actual\ forward\ budget_{dj} = \frac{(R'^{w}_{dj} + \sum_{i \in N}(1 - a_{id}) x'^{w}_{idj}) - R^{w}_{dj}}{\hat{R}^{w+}_{dj}} \qquad \forall d \in D, \forall j \in S \tag{3.18c}$$

The comparison of the planned performance between the minimum cost roster, which is constructed assuming a deterministic operating environment, and the robust roster, which is constructed assuming a stochastic operating environment, provides insight into the cost of robustness. This cost of robustness indicates the cost disadvantage that arises due to the application of proactive strategies in the tactical scheduling phase.

The comparison of the actual performance between these two types of rosters gives the value of robustness, i.e. the cost advantage obtained in the operational allocation phase by anticipating uncertainty in the tactical scheduling phase.

3.4 Computational experiments

In this section, we provide computational insight into the uncertainty set characterisation strategies utilised to embed the uncertain personnel demand and employee availability in a stochastic personnel shift scheduling model. We describe our parameter settings, proactive strategies and considered levels of operational flexibility and variability in Section 3.4.1. In Section 3.4.2, we study the performance of the individual and combined proactive strategies to provide guidelines about their applicability in a dynamic operating environment. In addition, we discuss the robustness of the strategies and elaborate on their applicability for different levels of operational variability and flexibility in Section 3.4.3. All tests were carried out on an Intel Core processor 2.5 GHz and 4 GB RAM.

3.4.1 Test design

In this section, we describe the parameter settings utilised in the tactical scheduling and operational allocation phases. We define our proactive tactical scheduling strategies and the considered levels of operational flexibility and variability. All test instances consist of 20 employees who possess one skill, three non-overlapping shifts of eight hours (l_j) with specific start and end times, and a planning horizon of 28 days. Note that similar results are obtained for smaller test

Table 3.2 The uncertainty set characterisation strategies

Deviation measure strategies	$\tilde{R}^w_{dj} \geq R^w_{dj} - \tilde{b}_{dj}\,\boldsymbol{\hat{R}^{w-}_{dj}}$	(eq. (3.10a))
	$\tilde{R}^w_{dj} \leq R^w_{dj} + \tilde{b}_{dj}\,\boldsymbol{\hat{R}^{w+}_{dj}}$	(eq. (3.10b))
Uncertainty budget strategies	$\sum_{d \in D} \sum_{j \in S} \tilde{b}_{dj} \leq \boldsymbol{\tilde{g}}$	(eq. (3.11a)
Uncertainty budget allocation strategies	$\tilde{b}_{dj} \leq \boldsymbol{\tilde{e}_{dj}}$	(eq. (3.11b))
	$\sum_{d \in D} \sum_{j \in S} \boldsymbol{\tilde{q}_{dj}}\,\tilde{R}^w_{dj}$	(objective function (3.9))

instances.

Expected staffing requirements

Vanhoucke and Maenhout (2009) propose three indicators to define the profile of the expected staffing requirements (R^w_{dj}) over the days and shifts of the planning horizon. We generate test instances with expected staffing requirements corresponding to a TCC-value of 0.30, 0.40 and 0.50, a DCD-value of 0.00, 0.25 and 0.50 and an SCD-value of 0.00, 0.25 and 0.50 (cf. Chapter 2). We generate expected staffing requirements for 27 (3×3×3) (TCC, DCD, SCD)-combinations.

Time-related constraints

In this chapter, we aim to achieve an unbiased insight in the robustness offered by the formulated proactive strategies. As such, we limit the number of time-related constraints.

We impose that each employee can only receive one duty per day (eq. (3.3)) and that two consecutive duties need to be separated by a minimum of 11 hours (eq. (3.4)). Each employee can also work a maximum of 160 hours ($\eta^{w,max}_i$ in eq. (3.5)), a minimum of 128 hours ($\eta^{w,min}_i$ in eq. (3.6)) and a maximum of 5 consecutive duties ($\theta^{w,max}_i$ in eq. (3.7)).

Objective function components

The objective function in the deterministic model to construct a baseline personnel shift roster is to minimise the costs. These costs consist of the following components, i.e.

- The wage cost of assigning an employee i to shift j on day d (c^w_{idj}) is 10.
- The preference penalty cost if an employee i receives a shift assignment j on day d (p_{idj}) is randomly generated in the range of 1 to 5.
- The cost of understaffing shift j on day d (c^{wu}_{dj}) is 20.

Proactive strategies to characterise the uncertainty set

We propose different strategies to define the robust counterpart of a stochastic personnel shift scheduling problem. The basis of this robust counterpart is determined by an uncertainty set. In this respect, we utilise the strategies in Table 3.2 to characterise the uncertainty set defined by model (3.9)-(3.12) in Section 3.2.2. We define these strategies as follows, i.e.

Deviation measure strategies (DM)

The deviation measure strategies represent the uncertainty of the staffing requirements in absolute numbers. We distinguish two strategies to determine the backward ($\hat{R}^{w-}_{dj}$) and forward ($\hat{R}^{w+}_{dj}$) deviation measures, i.e.

- Fixed ratio-based deviation measures (DM-FR): In Chapter 2, we showed that a fixed ratio of the expected staffing requirements is a good strategy to define reserve duty staffing requirements. In this chapter, we consider measure sizes given by a ratio Δ^{DM} of 0%, 25%, 50%, 75% and 100%. A 0% ratio represents the minimum cost roster.

  $$\hat{R}^{w-}_{dj} = \hat{R}^{w+}_{dj} = round[\Delta^{DM} \times R^{w}_{dj}] \qquad \forall d \in D, \forall j \in S \tag{3.19}$$

  In equation (3.19), we round the deviation measures to the closest integer. Note that this strategy installs a symmetric range forecast through the definition of the backward and forward deviation measures.

- Confidence interval-based deviation measures (DM-CI): Given that demand is often assumed to be Poisson distributed (Ahmed and Alkhamis, 2009; Yeh and Lin, 2007), we define confidence intervals based on the Poisson distribution. These intervals can be determined based on the Chi-square distribution (Dobson et al., 1991; Sahai and Khurshid, 1993). In correspondence with the first strategy, we experiment with measure sizes given by a ratio Δ^{DM} of 0%, 25%, 50% and 75% and approximately 100%. A ratio of 0% corresponds to the minimum cost roster.

$$\begin{aligned}\hat{R}^{w-}_{dj} = round\Bigg[&R^{w}_{dj} \times \big(1 + P(X=0)\big) - \\ &\frac{1}{2}\chi^2\bigg(\frac{1-\Delta^{DM}}{2}, 2 \times R^{w}_{dj} \times \big(1 + P(X=0)\big)\bigg)\Bigg] \forall d \in D, \forall j \in S\end{aligned} \tag{3.20a}$$

$$\begin{aligned}\hat{R}^{w+}_{dj} = round\Bigg[&\frac{1}{2}\chi^2\bigg(1 - \frac{1-\Delta^{DM}}{2}, 2 \times R^{w}_{dj} \times \big(1 + P(X=0)\big) + 2\bigg) \\ &- R^{w}_{dj} \times \big(1 + P(X=0)\big)\Bigg] \forall d \in D, \forall j \in S\end{aligned} \tag{3.20b}$$

  In equations (3.20), we multiply the expected staffing requirements (R^{w}_{dj}) with the basic probability of absenteeism ($P(X = 0)$). As such, the confidence interval considers the uncertainty corresponding to the personnel demand and employee availability. Moreover, we round the deviation measures to the closest integer and it is possible that, in this strategy, the range forecast is asymmetric.

Uncertainty budget strategies (UB)
The available uncertainty budget ($\tilde{g}$) can be defined based on two strategies. Both strategies define the available uncertainty budget based on the maximum possible budget ($|D| \times |S|$). The availability of this maximum budget entails that the stochastic staffing requirements ($\tilde{R}^{w}_{dj}$) may differ from their expected value (R^{w}_{dj}) during each shift and day and that the complete backward

($\hat{R}_{dj}^{w-}$) and forward ($\hat{R}_{dj}^{w+}$) deviation measures may be utilised in the 'requirements diminution and enlargement'.
We define the strategies as follows, i.e.

- Percentage based-uncertainty budget (UB-P): We define the available uncertainty budget as a percentage of the maximum possible budget ($\tilde{g} = \Delta^{UB} \times (|D| \times |S|)$). We investigate a percentage Δ^{UB} of 0%, 25%, 50%, 75% and 100%. Note that a percentage of 0% represents the minimum cost roster.

- Percentage and demand-based uncertainty budget (UB-PD): We adapt the first strategy (UB-P) by considering the demand profile to determine the uncertainty budget. Given that variability occurs during more days and shifts if the staffing requirements are more equally distributed, this strategy provides a higher budget if the DCD- and SCD-values are lower. Irrespective of the DCD- and SCD-values, a higher TCC-value leads to higher staffing requirements. These higher staffing requirements cause a larger variability during more days and shifts. Therefore, this strategy provides a larger uncertainty budget for higher TCC-values.

$$\tilde{g} = \Delta^{UB} \times (|D| \times |S|) \times \left(\frac{TCC}{TCC^{max}} \times (1 - DCD) \times (1 - SCD) \right) \tag{3.21}$$

  The parameter TCC^{max} represents the largest TCC-value in the test design. This parameter ensures that we can dispose of a full budget if Δ^{UB} in equation (3.21) is 100% and the (TCC, DCD, SCD)-combination is (0.50, 0.00, 0.00). For other (TCC, DCD, SCD)-combinations and percentages Δ^{UB}, the available budget is lower. We investigate a percentage Δ^{UB} of 0%, 25%, 50%, 75% and 100%. The first represents the minimum cost roster.

Uncertainty budget allocation strategies (UBA)
The allocation of the uncertainty budget to utilise the deviation measures can be guided by strategies that limit the scaled deviations (eq. (3.11b)) and by strategies that propose benefit values for the stochastic staffing requirements (eq. (3.9)), i.e.

- Scaled deviation strategies (UBA-SD): These strategies limit the utilisation of backward and forward deviation measures in the stochastic staffing requirements. In constraint (3.11b), the allowable scaled deviations ($\tilde{e}_{dj}$) determine the uncertainty budget that can be included per shift and day. We distinguish two strategies, i.e.
    - In a first strategy (UBA-SD1), the maximum allowable deviations equal the complete forward or backward deviation measures, i.e. $\tilde{e}_{dj} = 1$ ($\forall d \in D, \forall j \in S$). Therefore, this strategy reflects the standard scaled deviation strategy to allocate the uncertainty budget without any restrictions imposed.
    - In a second strategy (UBA-SD2), we consider the demand profile and limit the scaled

deviations based on the following equation, i.e.

$$\tilde{e}_{dj} = \frac{R^w_{dj}}{\max\limits_{d' \in D, j' \in S}(R^w_{d'j'})} \qquad \forall d \in D, \forall j \in S \qquad (3.22)$$

We assume that shifts with larger expected staffing requirements (R^w_{dj}) experience more uncertainty. Therefore, we prioritise the allocation of the uncertainty budget to these shifts over shifts with smaller expected staffing requirements. In this respect, we enable the utilisation of a greater percentage of the forward ($\hat{R}^{w+}_{dj}$) or backward ($\hat{R}^{w-}_{dj}$) deviation measures for shifts with larger staffing requirements.

- Benefit value strategies (UBA-BV): These strategies allocate the uncertainty budget based on the definition of specific benefit values for the stochastic staffing requirements ($\tilde{q}_{dj}$ in objective (3.9)). We distinguish two strategies to define these benefit values, i.e.
  - In the first strategy (UBA-BV1), we impose constant benefit values, i.e. $\tilde{q}_{dj}$ is Δ^{UBA-BV} for every day d and shift j. Hence, this strategy is the standard benefit value strategy to allocate the uncertainty budget. In order to investigate the trade-off with the wage and preference penalty cost, we experiment with a factor Δ^{UBA-BV} that ranges between 11 and 16.
  - In a second strategy (UBA-BV2), we consider the demand profile and determine the benefit value based on the ratio between the staffing requirements for a shift j and day d and the maximum staffing requirements over the total planning horizon (eq. (3.23)). In equation (3.23), we multiply this ratio with a factor Δ^{UBA-BV}, whose value ranges between 11 and 16.

$$\tilde{q}_{dj} = \frac{R^w_{dj}}{\max\limits_{d' \in D, j' \in S}(R^w_{d'j'})} \times \Delta^{UBA-BV} \qquad \forall d \in D, \forall j \in S \qquad (3.23)$$

Level of operational flexibility and variability

In the operational allocation phase, we investigate the utility of the proposed uncertainty set characterisation strategies under different levels of operational flexibility and variability, i.e.

- We distinguish different combinations of recovery options that correspond to different levels of operational flexibility (Table 3.3). A low operational flexibility means that the personnel planner only utilises cancellations as a recovery option. A medium operational flexibility also allows between-shift changes. A high operational flexibility means that the personnel planner can utilise cancellations and both types of reassignments, i.e. between-shift and day-off-to-work changes.

- We consider different levels of operational variability by investigating different basic probabilities of absenteeism ($P(X = 0)$). We distinguish a realistic ($P(X = 0) = 2.44\%$ cf. Section 2.5.1.2) and a high ($P(X = 0) = 10.00\%$) level of operational variability.

Table 3.3 The available recovery options according to different levels of operational flexibility

Level of operational flexibility	Cancellations	Reassignments	
		Between-shift changes	Day-off-to-work changes
Low	✓	X	X
Medium	✓	✓	X
High	✓	✓	✓

3.4.2 Analysis and comparison of uncertainty set characterisation strategies

In this section, we analyse the uncertainty set characterisation strategies based on their planned performance in the tactical scheduling phase and their actual performance in the operational allocation phase. In the operational allocation phase, we consider a realistic level of operational variability, i.e. $P(X = 0) = 2.44\%$, and a low level of operational flexibility (cf. Table 3.3). This low level of operational flexibility enables us to clearly discern the impact of the proposed strategies on the robustness of the baseline personnel shift rosters.

We analyse the performance of the deviation measure strategies in Section 3.4.2.1, the uncertainty budget strategies in Section 3.4.2.2 and the uncertainty budget allocation strategies in Section 3.4.2.3. Moreover, we investigate the impact of the combination of strategies in Section 3.4.2.4.

3.4.2.1 The impact of deviation measure strategies (DM)

In this section, we discuss the planned and actual performance of the fixed ratio-based deviation measures (DM-FR) and the confidence interval-based deviation measures (DM-CI). Moreover, we evaluate the impact of the ratio Δ^{DM} that determines the size of the deviation measures. Unless otherwise stated, our discussion is based on average results over the different uncertainty budget strategies and percentages Δ^{UB} and over the different uncertainty budget allocation strategies and factors Δ^{UBA-BV}.

Planned performance

Table 3.4 displays the average planned performance of the deviation measure strategies over the different (TCC, DCD, SCD)-combinations and the different ratios Δ^{DM}. The planned performance is characterised by the planned cost, the planned overstaffing and the planned uncertainty budget, i.e.

- The DM-CI strategy results in a larger planned cost due to higher shortages and a higher total assignment cost. This larger total assignment cost indicates that more duties have been included in the baseline personnel shift roster constructed with the DM-CI strategy than with the DM-FR strategy.
- For both strategies, the time-related constraints (eqs. (3.3)-(3.7)) make it inevitable that a certain degree of overstaffing cannot be eliminated by ‘requirements enlargement’. Nevertheless, the planned overstaffing significantly differs between both strategies. Although the DM-CI strategy includes more working assignments in the baseline personnel shift roster,

Table 3.4 The planned performance of the deviation measure strategies

Planned Performance	DM-FR	DM-CI
Planned cost	3,757.42	3,824.86
Shortages (in shifts)	0.05	0.15
Total assignment cost	3,756.42	3,821.86
Wage cost	3,235.61	3,285.01
Preference penalty cost	520.80	536.86
Planned overstaffing	47.38	38.14
Planned uncertainty budget		
Included uncertainty budget	26.11	24.52
Requirements diminution	$(3.96, 0.60, 0.13)$	$(6.15, 0.66, 0.17)$
Requirements enlargement	$(30.21, 2.03, 0.87)$	$(33.08, 2.36, 1.89)$

this strategy results in a lower amount of overstaffing. Hence, 'requirements enlargement' has converted these additional working assignments into personnel buffers on top of the expected staffing requirements.

- Table 3.4 clearly shows that the DM-CI strategy combines a smaller 'included uncertainty budget' with a higher 'requirements diminution' and especially 'requirements enlargement' than the DM-FR strategy. This can be clarified by the definition of both strategies. The fixed ratio-based deviation measures (DM-FR) remain limited to the expected staffing requirements (eq. (3.19)) but the confidence interval-based deviation measures (DM-CI) can result in forward deviation measures that exceed the expected staffing requirements (eq. (3.20)). Indeed, the latter is defined based on the Poisson-distribution, which is asymmetric and results in large deviation measures for high ratios Δ^{DM}.

In Table 3.5, we subdivide the planned uncertainty budget according to the ratio Δ^{DM}, which determines the size of the proposed deviation measures ($\hat{R}_{dj}^{w-}$ and $\hat{R}_{dj}^{w+}$). The table shows that smaller ratios Δ^{DM} result in less and smaller 'requirements diminutions and enlargements' than larger ratios Δ^{DM}. However, these diminutions and enlargements are more equally distributed over the different shifts for smaller ratios than for larger ratios, i.e. their variance is lower. Moreover, the 'included uncertainty budget' exhibits a concave behaviour and is maximal for a ratio Δ^{DM} of 50%. This indicates that a 50% ratio provides a high number of shifts for which a large portion of the backward and forward deviation measures has been utilised for 'requirements diminutions and enlargements'.

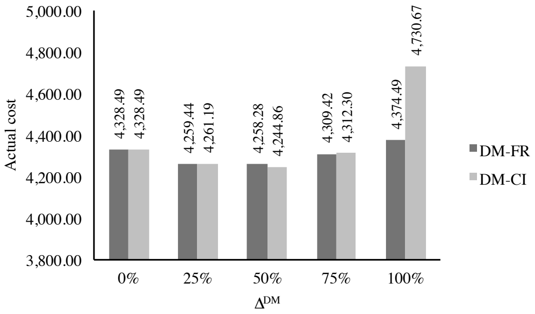
Table 3.5 The impact of the size of the deviation measures (Δ^{DM}) on the planned uncertainty budget

Δ^{DM}	0%	25%	50%	75%	100%
Planned uncertainty budget					
Included uncertainty budget	0.00	23.39	27.57	27.35	22.96
Requirements diminution	$(0.00, 0.00, 0.00)$	$(0.88, 0.22, 0.01)$	$(2.79, 0.44, 0.06)$	$(5.32, 0.67, 0.13)$	$(11.25, 1.20, 0.41)$
Requirements enlargement	$(0.00, 0.00, 0.00)$	$(23.76, 1.20, 0.07)$	$(32.69, 1.74, 0.39)$	$(34.15, 2.35, 1.00)$	$(35.99, 3.49, 4.05)$

Actual performance

Figure 3.2 displays the actual cost of the fixed ratio-based deviation measures (DM-FR) and the confidence interval-based deviation measures (DM-CI) for the different ratios Δ^{DM}. In general, the difference between both strategies is quite limited. However, the DM-FR strategy provides a significantly lower actual cost than the DM-CI strategy for a ratio Δ^{DM} of approximately 100%.[1] For this ratio, the significance level $(1 - \Delta^{DM})$ of the confidence interval (eq. (3.20)) is very small and results in large deviation measures that do not represent the variability encountered in the operational allocation phase.

Figure 3.2 The actual cost of the deviation measure strategies for the different ratios Δ^{DM}



Both strategies exhibit a convex behaviour over the different ratios Δ^{DM} and the minimum actual cost is obtained with a ratio Δ^{DM} of 50%. This is consistent with the evolution of the planned uncertainty budget. Hence, a more equal distribution of smaller ‘requirements diminutions and enlargements’ provides good results and can be obtained by limiting the size of the proposed deviation measures.

Thus, a ratio Δ^{DM} of 50% represents the best average actual cost over all simulation runs and (TCC, DCD, SCD)-combinations. However, it is important to note that as Δ^{DM} increases, the range between the minimal and maximal actual cost over all simulation runs also increases. As such, the actual performance of the personnel shift rosters becomes less stable and results in a larger variability for a ratio Δ^{DM} of 50% than a ratio Δ^{DM} of 0%, i.e. the the standard deviation and the minimal and maximal actual cost (σ, lb, ub) amount to (298.04, 3776.83, 4993.78) and (258.73, 3948.87, 4730.94) for a ratio Δ^{DM} of 50% and 0%, respectively.

3.4.2.2 The impact of uncertainty budget strategies (UB)

We analyse the planned and actual performance for the different uncertainty budget strategies and percentages Δ^{UB} in this section. We distinguish a percentage-based uncertainty budget strategy (UB-P) and a percentage and demand-based uncertainty budget strategy (UB-PD). Unless otherwise stated, this analysis is based on average results over the different deviation

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[1] p-value<0.01 (Mann-Whitney U test)

measure strategies and ratios Δ^{DM} and over the different uncertainty budget allocation strategies and factors Δ^{UBA-BV}.

Planned performance

We show the planned performance of the uncertainty budget strategies as an average over each (TCC, DCD, SCD)-combination and each percentage Δ^{UB} in Table 3.6. It is clear that, by definition (eq. (3.21)), the UB-P and UB-PD strategies differ in their planned cost, overstaffing and uncertainty budget as follows, i.e.

- The planned cost is significantly higher for the UB-P strategy than for the UB-PD strategy. This cost increase is due to a larger number of working assignments in the baseline personnel shift roster, i.e. the wage and preference penalty costs are higher.
- Irrespective of a higher number of working assignments for the UB-P strategy, the planned overstaffing is significantly larger for the UB-PD strategy. As such, the latter provides more implicit personnel buffers that are not positioned according to the 'requirements enlargement' provided by the proposed uncertainty set characterisation strategies but according to the wage and preference penalty costs of the available employees.
- The 'included uncertainty budget' for the UB-P strategy exceeds the 'included uncertainty budget' for the UB-PD strategy. This larger uncertainty budget results in more shifts with 'requirements diminutions and enlargements'.

Table 3.6 The planned performance of the uncertainty budget strategies

Planned Performance	UB-P	UB-PD
Planned cost	3,812.42	3,769.86
Shortages (in shifts)	0.04	0.16
Total assignment cost	3,811.57	3,766.71
Wage cost	3,272.06	3,248.57
Preference penalty cost	539.51	518.14
Planned overstaffing	34.92	50.60
Planned uncertainty budget		
Included uncertainty budget	31.67	18.97
Requirements diminution	(6.43, 0.75, 0.19)	(3.68, 0.51, 0.11)
Requirements enlargement	(37.46, 2.17, 1.44)	(25.83, 2.22, 1.32)

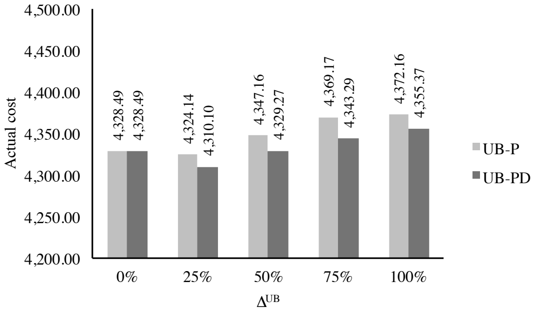
This observation is confirmed for the different (TCC, DCD, SCD)-combinations and percentages Δ^{UB}. Naturally, a rising percentage Δ^{UB} increases the 'included uncertainty budget', the number of shifts with 'requirements diminutions and enlargements', the average size of these diminutions and enlargements, and the variance of this size for both uncertainty budget strategies. As such, a higher percentage Δ^{UB} results in more, larger and more variable deviations between the stochastic and expected staffing requirements.

Actual performance

Figure 3.3 displays the actual cost of the uncertainty budget strategies for different percentages

Δ^{UB}. The figure represents the average actual cost over the different (TCC, DCD, SCD)-combinations and shows a slight advantage of the UB-PD strategy over the UB-P strategy. However, a comparison of the uncertainty budget strategies based on their interaction with deviation measure strategies and the corresponding ratios Δ^{DM} in Figure 3.4 provides a more nuanced conclusion. In fact, the UB-P strategy represents a smaller actual cost than the UB-PD strategy when the deviation measure ratio is small, i.e. Δ^{DM} is 25 or 50%. Hence, the UB-P (UB-PD) strategy is useful if there is limited (ample) opportunity to diminish or enlarge the staffing requirements in the baseline personnel shift roster. Moreover, these uncertainty budget strategies should be combined with the DM-FR strategy for small and large ratios Δ^{DM} and with the DM-CI strategy for medium ratios Δ^{DM}.

Figure 3.3 The actual cost of the uncertainty budget strategies for the different percentages Δ^{UB}

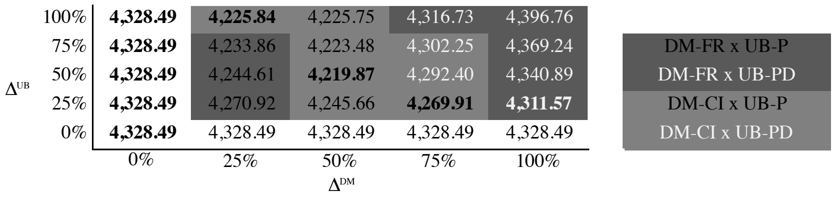


In Figure 3.3, we notice that the actual cost is minimal for an uncertainty budget percentage Δ^{UB} of 25%. This means that, in general, a positive but small available budget improves the actual cost. Similar to the impact of the ratio Δ^{DM}, we notice that the standard deviation and the range between the minimal and maximal actual cost over all simulation runs increases significantly as the percentage Δ^{UB} rises. Therefore, it is important to note that the best percentage Δ^{UB} is highly dependent on the deviation measure ratio Δ^{DM}. In Figure 3.4, we indicate in bold the best actual cost for each ratio Δ^{DM}. In order to control the variance in the 'requirements diminutions and enlargements', large uncertainty budgets (Δ^{UB}) should be combined with small deviation measures (Δ^{DM}). Small uncertainty budgets (Δ^{UB}) however, should be combined with large deviation measures (Δ^{DM}) to ensure a sufficient utilisation of 'requirements diminutions and enlargements'. Thus, the largest (i.e. high Δ^{DM} and Δ^{UB}) and smallest (i.e. low Δ^{DM} and Δ^{UB}) variability in the actual performance of personnel shift rosters should be avoided to obtain a good average actual cost.

3.4.2.3 The impact of uncertainty budget allocation strategies (UBA)

In this section, we evaluate the planned and actual performance of the uncertainty budget allocation strategies. We distinguish scaled deviation strategies (UBA-SD) and benefit value strategies

Figure 3.4 The actual cost of the best combination of deviation measure and uncertainty budget strategies for the different ratios Δ^{DM} and percentages Δ^{UB}



Δ^{UB} \ Δ^{DM}	0%	25%	50%	75%	100%
100%	**4,328.49**	**4,225.84**	4,225.75	4,316.73	4,396.76
75%	**4,328.49**	4,233.86	4,223.48	4,302.25	4,369.24
50%	**4,328.49**	4,244.61	**4,219.87**	4,292.40	4,340.89
25%	**4,328.49**	4,270.92	4,245.66	**4,269.91**	**4,311.57**
0%	**4,328.49**	4,328.49	4,328.49	4,328.49	4,328.49

(UBA-BV). For both strategies, we propose a standard allocation strategy and a strategy that considers the profile of the staffing requirements (cf. Section 3.4.1). Unless otherwise stated, the performance evaluation of both strategies is based on average results over the different deviation measure strategies and ratios Δ^{DM} and over the different uncertainty budget strategies and percentages Δ^{UB}.

Planned performance

We display the planned performance of the uncertainty budget allocation strategies in Table 3.7 as an average over each (TCC, DCD, SCD)-combination. We compare the strategies based on their planned cost, planned overstaffing and planned uncertainty budget as follows, i.e.

- The planned cost especially differs between the two benefit value strategies. This difference can be attributed to the number of scheduled working assignments, i.e. the wage cost. This number is larger but represents a smaller preference penalty cost for the UBA-BV1 strategy than for the UBA-BV2 strategy.
- Since the UBA-SD2 strategy limits the deviations between the stochastic and expected staffing requirements (cf. eq. (3.22)), the overstaffing or the implicit personnel buffers are larger for this strategy than for the standard scaled deviation strategy.
- We observe important divergences in the planned uncertainty budget between the scaled deviation strategies and between the benefit value strategies. The UBA-SD1 strategy is characterised by a higher 'included uncertainty budget', which results in more shifts with larger 'requirements diminutions and enlargements' than the UBA-SD2 strategy. The UBA-BV2 strategy provides a larger 'included uncertainty budget' than the UBA-BV1 strategy. This is reflected in 'requirements diminutions' for a larger number of shifts and in greater 'requirements enlargements' with a higher variance for a smaller number of shifts. Hence, the second benefit value strategy focuses its 'requirements enlargements' during specific shifts but simultaneously diminishes the requirements during more shifts.

The results in Table 3.7 are based on the average planned performance over all factors Δ^{UBA-BV}. A rise in this factor Δ^{UBA-BV} results in a marginal increase of the 'included uncertainty budget', which is combined with a smaller number of shifts with lower 'requirements diminutions' and more shifts with higher 'requirements enlargements'. As such, more working assignments

Table 3.7 The planned performance of the uncertainty budget allocation strategies

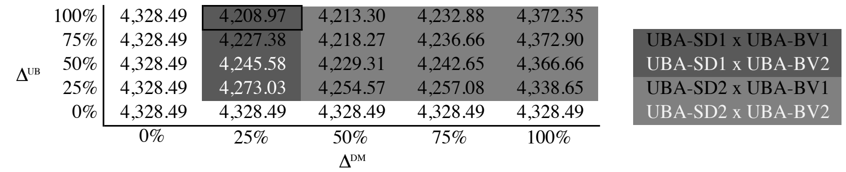
Planned Performance	UBA-SD1		UBA-SD2	
	UBA-BV1	UBA-BV2	UBA-BV1	UBA-BV2
Planned cost	3,850.24	3,762.91	3,805.01	3,746.40
Shortages (in shifts)	0.16	0.05	0.11	0.08
Total assignment cost	3,847.12	3,761.84	3,802.71	3,744.89
Wage cost	3,325.56	3,214.50	3,282.30	3,218.88
Preference penalty cost	521.56	547.33	520.41	526.01
Planned overstaffing	36.15	36.40	49.28	49.22
Planned uncertainty budget				
Included uncertainty budget	31.39	33.96	17.76	18.16
Requirements diminution	$(3.91, 0.67, 0.20)$	$(9.39, 0.78, 0.17)$	$(2.43, 0.59, 0.16)$	$(5.36, 0.59, 0.11)$
Requirements enlargement	$(38.30, 1.98, 0.94)$	$(30.46, 2.70, 2.14)$	$(31.35, 1.72, 0.72)$	$(24.74, 2.25, 1.53)$

are scheduled that become personnel buffers on top of the expected staffing requirements (R_{dj}^{w}).

Actual performance

In Figure 3.5, we display the combination of uncertainty budget allocation strategies that provides the best average actual cost over the different deviation measure and uncertainty budget strategies. The figure clearly shows that the best combination of strategies depends on the size of the deviation measures (Δ^{DM}) and the available uncertainty budget (Δ^{UB}). Small deviation measures and low uncertainty budgets require the second benefit value strategy (UBA-BV2) in combination with the standard scaled deviation strategy (UBA-SD1) to guide the allocation of the uncertainty budget to the most important shifts. Small deviation measures and high uncertainty budgets require standard uncertainty budget allocation strategies. However, we notice that for larger deviation measures, it is useful to apply the second scaled deviation strategy UBA-SD2. Hence, we can conclude that the non-standard uncertainty budget allocation strategies provide value when there is too little or too much liberty to diminish and enlarge the staffing requirements.

Figure 3.5 The actual cost of the best combination of scaled deviation and benefit value strategies for different ratios Δ^{DM} and percentages Δ^{UB}



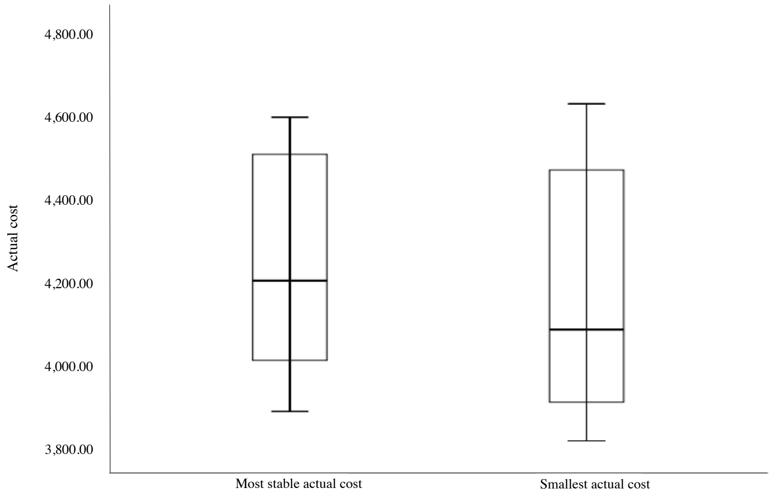
Δ^{UB}	0%	25%	50%	75%	100%
100%	4,328.49	4,208.97	4,213.30	4,232.88	4,372.35
75%	4,328.49	4,227.38	4,218.27	4,236.66	4,372.90
50%	4,328.49	4,245.58	4,229.31	4,242.65	4,366.66
25%	4,328.49	4,273.03	4,254.57	4,257.08	4,338.65
0%	4,328.49	4,328.49	4,328.49	4,328.49	4,328.49

3.4.2.4 The impact of the combination of strategies

In this section, we investigate whether the different strategies can be combined to deliver good and stable results. Figure 3.5 shows that the smallest actual cost can be obtained with the standard uncertainty budget allocation strategies and with a ratio Δ^{DM} of 25% and a percentage

Δ^{UB} of 100%. For this ratio and percentage, the best actual performance is obtained with the DM-CI deviation measure strategy and the UB-P uncertainty budget strategy (cf. Figure 3.4). Thus, this combination of strategies provides the smallest average actual cost over all simulation runs. However, this combination does not provide the most stable personnel shift rosters in the operational allocation phase. In order to improve the actual performance stability of the constructed personnel shift rosters, the percentage Δ^{UB} needs to be decreased to 50%. Figure 3.6 shows the comparison of the combination of strategies that provide the smallest actual cost and the most stable actual cost as an average over all Δ^{UBA-BV}. The figure clearly shows that the combination of strategies providing the most stable actual performance, offers a better maximal actual performance and a worse minimal actual performance. More specifically, the standard deviation and the minimal and maximal actual cost (σ, lb, ub) is (226.87, 3891.51, 4595.35) and (265.94, 3821.14, 4627.09) for the personnel shift rosters with the most stable and smallest actual cost, respectively. Furthermore, the average actual cost is significantly different[2] and entails that a premium cost of 72.07 is incurred for extra stability.

Figure 3.6 A comparison of the most stable and smallest actual performance in the operational allocation phase over all values for the factor Δ^{UBA-BV}



Irrespective of the value for the factor Δ^{UBA-BV}, these combinations of strategies actually provide the smallest and most stable actual cost. A rise in this factor results in a reduction of the actual shortages and an increase in the preference penalty cost and number of cancellations. Given that the cost reduction corresponding to the shortages outweighs the cost increase corresponding to the preference penalty cost and cancellations, the actual cost decreases as the factor Δ^{UBA-BV} rises.

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[2] p-value<0.01 (Mann-Whitney U test)

3.4.3 Validation of uncertainty set characterisation strategies

In this section, we validate the robustness of the baseline personnel shift rosters obtained through the application of uncertainty set characterisation strategies in the tactical scheduling phase (cf. Section 3.4.2). This validation is based on a comparison between the planned and actual performance of the robust rosters, which anticipate a stochastic environment in the operational allocation phase, and the minimum cost rosters, which assume a deterministic environment in the operational allocation phase. Unless otherwise stated, the uncertainty set underlying the best robust rosters is defined based on the following strategies, i.e.

- The confidence interval-based deviation measure strategy DM-CI with a ratio Δ^{DM} of 25%
- The percentage-based uncertainty budget strategy UB-P with a ratio Δ^{UB} of 100%
- The standard scaled deviation strategy UBA-SD1 and standard benefit value strategy UBA-BV1 with a factor Δ^{UBA-BV} of 16

Hence, we focus on the strategy combination that offers the smallest average actual cost to determine whether its stability in actual performance is acceptable in comparison to the minimum cost scheduling strategy.

Planned performance

Table 3.8 indicates the planned performance for two scheduling strategies, i.e. the minimum cost scheduling strategy and the robust scheduling strategy. The robust personnel shift rosters are, on average, obtained within 16.5 seconds and with an integrality gap of 0.03%, i.e. the relative difference between the solution value of the LP- and IP-solution.

The difference in the planned cost corresponding to these scheduling strategies expresses the cost of robustness. This cost equals 193.04 and is due to larger wage and preference penalty costs for the robust scheduling strategy, which indicates a higher number of working assignments. This additional cost enables the inclusion of a large uncertainty budget, which is smaller than the available budget (Δ^{UB}=100%) because certain (TCC, DCD, SCD)-combinations result in shifts that have an expected staffing requirement of zero. These shifts can therefore not possess backward and forward deviation measures differing from zero. Given the small size of the deviation measures for the other shifts (Δ^{DM}=25%), the size of the 'requirements diminutions and enlargements' remains limited. This size also exhibits a low variance because of the constant value for the factor Δ^{UBA-BV}.

Actual performance

In our analysis of the actual performance of the minimum cost and robust scheduling strategy, we distinguish different levels of operational flexibility. In correspondence to Table 3.3 (cf. Section 3.4.1), we distinguish a low, medium and high level of operational flexibility. Moreover, we distinguish two levels of operational variability based on the basic probability of absenteeism, i.e. $P(X = 0) = 2.44\%$ and $P(X = 0) = 10.00\%$.

Table 3.8 The planned performance corresponding to the minimum cost and robust scheduling strategies

Planned Performance	Minimum cost	Robust
Planned cost	3,672.15	3,865.19
Shortages (in shifts)	0.59	0.04
Total assignment cost	3,660.30	3,864.44
Wage cost	3,200.00	3,334.07
Preference penalty cost	460.30	530.37
Planned overstaffing	96.59	33.26
Planned uncertainty budget		
Included uncertainty budget	0.00	72.69
Requirements diminution	(0.00, 0.00, 0.00)	(0.70, 0.19, 0.00)
Requirements enlargement	(0.00, 0.00, 0.00)	(72.22, 1.07, 0.06)

Table 3.9 The actual performance for different tactical scheduling strategies subject to different levels of operational flexibility and variability

P(X=0)	2.44%					
Level of operational flexibility	Low		Medium		High	
Tactical scheduling strategy	Minimum cost	Robust	Minimum cost	Robust	Minimum cost	Robust
Actual cost	4,328.49	**4,154.30**	4,052.71	**3,958.43**	3,978.24	**3,913.86**
Shortages (in shifts)	37.63	23.55	17.03	9.99	6.25	5.84
Total assignment cost	3,575.86	3,683.32	3,606.73	3,679.18	3,688.95	3,676.56
Wage cost	3,126.19	3,123.60	3,126.19	3,124.46	3,139.57	3,135.59
Preference penalty cost	449.67	492.68	480.54	505.48	501.92	508.74
Number of duties cancelled	0.00	13.41	0.00	9.85	9.49	6.45
Number of duty changes	0.00	0.00	21.10	15.89	32.85	24.11
Between-shift changes	0.00	0.00	21.10	15.89	22.02	16.97
Day-off-to-work changes	0.00	0.00	0.00	0.00	10.83	7.14
Actual overstaffing	125.77	111.43	105.16	97.96	95.73	94.92
Actual uncertainty budget						
Included uncertainty budget	0.00	92.06	0.00	92.08	0.00	92.03
Requirements diminution	(0.00, 0.00, 0.00)	(19.89, 0.11, 0.01)	(0.00, 0.00, 0.00)	(19.93, 0.11, 0.01)	(0.00, 0.00, 0.00)	(20.22, 0.11, 0.01)
Requirements enlargement	(0.00, 0.00, 0.00)	(61.15, 0.17, 0.02)	(0.00, 0.00, 0.00)	(61.11, 0.17, 0.02)	(0.00, 0.00, 0.00)	(60.52, 0.17, 0.02)
P(X=0)	10.00%					
Level of operational flexibility	Low		Medium		High	
Tactical scheduling strategy	Minimum cost	Robust	Minimum cost	Robust	Minimum cost	Robust
Actual cost	4,255.56	**4,045.72**	3,960.96	**3,839.28**	3,867.56	**3,793.61**
Shortages (in shifts)	44.45	23.82	22.39	13.32	8.62	7.73
Total assignment cost	3,366.50	3,569.24	3,400.13	3,484.55	3,507.61	3,513.12
Wage cost	2,943.25	2,918.93	2,943.25	2,933.59	2,964.11	2,943.76
Preference penalty cost	423.25	492.59	456.88	484.29	484.62	490.77
Number of duties cancelled	0.00	31.54	0.00	13.34	11.77	15.72
Number of duty changes	0.00	0.00	22.59	17.67	37.50	25.18
Between-shift changes	0.00	0.00	22.59	17.67	23.64	18.50
Day-off-to-work changes	0.00	0.00	0.00	0.00	13.86	6.68
Actual overstaffing	114.30	91.24	92.24	82.20	80.55	77.62
Actual uncertainty budget						
Included uncertainty budget	0.00	50.78	0.00	95.33	0.00	95.35
Requirements diminution	(0.00, 0.00, 0.00)	(2.59, 0.07, 0.00)	(0.00, 0.00, 0.00)	(2.81, 0.07, 0.00)	(0.00, 0.00, 0.00)	(3.04, 0.07, 0.00)
Requirements enlargement	(0.00, 0.00, 0.00)	(78.93, 0.37, 0.06)	(0.00, 0.00, 0.00)	(79.30, 0.35, 0.05)	(0.00, 0.00, 0.00)	(79.04, 0.34, 0.05)

Table 3.9 provides the average actual performance of the minimum cost and robust scheduling strategies over each (TCC, DCD, SCD)-combination. In this table, we highlight the actual performance for different levels of operational flexibility and variability. It is clear that robust rosters generally provide a significantly smaller actual cost and are more robust than minimum cost rosters.[3] This cost advantage can be attributed to smaller shortages and changes. As such,

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[3] p-value$<$0.01 (Mann-Whitney U test)

the uncertainty set characterisation strategies provide baseline personnel shift rosters that are not only stable but also flexible in the operational allocation phase. Hence, the planned uncertainty budget enables the anticipation of a portion of the operational variability, which is represented by the actual uncertainty budget. Given this operational variability, the robust scheduling strategy does not only provide personnel shift rosters with a good average actual cost but also a more stable actual performance than the minimum cost scheduling strategy (cf. Figure 3.B.1 in Appendix 3.B).

The impact of the operational flexibility
The difference in the actual cost between the minimum cost and robust scheduling strategies is most significant for the lowest level of operational flexibility. For this level, we observe the largest absolute difference in terms of shortages and overstaffing. Given the limited 'requirements diminutions' in the baseline personnel shift roster (cf. Table 3.8), this indicates the proficiency of the uncertainty set characterisation strategies to introduce personnel buffers during the most appropriate shifts in the baseline personnel shift roster.

As the operational flexibility increases, more implicit adjustment possibilities become available. In order to control the number of cancellations in this case, smaller personnel buffers need to be present in the baseline personnel shift roster. This can be achieved by reducing the value of the factor Δ^{UBA-BV}, the ratio Δ^{DM} or the percentage Δ^{UB}. Nevertheless, we notice that the variability in the actual performance declines and that the beneficial impact of the uncertainty set characterisation strategies diminishes (cf. Figure 3.B.1 and Table 3.9). However, the number of changes remains lower than for the minimum cost rosters. Hence, the proposed uncertainty set characterisation strategies are not only capable of appropriately positioning the personnel buffers but also provide a fitting number of employees in these buffers.

The impact of the operational variability
The relative difference in robustness between the minimum cost and robust scheduling strategies increases as the uncertainty of capacity augments. In order to achieve this improvement, we need to include more and larger personnel buffers in the baseline personnel shift roster during the tactical scheduling phase. Therefore, the ratio Δ^{DM}, the percentage Δ^{UB} or the factor Δ^{UBA-BV} need to be augmented, which reduces the stability of the actual performance (cf. Figure 3.B.1). However, this facilitates the occurrence of a larger difference in shortages between the minimum cost and robust scheduling strategies but also causes a larger number of cancellations for the robust scheduling strategy. Since this larger number of cancellations does not offset the smaller shortage cost, the provided personnel buffers remain appropriate.

3.5 Conclusions

In this chapter, we aim to improve the robustness of baseline personnel shift rosters through the application of the principles of robust optimisation in the tactical scheduling phase. These principles are utilised to construct a baseline personnel shift roster based on the formulation of a robust counterpart. This counterpart is a deterministic formulation of a personnel shift scheduling problem that is uncertain in terms of the staffing requirements, which represent the required number of employees given the unknown personnel demand and employee availability. The deterministic formulation is based on the definition of an uncertainty set that determines the actual values the personnel demand and employee availability may obtain in the operational allocation phase.

The uncertainty set is characterised by deviation measures and an available uncertainty budget for which we define specific strategies. Moreover, we propose strategies to guide the allocation of the uncertainty budget to specific shifts, i.e. uncertainty budget allocation strategies. We validate the robustness of these strategies by comparing their performance to the performance of a deterministic minimum cost scheduling strategy in a three-step methodology. This methodology consists of roster construction in the tactical scheduling phase, day-by-day simulation of uncertainty and adjustment in the operational allocation phase, and robustness evaluation.

We can conclude that uncertainty set characterisation strategies certainly enable the construction of a baseline personnel shift roster in the tactical scheduling phase that is stable and flexible in the operational allocation phase. Generally, a combination of deviation measures defined based on confidence intervals, an uncertainty budget defined based on a percentage of the maximum possible budget and standard uncertainty budget allocation strategies provide the highest level of robustness. However, the combination of these strategies should be carefully considered such that a large number of small personnel buffers can be included.

3.A Appendix - Operational decision model

Notation

Sets

N	set of employees (index i)
S	set of shifts (index j)

General parameters

d	day under consideration in the operational planning horizon
c^{w}_{idj}	wage cost of assigning an employee i to shift j on day d
c^{wu}_{dj}	shortage cost for shift j on day d
p_{idj}	preference penalty cost if an employee i receives a shift assignment j on day d
l_j	duration of shift j
κ^{α}_{idj}	1 if employee i is allowed to receive an assignment during shift j on day d, 0 otherwise
κ^{f}_{id}	the total number of hours employee i has to receive on day d

Simulation parameters

a_{id}	1 if employee i is available on day d, 0 otherwise
R'^{w}_{dj}	simulated staffing requirements for shift j on day d

Roster change parameters

x'^{w}_{idj}	1 if employee i received a shift assignment j on day d in the baseline personnel shift roster, 0 otherwise
x'^{v}_{id}	1 if employee i received a day off on day d in the baseline personnel shift roster, 0 otherwise
$c^{w\delta}_{idj}$	roster change cost for assigning an employee i to shift j on day d with $c^{w\delta}_{idj} > 0$ if $x'^{w}_{idj} = 0$ $c^{w\delta}_{idj} = 0$ otherwise
c^{v}_{id}	duty cancellation cost for employee i on day d with $c^{v}_{id} > 0$ if $x'^{v}_{id} = 0$ and $a_{id} = 1$ $c^{v}_{id} = 0$ otherwise

Variables

x^{w}_{idj}	1 if employee i receives a shift assignment j on day d, 0 otherwise
x^{v}_{id}	1 if employee i receives a day off on day d, 0 otherwise
x^{wu}_{dj}	the shortage of employees for shift j on day d

Mathematical formulation

$$\min \quad \sum_{i\in N}\sum_{j\in S}(c^{w}_{idj} + c^{w\delta}_{idj} + p_{idj})x^{w}_{idj} + \sum_{i\in N} c^{v}_{id}x^{v}_{id} + \sum_{j\in S} c^{wu}_{dj}x^{wu}_{dj} \tag{3.A.1}$$

$$\sum_{i\in N} x^{w}_{idj} + x^{wu}_{dj} \geq R'^{w}_{dj} \qquad \forall j \in S \tag{3.A.2}$$

$$\sum_{j\in S} x^{w}_{idj} \leq a_{id} \qquad \forall i \in N \tag{3.A.3}$$

$$\sum_{j\in S} x^{w}_{idj} + x^{v}_{id} = 1 \qquad \forall i \in N \tag{3.A.4}$$

$$x^{w}_{idj} \leq \kappa^{\alpha}_{idj} \qquad \forall i \in N, \forall j \in S \tag{3.A.5}$$

$$\sum_{j\in S} l_j x^{w}_{idj} \geq \kappa^{f}_{id}a_{id} \qquad \forall i \in N \tag{3.A.6}$$

$$\begin{aligned} x^w_{idj} &\in \{0,1\} \quad \forall i \in N, \forall j \in S \\ x^v_{id} &\in \{0,1\} \qquad\quad \forall i \in N \\ x^{wu}_{dj} &\geq 0 \qquad\qquad\ \forall j \in S \end{aligned} \tag{3.A.7}$$

In the operational decision model, the objective (3.A.1) is to minimise the wage cost, roster change cost and preference penalty cost associated with an assignment of a duty. Moreover, the cancellation cost and the cost for understaffing needs to be optimised. The objective function weights are as follows, i.e.

- Every employee has a wage cost (c^w_{idj}) of 10.
- The roster change cost ($c^{w\delta}_{idj}$) and the duty cancellation cost (c^v_{id}) depend on the level of operational flexibility (Table 3.10). Note that the low and medium operational flexibility correspond to the fixed and adjustable reactive mechanism defined in Chapter 2.
- Every employee has a preference penalty cost for each duty (p_{idj}), which is randomly generated in the range of 1 to 5.
- The duty cancellation cost (c^v_{id}) amounts to 5.
- The cost of understaffing (c^{wu}_{dj}) is 20.

Table 3.10 The roster change cost and the duty cancellation cost according to different levels of operational flexibility

Level of operational flexibility	Cancellations	Reassignments	
		Between-shift changes	Day-off-to-work changes
Low	5	100	100
Medium	5	5	100
High	5	5	5

The staffing requirements are imposed in constraint (3.A.2). Each employee can receive a maximum of one shift assignment if (s)he is available (eq. (3.A.3)) and needs to receive a shift assignment or a day off (eq. (3.A.4)). Moreover, each assignment is subject to the time-related constraints in equations (3.4)-(3.7). The parameter κ^{α}_{idj} (eq. (3.A.5)) determines which assignments are feasible in terms of the minimum rest period (eq. (3.4)), the maximum number of hours that can be assigned to an employee (eq. (3.5)) and the maximum number of consecutive working assignments (eq. (3.7)). In equation (3.A.6), the parameter κ^{f}_{id} determines the number of hours the employee needs to work on day d to ensure a minimum number of hours over the complete planning horizon (eq. (3.6)). Note that we account for the availability of the employee (a_{id}) and diminish the minimum number of hours an employee needs to work if this employee is unavailable on a working day. As such, an absent employee does not need to catch up a duty because of an unavailability during a working day. Finally, equation (3.A.7) determines the integrality conditions of the operational variables.

3.B Appendix - Boxplot of the actual performance for different tactical scheduling strategies

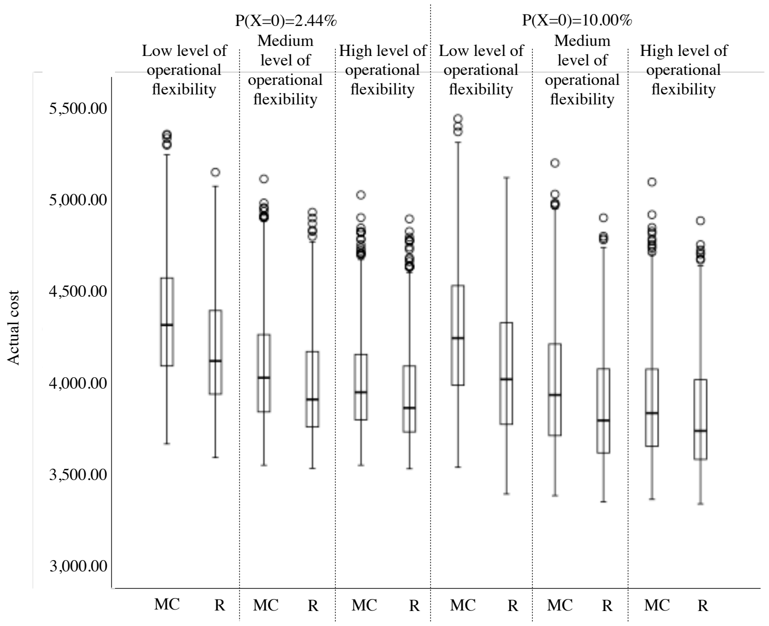


Figure 3.B.1 Boxplot of the actual performance for different tactical scheduling strategies subject to different levels of operational flexibility and variability (MC: minimum cost scheduling strategy, R: robust scheduling strategy)

4

Employee substitutability as a tool to improve the robustness in personnel scheduling

Organisations usually construct personnel rosters under the assumption of a deterministic operating environment. In the short term however, organisations operate in a stochastic environment as operational variability arises. This variability leads to the occurrence of unexpected events such as employee absenteeism and/or a demand for personnel that is higher or lower than expected. In order to deal with these uncertainties, organisations need to adopt proactive and reactive scheduling strategies to protect the personnel roster and to respond to this operational variability respectively. In this chapter, we discuss a proactive approach that exploits the concept of employee substitutability to improve the flexibility of a personnel shift roster to respond to schedule disruptions. We propose a preemptive programming approach to construct a medium-term personnel shift roster that maximises the employee substitutability value. Moreover, we assess different proactive strategies to introduce robustness with respect to the definition and formulation of employee substitutability and different reactive strategies that impact the decision freedom for schedule recovery. The robustness of the generated personnel shift rosters is evaluated using a three-step methodology of roster construction, daily simulation and optimisation, and evaluation.

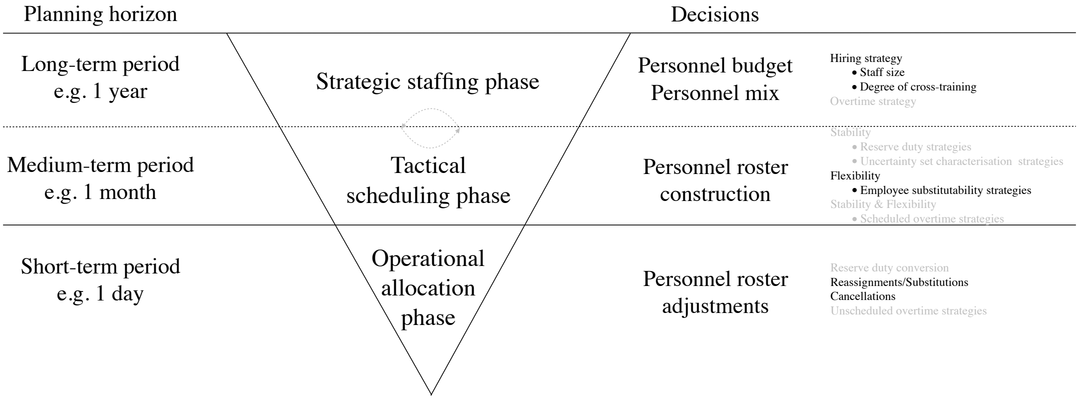


Figure 4.1 The research focus in Chapter 4

4.1 Introduction

In this chapter, we focus on the personnel shift scheduling problem, which encompasses the assignment of a multi-skilled workforce to cover the shift demand for a medium-term period. We investigate how the short-term adjustment capability or flexibility embedded in the personnel roster may be improved proactively by maximising the employee substitutability value. In general, a substitution exists if an employee can take over the skill-shift assignment of another employee on a particular day. In this respect, we discuss three mechanisms to perform a personnel substitution, i.e. a between-skill substitution, a within-skill substitution and a day-off-to-work substitution. We consider and analyse different ways to define, model and measure employee substitutability. As such, we define employee substitutability on the level of an individual employee and on the group level where the degree of cross-training of the workforce assigned to a particular shift is considered. Moreover, we propose and compare different strategies to model employee substitutability and to measure the number of substitution possibilities and the corresponding employee substitutability value.

We provide guidelines to define the tactical decision process as a better proxy for the short-term operating environment by considering the operational variability. Therefore, we validate the improved roster robustness that is achieved by maximising the employee substitutability value with a three-step methodology. In the *first* step, a personnel shift roster is constructed by applying a two-phase preemptive programming approach to evaluate the trade-off between robustness and the cost of robustness. In the first phase, we construct a personnel shift roster solely considering the objective of cost minimisation. In the second phase, a proactive strategy is applied to hedge against uncertainty as we maximise the employee substitutability value of the personnel shift roster given a restriction on the allowable cost increase on top of the minimum cost corresponding to the minimum cost personnel shift roster. The *second* step imitates the operational allocation phase for each day of the planning horizon and alternates between a simulation step and a reactive roster adjustment step in which certain types of personnel substitutions are executed. In a *third* step, the robustness of the original personnel shift roster is evaluated based on the planned and actual performance. The second and third step are repeated multiple times to obtain a clear idea of the obtained robustness. Computational experiments are conducted both in a real-life and an artificial setting to investigate the impact of different demand profiles.

The remainder of this chapter is organised as follows. In Section 4.2, we define the three-step methodology, which includes the tactical scheduling phase, the operational allocation phase and the robustness evaluation. We describe different types of employee substitutions and formulate the personnel shift scheduling problem to maximise the employee substitutability value on an individual and group level. In Section 4.3, we define and explain the two-phase preemptive approach to construct a medium-term personnel shift roster. The test design, test instances and computational experiments are discussed in Section 4.4. We provide conclusions in Section 4.5.

4.2 Problem definition, formulation and methodology

We investigate how the flexibility of a personnel shift roster may be increased as a result of improving the employee substitutability of a heterogeneous multi-skilled workforce in order to recover efficiently from schedule disruptions. In this study, we extend the idea of swaps in a personnel shift scheduling context. Ionescu and Kliewer (2011) and Shebalov and Klabjan (2006) study the concept of swaps in an airline scheduling context as a possible reaction to delays due to arrival uncertainty. A swap is a two-direction mechanism that exchanges duties between employees, i.e. employees that have a later assignment are assigned to an earlier assignment while the employees from a disrupted assignment are assigned to the later assignment. In airline scheduling, duties may have different start times and durations, which are subject to operational variability. Therefore, it can be useful to swap duties between employees. In the context of a shift scheduling problem, this practice is often not useful because the assigned duties have fixed start and end times. An unexpected increase in demand during a particular shift for example, does not benefit from a swap between employees because this would not resolve the understaffing. In this case, an employee needs to be reassigned from an overstaffed shift or from a day off to the understaffed shift, i.e. a substitution needs to be performed.

Therefore, we study how and to which extent employee substitutions can serve as a proactive methodology to improve the robustness of a personnel shift roster. In this respect, we maximise the employee substitutability value to add flexibility to the decision-making process in the operational allocation phase. The personnel scheduling problem under study, which aims to create an increased flexibility by including employee substitutability in the objective function, is relevant for many application areas to facilitate the recovery from disruptions. An important application lies in employee *self-scheduling* (Bailyn et al., 2007) where the management typically first proposes a feasible (minimum cost) roster indicating the line-of-work for each individual employee. Subsequently, employees are able to adapt their own schedule. Bailyn et al. (2007), however, identified that employees are not able to adapt their own schedule taking the stipulated (time-related) rules and regulations into account. In this way, self-scheduling may lead to conflicts or even infeasible schedules that need to be resolved by the employees or the management. The resolution of these conflicts can be executed more effectively and efficiently if employee substitutability is proactively considered when the personnel roster is constructed.

In this chapter, we utilise a three-step methodology to validate the impact of employee substitutability on the robustness of a personnel shift roster, i.e.

- In the first step, i.e. the (tactical) scheduling phase, a baseline personnel shift roster is constructed for a medium-term period based upon a proactive strategy. A new objective is included to optimise the employee substitutability. We provide the description and model formulation of the problem under study in Section 4.2.1.
- In the second step, we start from the baseline personnel shift roster and imitate the operational allocation phase for each day of the planning horizon (Section 4.2.2). This operational phase consists of a simulation component and a reactive adjustment component (cf. Chapter 2). Since the planning horizon is small, the adjustment capability heavily depends on the built-in robustness or flexibility, i.e. the employee substitutability.
- In the third step, we evaluate the robustness of the baseline personnel shift roster through a comparison of the planned and actual performance of the roster (Section 4.2.3).

The general methodology of validating the proactively obtained robustness by imitating the reactive operational phase is similar to the approach of Abdelghany et al. (2008) and Bard and Purnomo (2005a) and is based on the methodology in Chapter 2.

4.2.1 Tactical scheduling phase

We study a general tactical shift scheduling problem that assigns employees to shifts during the scheduling phase. The shift assignments cover multiple categorical skills, which means that employees are fully capable of executing a duty in correspondence with their skills and the individual skills cannot be hierarchically ranked (De Bruecker et al., 2015). The personnel characteristics and shift characteristics are common in personnel scheduling literature (Burke et al., 2004; Van den Bergh et al., 2013) and the model can be categorised as $AS\,2|RV\,|S||LRG$ according to the classification of De Causmaecker and Vanden Berghe (2011).

In this phase, we introduce the concept of employee substitutability on the level of an individual employee (Section 4.2.1.1) and on a group level (Section 4.2.1.2). Individual employee substitutability depends on the value and number of substitution possibilities while group employee substitutability depends on the degree of cross-training of the assigned workforce. Both problems are modelled with a multi-criteria objective function, simultaneously optimising the personnel assignment costs (i.e. the wage cost and preference penalty cost), the understaffing cost and the individual or group employee substitutability value. We utilise the following mathematical notation to include employee substitutability in personnel shift rosters, i.e.

Notation

Sets

G	set of skills (index m)
N	set of employees (index i)
D	set of days (index d)
S	set of shifts (index j)
$T^{'}_{dj}$	set of shifts that cannot be assigned the day before day d and shift assignment j (index s)
$T^{''}_{dj}$	set of shifts that cannot be assigned the day after day d and shift assignment j (index s)

Parameters

b_{im}	1 if employee i possesses skill m, 0 otherwise
R^{w}_{mdj}	minimum staffing requirements for shift j, day d and skill m
$R^{w,+}_{mdj}$	additional group staffing requirements for shift j, day d and skill m
c^{w}_{imdj}	wage cost of assigning an employee i to shift j, day d and skill m
c^{wu}_{mdj}	shortage cost of the minimum staffing requirements for shift j, day d and skill m
$c^{wu,+}_{mdj}$	shortage cost of the group staffing requirements for shift j, day d and skill m
p_{idj}	preference penalty cost if an employee i receives a shift assignment j on day d
γ_{mdj}	the benefit value of a substitution possibility for shift j, day d and skill m
l_j	duration of shift j
$\eta^{w,min}_{i}$	minimum number of hours that need to be assigned to employee i
$\eta^{w,max}_{i}$	maximum number of hours that can be assigned to employee i
$\theta^{w,max}_{i}$	maximum number of consecutive working assignments for employee i
ϵ	maximum number of substitution possibilities an employee can offer on a day
β_{mdj}	maximum number of substitution possibilities for shift j, day d and skill m

Variables

x^{w}_{imdj}	1 if employee i receives a shift assignment j for skill m on day d, 0 otherwise
x^{v}_{id}	1 if employee i receives a day off on day d, 0 otherwise
x^{wu}_{mdj}	the shortage of employees for shift j, day d and skill m (w.r.t. the minimum staffing requirements (eq. (4.2)))
$x^{wu,+}_{mdj}$	the shortage of employees for shift j, day d and skill m (w.r.t. the group staffing requirements (eq. (4.14)))
z_{imdj}	1 if a substitution possibility exists for employee i for shift j, day d and skill m, 0 otherwise

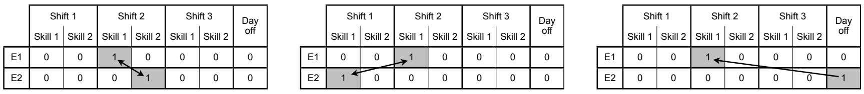
4.2.1.1 Individual employee substitutability

The consideration of employee substitutability on an individual employee level during the construction of a personnel shift roster entails a maximisation of the weighted sum of the value of substitution possibilities between employees. This means that not only the number of substitution possibilities is taken into account but also the weight of the specific (skill, day, shift)-combinations in the personnel shift roster. Hence, we aim to optimise the number and position of the introduced substitution possibilities for a particular skill.

A substitution is defined as the possibility of an employee to take over the assignment of another employee on the same day. We distinguish and investigate three different types of substitutions, which are illustrated in Figure 4.2. These types are defined as follows:

(a) A *between-skill substitution* indicates that a working employee can be reassigned to another skill during the same or another shift on the same day (Figure 4.2(a)).

(b) A *within-skill substitution* is the potential reassignment of an employee from one shift to another shift within the same skill category on the same day (Figure 4.2(b)).



	Shift 1		Shift 2		Shift 3		Day off
	Skill 1	Skill 2	Skill 1	Skill 2	Skill 1	Skill 2	
E1	0	0	1	0	0	0	0
E2	0	0	0	1	0	0	0

(a) *Between-skill substitutions*

	Shift 1		Shift 2		Shift 3		Day off
	Skill 1	Skill 2	Skill 1	Skill 2	Skill 1	Skill 2	
E1	0	0	1	0	0	0	0
E2	1	0	0	0	0	0	0

(b) *Within-skill substitutions*

	Shift 1		Shift 2		Shift 3		Day off
	Skill 1	Skill 2	Skill 1	Skill 2	Skill 1	Skill 2	
E1	0	0	1	0	0	0	0
E2	0	0	0	0	0	0	1

(c) *Day-off-to-work substitutions*

Figure 4.2 Substitution types

(c) A *day-off-to-work substitution* consists of the potential reassignment of an employee with a day off to a working shift subject to his/her competencies (Figure 4.2(c)).

These substitution types are taken into account in the following mathematical model, i.e.

$$\min \sum_{i\in N}\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}(c^{w}_{imdj}+p_{idj})x^{w}_{imdj}+\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}c^{wu}_{mdj}x^{wu}_{mdj} \tag{4.1.1}$$

$$+\sum_{i\in N}\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}\gamma_{mdj}(1-z_{imdj}) \tag{4.1.2}$$

In this study, we optimise the personnel assignment costs and the understaffing of the minimum staffing requirements in objective (4.1.1). Simultaneously, we maximise the employee substitutability value on an individual employee level in objective (4.1.2). The personnel assignment costs include the wage cost (c^{w}_{imdj}) and preference penalty cost (p_{idj}) associated with the assignment of an employee to a duty.

$$\sum_{i\in N} b_{im}x^{w}_{imdj}+x^{wu}_{mdj}\geq R^{w}_{mdj} \qquad \forall m\in G, \forall d\in D, \forall j\in S \tag{4.2}$$

The staffing requirements in equation (4.2) define the number of employees that are required to meet the demand for every skill category, day and shift. This constraint is relaxed by allowing understaffing.

$$\sum_{m\in G}\sum_{j\in S}x^{w}_{imdj}+x^{v}_{id}=1 \qquad \forall i\in N, \forall d\in D \tag{4.3}$$

$$\sum_{m\in G}x^{w}_{imdj}+\sum_{m\in G}\sum_{s\in T''_{dj}}x^{w}_{im(d+1)s}\leq 1 \qquad \forall i\in N, \forall d\in D, \forall j\in S \tag{4.4}$$

Equation (4.3) postulates that an employee should be assigned to either a duty or a day off. The minimum rest period between consecutive shift assignments is ensured by constraint (4.4).

$$\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}l_{j}x^{w}_{imdj}\leq \eta^{w,max}_{i} \qquad \forall i\in N \tag{4.5}$$

$$\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}l_{j}x^{w}_{imdj}\geq \eta^{w,min}_{i} \qquad \forall i\in N \tag{4.6}$$

$$\sum_{d'=d}^{d+\theta^{w,max}_{i}}(1-x^{v}_{id'})\leq \theta^{w,max}_{i} \qquad \forall i\in N, \forall d\in D \tag{4.7}$$

The other time-related constraints include the maximum (eq. (4.5)) and minimum (eq. (4.6)) number of hours for every employee. Furthermore, the number of consecutive working assignments is limited by equation (4.7).

$$x^{w}_{imdj} + z_{imdj} \leq b_{im} \qquad \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \tag{4.8}$$

Equation (4.8) ensures that a substitution possibility only exists if the employee possesses the required skill and if the employee does not work that particular assignment.

$$z_{imdj} \leq 1 - \sum_{m' \in G} \sum_{s \in T''_{dj}} x^{w}_{im'(d+1)s} \qquad \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \tag{4.9a}$$

$$z_{imdj} \leq 1 - \sum_{m' \in G} \sum_{s \in T'_{dj}} x^{w}_{im'(d-1)s} \qquad \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \tag{4.9b}$$

Equation (4.9) ensures that there is no substitution possibility with another shift that would violate the minimum rest period with the assignment on the next day (eq. (4.9a)) or the assignment on the previous day (eq. (4.9b)).

$$\sum_{m \in G} \sum_{j \in S} z_{imdj} \leq \epsilon \qquad \forall i \in N, \forall d \in D \tag{4.10}$$

Equation (4.10) limits the daily number of substitution possibilities each employee has to offer to ϵ and helps to model the way the substitution possibilities are counted.

$$\sum_{i \in N} z_{imdj} \leq \beta_{mdj} \qquad \forall m \in G, \forall d \in D, \forall j \in S \tag{4.11}$$

Equation (4.11) is a constraint that limits the number of substitution possibilities with respect to a particular shift, day and skill. This constraint helps to model how substitution possibilities should be included in the baseline personnel shift roster.

$$\begin{aligned} x^{w}_{imdj} &\in \{0,1\} && \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \\ x^{v}_{id} &\in \{0,1\} && \forall i \in N, \forall d \in D \\ x^{wu}_{mdj} &\geq 0 && \forall m \in G, \forall d \in D, \forall j \in S \\ z_{imdj} &\in \{0,1\} && \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \end{aligned} \tag{4.12}$$

Constraints (4.12) embody the integrality conditions.

The algorithm to optimise this problem is discussed in Section 4.3. In order to optimise the employee substitutability for the individual and combined substitution types, we propose value strategies to model objective (4.1.2) and counting strategies to model constraints (4.10)-(4.11). More information on how these strategies are employed to increase the employee substitutability and to obtain a good proxy of the operational decision phase in the tactical scheduling phase is provided in Section 4.4.1.

4.2.1.2 Group employee substitutability

We investigate group employee substitutability by requiring an additional number of employees to be on duty that are able to carry out a specific skill on top of the minimum staffing requirements during a particular shift. In line with the research of Campbell (1999) and Olivella and Nembhard (2016), employee substitutability on a group level is highly impacted by the degree of cross-training of the working employees. In this respect, Campbell (1999) investigates the utility of cross-utilisation of cross-trained employees over different departments to reactively deal with demand variability at the start of a shift while Olivella and Nembhard (2016) aim to proactively determine the optimal level of cross-training in work teams to deal with demand variability and employee absenteeism.

In this chapter, we aim to proactively schedule a higher number of skilled workers on duty, which can be obtained in different ways. First, more skilled employees are assigned to the specific duty than minimum required and a capacity buffer is created. The appropriate buffer size and positioning of capacity buffers has been investigated in Chapter 2. Second, a higher number of skilled employees is attained by assigning multi-skilled employees to other skill-duties during the same shift. The selected option is dependent upon the imposed constraints and the trade-off in the objective function, i.e. the extra wage cost for scheduling an additional duty on top of the minimum staffing requirements versus the cost of scheduling a more expensive multi-skilled worker. As such, the provided group employee substitutability gives an indication of the available between-skill, within-skill and day-off-to-work substitutions in the operational allocation phase. When the additional required number of skilled employees (i.e. the group staffing requirements) is low, a smaller number of workers are on duty and/or the more expensive multi-skilled employees are preferably assigned to a day off. As such, the baseline personnel shift roster embeds a higher flexibility in terms of day-off-to-work substitutions. When the additional required number of skilled employees increases, a higher number of multi-skilled workers is assigned and a higher flexibility is introduced in terms of between- and within-skill substitutions.

We formulate group employee substitutability in the following mathematical model, i.e.

$$\min \quad \sum_{i\in N}\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}(c^{w}_{imdj}+p_{idj})x^{w}_{imdj}+\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}c^{wu}_{mdj}x^{wu}_{mdj} \tag{4.13.1}$$

$$+\sum_{m\in G}\sum_{d\in D}\sum_{j\in S}c^{wu,+}_{mdj}x^{wu,+}_{mdj} \tag{4.13.2}$$

Similar to the problem that considers individual employee substitutability (Section 4.2.1.1), we optimise the personnel assignment costs and the understaffing of the staffing requirements in objective (4.13.1). Additionally, we minimise the deviation from the desired degree of group employee substitutability, i.e. the shortage in the desired number of employees on duty able to carry out a specific skill in objective (4.13.2).

$$\sum_{i \in N} b_{im} \sum_{m' \in G} x^{w}_{im'dj} + x^{wu,+}_{mdj} \geq R^{w}_{mdj} + R^{w,+}_{mdj} \qquad \forall m \in G, \forall d \in D, \forall j \in S \tag{4.14}$$

Equation (4.14) determines the required number of employees on duty with a skill m during shift j on day d, which is the sum of the minimum staffing requirements and the additional group staffing requirements. In contrast to the minimum staffing requirements (eq. (4.2)), where each employee is accounted to carry out a single duty and skill along with his assignment, the group staffing requirements (eq. (4.14)) consider the overall number of skilled employees on duty during a particular shift since employees with multiple skills contribute to multiple skill categories. As such, this equation specifies the employee substitutability on a group level and the available degree of cross-training of the assigned workforce.

Note that this equation, which aims to improve the flexibility of the personnel shift roster in a multi-skilled operating environment via an increase of the staffing requirements for a specific skill, is conceptually based on the work in Chapter 2.

Apart from the cross-training objective and related constraints, we also include the minimum staffing requirements (eq. (4.2)) and the time-related constraints imposed on an employee schedule (eqs. (4.3)-(4.7)) in this model.

$$\begin{aligned} x^{w}_{imdj} &\in \{0,1\} & \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \\ x^{v}_{id} &\in \{0,1\} & \forall i \in N, \forall d \in D \\ x^{wu}_{mdj} &\geq 0 & \forall m \in G, \forall d \in D, \forall j \in S \\ x^{wu,+}_{mdj} &\geq 0 & \forall m \in G, \forall d \in D, \forall j \in S \end{aligned} \tag{4.15}$$

Constraints (4.15) embody the integrality conditions.

We optimise this problem by applying the commercial software package Gurobi (Gurobi Optimization, Inc., 2015).

4.2.2 Operational allocation phase

In this short-term phase, we consider the baseline personnel shift roster on a day-by-day basis (cf. Chapter 2). In order to imitate this phase, we first simulate the ad-hoc variability for one particular day, which potentially leads to unexpected changes in the minimum staffing requirements and employee availability. Second, the simulated variability may require adjustments to the baseline personnel shift roster to restore its workability and/or feasibility. The mathematical formulation of this operational decision model is added in appendix 4.A. The recourse structure to deal with schedule disruptions contains the following reactive strategies:

- The *reassignment* of duties scheduled in the baseline personnel shift roster is the common reactive allocation strategy that is applied for schedule recovery. We consider the following types of reassignments: (a) a between-skill substitution, (b) a within-skill substitution and (c) a day-off-to-work substitution.

- The *cancellation* of duties that are superfluous on top of the actual demand for staff.

4.2.3 Robustness evaluation

We evaluate the robustness of the personnel shift rosters based on the planned and actual performance (cf. Chapter 2). Table 4.1 displays the different components and subcomponents based upon which the planned and actual performance is assessed.

Table 4.1 Building blocks of the planned and actual performance

PLANNED PERFORMANCE	ACTUAL PERFORMANCE
Planned cost	**Actual cost**
Shortages (in shifts)	Shortages (in shifts)
Total assignment cost	Total assignment cost
Wage cost	Wage cost
Preference penalty cost	Preference penalty cost
	Number of duties cancelled
Planned substitutability value	**Utilised substitutions**
Between-skill substitutions	Between-skill substitutions
Within-skill substitutions	Within-skill substitutions
Day-off-to-work substitutions	Day-off-to-work substitutions
Planned cross-training	
Available cross-training (CT)	
Cross-training surplus (CT^+)	
Cross-training shortage (CT^-)	
Planned overstaffing	**Actual overstaffing**

The planned performance is defined by the planned cost, the planned substitutability value or the planned cross-training, and the planned overstaffing corresponding to the personnel shift roster constructed in the tactical scheduling phase (Section 4.2.1). The planned substitutability value is determined by the value and number of between-skill, within-skill and day-off-to-work substitution possibilities. We define the available cross-training, cross-training surplus and cross-training shortage in equations (4.16), (4.17) and (4.18) respectively. The cross-training surplus (shortage) indicates the surplus (shortage) of skills of the working employees compared to the minimum staffing requirements.

$$CT = \sum_{i \in N} \sum_{m \in G} b_{im} \sum_{d \in D} (1 - x^v_{id}) \tag{4.16}$$

$$CT^{+} = \sum_{m \in G} \sum_{d \in D} \sum_{j \in S} max(\sum_{i \in N} b_{im} \sum_{m' \in G} x^{w}_{im'dj} - R^{w}_{mdj}, 0) \tag{4.17}$$

$$CT^{-} = \sum_{m \in G} \sum_{d \in D} \sum_{j \in S} max(R^{w}_{mdj} - \sum_{i \in N} b_{im} \sum_{m' \in G} x^{w}_{im'dj}, 0) \tag{4.18}$$

The building blocks comprising the actual performance are similar to those of the planned performance but there are important differences. First, the cancelled duties are incorporated in the actual cost. Second, instead of listing the number of substitution possibilities for the planned performance, we list the number of adjustments or the utilised substitution possibilities in the operational phase. Since adjustments are made on the level of an individual employee in the operational phase, these metrics are also used to evaluate the personnel shift rosters with optimised substitutability on the group level.

4.3 A two-phase preemptive programming approach for individual employee substitutability

Many exact and heuristic solution methodologies have been proposed in literature to construct personnel shift rosters (Burke et al., 2004; Ernst et al., 2004a,b; Van den Bergh et al., 2013). These solution methodologies often utilise some type of decomposition to reduce the size and complexity of the problem (Van den Bergh et al., 2013). In this respect, a complex problem can be partitioned into a series of smaller and more manageable (sub-)problems or phases. These phases solve distinct parts of the original problem and differ in the constraints and/or objectives they consider. Decomposition is applied in solution methodologies such as preemptive programming (Shebalov and Klabjan, 2006; Topaloglu and Ozkarahan, 2004), branch-and-bound (Trivedi and Warner, 1976), column generation (Bard and Purnomo, 2005b) and branch-and-price (Maenhout and Vanhoucke, 2010). In this respect, Shebalov and Klabjan (2006) build robust rosters using a two-phase approach by minimising the crew costs in a first phase and maximising the number of move-up crews within the limits of an allowable cost increase in a second phase.
In the tactical scheduling phase, we propose a preemptive programming approach to construct medium-term personnel shift rosters. In this respect, we utilise a two-phase methodology to solve model (4.1)-(4.12) as follows, i.e.

- In the first phase, we focus solely on cost minimisation. As such, we construct the minimum cost personnel shift roster by solving the model comprising the cost objective (4.1.1), the minimum staffing requirements (eq. (4.2)) and the time-related constraints (eqs. (4.3)-(4.7)). Equation (4.1.1) determines the total cost (c^{total}) of the constructed roster.
- In the second phase, we adapt the minimum cost personnel shift roster and focus on the maximisation of the value of individual employee substitutability. We solve a model that

consists of the substitutability objective (4.1.2), minimum staffing requirements (eq. (4.2)), time-related constraints (eqs. (4.3)-(4.7)) and substitutability constraints (eqs. (4.8)-(4.11)). Moreover, we add a constraint to limit the total cost (eq. (4.19)), i.e. the cost of the robust personnel shift roster may not exceed $c^{total} \times (1 + \tau)$. This cost constraint enables the management of the allowable cost increase (τ) and facilitates the investigation of the trade-off between the additional flexibility and the cost of this extra robustness, i.e. the cost difference between the minimum cost personnel shift roster (first phase) and the more robust personnel shift roster (second phase) (Bertsimas and Sim, 2004). Hence, we utilise the minimum cost personnel shift roster as an initial solution and expand it to construct a roster with optimised employee substitutability within a given cost constraint.

$$\sum_{i \in N} \sum_{m \in G} \sum_{d \in D} \sum_{j \in S} (c^{w}_{imdj} + p_{idj}) x^{w}_{imdj} + \sum_{m \in G} \sum_{d \in D} \sum_{j \in S} c^{wu}_{mdj} x^{wu}_{mdj} \leq c^{total} \times (1 + \tau) \tag{4.19}$$

In correspondence to Shebalov and Klabjan (2006), these two phases are solved separately and result in a personnel shift roster with a maximal employee substitutability value for a given cost of robustness. The first phase is solved to optimality by applying the standard branch-and-bound procedure comprised in the commercial optimisation software Gurobi (Gurobi Optimization, Inc., 2015). For the second phase, we employ a dedicated procedure to maximise employee substitutability since commercial software fails to provide optimal solutions within a reasonable timeframe (3600s). This is due to the symmetries in the problem structure, which make it difficult for a non-dedicated procedure to prove optimality of a solution. The applied procedure consists of a truncated branch-and-bound procedure followed by a branch-and-price procedure, i.e.

- The truncated branch-and-bound procedure aims to improve the minimum cost roster by constructing a better incumbent solution in terms of employee substitutability. In this respect, a standard branch-and-bound thriving on commercial software (Gurobi Optimization, Inc., 2015) is truncated after reaching a time limit of 120s. This results in a strong upper bound, which is input to the branch-and-price procedure to speed up its performance.
- Next, the employee schedules obtained with the truncated branch-and-bound are utilised as the initial solution in a branch-and-price procedure (Maenhout and Vanhoucke, 2010). This procedure is applied to improve the MIP-gap between the lower and upper bound. This procedure is stopped when the MIP-gap shrinks to a value below 0.1%. Since this gap is very small, we are able to report optimal and/or near-optimal solutions in a reasonable timeframe.

4.4 Computational experiments

In this section, we provide computational insight into our methodology to improve the robustness of personnel shift rosters. In Section 4.4.1, we describe the characteristics of the test problem instances and discuss the parameter settings of the optimisation models. We evaluate the planned and actual performance of different strategies to model employee substitutability in Section 4.4.2. In this section, we consider a test design that consists of a set of artificially generated test instances. As such, we analyse the impact of different demand profiles and skill possession settings of the workforce to provide specific guidelines to increase the robustness of a personnel shift roster. In Section 4.4.3, we investigate the variability of the obtained results to validate the robustness provided by three scheduling strategies for constructing a baseline personnel shift roster, i.e. the minimum cost, individual employee substitutability and group employee substitutability strategy. In order to confirm the delivered robustness in a real-life setting, we focus on test instances with a real-life demand profile. All tests were carried out on an Intel Core processor 2.5 GHz and 4GB RAM. Model (4.1)-(4.12) is solved by the proposed preemptive programming approach to construct a personnel shift roster with an optimised individual employee substitutability. The average CPU time is 354 seconds with a median of 3.13 seconds. This means that we observe a number of outliers that require a high CPU time to obtain an average optimality gap of 0.0015%. Other models were solved to optimality within smaller CPU times.

4.4.1 Test design

In this section, we provide detailed information on the design of the generated problem instances (*Artificial set*) and the characteristics of a set of instances based on a real-life demand profile (*Real-life set*). In addition, we discuss the underlying parameter settings of the personnel and shift characteristics (Section 4.4.1.1), the constraints (Section 4.4.1.2) and the objectives (Section 4.4.1.3) of the different models.

4.4.1.1 Personnel and shift characteristics

The artificial test instances consist of 20 employees and have a planning horizon of 28 days while the real-life test instances consist of a varying number of employees and a 30-day planning horizon.

Personnel characteristics - Both sets of instances contain a maximum of 2 skills for each employee. In total, we categorise 11 skill possession settings according to the triplet (m_1% - m_2% - $m_{1,2}$%). This triplet indicates the percentage of employees who uniquely possess skill 1 (m_1%), skill 2 (m_2%) or both skills ($m_{1,2}$%). We distinguish skill possession settings varying between (50%, 50%, 0%) and (0%, 0%, 100%) with intervals of 5% for m_1 and m_2 and an interval of 10% for $m_{1,2}$. In this respect, $m_{1,2}$% represents the degree of cross-training of the workforce.

Shift characteristics - The artificial and real-life problem instances are characterised by three non-overlapping shifts with specific start and end times. These shifts have a fixed duration of 8 hours and respectively start at 6 a.m., 2 p.m. and 10 p.m.

4.4.1.2 Constraint parameter settings

Time-related constraints - All personnel members can perform only a single working assignment per day (eq. (4.3)). There is also a minimum rest period of 11 hours imposed between two working assignments (eq. (4.4)). Furthermore, we include the following time-related constraints with their corresponding parameter values:

- The maximum number of working hours ($\eta_i^{w,max}$) is 160 and 168 (eq. (4.5)) for the artificial and real-life set, respectively.
- The minimum number of working hours ($\eta_i^{w,min}$) is 128 and 136 (eq. (4.6)) for the artificial and real-life set, respectively.
- The maximum number of consecutive working assignments per employee ($\theta_i^{w,max}$) is 5 (eq. (4.7)) for the artificial and real-life set.

Note that, in this research study, we want to obtain an unbiased insight in the impact of substitutability on the robustness of a personnel shift roster. To that purpose, we do not consider other time-related constraints in both optimisation models (4.1)-(4.12) and (4.13)-(4.15) to have some extra scheduling flexibility and a better match with the minimum staffing requirements.

Minimum staffing requirements of the artificial set - We obtain different demand profiles by using the complexity indicators designed for the nurse shift scheduling problem (Vanhoucke and Maenhout, 2009). We generate instances with a TCC-value of 0.30, 0.40 and 0.50 and distribute these staffing requirements evenly over the two skill categories. Additionally, we investigate test instances with a DCD-value of 0.00, 0.25 and 0.50 and SCD-values of 0.00, 0.25 and 0.50.
Each combination of values for these complexity indicators leads to a specific demand profile. We generate instances for 27 (3×3×3) (TCC, DCD, SCD)-combinations and 11 skill possession settings, which implies that we discuss the most significant results over 297 (=27×11) artificial instances.

Minimum staffing requirements of the real-life set - We highlight the practical relevance of employee substitutability by extending our test set with a number of instances that consider real-life demand profiles. These profiles are based on the data set of Ikegami and Niwa (2003), which is also available as a personnel rostering benchmark problem (Brucker et al., 2010).[1] The authors investigate the nurse shift scheduling problem in Japan and provide lower and upper bounds on the staffing requirements for two teams of nurses with different skills. These lower and upper bounds provide real-life demand profiles for which we investigate different staffing levels. These staffing levels vary between 27 and 67 workers and are based on a TCC-value ranging between 0.30 and 0.50. These demand profiles are investigated in conjunction with 11 different skill possession settings, which results in a total of 187 realistic test instances.

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[1] `http://www.cs.nott.ac.uk/~psztc/NRP/`

Additional group staffing requirements - The parameter $R^{w,+}_{mdj}$ defines the number of skilled employees that are required to be on duty on top of the minimum staffing requirements. This parameter gives an indication of the group employee substitutability as explained in Section 4.2.1.2. In correspondence to Chapter 2, we define $R^{w,+}_{mdj}$ as a fixed ratio of the minimum staffing requirements (R^{w}_{mdj}) and distinguish a ratio of 0%, 25%, 50%, 75% and 100%.

Individual employee substitutability constraints - Constraints (4.8)-(4.11) define the employee substitutability in the tactical decision phase. These constraints impose restrictions on the counted number of substitution possibilities in order to obtain a better estimate of the real substitutability in the operational decision phase. The right-hand side parameters ϵ and β_{mdj} in constraints (4.10) and (4.11) allow the definition of different strategies to count the number of substitution possibilities and to position them appropriately. By experimenting with these strategies, we aim to include the best proxy of the operational allocation phase in the tactical decision process to construct a baseline personnel shift roster that embeds a higher flexibility. Constraint (4.10) reflects the number of substitution possibilities an employee may offer on a single day and ϵ may be defined as follows:

- $\epsilon = 1$: As a single employee on a particular day may only be reassigned to one other duty in the operational allocation phase, at most one substitution possibility is counted in the tactical scheduling phase.
- $\epsilon = |G| \times |S|$: A single duty or day-off assignment for an employee may offer multiple substitution possibilities to each other feasible (skill, shift)-combination. The maximum number of substitution possibilities is equal to the number of (skill, shift)-combinations.

Constraint (4.11) reflects the number of substitution possibilities that may be counted per duty, i.e. for a single (skill, day, shift)-combination, and β_{mdj} may be defined as follows:

- $\beta_{mdj} = R^{w}_{mdj}$: The number of substitution possibilities is limited to the minimum staffing requirements of the particular duty. This implies that the scheduling of additional and/or excess employees will have no impact on the number of substitution possibilities.
- $\beta_{mdj} = \sum_{i \in N} x^{w}_{imdj}$: The number of substitution possibilities is limited to the number of assigned employees. Hence, each scheduled employee can be substituted by a maximum of one other employee.
- $\beta_{mdj} = M$: No limitation is imposed on the number of substitution possibilities (with $M =$ a very large number). Each assigned employee can be substituted by an unlimited number of other employees.

Based upon the parameter values of ϵ and β_{mdj}, different strategies can be defined to count the number of substitution possibilities and to mimic the substitutability in the operational phase. The different counting strategies refer to the set of potential substitution possibilities each assignment for a specific employee may offer. These strategies are displayed in Table 4.2 and are assessed in Section 4.4.2 for different values of the allowable cost increase (τ) to obtain insight into the cost of robustness.

Table 4.2 An overview of the substitution counting strategies

Counting strategy	ϵ	β_{mdj}
Strategy 1.1	1	R^w_{mdj}
Strategy 1.2	1	$\sum_{i \in N} x^w_{imdj}$
Strategy 1.3	1	M
Strategy 2.1	$\lvert G \rvert \times \lvert S \rvert$	R^w_{mdj}
Strategy 2.2	$\lvert G \rvert \times \lvert S \rvert$	$\sum_{i \in N} x^w_{imdj}$
Strategy 2.3	$\lvert G \rvert \times \lvert S \rvert$	M

4.4.1.3 Objective function parameter settings

General objective function - The general objective in models (4.1)-(4.12) and (4.13)-(4.15) is to optimise the costs, i.e. the personnel assignment costs and the understaffing of the minimum staffing requirements in objectives (4.1.1) and (4.13.1). For these *general objective function components*, we define the following parameter values, i.e.

- Every employee has a wage cost (c^w_{imdj}) depending on the number of skills this employee possesses (De Bruecker et al., 2015), i.e. $10 \times 1.2^{\sum_{m \in G} b_{im} - 1}$.
- Every employee has a preference penalty cost (p_{idj}) that is randomly generated in the range of 1 to 5.
- The shortage cost (c^{wu}_{mdj}) is fixed at 20.

The objective to optimise the individual employee substitutability - In order to optimise the employee substitutability on the level of the individual employee, we define a value for the objective function coefficient γ_{mdj} in objective (4.1.2) based on three different strategies, i.e.

- Strategy 1: Each substitution embodies a fixed value (γ) independent of the skill category, day and shift (eq. (4.20)).

$$\gamma^1_{mdj} = \gamma \qquad \forall m \in G, \forall d \in D, \forall j \in S \tag{4.20}$$

- Strategy 2: The weight for a substitution possibility associated with a shift and skill category depends on how the staffing requirements are related to the maximum staffing requirements over all shifts for the skill category on that day (eq. (4.21)).

$$\gamma^2_{mdj} = \frac{R^w_{mdj}}{\max_{s \in S} R^w_{mds}} \times \gamma \qquad \forall m \in G, \forall d \in D, \forall j \in S \tag{4.21}$$

- Strategy 3: This strategy calculates the ratio of the staffing requirements for a shift, day and skill category to the maximum staffing requirements over the total planning horizon for the skill category (eq. (4.22)).

$$\gamma^3_{mdj} = \frac{R^w_{mdj}}{\max_{d' \in D, s \in S} R^w_{md's}} \times \gamma \qquad \forall m \in G, \forall d \in D, \forall j \in S \tag{4.22}$$

These value strategies try to value the substitution possibilities in such a way that the most valuable, in terms of the performance in the operational allocation phase, are added during the construction of the personnel shift roster in the tactical scheduling phase.

The objective to optimise the group employee substitutability - In order to optimise the employee substitutability on the group level, we define a value for the objective function coefficient $c_{mdj}^{wu,+}$ that amounts to 25 in objective (4.13.2). This value exceeds the cost of a shortage of employees for a skill, day and shift (c_{mdj}^{wu}) to express that it is worse to have a shortage of skills on a group level rather than on the level of an individual employee. Hence, this value accommodates the satisfaction of the additional group staffing requirements defined in equation (4.14).

4.4.2 Comparison of strategies to model employee substitutability

In this section, we discuss and compare the planned and actual performance of different proactive scheduling strategies for the artificial set of instances. In Section 4.4.2.1, we consider employee substitutability on the level of the individual employee and analyse the impact of the allowable cost increase (τ) for different counting strategies, the value strategies and the substitution types. In Section 4.4.2.2, we consider employee substitutability on a group level and assess the benefits of different sizes of the group staffing requirements.

4.4.2.1 Employee substitutability on an individual employee level

The constructed personnel shift rosters differ based on the allowable cost increase (τ) and the applied counting strategies, the value strategies and the individual substitution types. Unless otherwise stated, we discuss the average results over all demand profiles, skill possession settings and substitution types.

Impact of the counting strategy and the cost of robustness
Table 4.3 indicates over all value strategies and substitution benefit values (γ) the planned cost of the counting strategies given different values for the allowable cost increase (τ). These counting strategies include the most and least restrictive method to count substitution possibilities, i.e. strategy 1.1 and 2.3 respectively. Moreover, we distinguish the counting strategy with the best actual performance in the operational allocation phase, i.e. strategy 1.2. Note that all other counting strategies exhibit a performance that can be situated between strategies 1.1 and 2.3.
It is clear that the impact of the allowable cost increase (τ) on the planned cost strongly depends on the way the substitution possibilities are counted. In this respect, the planned cost associated to strategy 2.3 rises with the allowed cost increase while strategy 1.1 results in a stable planned cost. This difference is due to the definition of the corresponding values for ϵ and β_{mdj} in constraints (4.10) and (4.11) (cf. Table 4.2). Counting strategy 2.3 does not impose a limit on the counted number of substitution possibilities for each employee and duty in the constructed baseline personnel shift roster. This number can only be increased by changing the position of surplus duties between shifts or by creating more planned overstaffing or capacity buffers. This

results in an increase of the preference penalty cost and/or wage cost and in a larger cost of robustness, i.e. the cost difference between the minimum cost and the more robust personnel shift roster. Counting strategy 1.1 in contrast, has a stringent upper limit on the counted number of substitution possibilities and is therefore not affected by an increase of the allowable cost. As such, the maximum number of substitution possibilities that can be introduced is significantly smaller than for strategy 2.3 and the planned cost remains relatively equal to the cost of the minimum cost personnel shift roster, i.e. $\tau = 0\%$. Given the extra flexibility offered by strategy 1.2, the cost of robustness rises due to a small increase in the total assignment cost. This is the result of a change in the assignments to increase the number of substitution possibilities.

Table 4.3 The planned cost for the counting strategies and allowable cost increases (τ)

Counting strategy	$\tau = 0\%$	$\tau = 1\%$	$\tau = 2\%$	$\tau = 3\%$
Strategy 1.1	3,981.29	3,981.65	3,981.65	3,981.65
Strategy 1.2	3,981.29	3,986.43	3,986.52	3,986.53
Strategy 1.3	3,981.29	3,999.41	4,001.06	4,001.14
Strategy 2.1	3,981.29	3,981.57	3,981.57	3,981.57
Strategy 2.2	3,981.29	3,986.48	3,986.56	3,986.56
Strategy 2.3	3,981.29	4,007.95	4,021.05	4,024.73

The allowable cost increase and the counting strategy have a substantial impact on the performance of the personnel shift roster in the operational allocation phase. We display the actual cost of the counting strategies in Table 4.4 as an average over all value strategies and substitution benefit values (γ). Given that the actual cost of the minimum cost personnel shift roster equals 4,193.61, this table shows that only counting strategies 1.2 and 2.2 are able to provide an improvement when the cost of robustness is 0, i.e. $\tau = 0\%$.

Hence, the actual cost depends on the selected counting strategy. Irrespective of the allowable cost increase (τ), it is clear that the average actual cost and the associated shortages, changes and cancellations are lower when the maximum number of substitution possibilities per duty (β_{mdj}) are restricted to the number of scheduled employees (strategy 1.2 and 2.2). For these strategies, the actual cost does not significantly change when the allowable cost increase (τ) rises.[2] It is therefore important to limit the cost of robustness. A more stringent restriction on the maximum number of substitution possibilities per duty (strategy 1.1) provides the smallest actual cost when the staffing requirements exhibit a small variability over the different days (i.e. a small DCD-value), which is the case for the real-life staffing requirements. In this respect, counting strategy 1.1 ensures that a lower number of substitution possibilities can be better distributed over the days and shifts to enhance their position and to achieve a smaller number of shortages and cancellations in the operational allocation phase. As such, counting strategy 2.3 always leads to a significantly higher actual cost[3] due to a poor positioning of an excessive number of substitution possibilities leading to a high number of shortages, changes and cancellations, which

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[2] p-value>0.05 (Kruskal-Wallis H test)

[3] p-value<0.01 (Mann-Whitney U test)

is especially true for a rising allowable cost increase (τ).

The number of substitution possibilities that may be counted for one employee per day (ϵ) should generally be restricted. However, this number actually depends on the degree of cross-training of the workforce and the potential number of substitution possibilities available in the constructed personnel shift roster, i.e. the implicit flexibility. For a high number of multi-skilled employees, it is important to limit the number of substitution possibilities that may be counted per employee, i.e. $\epsilon = 1$. This limitation facilitates a focus on the definition of the most important substitution possibilities in terms of their position in the personnel shift roster. In contrast, a low number of multi-skilled employees represents less implicit flexibility and more substitution possibilities should be added.

Therefore, a sufficient number of substitution possibilities should always be available in the operational allocation phase. This can be ensured by applying the adequate counting strategy and allowable cost increase (τ). In this respect, a lower number of multi-skilled employees and a smaller number of implicit substitution possibilities require a less restrictive counting strategy (strategy 2.2) in combination with a higher allowable cost increase (τ). As more substitution possibilities are implicitly available, the focus should move to a counting strategy that better reflects the actual situation in the operational allocation phase, i.e. each scheduled employee can be substituted by maximum one other employee and a single employee may only be reassigned to one other duty in the operational allocation phase (strategy 1.2).

Table 4.4 The actual cost for the counting strategies and allowable cost increases (τ)

Counting strategy	$\tau = 0\%$	$\tau = 1\%$	$\tau = 2\%$	$\tau = 3\%$
Strategy 1.1	4,194.84	4,194.58	4,194.58	4,194.59
Strategy 1.2	4,185.23	4,185.03	4,185.06	4,185.06
Strategy 1.3	4,193.74	4,203.65	4,204.74	4,204.81
Strategy 2.1	4,195.48	4,195.30	4,195.31	4,195.31
Strategy 2.2	4,185.91	4,185.80	4,185.82	4,185.82
Strategy 2.3	4,194.56	4,214.54	4,223.15	4,225.30

The impact of the value strategies

Table 4.5 shows the planned performance corresponding to the defined value strategies (cf. eqs. (4.20)-(4.22)) over counting strategies 1.1, 1.2, 2.1 and 2.2, over all allowable cost increases τ and all substitution benefit values γ. We discard counting strategies 1.3 and 2.3 because of their bad planned and actual performance (cf. supra).

Value strategy 1, which defines constant substitution benefit values, exhibits the smallest planned cost and the highest substitutability value and number of substitution possibilities. In contrast, value strategy 2 leads to the largest planned cost. Note that for all value strategies, a smaller (larger) substitution benefit value γ leads to a lower (higher) planned cost and planned substitutability value.

Table 4.5 The planned performance for different value strategies

	Strategy 1	Strategy 2	Strategy 3
Planned cost	3,981.53	3,984.68	3,983.19
Shortages	0.02	0.02	0.02
Total assignment cost	3,981.19	3,984.34	3,982.85
Wage cost	3,520.00	3,520.00	3,520.00
Preference penalty cost	461.19	464.34	462.85
Planned substitutability value	337.15	286.05	211.42
Between-skill substitutions	65.93	64.39	61.20
Within-skill substitutions	100.80	96.42	99.04
Day-off-to-work substitutions	102.91	100.14	99.28
Planned overstaffing	96.02	96.02	96.02

Figure 4.3 displays the actual cost for the different value strategies. Value strategies 2 and 3 are significantly better than the first strategy[4], which confirms that the position of the substitution possibilities is of major importance. Moreover, strategy 3 provides a significantly better actual cost than strategy 2.[5] Hence, the demand profile over the complete planning horizon should be considered. More specifically, value strategy 3 leads to the best positions and is especially useful when the staffing requirements exhibit a large variability over the days in the planning horizon (i.e. a large DCD-value). In contrast, strategy 2 provides similar results when the staffing requirements exhibit a small variability over the days in the planning horizon (cf. Figure 4.B.1 in Appendix 4.B), which is the case for the real-life staffing requirements.

It is interesting to note that the value strategies only exhibit a significantly different actual performance in combination with counting strategies 1.2 and 2.2. Counting strategies 1.1 and 2.1 limit the number of substitution possibilities per duty (β_{mdj}) to the staffing requirements. Therefore, the substitution possibilities cannot be increased at a certain valuable position but are automatically distributed over all positions.

Moreover, the substitution benefit value γ does not have a significant impact on the weighting and positioning of the substitution possibilities and, consequently, on the actual performance of a baseline personnel shift roster.[6]

The impact of the individual substitution types

The employee substitutability optimisation model (eqs. (4.1)-(4.12)) concurrently considers the three types of substitutions. However, it is possible to adapt this model by adding constraint (4.23a), constraint (4.23b) or constraint (4.23c) to optimise employee substitutability solely based on between-skill substitutions, within-skill substitutions or day-off-to-work substitutions, respec-

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[4] p-value<0.01 (Mann-Whitney U test)

[5] p-value<0.01 (Mann-Whitney U test)

[6] p-value>0.05 (Kruskal-Wallis H test)

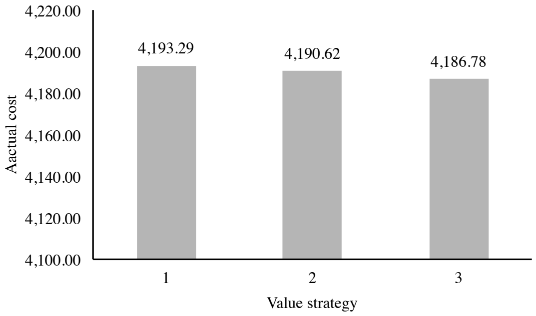

Figure 4.3 The actual cost for different value strategies

tively.

$$z_{imdj} \leq \sum_{m' \in G \setminus \{m\}} \sum_{j' \in S} x^{w}_{im'dj'} \qquad \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \tag{4.23a}$$

$$z_{imdj} \leq \sum_{j' \in S \setminus \{j\}} x^{w}_{imdj'} \qquad \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \tag{4.23b}$$

$$z_{imdj} \leq x^{v}_{id} \qquad \forall i \in N, \forall m \in G, \forall d \in D, \forall j \in S \tag{4.23c}$$

Regardless of the considered substitution types, the general conclusions concerning the cost of robustness, counting strategies and value strategies remain valid. However, as a result of restricting the substitution types, the potential number of substitution possibilities available in the baseline personnel shift roster, i.e. the implicit flexibility, is smaller. In this respect, more substitution possibilities need to be explicitly added by applying counting strategy 2.2.

The individual substitution types only affect the planned performance of personnel shift rosters in terms of planned substitutability value and the associated number of substitution possibilities. However, they do have an important impact on the actual performance in the operational allocation phase. In this phase, a change cost $c^{w\delta}_{imdj}$ is accounted if an assignment is adjusted, i.e. a substitution is executed. In Table 4.6, we distinguish four operational change cost scenarios in relation to the considered substitution types in the tactical scheduling phase. Note that a change cost of 100 implies that this substitution type is never executed in the operational allocation phase.
Figure 4.4 displays the actual cost corresponding to the best parameter settings for the individual and combined substitution types as a function of the degree of cross-training of the workforce (cf. supra). It is clear that the lowest actual cost and, hence, the highest flexibility is obtained when all substitution types are considered, which indicates that the substitution types are complementary. When we analyse the results in relation to the degree of cross-training, we observe

Table 4.6 The change cost ($c_{imdj}^{w\delta}$) for the different substitution types

	$c_{imdj}^{w\delta}$	Between-skill substitution	Within-skill substitution	Day-off-to-work substitution
Considered substitutions	All	1	1	1
	Between-skill	1	100	100
	Within-skill	100	1	100
	Day-off-to-work	100	100	1

that for a low number of multi-skilled employees, the within-skill substitutions offer the highest flexibility and have a better actual cost than the between-skill and day-off-to-work substitutions. As the number of multi-skilled employees rises however, the between-skill substitutions offer the lowest actual cost and highest flexibility.
The figure clearly shows that the between-skill substitutions are unable to provide a good actual performance when the degree of cross-training is less than 50%. This is due to the high number of staff shortages. As the degree of cross-training increases, the number of staff shortages decreases. However, since multi-skilled employees receive a higher wage than single-skilled employees, there is an increase in the actual cost when the degree of cross-training is more than 50%. In contrast, when considering within-skill substitutions only, the actual cost steadily increases with a higher number of multi-skilled workers whereas the number of shortages remains rather constant. When focusing solely on day-off-to-work substitutions, the increase of the actual cost is smaller due to the decrease of the number of shortages when the degree of cross-training increases. The latter is the result of the larger flexibility embedded in the day-off assignments of employees, who can be reassigned to each duty, regardless of the corresponding skill. As such, the day-off-to-work substitutions outperform the within-skill substitutions for a large number of multi-skilled employees.

In general, we can conclude that it is important to exploit the multi-skilled nature of employees by considering between-skill and day-off-to-work substitutions. Additionally, within-skill substitutions provide extra flexibility in the personnel shift roster and are complementary to the other two substitution types.

4.4.2.2 Employee substitutability on a group level

Table 4.7 shows the planned performance associated to different sizes of the additional group staffing requirements $R_{mdj}^{w,+}$ calculated as a fixed ratio (i.e. 0%, 25%, 50%, 75% and 100%) of the minimum staffing requirements R_{mdj}^{w}. In this respect, a ratio of 0% results in a personnel shift roster that represents the same planned cost as the minimum cost personnel shift roster (cf. Table 4.3). As the additional group staffing requirement rise however, the planned cost increases due to a higher total assignment cost as a result of a higher number of working assignments and/or a higher number of assignments for multi-skilled workers. Note that the reported results in Table 4.7 represent the average over all skill possession settings and demand profiles (cf. Section 4.4.1). As such, it is not possible to avoid a certain level of understaffing (0.02) in some cases, which is

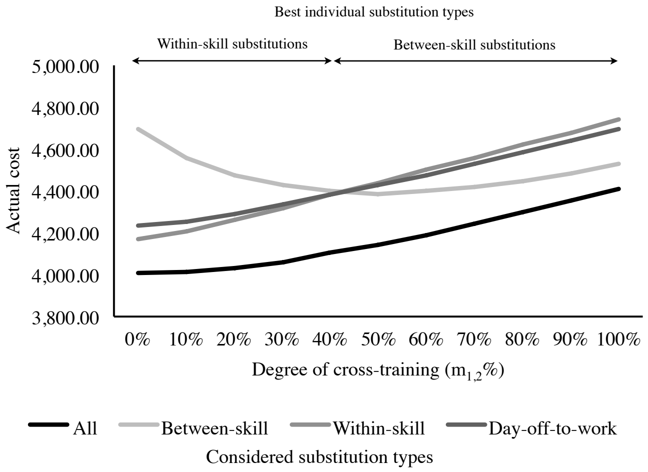

Figure 4.4 The evolution of the actual cost for different substitution types and degrees of cross-training of the workforce

caused by the hard minimum rest constraint (cf. eq. (4.4)). An increase in the group staffing requirements actually slightly increases the understaffing and the cross-training shortage (CT^-). As the group staffing requirements increase, they become more difficult to satisfy given the hard time-related constraints. Therefore, a shortage can be created at a certain position to satisfy the (group) staffing requirements at another position.

Figure 4.5 shows that the overstaffing increases as the desired group staffing requirements increase. This is due to the fact that the group staffing requirements cannot be entirely satisfied by scheduling multi-skilled employees, especially if the number of multi-skilled employees is low. As such, additional working assignments are scheduled and create capacity buffers that augment the available cross-training (CT) and cross-training surplus (CT^+).

Table 4.7 The planned performance for different sizes of the additional group staffing requirements

	0%	25%	50%	75%	100%
Planned cost	3,981.29	4,037.27	4,178.86	4,363.30	4,449.72
Shortages	0.02	0.02	0.03	0.02	0.04
Total assignment cost	3,980.96	4,036.86	4,178.26	4,362.83	4,448.98
Wage cost	3,520.00	3,540.65	3,631.49	3,751.10	3,808.90
Preference penalty cost	460.96	496.21	546.76	611.73	640.08
Planned cross-training					
CT	480.00	482.27	493.92	510.46	519.21
CT^+	256.02	258.29	269.94	286.48	295.24
CT^-	0.02	0.02	0.02	0.02	0.03
Planned overstaffing	96.02	98.03	106.49	117.30	122.35

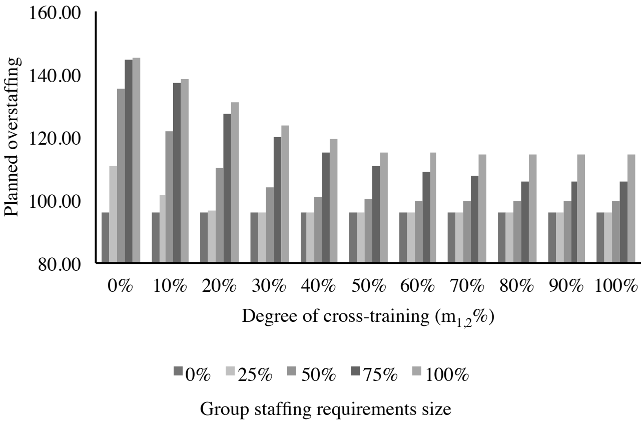

Figure 4.5 The evolution of planned overstaffing for different sizes of the additional group staffing requirements and degrees of cross-training of the workforce

The actual performance of personnel shift rosters for different sizes of the additional group staffing requirements is displayed in Table 4.8. In this table, we report the results obtained by considering all substitution types (cf. Table 4.6) and show that the best performance is obtained with an additional group staffing requirement ratio of 0% and especially 25%[7], for which we observe the smallest number of shortages and cancellations. Larger ratios provide an actual performance that is less stable with a larger range between the minimal and maximal actual cost over all simulation runs. Moreover, these ratios result in more shortages and cancellations, which are especially large for high group staffing requirements in combination with a small number of multi-skilled employees. This indicates a poor positioning of excessive capacity buffers. Moreover, the number of utilised substitutions decreases as the size of the additional group staffing requirements rises, which is mainly due to a reduction in the day-off-to-work substitutions.

Similar to the observations in Chapter 2, it is important to note that the size of the additional group staffing requirements depends on the ratio between the total minimum staffing requirements (R^w_{mdj}) and the total number of hired employees, i.e. the TCC-value. As the total staffing requirements rise or the number of hired employees declines, the optimal size actually decreases because a large TCC-value in combination with a high desired size actually results in substantial group staffing requirements ($R^{w,+}_{mdj}$). This is especially problematic for a low number of multi-skilled employees. In this case, the group staffing requirements need to be primarily satisfied by capacity buffers. However, the total capacity buffer that can be scheduled is limited by the total minimum staffing requirements (R^w_{mdj}) and the time-related constraints. As such, it becomes difficult to appropriately position the capacity buffers (cf. Chapter 2).

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[7] p-value<0.01 (Mann-Whitney U test)

Table 4.8 The actual performance for different sizes of the additional group staffing requirements

	0%	25%	50%	75%	100%
Actual cost	4,193.99	4,188.43	4,220.95	4,277.24	4,306.36
Shortages	3.91	3.75	3.84	4.07	4.16
Total assignment cost	4,055.88	4,057.50	4,091.98	4,145.65	4,173.18
Wage cost	3,455.75	3,452.42	3,446.72	3,439.47	3,436.98
Preference penalty cost	530.24	538.48	551.65	570.69	578.87
Number of duties cancelled	13.98	13.32	18.72	27.10	31.47
Utilised substitutions	59.91	55.90	52.10	50.12	50.01
Between-skill substitutions	19.47	20.47	20.97	21.06	21.18
Within-skill substitutions	24.86	22.79	21.86	22.65	23.20
Day-off-to-work substitutions	15.59	12.64	9.27	6.40	5.64
Actual overstaffing	93.58	93.09	92.65	92.22	92.08

4.4.3 Robustness validation of employee substitutability

In this section, we compare the resulting robustness for different proactive scheduling strategies applied to construct a personnel shift roster for the real-life staffing requirements. In this respect, we consider the results over all hiring levels for the minimum cost roster without an increased employee substitutability ('MC'), the personnel shift roster with group employee substitutability ('GS') and a personnel shift roster with individual employee substitutability ('IS').

Figure 4.6 represents the variability of the actual cost corresponding to the different scheduling strategies and substitution types (cf. Table 4.6) for the real-life demand profile with the highest minimum staffing requirements. In this respect, we notice that the group substitutability strategy is especially useful and provides a significantly lower actual cost than the minimum cost strategy if only between-skill substitutions are considered.[8] Moreover, the individual substitutability strategy ensures a significantly better[9] and more stable actual performance than the minimum cost strategy if the between-skill, within-skill and day-off-to-work substitutions are considered individually.

We display the maximal actual cost over all simulation runs as an average over all hiring levels for the between-skill substitutions according to the different scheduling strategies in Figure 4.7. This figure confirms the dominance of including some substitutability in the optimisation model over the minimum cost strategy for different degrees of cross-training of the workforce. Moreover, Figure 4.7 indicates that the overall performance of the group substitutability actually stems from the beneficial impact of the created capacity buffers for a low number of multi-skilled employees in the tactical scheduling phase (cf. Figure 4.5). As the number of multi-skilled employees rises, the individual employee substitutability strategy provides the best results. In order to compensate the poor performance of individual employee substitutability for a small number

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[8] p-value<0.01 (Mann-Whitney U test)

[9] p-value<0.01 (Mann-Whitney U test)

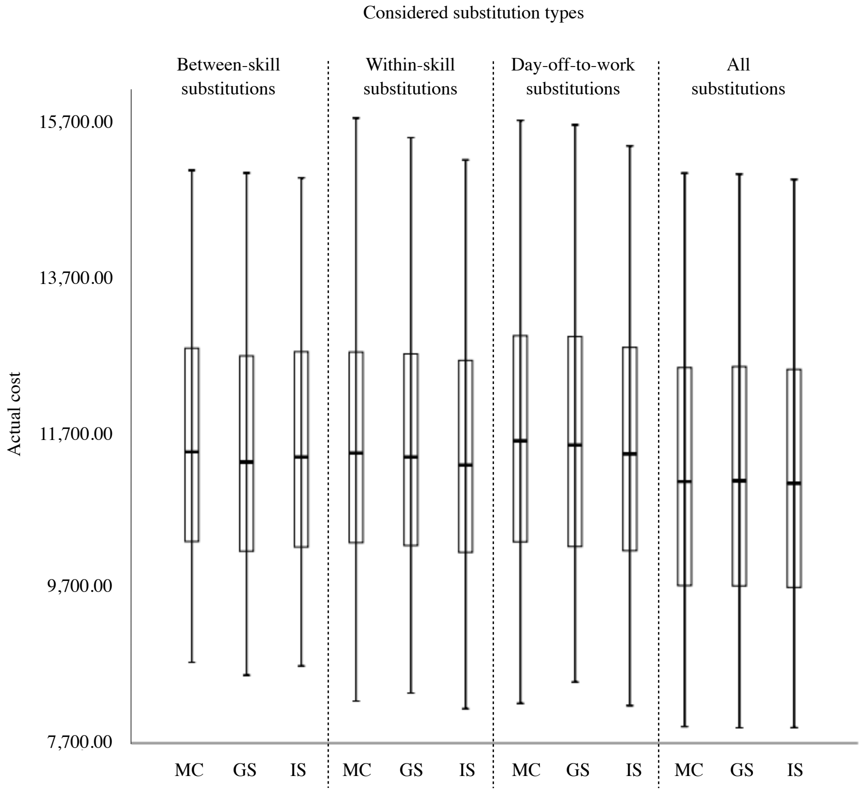


Figure 4.6 The actual cost over all hiring levels for the different scheduling strategies and considered substitution types (MC: minimum cost scheduling strategy, GS: group substitutability scheduling strategy, IS: individual employee substitutability scheduling strategy)

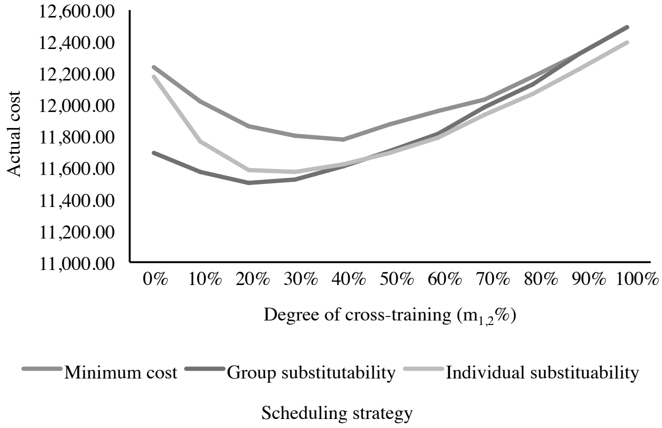


Figure 4.7 The maximal actual cost over all simulation runs as an average over all hiring levels for the between-skill substitutions

of multi-skilled employees, the value of within-skill and day-off-to-work substitutions should be optimised. In this respect, when all substitution types are considered, we notice that individual employee substitutability continuously outperforms group employee substitutability for all skill possession settings.

Regardless of the hiring level and the artificial and real-life demand profiles, employee substitutability on the level of the individual employee provides personnel shift rosters that exhibit a smaller variability in the operational allocation phase. In fact, personnel shift rosters with individual employee substitutability are actually able to achieve a similar or better actual performance in terms of shortages with a smaller number of multi-skilled employees in comparison to the minimum cost personnel shift roster. In this respect, the personnel shift rosters with employee substitutability require less implicit flexibility in terms of multi-skilled employees to achieve the same or a higher flexibility (cf. Figure 4.B.2 in Appendix 4.B). However, it is important to facilitate this flexibility in the operational allocation phase by limiting the change cost ($c^{w\delta}_{imdj}$). Otherwise, the number of executed substitutions decreases and the individual employee substitutability strategy loses its value. In this case, personnel shift rosters with capacity buffers obtained by considering group employee substitutability, provide a better actual performance.

4.5 Conclusions

In this chapter, we consider a bi-objective personnel scheduling model that not only minimises the personnel assignment cost but also maximises the employee substitutability to improve the robustness of a personnel shift roster. We define employee substitutability on the group level and on the level of the individual employee. Employee substitutability on the group level is characterised by group staffing requirements that determine the desired number of employees to be on duty able to carry out a specific skill. In order to maximise the individual employee substitutability of a tactical personnel shift roster, we consider three types of substitution possibilities between personnel members, i.e. between-skill, within-skill and day-off-to-work substitutions. Since we want to investigate the cost of robustness, we propose a two-phase preemptive programming approach.

The best strategy to improve the flexibility and the corresponding robustness of a tactical personnel shift roster is to define employee substitutability on the level of the individual employee. In the design of the proactive strategy as a proxy for the uncertainty and variability in the operational allocation phase, it is important to apply the correct strategy to count the number of substitution possibilities and to define the associated objective function structure in the tactical scheduling phase. The best strategy and objective function structure should depend on the variability of the minimum staffing requirements and the implicitly available flexibility, i.e. the number of multi-skilled employees and the considered substitution types, such that a sufficient but not excessive number of substitution possibilities is available and appropriately positioned. In this respect, the number of substitution possibilities that may be counted per duty should be limited to the minimum staffing requirements or to the number of scheduled duties in case of a small and large variability of the minimum staffing requirements, respectively. Moreover, a low implicit flexibility requires the definition of more additional substitution possibilities than a high implicit flexibility. This means that a low implicit flexibility requires a counting strategy that allows the definition of an unrestricted number of substitution possibilities per employee. In contrast, a higher implicit flexibility benefits from a more restrictive counting strategy. As such, the strategy should reflect the actual situation in the operational allocation phase where a single employee may only be reassigned to one other duty. In order to facilitate the availability and fitting positioning of these substitution possibilities, an appropriate cost of robustness and substitutability value should be defined. The cost increase associated with the addition of more substitution possibilities should remain limited in the tactical scheduling phase. Furthermore, the position of the substitution possibilities should be carefully considered by determining the associated value based on the structure of the minimum staffing requirements.

4.A Appendix - Operational allocation model

Notation

Sets

G	set of skills (index m)
N	set of employees (index i)
S	set of shifts (index j)

General parameters

d	day under consideration in the operational planning horizon
b_{im}	1 if employee i possesses skill m, 0 otherwise
c^{w}_{imdj}	wage cost of assigning an employee i to shift j, day d and skill m
c^{wu}_{mdj}	shortage cost for shift j, day d and skill m
p_{idj}	preference penalty cost if an employee i receives a shift assignment j on day d
l_j	duration of shift j
κ^{α}_{idj}	1 if employee i is allowed to receive an assignment during shift j on day d, 0 otherwise
κ^{f}_{id}	the total number of hours employee i has to receive on day d

Simulation parameters

a_{id}	1 if employee i is available on day d, 0 otherwise
R'^{w}_{mdj}	simulated staffing requirements for shift j, day d and skill m

Roster change parameters

x'^{w}_{imdj}	1 if employee i received a shift assignment j for skill m on day d in the baseline personnel shift roster, 0 otherwise
$c^{w\delta}_{imdj}$	roster change cost for assigning an employee i to shift j, day d and skill m with $c^{w\delta}_{imdj} > 0$ if $x'^{w}_{imdj} = 0$ $c^{w\delta}_{imdj} = 0$ otherwise
c^{v}_{id}	duty cancellation cost for employee i on day d with $c^{v}_{id} > 0$ if $\sum_{m \in G} \sum_{j \in S} x'^{w}_{imdj} = 1$ and $a_{id} = 1$ $c^{v}_{id} = 0$ otherwise

Variables

x^{w}_{imdj}	1 if employee i receives a shift assignment j for skill m on day d, 0 otherwise
x^{v}_{id}	1 if employee i receives a day off on day d, 0 otherwise
x^{wu}_{mdj}	the shortage of employees for shift j, day d and skill m

Mathematical formulation

$$\min \quad \sum_{i \in N} \sum_{m \in G} \sum_{j \in S} (c^{w}_{imdj} + c^{w\delta}_{imdj} + p_{idj}) x^{w}_{imdj} + \sum_{i \in N} c^{v}_{id} x^{v}_{id} + \sum_{m \in G} \sum_{j \in S} c^{wu}_{mdj} x^{wu}_{mdj} \tag{4.A.1}$$

$$\sum_{i \in N} b_{im} x^{w}_{imdj} + x^{wu}_{mdj} \geq R'^{w}_{mdj} \qquad \forall m \in G, \forall j \in S \tag{4.A.2}$$

$$\sum_{m \in G} \sum_{j \in S} x^{w}_{imdj} \leq a_{id} \qquad \forall i \in N \tag{4.A.3}$$

$$\sum_{m \in G} \sum_{j \in S} x^{w}_{imdj} + x^{v}_{id} = 1 \qquad \forall i \in N \tag{4.A.4}$$

$$\sum_{m \in G} x^{w}_{imdj} \leq \kappa^{\alpha}_{idj} \qquad \forall i \in N, \forall j \in S \tag{4.A.5}$$

$$\sum_{m \in G} \sum_{j \in S} l_j x^{w}_{imdj} \geq \kappa^{f}_{id} a_{id} \qquad \forall i \in N \tag{4.A.6}$$

$$\begin{aligned} x^{w}_{imdj} &\in \{0,1\} \quad && \forall i \in N, \forall m \in G, \forall j \in S \\ x^{v}_{id} &\in \{0,1\} && \forall i \in N \\ x^{wu}_{mdj} &\geq 0 && \forall m \in G, \forall j \in S \end{aligned} \tag{4.A.7}$$

The objective function (4.A.1) minimises the wage cost, roster change cost, preference penalty cost, cancellation cost and the cost for understaffing. The objective function weights are as follows, i.e.

- Every employee has a wage cost (c^{w}_{imdj}) of 10×$1.2^{\sum_{m \in G} b_{im} - 1}$.
- The roster change cost ($c^{w\delta}_{imdj}$) depends on the chosen scenario (cf. Table 4.6).
- Every employee has a preference penalty cost (p_{idj}) that is randomly generated in the range of 1 to 5.
- The duty cancellation cost (c^{v}_{id}) is 5.
- The shortage cost (c^{wu}_{mdj}) is fixed at 20.

Constraint (4.A.2) imposes the staffing requirements and every employee can only receive a shift assignment if (s)he is available (eq. (4.A.3)). Constraint (4.A.4) ensures that every employee receives either a shift assignment or a day off. The shifts assigned to the employees need to satisfy the time-related constraints (eqs. (4.4)-(4.7)). The satisfaction of the minimum rest period (eq. (4.4)), maximum number of hours that can be assigned (eq. (4.5)) and maximum consecutive working assignments (eq. (4.7)) is ensured through the definition of κ^{α}_{idj} in constraint (4.A.5). Finally, constraint (4.A.6) ensures that every employee works a minimum number of hours over the complete planning period (eq. (4.6)). Note that if an employee is unavailable on a working day (a_{id}=0), we adapt the minimum number of hours for this employee such that the employee does not have to catch up this duty. We define the integrality conditions in equations (4.A.7).

4.B Appendix - Supporting figures

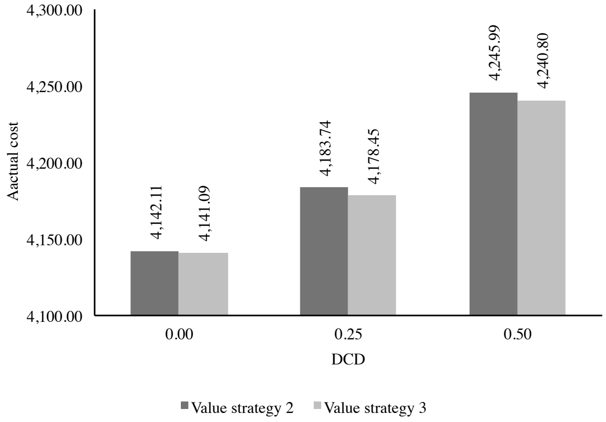


Figure 4.B.1 The actual cost corresponding to different value strategies and DCD-values

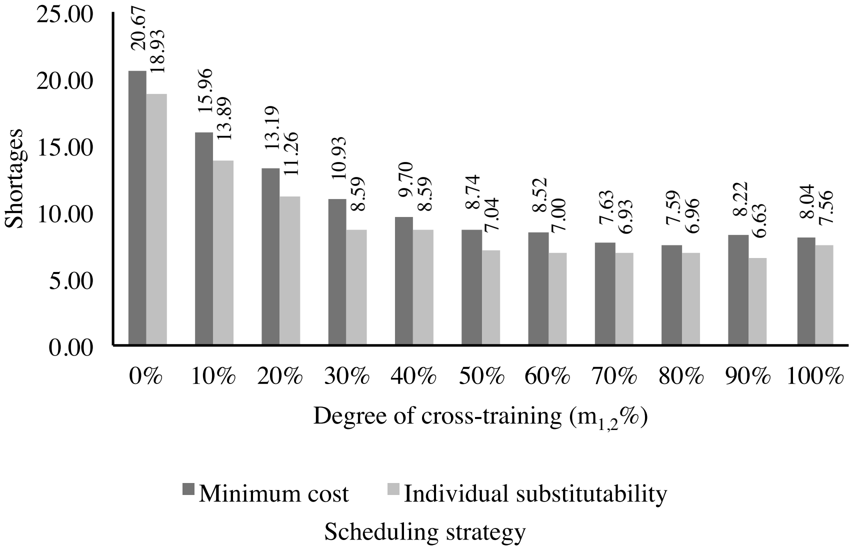


Figure 4.B.2 The actual shortages corresponding to different degrees of cross-training of the workforce for the artificial demand profiles

5

The impact of overtime as a time-based proactive scheduling and reactive allocation strategy on the robustness of a personnel shift roster

The workforce size and the overtime budget have an important impact on the total personnel costs of an organisation. Since the personnel costs represent a significant fraction of the operating costs, it is important to define an appropriate hiring and overtime policy. Overtime is defined as an extension of the daily working time or the total working time over the planning period. In this chapter, we make the distinction between scheduled and unscheduled overtime when we define the overtime policy. Scheduled overtime is proactively assigned in the baseline personnel roster whereas unscheduled overtime is allocated as a reactive strategy to overcome operational disruptions. The hiring and overtime policy undoubtedly influence the robustness included in a personnel roster, i.e. the capability of an organisation to deal with roster disruptions at an acceptable cost. In this chapter, we investigate the trade-off between the hiring budget and the overtime budget and the way overtime should be allocated in the personnel planning process. The latter comprehends a trade-off between the proactive scheduling of overtime and the reservation of overtime to balance supply and demand in response to operational variability. Insights are obtained by exploring three different strategies to compose a baseline personnel shift roster. We verify the robustness of each of these strategies by applying a three-step methodology that thrives on optimisation and simulation and we formulate some managerial guidelines to define an appropriate hiring and overtime policy.

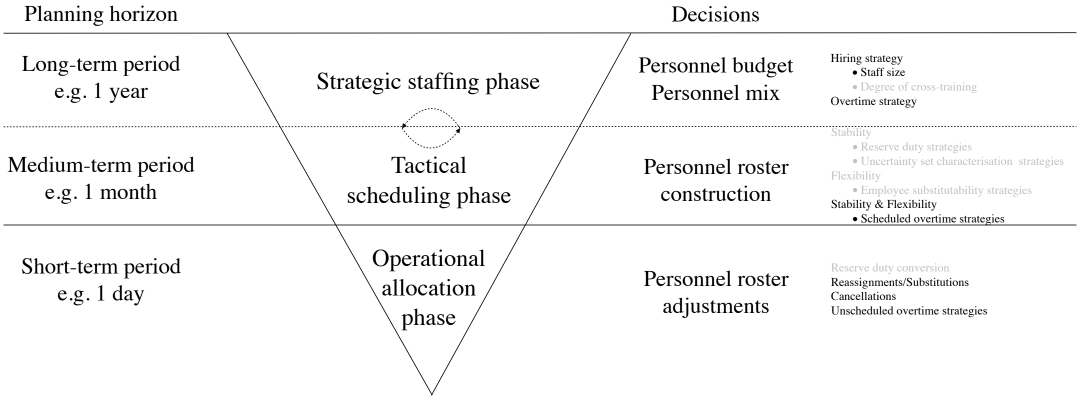


Figure 5.1 The research focus in Chapter 5

5.1 Introduction

In organisations, the personnel cost is typically one of the largest operating costs (Ernst et al., 2004b; Van den Bergh et al., 2013). An appropriate personnel planning process is therefore indispensable to manage these costs. The use of overtime is one strategy that is frequently applied in different phases of the personnel planning process to reduce the costs. In the higher-level hierarchical phases of staff budgeting and personnel scheduling, there is a clear interrelationship between the hiring strategy and the overtime strategy as the number of hired employees defines both the degree in which overtime is required and the ability to include overtime in the personnel roster (Li and Li, 2000). A lower number of hired employees may lead to a higher number of overtime duties to satisfy the staffing requirements. This practice impacts the flexibility and ability to make decisions in the lower-level operational planning phase as the structure of a line-of-work becomes more rigid. In the latter phase, operational variability arises and the (deterministic) assumptions made in higher-level phases are not able to perfectly represent the operating environment. As a result, the personnel planner should cope with these schedule disruptions on a day-by-day basis by changing the timing and duration of duties, i.e. by reassigning employees and by assigning overtime ad hoc.

In this chapter, we define different time-based proactive and reactive strategies, which include overtime, in order to improve the robustness of a personnel shift roster. For all strategies, we study their impact on both the absorption and adjustment capability of the personnel shift roster in the operational allocation phase. The proactive scheduling strategy consists of the possibility of assigning employees to overtime during a prior shift extension and a subsequent shift extension or to a complete shift during the construction of the deterministic baseline roster. Since these shift extensions may increase the daily working time, the employees may receive a smaller number of daily assignments. In this perspective, we investigate the trade-off between the workforce size and overtime included in the deterministic baseline personnel roster, in relation to the quality of the personnel roster that is subject to operational variability. As it is not possible to ensure a perfect absorption of the operational variability at a reasonable cost based on the proactively scheduled overtime, a reactive allocation decision model is formulated to make adaptations to the baseline personnel roster in the short-term operational allocation phase. In this reactive decision model, we include the strategy to assign employees to unscheduled overtime as an extension of the daily and total working time. However, the ability to reactively introduce unscheduled overtime strongly depends on the total overtime that has been proactively scheduled. As more overtime is scheduled before the start of the operational allocation phase, the opportunity to use overtime as a reactive allocation strategy decreases. In this context, we investigate the trade-off between scheduling overtime in the baseline personnel roster and allocating overtime in response to operational variability in the operational allocation phase.

We evaluate the trade-offs and the performance of the different strategies using a three-step methodology, which is similar to the approaches of Bard and Purnomo (2005a) and Abdelghany et al. (2008) and the methodology in Chapter 2. Based on the performance evaluation, we formulate managerial guidelines about the required workforce size and the different overtime strategies and highlight their impact on the personnel roster robustness.

The remainder of this chapter is organised as follows. In Section 5.2, we describe the context of the problem under study. In Section 5.3, we discuss the research methodology by formulating the different optimisation and simulation models and by defining the time-based proactive and reactive strategies. In Section 5.4, we define the test design and discuss the computational experiments and results. Conclusions are given in Section 5.5.

5.2 Problem description

In this chapter, we focus on different time-based strategies to include overtime in the personnel shift roster and investigate their impact on the personnel roster robustness. A general overview of the problem context is provided in Figure 5.2 which is explained below.

The baseline personnel roster

For the problem under study, we first determine the personnel budget and a baseline personnel shift roster simultaneously. The integration of the strategic staffing phase and the tactical scheduling phase allows the realisation of a trade-off between the hiring budget and the overtime budget. The baseline roster assigns the personnel members on each day to a working shift or to a day off. The standard shifts have a fixed start time and a fixed duration and they are defined based upon non-overlapping demand periods of four hours. As such, each day is comprised of six demand periods in total. In Figure 5.2, we define three standard shifts with a fixed duration of eight hours that cover two consecutive demand periods, i.e. shift 1 (start time: 12 p.m.), shift 2 (start time: 8 a.m.) and shift 3 (start time: 4 p.m.). In order to investigate the impact of time-based strategies on the robustness of a personnel shift roster, we distinguish different types of overtime that extend the working time per employee as follows, i.e.

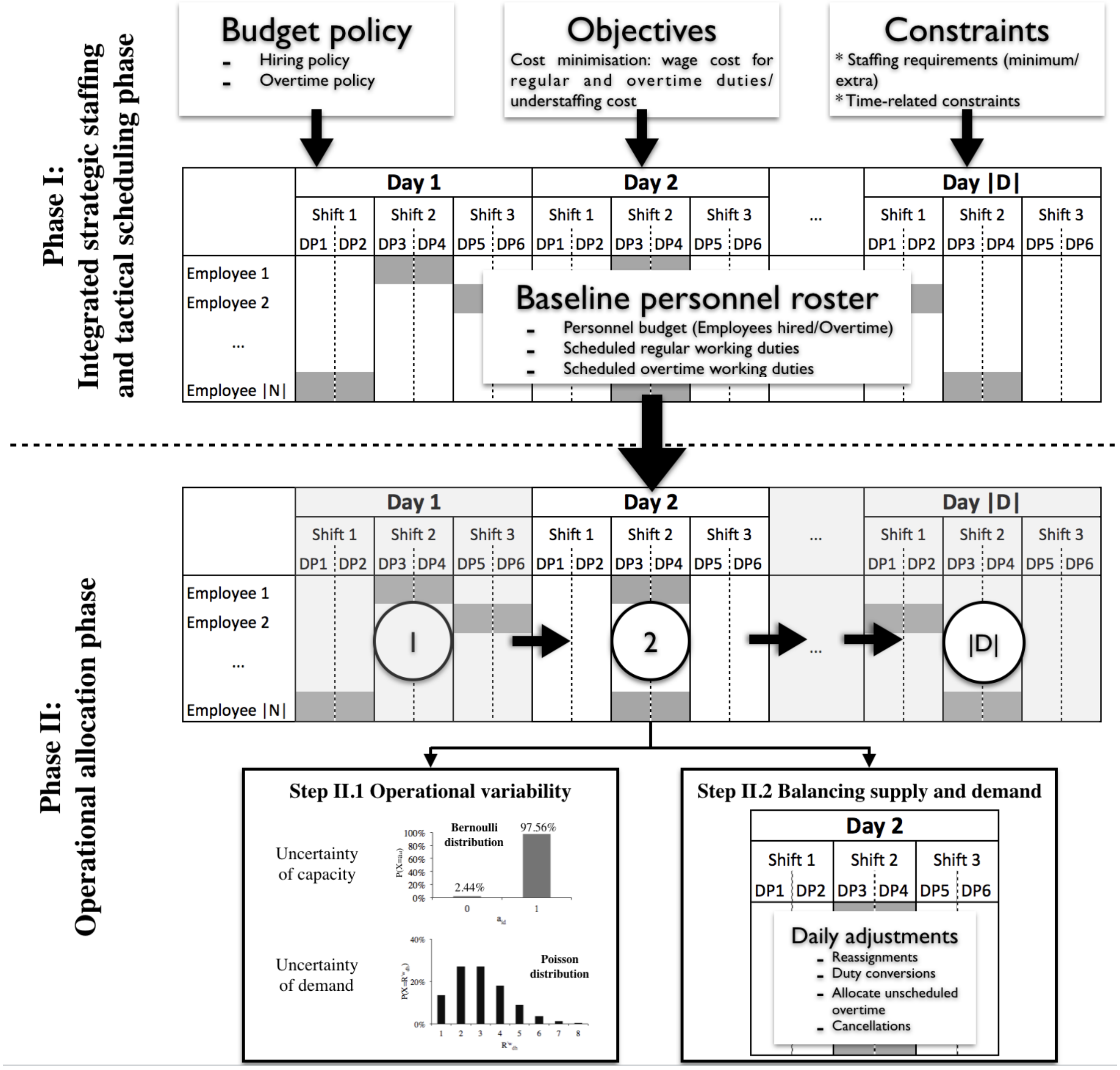


Figure 5.2 Problem description

- *Daily working time extension*: The extension of a standard shift increases the daily working time, which is defined as overtime. This type of overtime is the result of a *prior shift extension* or a *subsequent shift extension*. A prior shift extension and a subsequent shift extension respectively include the demand period immediately before and after the demand periods corresponding to a standard shift assigned to an employee. Figure 5.3 displays the resulting duty types for a particular day d. Each of the standard shifts can be extended with a prior shift extension and a subsequent shift extension. These prior and subsequent shift extensions of a standard shift comprehend an overlap with the corresponding previous and next standard shifts. The prior shift extension of shift 1 and the subsequent shift extension of shift 3 overlap with day $d-1$ and day $d+1$ respectively.

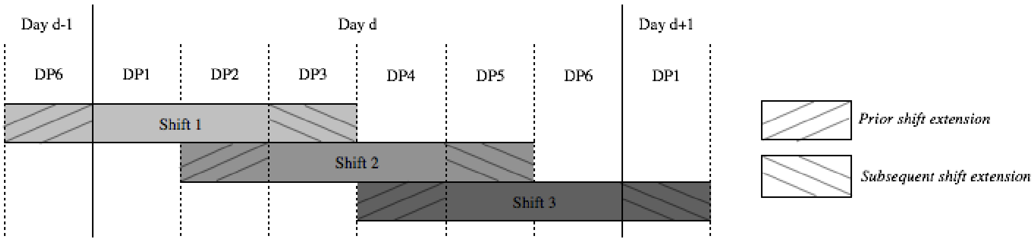


Figure 5.3 Illustration of the daily working time extensions

- *Total working time extension*: The assignment of a worker to a standard shift may extend the total working time on top of the maximum number of regular working hours a worker may be allocated to. This type of overtime extends the total working time of a single employee. Note that a daily working time extension may also comprehend an extension of the total working time.

The baseline personnel roster is constructed based on a set of inputs and parameters concerning the personnel budget policy, the performance measures and the constraints (cf. Figure 5.2), i.e.

- The personnel budget policy is characterised by the employee hiring policy and the overtime policy. These policies determine the characteristics of the hired personnel mix, the budget for overtime hours versus the number of regular hours and the amount of scheduled overtime hours to compose the baseline personnel roster. Hence, these inputs determine the balance between the workforce size employed and the number of overtime hours included in the baseline personnel roster.

- The objective function of this integrated staffing and scheduling problem involves the minimisation of different personnel costs, i.e. a wage cost for regular and overtime duties and an understaffing cost.

- In order to construct the personnel roster, staffing requirements and time-related constraints are imposed. The staffing requirements stipulate the number of required workers per demand period. On top of these regular staffing requirements, extra staffing requirements may be imposed that indicate the number of workers that need to be assigned to a prior shift extension or a subsequent shift extension. As such, these extra staffing requirements enable the inclusion of a capacity buffer to proactively anticipate operational variability.

The output of this integrated staffing and scheduling problem is a baseline personnel roster that stipulates the set of employees hired with a number of scheduled regular and overtime duties. The total scheduled overtime determines the overtime budget that has already been consumed and that is no longer available in the operational allocation phase.

Reactive balancing of personnel demand and supply

The baseline personnel roster is then input to the operational allocation phase (cf. Figure 5.2), in which adjustments may be necessary to restore the workability and/or the feasibility of the baseline roster. In order to enable a good balance between the demand and supply of employees during each demand period, we can apply different reactive allocation strategies, i.e.

- The *reassignment* of the regular and overtime duties, the *conversion* of a day off to a duty and the *cancellation* of regular or overtime duties are the common strategies (cf. previous chapters).
- The allocation of *unscheduled overtime* is a time-based reactive allocation strategy, which improves the reactive flexibility through assigning overtime duties.

5.3 Methodology

The objective in this chapter is to determine the impact of time-based proactive and reactive strategies on the robustness of personnel shift rosters. For this purpose, we use a methodology that consists of three steps:

- In the first step, a baseline personnel roster and the staffing budget are determined by integrating *the strategic staffing phase and the tactical scheduling phase.* In this step, different proactive time-based strategies are introduced in the baseline personnel roster to hedge against operational variability (Section 5.3.1).
- In the second step, we start from the baseline personnel shift roster and imitate the *operational allocation phase* (Section 5.3.2), which includes a day-by-day simulation of the operational variability and an adjustment decision model to balance the supply and demand for staff (cf. Figure 5.2).
- In the third step, we *evaluate the robustness* of the baseline personnel shift roster by assessing its planned and actual performance (Section 5.3.3).

This methodology of validating robustness through an imitation of the operational phase is similar to the approaches of Bard and Purnomo (2005a) and Abdelghany et al. (2008) and the methodology in Chapter 2, and is discussed in detail in the following sections.

Note that we repeat this methodology for multiple baseline personnel shift rosters, which differ based on the applied proactive and reactive strategies. As such, we can determine the baseline personnel shift roster and the corresponding proactive and reactive strategies that provide the highest level of robustness.

5.3.1 The integrated strategic staffing and tactical scheduling phase

We study a personnel shift scheduling problem, which entails the assignment of employees to working shifts over a planning period of multiple days subject to general personnel information, objectives and constraints (Burke et al., 2004; Van den Bergh et al., 2013). In contrast to Chapters 2, 3 and 4, we integrate the strategic staffing decision in the tactical scheduling phase in order to simultaneously determine the personnel budget and a baseline personnel roster (Maenhout and Vanhoucke, 2013). The overall problem under investigation can be categorised as $AS1|RV|S||LXRG$ according to the classification of De Causmaecker and Vanden Berghe (2011). Given the high level of uncertainty, we assume that this problem is stochastic in terms of the personnel demand and employee availability.

In the integrated staffing and scheduling step, the workforce size and the allowance of overtime are interrelated by the stipulated time-based robustness strategy. In principle, the number of hired employees is a proactive scheduling strategy since a buffer against uncertainty is created (Koutsopoulos and Wilson, 1987). However, since personnel costs significantly contribute to the total operating costs of an organisation (Ernst et al., 2004b; Van den Bergh et al., 2013), organisations may opt to limit the workforce size in favour of overtime (Lobo et al., 2013). Hence, there is a trade-off between the workforce size and the budget for overtime expenses in the integrated strategic staffing and tactical scheduling phase. In this perspective, the number of overtime hours employees can work, has a significant impact on the number of employees that need to be hired and the applicability of time-based robustness strategies. A lower number of allowed overtime hours may increase the required workforce size to cover the staffing requirements and vice versa.

The mathematical formulation of the integrated staffing and scheduling problem under study is the following:

Notation

Sets

N	set of employees (index i)
D	set of days (index d)
H	set of demand periods per day (index h)
S	set of shifts (index j)
S^-	set of shifts for which the prior shift extension covers a demand period on the previous day
S^+	set of shifts for which the subsequent shift extension covers a demand period on the next day
$T_{dj}^{\circ\vert\circ}$	set of shifts that cannot be assigned the day after day d and shift j (index s)
$T_{dj}^{\circ\vert'}$	set of prior shift extensions that cannot be assigned after day d and shift j (index s) with $T_{dj}^{\circ\vert\circ} \cap T_{dj}^{\circ\vert'} = \varnothing$
$T_{dj}^{\circ\vert''}$	set of subsequent shift extensions that cannot be assigned after day d and shift j (index s) with $(T_{dj}^{\circ\vert\circ} \cup T_{dj}^{\circ\vert'}) \cap T_{dj}^{\circ\vert''} = \varnothing$
$T_{dj}^{''\vert\circ}$	set of shifts that cannot be assigned the day after a subsequent shift extension on day d and shift j (index s)
$T_{dj}^{''\vert'}$	set of prior shift extensions that cannot be assigned the day after a subsequent shift extension on day d and shift j (index s) with $T_{dj}^{''\vert\circ} \cap T_{dj}^{''\vert'} = \varnothing$
$T_{dj}^{''\vert''}$	set of subsequent shift extensions that cannot be assigned the day after a subsequent shift extension on day d and shift j (index s) with $(T_{dj}^{''\vert\circ} \cup T_{dj}^{''\vert'}) \cap T_{dj}^{''\vert''} = \varnothing$

Deterministic parameters

l	duration of a demand period
β_j	number of demand periods in shift j
$\beta_j^{'}$	number of demand periods in the prior shift extension of shift j
$\beta_j^{''}$	number of demand periods in the subsequent shift extension of shift j

z_{jh}	1 if shift j covers demand period h, 0 otherwise
z'_{jh}	1 if the prior shift extension of shift j covers demand period h, 0 otherwise
z''_{jh}	1 if the subsequent shift extension of shift j covers demand period h, 0 otherwise
c^{w}	hourly regular wage cost
c^{wo}	hourly overtime wage cost
c^{wu}	cost for understaffing a demand period
R^{w}_{dh}	expected staffing requirement for demand period h on day d
$R^{w,extra}_{dh}$	extra staffing requirement for demand period h on day d
a_{id}	expected availability of employee i on day d, 1 if the employee is available and 0 otherwise
l^{max}_{id}	maximum number of regular and overtime hours that can be assigned to employee i on day d
$l^{w,max}_{id}$	maximum number of regular hours that can be assigned to employee i on day d
$l^{wo,max}_{id}$	maximum number of overtime hours that can be assigned to employee i on day d
η^{min}_{i}	minimum number of regular and overtime hours that have to be assigned to employee i
$\eta^{w,max}_{i}$	maximum number of regular hours that can be assigned to employee i
$\eta^{wo,max}_{i}$	maximum number of overtime hours that can be assigned to employee i
$\theta^{w,max}_{i}$	maximum number of consecutive working assignments for employee i

Stochastic parameters

$\tilde{R}^{w}_{dh}$	stochastic staffing requirement for demand period h on day d
$\tilde{a}_{id}$	stochastic availability of employee i on day d; 1 if the employee is available, 0 otherwise

Variables

ζ_i	1 if employee i is hired, 0 otherwise
x^{w}_{idj}	1 if employee i is assigned to shift j on day d, 0 otherwise
x^{wo}_{idj}	1 if employee i is assigned to overtime during a complete shift j on day d, 0 otherwise
$x^{wo'}_{idj}$	1 if employee i is assigned to overtime during a prior shift extension of shift j on day d, 0 otherwise
$x^{wo''}_{idj}$	1 if employee i is assigned to overtime during a subsequent shift extension of shift j on day d, 0 otherwise
x^{v}_{id}	1 if employee i receives a day off on day d, 0 otherwise
x^{wu}_{dh}	the shortage of employees during demand period h on day d

Mathematical formulation

$$Minimise \quad \sum_{i\in N}\sum_{d\in D}\sum_{j\in S} l\beta_j c^{w} x^{w}_{idj} + \sum_{d\in D}\sum_{h\in H} c^{wu} x^{wu}_{dh} \tag{5.1a}$$

$$+\sum_{i\in N}\sum_{d\in D}\sum_{j\in S} l\beta_j c^{wo} x^{wo}_{idj} + \sum_{i\in N}\sum_{d\in D}\sum_{j\in S} l\beta'_j c^{wo} x^{wo'}_{idj} + \sum_{i\in N}\sum_{d\in D}\sum_{j\in S} l\beta''_j c^{wo} x^{wo''}_{idj} \tag{5.1b}$$

subject to

$$\begin{aligned}&\sum_{i\in N}\sum_{j\in S}\tilde{a}_{id} z_{jh}(x^{w}_{idj}+x^{wo}_{idj}) + \sum_{i\in N}\sum_{j\in S\setminus S-}\tilde{a}_{id} z'_{jh} x^{wo'}_{idj} + \sum_{i\in N}\sum_{j\in S-}\tilde{a}_{id} z'_{jh} x^{wo'}_{id+1j}\\ &+\sum_{i\in N}\sum_{j\in S\setminus S+}\tilde{a}_{id} z''_{jh} x^{wo''}_{idj} + \sum_{i\in N}\sum_{j\in S+}\tilde{a}_{id} z''_{jh} x^{wo''}_{id-1j} + x^{wu}_{dh} \geq \tilde{R}^{w}_{dh} + R^{w,extra}_{dh} \qquad \forall d\in D, \forall h \in H\end{aligned} \tag{5.2}$$

$$\sum_{j\in S}(x^{w}_{idj}+x^{wo}_{idj}) + x^{v}_{id} = \zeta_i \qquad \forall i\in N, \forall d \in D \tag{5.3}$$

$$x^{wo'}_{idj} \leq x^{w}_{idj} + x^{wo}_{idj} \qquad \forall i\in N, \forall d\in D, \forall j \in S \tag{5.4a}$$

$$x^{wo''}_{idj} \leq x^{w}_{idj} + x^{wo}_{idj} \qquad \forall i\in N, \forall d\in D, \forall j \in S \tag{5.4b}$$

$$x^{w}_{idj} + x^{wo}_{idj} + \sum_{s \in T^{\circ|'}_{dj}} x^{wo'}_{id+1s} + \sum_{s \in T^{\circ|\circ}_{dj}} (x^{w}_{id+1s} + x^{wo}_{id+1s}) + \sum_{s \in T^{\circ|''}_{dj}} x^{wo''}_{id+1s} \leq 1 \qquad \forall i \in N, \forall d \in D, \forall j \in S \quad (5.5a)$$

$$x^{wo''}_{idj} + \sum_{s \in T^{''|'}_{dj}} x^{wo'}_{id+1s} + \sum_{s \in T^{''|\circ}_{dj}} (x^{w}_{id+1s} + x^{wo}_{id+1s}) + \sum_{s \in T^{''|''}_{dj}} x^{wo''}_{id+1s} \leq 1 \qquad \forall i \in N, \forall d \in D, \forall j \in S \quad (5.5b)$$

$$\sum_{j \in S} \left(l\beta_j (x^{w}_{idj} + x^{wo}_{idj}) + l\beta^{'}_{j} x^{wo'}_{idj} + l\beta^{''}_{j} x^{wo''}_{idj} \right) \leq l^{max}_{id} \qquad \forall i \in N, \forall d \in D \quad (5.6a)$$

$$\sum_{j \in S} l\beta_j x^{w}_{idj} \leq l^{w,max}_{id} \qquad \forall i \in N, \forall d \in D \quad (5.6b)$$

$$\sum_{j \in S} \left(l\beta_j x^{wo}_{idj} + l\beta^{'}_{j} x^{wo'}_{idj} + l\beta^{''}_{j} x^{wo''}_{idj} \right) \leq l^{wo,max}_{id} \qquad \forall i \in N, \forall d \in D \quad (5.6c)$$

$$\sum_{d \in D} \sum_{j \in S} \left(l\beta_j (x^{w}_{idj} + x^{wo}_{idj}) + l\beta^{'}_{j} x^{wo'}_{idj} + l\beta^{''}_{j} x^{wo''}_{idj} \right) \geq \eta^{min}_{i} \zeta_i \qquad \forall i \in N \quad (5.7)$$

$$\sum_{d \in D} \sum_{j \in S} l\beta_j x^{w}_{idj} \leq \eta^{w,max}_{i} \zeta_i \qquad \forall i \in N \quad (5.8a)$$

$$\sum_{d \in D} \sum_{j \in S} (l\beta_j x^{wo}_{idj} + l\beta^{'}_{j} x^{wo'}_{idj} + l\beta^{''}_{j} x^{wo''}_{idj}) \leq \eta^{wo,max}_{i} \zeta_i \qquad \forall i \in N \quad (5.8b)$$

$$\sum_{\underline{d}=d}^{d+\theta^{w,max}_{i}} (1 - x^{v}_{i\underline{d}}) \leq \theta^{w,max}_{i} \qquad \forall i \in N, \forall d \in D \quad (5.9)$$

$$\begin{aligned}
\zeta_i &\in \{0,1\} && \forall i \in N \\
x^{w}_{idj} &\in \{0,1\} && \forall i \in N, \forall d \in D, \forall j \in S \\
x^{wo}_{idj} &\in \{0,1\} && \forall i \in N, \forall d \in D, \forall j \in S \\
x^{wo'}_{idj} &\in \{0,1\} && \forall i \in N, \forall d \in D, \forall j \in S \quad (5.10) \\
x^{wo''}_{idj} &\in \{0,1\} && \forall i \in N, \forall d \in D, \forall j \in S \\
x^{v}_{id} &\in \{0,1\} && \forall i \in N, \forall d \in D \\
x^{wu}_{dh} &\geq 0 && \forall d \in D, \forall h \in H
\end{aligned}$$

We solve a deterministic version of this stochastic model formulation by assuming that the stochastic staffing requirements equal the expected staffing requirements ($\tilde{R}^{w}_{dh} = R^{w}_{dh}$) and that the employees are available on each day ($\tilde{a}_{id} = a_{id} = 1$). We obtain a baseline personnel shift roster by solving this model with the commercial software package Gurobi (Gurobi Optimization, Inc., 2015).

The general objective in model (5.1)-(5.10) is to minimise the employee wage costs and the cost for understaffing particular demand periods. Objective (5.1a) minimises the wage costs for assigning workers to regular duties and the understaffing costs. Objective (5.1b) minimises the cost for assigning overtime during complete shifts and during a prior shift extension or a subsequent shift extension.

In order to construct a workable personnel shift roster, a specified number of employees should be scheduled for every demand period (eq. (5.2)). The staffing requirements include the minimum staffing requirements for duties ($\tilde{R}^{w}_{dh}$) and the staffing requirements for extra duties ($R^{w,extra}_{dh}$). Note that the staffing requirements $R^{w,extra}_{dh}$ are only employed for certain time-based buffer strategies (cf. Section 5.4). The staffing constraints are relaxed since understaffing is allowed.

We impose different time-related constraints on the schedule of a single employee. Equation (5.3) stipulates that each hired employee receives an assignment on each day. A prior and subsequent daily shift time extension of a regular duty can only be assigned to employees who work the corresponding shift (eq. (5.4a) and (5.4b)). Moreover, a minimum rest period between two consecutive duties is imposed by the constraints (5.5a) and (5.5b). Equation (5.5a) is the common constraint that prohibits certain duty assignments to succeed a particular duty. This constraint does not completely ensure the satisfaction of the minimum rest period since a shift can be extended by an overtime subsequent shift extension, which is considered by equation (5.5b). Notice that the succession constraint that restricts the type of duty that follows a prior shift extension, is implicitly modelled by constraint (5.5a). Furthermore, every employee can work a maximum number of hours per day (eq. (5.6a)). This maximum is further refined into two constraints that impose a maximum on the number of regular hours per day (eq. (5.6b)) and a maximum on the number of overtime hours per day (eq. (5.6c)). We also impose constraints on the number of hours assigned over the total planning period. Every hired employee has to work a minimum number of hours (eq. (5.7)). Equation (5.8) imposes a maximum on the number of hours an employee can work. Constraints (5.8a) and (5.8b) respectively impose a maximum on the number of regular hours and overtime hours for a single employee over the total planning period. Irrespective of the duration of the assignments, equation (5.9) ensures that the number of consecutive duties is limited for every employee.

Equation (5.10) defines the integrality conditions for each variable.

5.3.2 Operational allocation phase

The operational allocation phase considers the baseline personnel roster on a day-by-day basis (cf. Chapter 2) and comprehends a simulation component (Section 5.3.2.1) and an adjustment component (Section 5.3.2.2).

5.3.2.1 Simulation of operational variability

The simulation component simulates the uncertainty of capacity (a_{id}) for each employee. The uncertainty of demand is simulated for every demand period of the day under consideration. This means that the demand for employees is independent between demand periods, which is based on the assumption of an exogenous demand characterised by independent and identically distributed inter-arrival and service times (de Bruin et al., 2010; Paul and Lin, 2012). Hence, the demand can differ between demand periods within a single standard shift. In order to avoid too large differences in our simulation experiment and to control the demand variability in general, we impose a lower and upper bound on the possible values for the staffing requirements (eqs. (5.11) and (5.12)). In these equations, the variability is expressed with a parameter (σ) that ranges from one (low variability) to $+\infty$ (high variability). Given the assumption that the demand is Poisson distributed (Ahmed and Alkhamis, 2009; Yeh and Lin, 2007), we simulate the demand uncertainty using the staffing requirement R^w_{dh} as the mean and accept the simulated staffing requirements ($R^{'w}_{dh}$) if equation (5.13) is satisfied. Otherwise, we repeat the simulation of the demand uncertainty.

$$LB_h = max(R^w_{dh} - \sigma, 0) \qquad \forall h \in H \tag{5.11}$$

$$UB_h = R^w_{dh} + \sigma \qquad \forall h \in H \tag{5.12}$$

$$R^{'w}_{dh} \in [LB_h, UB_h] \qquad \forall h \in H \tag{5.13}$$

5.3.2.2 Balancing supply and demand

As a result of the new information obtained by the simulation of supply and demand, the personnel planner needs to evaluate whether the day roster needs to be adjusted. These adjustments are guided by the reactive allocation strategies, which include the allocation of unscheduled overtime (Bard and Purnomo, 2005a). The mathematical formulation of the operational allocation problem under study is given below. Note that we only define those sets, parameters and variables that are specific to the operational allocation phase to avoid duplication.

Notation

Sets

B_{id}	set of shifts that cannot be assigned to employee i on day d (index j)
$B^{'}_{id}$	set of shifts for which the prior shift extension cannot be assigned to employee i on day d (index j)
$B^{''}_{id}$	set of shifts for which the subsequent shift extension cannot be assigned to employee i on day d (index j)
$\underline{T}^{''\vert'}_{d+1j}$	set of subsequent shift extensions that cannot be assigned the day before a prior shift extension on day $d+1$ and shift j (index s)

General parameters

d	day under consideration in the operational planning horizon
M	a large number
$\kappa^{\alpha'}_{idj}$	1 if employee i is allowed to receive a prior shift extension corresponding to shift $j \in S^-$ on day $d+1$, 0 otherwise
$\kappa^{\alpha''}_{idj}$	1 if employee i is allowed to receive a subsequent shift extension corresponding to shift $j \in S^+$ on day $d-1$, 0 otherwise
κ^{f}_{idj}	1 if employee i is forced to work shift $j \in S^-$ on day d, 0 otherwise
κ^{min}_{id}	the total number of hours employee i is forced to work on day d

$\kappa_{id}^{w,max}$	the maximum number of regular hours employee i can work on day d
$\kappa_{id}^{wo,max}$	the maximum number of overtime hours employee i can work on day d

Simulation parameters

a_{id}	1 if employee i is available on day d, 0 otherwise
$R_{dh}^{'w}$	simulated staffing requirement for demand period h on day d

Roster change parameters

$\bar{x}_{idj}^{w}$	1 if employee i was assigned to shift j on day d in the baseline personnel roster, 0 otherwise
$\bar{x}_{idj}^{wo}/\bar{x}_{idj}^{wo'}/\bar{x}_{idj}^{wo''}$	1 if employee i was assigned to overtime during shift j/prior shift extension of shift j/ subsequent shift extension of shift j on day d in the baseline personnel roster, 0 otherwise
$\bar{x}_{id}^{v}$	1 if employee i received a day off on day d in the baseline personnel roster, 0 otherwise
$c_{idj}^{w\delta}/c_{idj}^{w\delta'}/c_{idj}^{w\delta''}$	roster change cost for assigning employee i to shift j/prior shift extension of shift j/ subsequent shift extension of shift j on day d with $c_{idj}^{w\delta} > 0$ if $\bar{x}_{idj}^{w} + \bar{x}_{idj}^{wo} = 0$ with $c_{idj}^{w\delta'} \geq 0$ if $\bar{x}_{idj}^{wo'} = 0$ and $c_{idj}^{w\delta'} = 0$ otherwise with $c_{idj}^{w\delta''} \geq 0$ if $\bar{x}_{idj}^{wo''} = 0$ and $c_{idj}^{w\delta''} = 0$ otherwise
c_{id}^{v}	cancellation cost for employee i on day d with $c_{id}^{v} > 0$ if $\bar{x}_{id}^{v} = 0 \wedge a_{id} = 1$ and $c_{id}^{v} = 0$ otherwise

Mathematical formulation

$$\begin{aligned}
\textit{Minimise} \quad & \sum_{i\in N}\sum_{j\in S}\Big((l\beta_j c^{w} + c_{idj}^{w\delta})x_{idj}^{w} + (l\beta_j c^{wo} + c_{idj}^{w\delta})x_{idj}^{wo}\Big) + \sum_{i\in N} c_{id}^{v}x_{id}^{v} + \sum_{h\in H} c^{wu}x_{dh}^{wu} \\
& + \sum_{i\in N}\Big(\sum_{j\in S\setminus S^-}(l\beta_j^{'}c^{wo} + c_{idj}^{w\delta'})x_{idj}^{wo'} + \sum_{j\in S^-}(l\beta_j^{'}c^{wo} + c_{id+1j}^{w\delta'})x_{id+1j}^{wo'}\Big) \\
& + \sum_{i\in N}\Big(\sum_{j\in S\setminus S^+}(l\beta_j^{''}c^{wo} + c_{idj}^{w\delta''})x_{idj}^{wo''} + \sum_{j\in S^+}(l\beta_j^{''}c^{wo} + c_{id-1j}^{w\delta''})x_{id-1j}^{wo''}\Big)
\end{aligned} \tag{5.14}$$

subject to

$$\begin{aligned}
& \sum_{i\in N}\sum_{j\in S} z_{jh}(x_{idj}^{w} + x_{idj}^{wo}) + \sum_{i\in N}\sum_{j\in S\setminus S^-} z_{jh}^{'}x_{idj}^{wo'} + \sum_{i\in N}\sum_{j\in S^-} z_{jh}^{'}x_{id+1j}^{wo'} + \sum_{i\in N}\sum_{j\in S\setminus S^+} z_{jh}^{''}x_{idj}^{wo''} \\
& \qquad + \sum_{i\in N}\sum_{j\in S^+} z_{jh}^{''}x_{id-1j}^{wo''} + x_{dh}^{wu} \geq R_{dh}^{'w} \qquad \forall h \in H
\end{aligned} \tag{5.15}$$

$$\sum_{j\in S}(x_{idj}^{w} + x_{idj}^{wo}) + x_{id}^{v} = 1 \qquad \forall i \in N \tag{5.16}$$

$$x_{idj}^{wo'} \leq x_{idj}^{w} + x_{idj}^{wo} \qquad \forall i \in N, \forall j \in S \setminus S^- \tag{5.17a}$$

$$x_{id+1j}^{wo'} \leq \kappa_{idj}^{\alpha'}(1 - \sum_{s\in \underline{T}_{d+1j}^{''|'}} x_{ids}^{wo''}) \qquad \forall i \in N, \forall j \in S^- \tag{5.17b}$$

$$x_{idj}^{wo''} \leq x_{idj}^{w} + x_{idj}^{wo} \qquad \forall i \in N, \forall j \in S \setminus S^+ \tag{5.17c}$$

$$x^{wo''}_{id-1j} \leq \kappa^{\alpha''}_{idj}(1 - \sum_{s \in T^{''|'}_{d-1j}} x^{wo'}_{ids}) \qquad \forall i \in N, \forall j \in S^{+} \quad (5.17d)$$

$$\sum_{j \in B_{id}} (x^{w}_{idj} + x^{wo}_{idj}) + \sum_{j \in B^{'}_{id}} x^{wo'}_{idj} + \sum_{j \in B^{''}_{id}} x^{wo''}_{idj} = 0 \qquad \forall i \in N \quad (5.18)$$

$$\sum_{j \in S}(x^{w}_{idj} + x^{wo}_{idj}) + \sum_{j \in S \setminus S^{-}} x^{wo'}_{idj} + \sum_{j \in S^{-}} x^{wo'}_{id+1j} + \sum_{j \in S \setminus S^{+}} x^{wo''}_{idj} + \sum_{j \in S^{+}} x^{wo''}_{id-1j} \leq Ma_{id} \qquad \forall i \in N \quad (5.19)$$

$$x^{w}_{idj} + x^{wo}_{idj} \geq \kappa^{f}_{idj} a_{id} \qquad \forall i \in N, \forall j \in S^{-} \quad (5.20)$$

$$\sum_{j \in S} \left(l\beta_j (x^{w}_{idj} + x^{wo}_{idj}) + l\beta^{'}_j x^{wo'}_{idj} + l\beta^{''}_j x^{wo''}_{idj} \right) \geq \kappa^{min}_{id} a_{id} \qquad \forall i \in N \quad (5.21)$$

$$\sum_{j \in S} \left(l\beta_j (x^{w}_{idj} + x^{wo}_{idj}) + l\beta^{'}_j x^{wo'}_{idj} + l\beta^{''}_j x^{wo''}_{idj} \right) \leq l^{max}_{id} \qquad \forall i \in N \quad (5.22a)$$

$$\sum_{j \in S} l\beta_j x^{w}_{idj} \leq \kappa^{w,max}_{id} \qquad \forall i \in N \quad (5.22b)$$

$$\sum_{j \in S} (l\beta_j x^{wo}_{idj} + l\beta^{'}_j x^{wo'}_{idj} + l\beta^{''}_j x^{wo''}_{idj}) \leq \kappa^{wo,max}_{id} \qquad \forall i \in N \quad (5.22c)$$

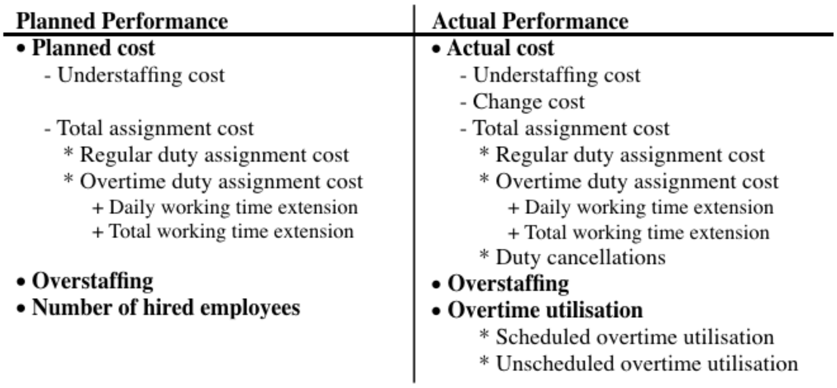
$$\begin{aligned}
x^{w}_{idj} &\in \{0,1\} & \forall i \in N, \forall j \in S \\
x^{wo}_{idj} &\in \{0,1\} & \forall i \in N, \forall j \in S \\
x^{wo'}_{idj} &\in \{0,1\} & \forall i \in N, \forall j \in S \setminus S^{-} \\
x^{wo'}_{id+1j} &\in \{0,1\} & \forall i \in N, \forall j \in S^{-} \\
x^{wo''}_{idj} &\in \{0,1\} & \forall i \in N, \forall j \in S \setminus S^{+} \quad (5.23) \\
x^{wo''}_{id-1j} &\in \{0,1\} & \forall i \in N, \forall j \in S^{+} \\
x^{v}_{id} &\in \{0,1\} & \forall i \in N \\
x^{wu}_{dh} &\geq 0 & \forall h \in H
\end{aligned}$$

We assign regular shifts, overtime duties and days off by solving this operational allocation model (eqs. (5.14)-(5.23)) with the commercial optimisation package Gurobi (Gurobi Optimization, Inc., 2015). The objective function (5.14) minimises the actual total cost that arises on day d in the operational allocation phase. This cost is comprised of the wage cost for regular and overtime duties, the change and cancellation cost for duties assigned in the baseline personnel roster and the cost for understaffing particular demand periods.

The staffing requirements (eq. (5.15)) postulate that a sufficient number of employees is present during each demand period, which ensures that the workers are assigned to the right demand periods to cover the demand for staff. This constraint is relaxed as understaffing is allowed, which is penalised in the objective function. Note that the staffing requirements $R^{'w}_{dh}$ in this operational decision model are the result of the simulated operational variability and can differ from the expected demand for staff (R^{w}_{dh}) used to construct the baseline personnel roster ($\tilde{R}^{w}_{dh} = R^{w}_{dh}$). The other constraints embody time-related rules imposed on the duty roster of a single employee. Equation (5.16) stipulates that each employee needs to receive either a shift assignment or a day off on day d. A standard shift may be extended with a prior shift extension (eqs. (5.17a)-(5.17b)) or a subsequent shift extension (eqs. (5.17c)-(5.17d)) only if the worker is assigned to the corresponding standard shift. Equations (5.17a) and (5.17c) regulate the daily shift extensions corresponding to standard shifts on day d. Equations (5.17b) and (5.17d) respectively determine whether a prior shift extension corresponding to a shift on day $d + 1$ and a subsequent shift extension corresponding to a shift on day $d - 1$ can be assigned on day d. Moreover, these equations ensure that we do not violate the minimum rest period between assignments that cover demand periods on day d. However, we also need to impose the minimum rest period between assignments that cover demand periods on different days. Constraint (5.18) ensures that the sequence and series constraints imposed on the baseline personnel roster (cf. eqs. (5.5a), (5.5b) and (5.9)) are satisfied given the assignments on the other days of the planning horizon by the definition of the sets B_{id}, $B^{'}_{id}$ and $B^{''}_{id}$. Equation (5.19) imposes that an employee can only receive work duties when he is available. The parameter a_{id} indicates for each employee whether this employee is available to perform a duty on day d and is the result of the simulated operational variability. Given that the prior shift extension corresponding to a standard shift on day d can cover a demand period on day $d - 1$, an employee should be assigned to that standard shift if this extension was assigned to that employee on day $d - 1$ (eq. (5.20)). In this respect, there is a dependency between day $d - 1$ and day d that is penalised as overstaffing in case of smaller-than-expected demand during the corresponding shift on day d. However, it is important to note that this prior shift extension will not have been assigned on day $d - 1$ unless the employee was scheduled to work the corresponding shift on day d in the baseline personnel shift roster (cf. $\kappa^{\alpha'}_{idj}$ in eq. (5.17b)). Based on the assignments on the other days, each employee needs to work a minimum number of hours on day d if available (eq. (5.21)) in order to respect the minimum number of working hours over the complete planning horizon. Equation (5.22a) limits the number of hours an employee can work on one particular day. Equations (5.22b) and (5.22c) impose a restriction upon the allowed number of regular and overtime hours on day d in order to satisfy the maximum number of regular and overtime hours over the complete planning horizon. Constraint (5.23) defines the integrality conditions of the variables that correspond to demand periods that lie within the planning period.

5.3.3 Robustness evaluation

We evaluate the personnel shift rosters based on their planned and actual performance. Figure 5.4 gives a brief overview of the different components of both performance measures.



Planned Performance	**Actual Performance**
• Planned cost	**• Actual cost**
- Understaffing cost	- Understaffing cost
	- Change cost
- Total assignment cost	- Total assignment cost
* Regular duty assignment cost	* Regular duty assignment cost
* Overtime duty assignment cost	* Overtime duty assignment cost
+ Daily working time extension	+ Daily working time extension
+ Total working time extension	+ Total working time extension
	* Duty cancellations
• Overstaffing	**• Overstaffing**
• Number of hired employees	**• Overtime utilisation**
	* Scheduled overtime utilisation
	* Unscheduled overtime utilisation

Figure 5.4 The building blocks of the planned and actual performance

The planned performance reflects the planned cost, the existent overstaffing of the baseline personnel roster and the number of hired employees. The planned cost comprises the understaffing cost and the total assignment cost, i.e. the wage cost and the overtime wage cost.
The actual performance evaluates the actual cost, the overstaffing and the overtime utilisation of the eventual personnel roster. The actual cost includes the cost for understaffing, the change cost and the total assignment cost, i.e. the wage cost, the overtime wage cost and the cancellation cost. The overtime utilisation comprises the scheduled overtime utilisation and the unscheduled overtime duties. The scheduled overtime utilisation reports the percentage of scheduled overtime periods that are actually utilised in the operational allocation phase. Furthermore, we identify the number of demand periods during which unscheduled overtime is reactively allocated.

5.4 Computational experiments

In this section, we provide insight into our computational experiments and the robustness of the time-based proactive and reactive strategies. In Section 5.4.1, we describe our test design and parameter settings and we define our time-based proactive scheduling and reactive allocation strategies. We outline the different experiments and their results in Section 5.4.2. All tests were carried out on an Intel Core processor 2.5 GHz and 4 GB RAM.

5.4.1 Test design

In this section, we describe the parameter settings of the test instances for our computational experiments and we formulate our time-based strategies. Note that all test instances have a planning period of 7 days.

Shift characteristics
Each day consists of six demand periods with a duration of 4 hours (l). Hence, each day contains

three non-overlapping shifts with a duration of 8 hours ($l \times \beta_j$) and three prior shift extensions ($l \times \beta_j^{'}$) and subsequent shift extensions ($l \times \beta_j^{''}$) with a duration of 4 hours (cf. Figure 5.3).

Staffing requirements
We generate staffing requirements based on three indicators defined in literature assuming a fixed hiring level of 10 employees. Vanhoucke and Maenhout (2009) define the *total coverage constrainedness (TCC)*, the *day coverage distribution (DCD)* and the *shift coverage distribution (SCD)* to characterise the staffing requirements (cf. Chapter 2). We consider *TCC-values* of 0.30, 0.40 and 0.50, *DCD-values* of 0.00, 0.25 and 0.50 and *SCD-values* of 0.00, 0.25 and 0.50. Since the basic assignments in the integrated strategic staffing and tactical scheduling phase (Section 5.3.1) occur on a shift level, we generate the staffing requirements per shift. Next, we transfer these requirements to the demand periods that correspond to that shift (R_{dh}^{w}).

Time-related constraints
We define the parameter values for the time-related constraints below, i.e.

- The maximum number of regular and overtime hours that can be assigned to employee i on day d (l_{id}^{max}) is 12.
- The maximum number of regular hours that can be assigned to employee i on day d ($l_{id}^{w,max}$) is 8.
- The maximum number of overtime hours that can be assigned to employee i on day d ($l_{id}^{wo,max}$) is 12.
- The minimum number of regular and overtime hours that have to be assigned to employee i (η_i^{min}) is 32.
- The maximum number of regular hours that can be assigned to employee i ($\eta_i^{w,max}$) is 40.
- The maximum number of overtime hours that can be assigned to employee i ($\eta_i^{wo,max}$) is 12.
- The maximum number of consecutive assignments for employee i ($\theta_i^{w,max}$) is 5.

Objective function
The objective function coefficients used during the integrated strategic staffing and tactical scheduling phase (eqs. (5.1)-(5.10)) and the operational allocation phase (eqs. (5.14)-(5.23)) are defined as follows:

- General objective function coefficients
  - The hourly regular wage cost (c^{w}) is 1.25.
  - The hourly overtime wage cost (c^{wo}) is 1.875.
  - The cost for understaffing a demand period (c^{wu}) is 10.
- Objective function coefficients specific to the operational allocation phase
  - The roster change cost for assigning employee i to shift j on day d ($c_{idj}^{w\delta}$) is 5 if the employee was not originally assigned to shift j or 2.5 if the employee was originally assigned to a prior or subsequent shift extension that overlaps with shift j. Otherwise, this cost equals 0.

- The roster change cost for assigning employee i to a prior shift extension/a subsequent shift extension of shift j on day d ($c_{idj}^{w\delta'}/c_{idj}^{w\delta''}$) is 2.5. This cost is 0 if the employee was originally assigned to this extension or if the extension overlaps with a shift the employee was originally assigned to.
- The cancellation cost for employee i on day d (c_{id}^{v}) is 5.

Time-based proactive scheduling and reactive allocation strategies
We consider three types of baseline personnel rosters that differ based on the applied time-based proactive scheduling and reactive allocation strategy, i.e.

- The *basic baseline roster* does not include any (un)scheduled overtime. This means that we do not allow employees to be assigned to overtime duties in the integrated strategic staffing and tactical scheduling phase or in the operational allocation phase, i.e. $l_{id}^{wo,max} = \eta_i^{wo,max} = 0$. As such, employees cannot be assigned to prior or subsequent shift extensions. Moreover, we do not require extra staffing requirements in the tactical scheduling phase, i.e. $R_{dh}^{w,extra} = 0$ ($\forall d \in D, \forall j \in S$).
- The *minimum cost baseline roster* does include scheduled and unscheduled overtime. Hence, employees can extend their daily working time and total working time by overtime duties in the integrated strategic staffing and tactical scheduling phase and in the operational allocation phase, i.e. $l_{id}^{wo,max} = \eta_i^{wo,max} = 12$. However, we do not require extra duties to be scheduled in the tactical scheduling phase, i.e. $R_{dh}^{w,extra} = 0$ ($\forall d \in D, \forall j \in S$).
- The *time buffer baseline roster* is very similar to the *minimum cost baseline roster* in terms of overtime, i.e. $l_{id}^{wo,max} = \eta_i^{wo,max} = 12$. The sole difference is that we introduce specific staffing requirements in the integrated strategic staffing and tactical scheduling phase on top of the minimum staffing requirements (R_{dh}^{w}). These requirements entail staffing requirements for extra duties ($R_{dh}^{w,extra}$). Hence, a capacity buffer is installed based upon the number of available employees and the intrinsic time buffer, which is determined by the overtime policy and the time-related restrictions imposed on the schedule of a single worker. The capacity buffer is defined based upon the research in Chapter 2 that identifies a fixed ratio of the minimum staffing requirements as a good strategy to install a buffer capacity. We round the staffing requirements for the extra duties to the nearest integer (eq. (5.24)).

$$R_{dh}^{w,extra} = round[0.25 \times R_{dh}^{w}] \qquad \forall d \in D, \forall h \in H \qquad (5.24)$$

  These extra staffing requirements ($R_{dh}^{w,extra}$) and the expected staffing requirements (R_{dh}^{w}) are available online.[1]

Note that each of these rosters is obtained by solving model (5.1)-(5.10) with specific values for the maximum number of overtime hours that can be assigned per day ($l_{id}^{wo,max}$ in eq. (5.6c)) and over the planning horizon ($\eta_i^{wo,max}$ in eq. (5.8b)), and for the extra staffing requirements ($R_{dh}^{w,extra}$ in eq. (5.2)).

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[1] `http://www.projectmanagement.ugent.be/?q=research/rps`

5.4.2 Computational results

In this section, we describe the computational experiments and results. Unless otherwise stated, the computational results are averaged over all settings in the test design (Section 5.4.1).
We construct the three types of baseline personnel rosters by solving model (5.1)-(5.10) with Gurobi (Gurobi Optimization, Inc., 2015). Given that certain instances could not be solved within a reasonable time however, we impose an MIP-gap of 5%. This MIP-gap enables us to construct baseline personnel shift rosters, which require an average CPU-time of 0.658 seconds. In order to solve the operational allocation model (eqs. (5.14)-(5.23)) with Gurobi (Gurobi Optimization, Inc., 2015), we do not impose an MIP-gap because the solution times are negligible. Section 5.4.2.1 reveals the benefits in terms of planned performance corresponding to the introduction of scheduled overtime in the baseline personnel roster. In Section 5.4.2.2, we assess the impact of overtime on the actual performance in the operational allocation phase and investigate the trade-off between the hiring policy and the overtime policy from different perspectives. In Section 5.4.2.3, we determine the extra number of employees required to improve the effectivity of the time buffer baseline roster. The impact of the variability of demand on the actual performance is studied in Section 5.4.2.4.

5.4.2.1 The impact of overtime on the planned performance

The total available overtime budget is calculated based on the number of hired employees ($\sum_{i\in N}\zeta_i$), the number of overtime hours employees are allowed to work ($\eta_i^{wo,max}$) and the hourly overtime wage cost (c^{wo}), i.e. the overtime budget $= c^{wo} \times \sum_{i\in N}\zeta_i \times \eta_i^{wo,max}$. This budget can be distributed over scheduled and unscheduled overtime in the integrated strategic staffing and tactical scheduling phase and the operational allocation phase respectively. In order to control this distribution, an additional constraint (eq. (5.25)) is imposed on the integrated strategic staffing and tactical scheduling decision model (eqs. (5.1)-(5.10)). This constraint ensures the construction of a baseline personnel roster where only a fraction f of the overtime budget may be used for scheduled overtime, i.e. $f = (Scheduled\,OT)\%$.

$$\sum_{i\in N}\sum_{d\in D}\sum_{j\in S}(l\beta_j c^{wo} x^{wo}_{idj} + l\beta_j^{'} c^{wo} x^{wo'}_{idj} + l\beta_j^{''} c^{wo} x^{wo''}_{idj}) \leq f \times overtime\ budget \tag{5.25}$$

In Figure 5.5, we show the impact on the planned cost for different values for the parameter f, i.e. 0.00, 0.20, 0.40, 0.60, 0.80 and 1.00. This parameter determines the percentage of the overtime budget that is available in the integrated strategic staffing and tactical scheduling phase for scheduled overtime. Figure 5.5 shows that this parameter f has no impact on the basic baseline roster as overtime is not incorporated. The planned cost for the minimum cost baseline roster and the time buffer baseline roster shows a steady decrease as the budget for scheduled overtime increases. Allotting the overtime completely in the integrated staffing and scheduling phase, i.e. (Scheduled OT)% = 100%, results in an average cost decrease of 5.2% compared to the

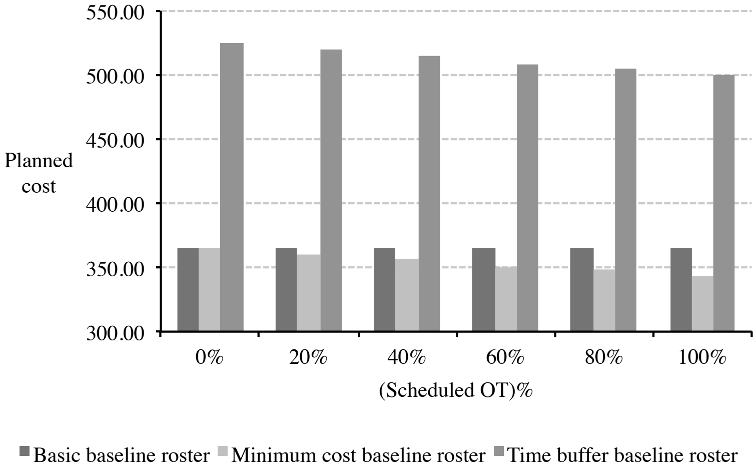

Figure 5.5 The impact of overtime on the planned cost for a fixed hiring level of 4 employees

scenario without scheduled overtime, i.e. (Scheduled OT)% = 0%. This improvement is due to the decrease in the understaffing, which compensates the rise in the wage cost for overtime, which is shown in Table 5.1. Note that the results in Figure 5.5 and Table 5.1 are obtained with a fixed hiring level of 4 employees. We observe the same trend for a higher number of hired employees but as the number of hired employees increases, the impact of scheduled overtime decreases.

Table 5.1 The evolution of the average planned performance metrics over the minimum cost and time buffer baseline roster for a fixed hiring level of 4 employees

	(Scheduled OT)%					
	0%	20%	40%	60%	80%	100%
Hired employees	4.00	4.00	4.00	4.00	4.00	4.00
Understaffing	18.47	17.06	15.84	13.98	12.89	11.56
Overstaffing	0.02	0.04	0.01	0.00	0.01	0.01
Overtime	0.00	2.00	3.80	6.27	7.71	9.43

The incorporation of scheduled overtime in the minimum cost and time buffer baseline roster leads to an increased assignment flexibility, which is not existent in the basic baseline roster. This flexibility results in a planned cost that is smaller for the minimum cost baseline roster than for the basic baseline roster. The planned cost for the time buffer baseline roster is significantly higher compared to the basic and minimum cost baseline roster. This is the result from the definition of specific staffing requirements (eq. (5.24)) on top of the minimum staffing requirements, which causes a higher number of regular and overtime assignments and a higher understaffing of demand periods. This higher understaffing significantly decreases as the number of hired employees increases and facilitates a drop in the cost premium of the time buffer baseline roster (Figure 5.6).

The additional assignment flexibility introduced by allowing overtime also influences the optimal hiring level when the planned cost is minimised (Figure 5.6). The optimal hiring level for a basic baseline roster is 7 employees whereas the optimal hiring level for the minimum cost baseline roster is 6 employees. Moreover, the planned cost for the minimum cost baseline roster with 6 hired employees is better than the planned cost for the basic baseline roster with 7 hired employees. Hence, the flexibility offered by scheduling overtime allows the organisation to hire a lower number of employees without repercussions in terms of the planned cost. This observation is not valid for the time buffer baseline roster because of the definition of the extra staffing requirements (eq. (5.24)). For this roster type, the optimal hiring level is 8 employees.

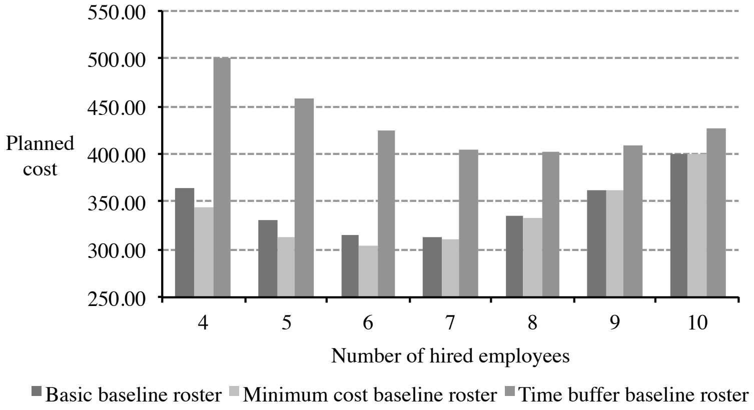

Figure 5.6 The impact of overtime on the optimal hiring level in terms of planned cost

Figure 5.6 indicates the minimal planned cost for each hiring level and type of baseline roster. These minima are obtained for varying percentages of the overtime budget available in the integrated strategic staffing and tactical scheduling phase. Table 5.2 indicates, for each hiring level, the percentage of the overtime budget that should be available in the integrated strategic staffing and tactical scheduling phase to obtain the best results that are displayed in Figure 5.6.[2] Since the basic baseline roster does not allow overtime, the optimal percentage of scheduled overtime is always 0%. For the minimum cost and time buffer baseline roster, we observe a decreasing trend. As more employees are hired, the percentage of scheduled overtime required to obtain a minimal planned cost decreases. This decrease is more pronounced for the minimum cost baseline roster than for the time buffer baseline roster. As mentioned before, this is due to the definition of the extra staffing requirements (eq. (5.24)), which increases the number of required duties.

5.4.2.2 The trade-off between hiring and overtime

In this section, we investigate the trade-off between the hiring budget and the overtime budget based on the performance of the baseline personnel rosters in the operational allocation phase.

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[2] Larger percentages do not provide a significantly lower planned cost (Mann-Whitney U test)

Table 5.2 The optimal percentage of scheduled overtime for the different hiring levels

	Number of hired employees						
	4	5	6	7	8	9	10
Basic baseline roster	0%	0%	0%	0%	0%	0%	0%
Minimum cost baseline roster	100%	100%	60%	40%	20%	20%	0%
Time buffer baseline roster	100%	100%	100%	100%	80%	60%	40%

In order to obtain insight in this trade-off, we performed two experiments that differ in the determination of the total personnel budget.

In the first experiment, we assess the impact of the distribution of the overtime budget between scheduled and unscheduled overtime for several fixed workforce sizes. This experiment approaches the trade-off from the perspective of a *varying personnel budget*, i.e. the hiring budget and the overtime budget are determined by the fixed workforce sizes. In the second experiment, we investigate the distribution of a *fixed personnel budget* over the hiring budget and the overtime budget.

Scheduled versus unscheduled overtime

We set the number of hired employees fixed and investigate the impact of different distributions between the allowed scheduled and unscheduled overtime. In order to fix the workforce size, we additionally impose equation (5.26) on the integrated strategic staffing and tactical scheduling decision model (eqs. (5.1)-(5.10)) to construct a baseline personnel roster. The fixed workforce sizes are varied in the experiment and range between 4 and 10 workers. The minimum hiring level is determined based on the minimum value of the TCC-indicator in the test design. A minimum TCC-value of 0.30 corresponds to a total staff demand of 21 duties and 168 hours (= $21 \times l\beta_j$). A workforce size of 4 employees may cover a total of 160 hours (= $4 \times \eta_i^{w,max}$) in regular time. A lower hiring level would create too much understaffing. Since we generate the staffing requirements assuming that 10 employees are available, a maximum hiring level of 10 workers is considered.

$$\sum_{i \in N} \zeta_i = fixed\ workforce\ size \tag{5.26}$$

In order to distribute the overtime budget into scheduled and unscheduled overtime in a controlled manner, we also add equation (5.25) to the integrated staffing and scheduling decision model (eqs. (5.1)-(5.10)) to construct a baseline roster. As a result of this constraint, we may have some flexibility to allocate unscheduled overtime in the operational allocation phase. In order to ensure that we do not exceed the total overtime budget, we impose equation (5.27) on the operational allocation decision model (eqs. (5.14)-(5.23)) that considers day d in the planning horizon. Note that this equation takes the unscheduled overtime that was already allotted before day d into account.

$$\begin{aligned}&\sum_{\underline{d}=1}^{d-1} utilised\ unscheduled\ budget_{\underline{d}} + \sum_{i \in N}\sum_{j \in S} l\beta_j c^{wo} x_{idj}^{wo}(1-\bar{x}_{idj}^{wo}) \\ &+\sum_{i \in N}\sum_{j \in S \setminus S^-} l\beta_j^{'} c^{wo} x_{idj}^{wo^{'}}(1-\bar{x}_{idj}^{wo^{'}}) + \sum_{i \in N}\sum_{j \in S^-} l\beta_j^{'} c^{wo} x_{id+1j}^{wo^{'}}(1-\bar{x}_{id+1j}^{wo^{'}}) \\ &+\sum_{i \in N}\sum_{j \in S \setminus S^+} l\beta_j^{''} c^{wo} x_{idj}^{wo^{''}}(1-\bar{x}_{idj}^{wo^{''}}) + \sum_{i \in N}\sum_{j \in S^+} l\beta_j^{''} c^{wo} x_{id-1j}^{wo^{''}}(1-\bar{x}_{id-1j}^{wo^{''}}) \\ &\leq (1-f) \times overtime\ budget \qquad (5.27)\end{aligned}$$

Figure 5.7 displays the impact of the distribution of scheduled and unscheduled overtime ((Scheduled OT)%-(Unscheduled OT)%) on the actual cost for several workforce sizes. Since the basic baseline roster does not consider overtime, the presented results are the average actual cost over the minimum cost and time buffer baseline roster. The chart shows that for small workforce sizes the actual cost decreases as more overtime is scheduled in the baseline personnel roster. In fact, at least 40% of the overtime budget needs to be scheduled proactively to obtain a significantly smaller actual cost in comparison the case without scheduled overtime.[3] As the number of hired employees increases, the impact of including scheduled overtime reduces because the need for overtime diminishes. In this respect, the overtime that should be scheduled proactively to significantly improve the actual cost decreases to 20%.[4] Thus, starting from a moderate workforce size, the actual cost is minimal if a combination of scheduled and unscheduled overtime is utilised. Thus, for a low number of employees, it is important to include as much overtime as possible

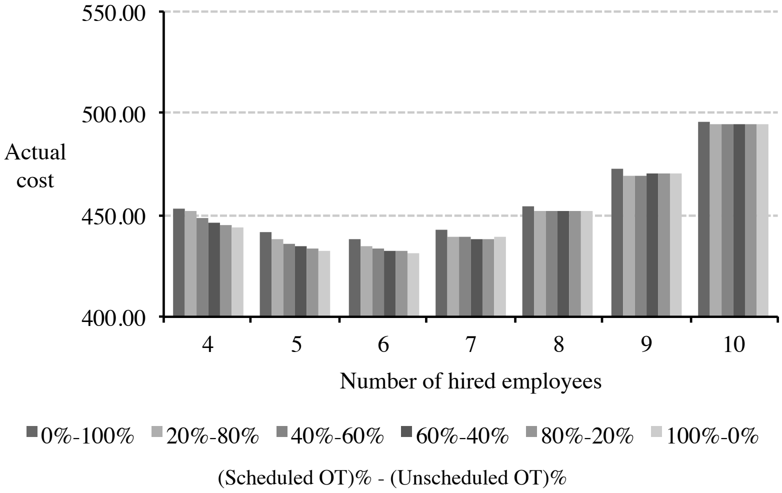

Figure 5.7 The impact of scheduled versus unscheduled overtime on the actual cost

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[3] p-value<0.01 (Mann-Whitney U test)

[4] p-value<0.01 (Mann-Whitney U test

in the integrated strategic staffing and tactical scheduling phase to reduce the planned understaffing. A higher number of hired employees, however, automatically results in less scheduled overtime and it is therefore beneficial to reserve a fraction of the overtime budget for allocating unscheduled overtime in the operational allocation phase.

We provide the results of the individual components of the actual performance in Table 5.3 averaged over all hiring levels. The table indicates the evolution of the understaffing, overstaffing, changes, cancellations and overtime for the different overtime distributions. The table reveals that the understaffing and overstaffing are minimal for a combination of both scheduled and unscheduled overtime. The number of cancellations is minimal if the complete overtime budget is reserved for the operational allocation phase. As more overtime budget is reserved for the operational allocation phase, the reactive flexibility increases, which results in a higher number of changes. Even though the scheduled overtime utilisation increases if more overtime is scheduled in the integrated strategic staffing and tactical scheduling phase, the number of overtime periods is maximal if a combination of scheduled and unscheduled overtime is utilised.

Table 5.3 The evolution of actual performance metrics

	(Scheduled OT)% - (Unscheduled OT)%					
	0% − 100%	20% − 80%	40% − 60%	60% − 40%	80% − 20%	100% − 0%
Understaffing (in periods)	11.72	11.38	11.17	11.11	11.35	12.65
Overstaffing (in periods)	16.44	16.31	16.27	16.30	16.35	16.50
Changes (in periods)	8.82	8.29	7.83	7.41	6.87	5.34
Cancellations (in days)	1.28	1.30	1.34	1.39	1.41	1.36
Overtime (in periods)	3.24	3.63	4.01	4.17	3.95	2.59
Scheduled OT utilisation	*NA*	*57.37%*	*60.75%*	*61.48%*	*61.65%*	*63.59%*
Unscheduled OT utilisation (in periods)	*3.24*	*3.06*	*2.95*	*2.72*	*2.12*	*0.00*

We show the impact of the workforce size on the actual cost in Figure 5.8. The figure shows the average results over all possible distributions for the scheduled and unscheduled overtime. It is clear that, for each baseline roster type (basic, minimum cost and time buffer), a workforce size that is either too low or too high leads to a deterioration in the actual cost. Based on the obtained results, we can conclude that the optimal hiring level is 6 employees. Note that this level represents an average result. This optimum certainly depends on the total staffing requirements, i.e. the TCC-value. A TCC-value of 0.30, 0.40 and 0.50 respectively leads to an optimal hiring level of 5, 6 and 8 employees (cf. Figure 5.A.1 in Appendix 5.A).

Moreover, Figure 5.8 reveals that it is most beneficial to employ the minimum cost baseline roster for a low workforce size and the time buffer baseline roster for a high workforce size.[5] As more employees are hired, the flexibility in the personnel roster construction increases and the personnel planner is better able to satisfy the extra staffing requirements (eq. (5.24)). This reduces the planned understaffing and facilitates the performance of the baseline personnel roster in the operational allocation phase. This improved actual performance is expressed by a smaller understaffing with a lower number of changes and unscheduled overtime periods. Furthermore, it is interesting to note that the actual performance stabilises as the number of hired employees increases, i.e. the standard deviation and the range between the minimal and maximal actual cost over all simulation runs decline.

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[5] p-value<0.01 (Mann-Whitney U test)

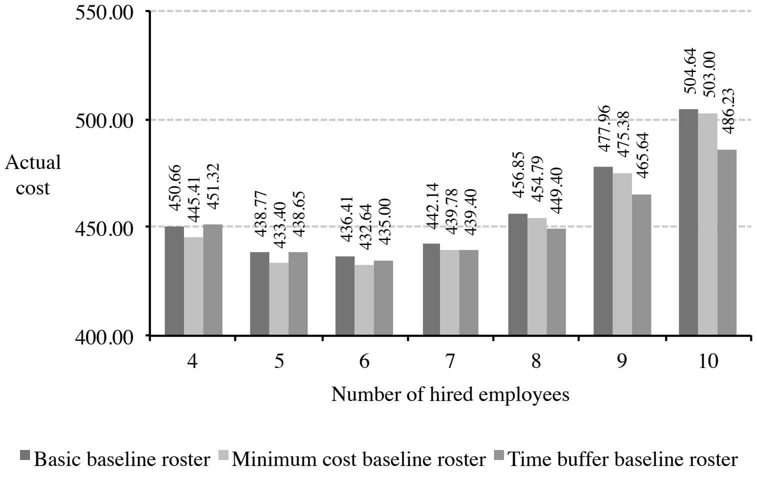

Figure 5.8 The impact of the workforce size on the actual cost

The hiring and overtime budget

In the second experiment, we assume a fixed number of budgeted working hours and discuss the impact of the distribution of this budget over regular and overtime hours, which correspond to the hiring and overtime budget. The fixed number of budgeted working hours is determined based upon the minimum required number of working hours and the additional workforce buffer to cope with the operational variability, i.e. $(1+buffer\ ratio)\times\sum_{d\in D}\sum_{h\in H} lR^w_{dh}$. The incorporation of the buffer ratio to determine the budgeted working hours facilitates the inclusion of an implicit buffer and an explicit buffer to hedge the personnel roster against operational variability. The buffer ratio enables an implicit buffer for the basic and minimum cost baseline rosters because it allows that a higher number of working hours may be assigned than required. It creates an explicit buffer for the time buffer baseline roster because it allows this roster to better satisfy the extra staffing requirements ($R^{w,extra}_{dh}$).

The number of required working hours may be distributed over the hiring budget and the overtime budget and in our computational experiments we explore three different scenarios, i.e.

- *Scenario 1*: We assign the complete budget to the hiring budget, i.e. we hire a maximum number of employees that only work during regular time. Given the allowed number of working hours $\eta^{w,max}_i$ for a single worker, we can calculate the required number of full-time equivalents ζ^{max} to execute all duties in regular time without overtime.
- *Scenario 2*: We distribute the budget between the hiring budget and overtime budget by reducing the hiring budget determined in scenario 1 by a full-time equivalent, i.e. the hiring budget drops to $\zeta^{max}-1$ workers, and by increasing the allowed number of overtime hours with $\eta^{w,max}_i$ hours.

- *Scenario 3*: We distribute the budget between the hiring budget and overtime budget by reducing the hiring budget determined in scenario 1 by two full-time equivalents, i.e. the hiring budget drops to $\zeta^{max} - 2$ workers, and by increasing the allowed number of overtime hours with $2 \times \eta_i^{w,max}$ hours.

Hence, we add equations (5.25) and (5.26) to the integrated strategic staffing and tactical scheduling decision model (eqs. (5.1)-(5.10)) for each of these scenarios. The overtime budget is determined as the allowed number of overtime hours times the overtime cost per hour.

Table 5.4 compares the average planned and actual performance for the three scenarios starting from the minimum cost roster as baseline roster. The table reveals that scenario 2 outperforms the other scenarios for both the planned and actual performance.[6] Hence, it is useful to include a limited budget for overtime at the expense of the hiring budget. More specifically, situations in which more employees are hired without overtime (scenario 1) and situations in which a low number of employees can work a high number of overtime hours (scenario 3) should be avoided and lead to inferior results.

When the hiring budget is reduced in the integrated strategic staffing and tactical scheduling phase, the table reveals that the number of hired employees and the overstaffing decrease. The impact on the planned understaffing is rather ambiguous and is lowest for scenario 2. The actual performance in the operational allocation phase shows that the understaffing is again the lowest for scenario 2, whereas the number of performed changes is the highest. The number of overstaffed demand periods decreases while the overtime budget increases as less employees are hired. Remarkably, this lower number of employees does not result in a lower number of cancellations. On the contrary, the number of cancellations increases when the workforce size decreases. This is due to the fact that more overtime is proactively scheduled. As such, smaller-than-expected staffing requirements in the operational allocation phase can lead to more cancellations without the violation of the minimum number of hours constraint (η_i^{min}).

Table 5.4 The evolution of planned and actual performance metrics for the minimum cost baseline roster

	Planned performance		
	Scenario 1	Scenario 2	Scenario 3
Planned cost	291.48	288.93	298.76
Understaffing (in periods)	1.34	0.89	2.08
Total assignment cost	278.07	280.03	277.97
Regular duty assignment cost	278.07	270.80	253.37
Overtime duty assignment cost	0.00	9.23	24.60
Overstaffing (in periods)	0.99	0.29	0.04
Hired employees	6.69	6.31	5.33

	Actual performance		
	Scenario 1	Scenario 2	Scenario 3
Actual cost	427.81	424.84	426.82
Understaffing (in periods)	14.05	12.34	13.89
Changes (in periods)	5.22	6.45	5.67
Total assignment cost	274.24	285.27	273.73
Regular duty assignment cost	271.43	258.85	233.44
Overtime duty assignment cost	0.00	21.33	31.93
Duty cancellations	0.56	1.02	1.67
Overstaffing (in periods)	12.36	10.95	8.75
Overtime (in periods)	0.00	2.84	4.26
Scheduled OT utilisation	*NA*	*64.21%*	*58.01%*
Unscheduled OT utilisation (in periods)	*0.00*	*2.70*	*3.62*

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[6] p-value<0.01 (Mann-Whitney U test)

5.4.2.3 The optimal workforce buffer for the time buffer baseline roster

In the two experiments in Section 5.4.2.2, we imposed the same personnel budget restrictions for the basic, minimum cost and time buffer baseline roster. However, the time buffer baseline roster installs a capacity buffer on top of the minimum number of staffing requirements. In order to meet the larger demand for staff, this time buffer baseline roster requires extra personnel budget, i.e. hiring budget and overtime budget. This extra budget enables the time buffer baseline roster to become an effective strategy and outperform the basic and minimum cost baseline roster in terms of actual cost. In this experiment, the objective is to determine how large the available personnel budget for the time buffer baseline roster should be relative to the personnel budget for the basic and minimum cost baseline roster. Hence, we aim to determine the optimal level of additional employees, i.e. hiring and overtime budget, needed for the time buffer baseline roster. In order to avoid that an implicit buffer could be created for the basic and minimum cost baseline roster, the starting point in this analysis is the minimum required number of employees to cover the minimum staffing requirements (R^w_{dh}). This can be calculated based on the minimum required number of working hours and the allowed number of working hours for a single employee in regular time and in overtime, i.e. $\lceil \sum_{d \in D} \sum_{h \in H} l R^w_{dh} / (\eta_i^{w,max} + \eta_i^{wo,max}) \rceil$. Hence, a TCC value of 0.30, 0.40 or 0.50 leads to a minimum of 4, 5 or 6 employees required, respectively. In our experiments, we vary the additional number of employees that are hired on top of this minimum number and we determine the impact of the number of additional employees on the actual cost. Note that we do not impose restrictions on the distribution of overtime over the integrated strategic staffing and tactical scheduling phase (eq. (5.25)) and the operational allocation phase (eq. (5.27)). The results are displayed in Figure 5.9, which shows the results for the basic baseline roster, the minimum cost baseline roster and the time buffer baseline roster with a different number of extra employees for different values of the TCC-indicator.

Figure 5.9 expresses the trade-off between hiring extra employees and the actual cost for the time buffer baseline roster. Figure 5.9(a) shows the evolution for a varying employee buffer size and a TCC-value of 0.30 and indicates that the best results are obtained without a buffer. For this case, the minimum cost and time buffer baseline roster provide a comparable actual performance[7] and outperform the basic baseline roster[8]. The optimal buffer for a TCC-value of 0.40 and 0.50 is respectively 1 and 2 employees and the actual cost for the time buffer baseline roster is smaller than that for the minimum cost baseline roster. However, this smaller actual cost is not significant. Nevertheless, it is useful and necessary to hire additional employees such that the time buffer baseline roster provides an actual performance that is significantly better and comparable to the basic baseline roster and minimum cost baseline roster, respectively. These additional employees facilitate an actual performance that exhibits a larger stability, i.e. the standard deviation and the range between the minimal and maximal actual performance is smallest for the time buffer baseline roster (cf. Table 5.5).

Table 5.6 reveals the results for the individual components of the planned and actual performance. As the number of employees increases, the understaffing of staffing requirements decreases in the

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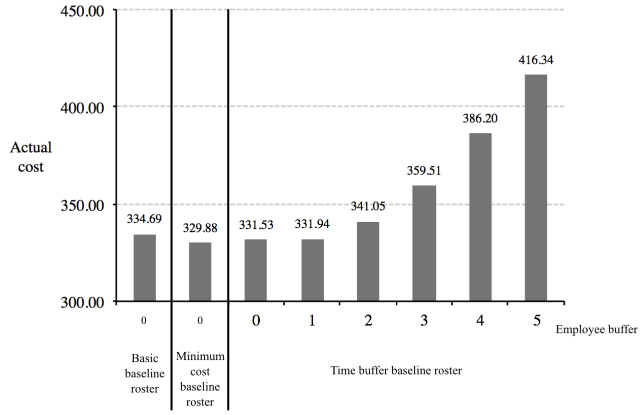
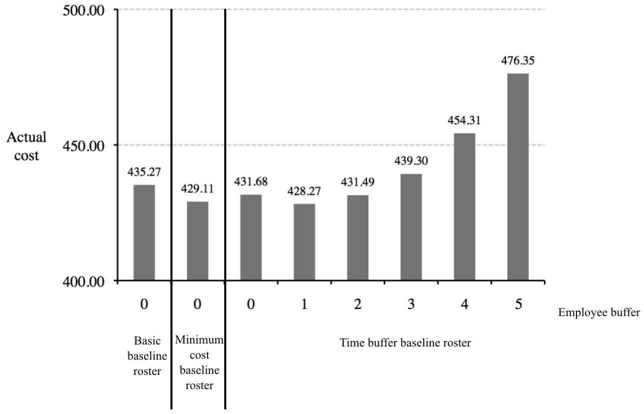
[7] p-value>0.5 (Mann-Whitney U test)

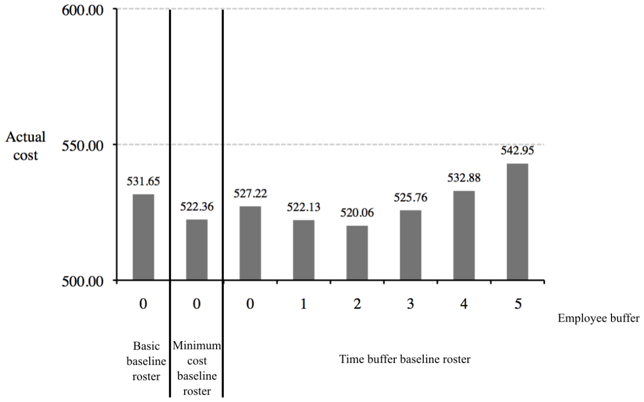
[8] p-value<0.01 (Mann-Whitney U test)



(a) TCC=0.30



(b) TCC=0.40



(c) TCC=0.50

Figure 5.9 The impact of an employee buffer for the time buffer baseline roster integrated strategic staffing and tactical scheduling phase. Naturally, we also observe a reduction in the scheduled overtime and an augmentation in the overstaffing. This results in a reduction of the actual understaffing in the operational allocation phase. Similarly, the availability of extra employees decreases the need for unscheduled overtime and reduces the number of cancellations in the operational allocation phase. However, these advantages of additional employees are negated through an increase in the overstaffing and the number of changes.

Table 5.5 The standard deviation (σ) and minimal (*lb*) and maximal (*ub*) actual performance of the different types of baseline rosters (σ, *lb*, *ub*)

	TCC 0.30	TCC 0.40	TCC 0.50
Basic baseline roster	(48.69, 192.00, 564.00)	(56.02, 265.00, 666.00)	(61.86, 297.00, 770.00)
Minimum cost baseline roster	(46.71, 192.00, 537.00)	(54.78, 255.00, 652.00)	(59.21, 327.50, 750.50)
Time buffer baseline roster	(46.39, 194.00, 514.00)	(49.18, 276.00, 646.00)	(51.65, 366.00, 732.50)

Table 5.6 The evolution of planned and actual performance metrics for the time buffer baseline roster

	Planned performance					
	Employee buffer					
	0	1	2	3	4	5
Hired employees	5.00	6.00	7.00	8.00	9.00	10.00
Understaffing (in periods)	6.12	2.27	0.66	0.24	0.09	0.01
Overstaffing (in periods)	0.02	0.07	0.48	1.83	5.04	9.95
Overtime (in periods)	12.38	10.96	8.02	4.84	2.87	1.63

	Actual performance					
	Employee buffer					
	0	1	2	3	4	5
Understaffing (in periods)	13.67	10.21	7.97	6.76	5.81	4.90
Overstaffing (in periods)	8.23	10.83	13.97	17.88	22.94	28.83
Changes (in periods)	4.82	5.21	5.95	6.72	7.35	8.27
Cancellations (in days)	2.48	2.93	2.86	2.24	1.53	0.75
Overtime (in periods)	6.40	5.77	5.07	4.25	3.52	2.63
Scheduled OT utilisation	*48.45%*	*45.27%*	*46.95%*	*53.90%*	*66.78%*	*74.90%*
Unscheduled OT utilisation (in periods)	*3.39*	*3.27*	*3.07*	*2.68*	*2.27*	*1.79*

5.4.2.4 The impact of the variability of demand

The experiments in sections 5.4.2.2 and 5.4.2.3 are based on the assumption that the demand variability per demand period is not restricted and can range up to a value of $+\infty$ (eqs. (5.11) - (5.13)). In this section, we investigate the impact of different degrees of variability of demand by varying the value of parameter σ in equations (5.11) and (5.12). We distinguish 6 uncertainty scenarios that differ in the degree of variability, i.e. $\sigma \in \{1, 2, 3, 4, 5, +\infty\}$. Moreover, the demand variability is simulated according to two simulation settings, i.e.

- In the first setting the demand variability is simulated for the defined *standard shifts*, which consist of two consecutive demand periods (cf. Figure 5.3). Hence, the simulated staffing requirements remain the same for both demand periods within a standard shift.
- The second setting simulates the staffing requirements per *demand period* and includes a higher intrinsic variability because the staffing requirements can differ from demand period to demand period.

Figure 5.10 displays the actual cost for the different uncertainty scenarios and the two simulation settings. When the uncertainty rises, the actual cost displays a convex behaviour, i.e. the actual cost increases at a decreasing rate. The main difference between the two simulation settings is the height of the actual cost. Since the first setting comprises a lower overall variability, the actual cost is lower than for the second setting.

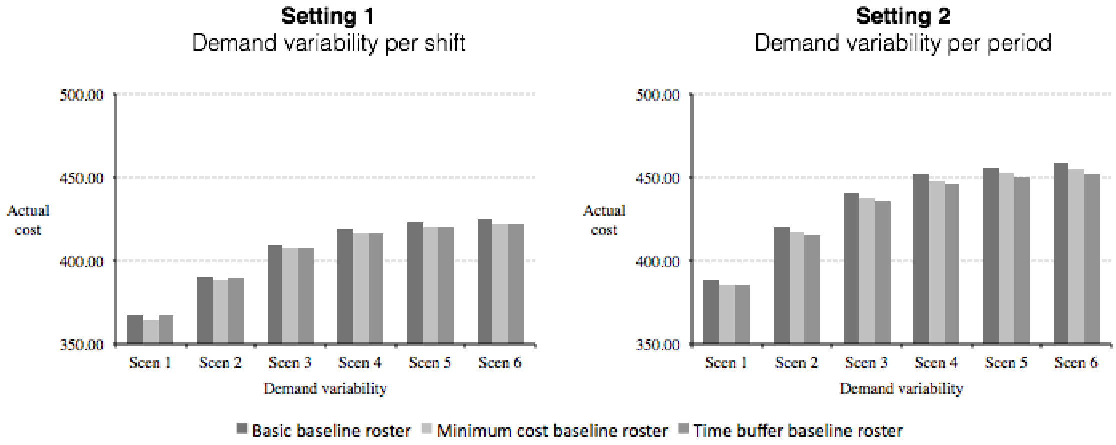


Figure 5.10 The impact of the variability of demand

In order to obtain further insight in the impact of the uncertainty scenarios for the two simulation settings, we repeat the experiments of sections 5.4.2.2 and 5.4.2.3. In general, the findings of these experiments are confirmed for different degrees of variability. The detailed results of the actual cost for different variability degrees and hiring levels are displayed in Table 5.7 for both the minimum cost baseline roster and the time buffer baseline roster. The coloured cells indicate whether the minimum cost baseline roster or the time buffer baseline roster performs significantly better for a given combination of demand variability and workforce size.[9] In general, the construction of a time buffer baseline roster is beneficial when the workforce size is relatively high and the demand variability increases. This is in particular the case when the demand is simulated per demand period (cf. Figure 5.10). When the number of employees is relatively low, the personnel planner should not introduce additional staffing requirements on top of the minimum staffing requirements as the minimum cost baseline roster performs best.

The inclusion of a small workforce buffer on top of the minimally required number of employees leads to an actual cost improvement for the time buffer baseline roster. However, the construction of a time buffer baseline roster is only beneficial when the total staffing requirements and the degree of variability are sufficiently high in comparison to the minimum cost baseline roster.

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[9] p-value$<$0.05 (Mann-Whitney U test)

Table 5.7 The actual cost for different variability degrees and hiring levels

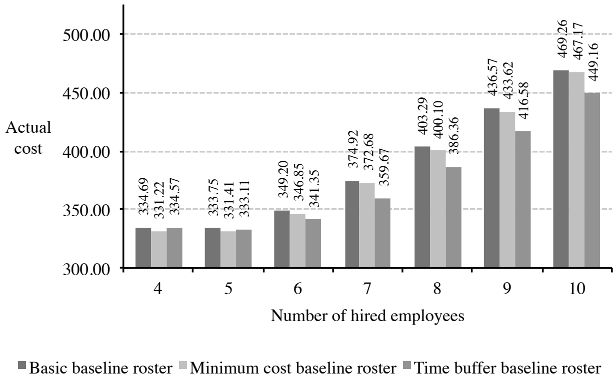
			Roster type											
			Minimum cost baseline roster						Time buffer baseline roster					
			Demand variability						Demand variability					
			Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6
Setting 1 Demand variability per shift	Hired employees	4 empl	359.93	383.85	406.71	417.74	421.99	424.63	366.03	389.22	411.40	422.49	426.72	429.27
		5 empl	341.33	369.56	391.39	401.99	405.81	407.82	348.54	375.03	396.00	406.38	410.23	412.12
		6 empl	336.32	363.94	385.83	396.15	399.50	401.50	342.35	367.86	388.78	398.70	401.88	403.77
		7 empl	343.30	369.74	390.13	399.32	402.66	404.72	348.50	372.67	391.27	399.96	403.41	405.44
		8 empl	362.05	385.61	402.69	412.14	415.91	417.80	364.15	385.93	401.46	410.42	413.85	415.57
		9 empl	387.40	407.75	422.97	431.14	434.52	436.50	386.95	405.65	419.07	426.55	429.73	431.66
		10 empl	420.58	438.40	451.62	458.65	461.59	463.08	415.57	431.71	443.55	450.05	452.94	454.46
Setting 2 Demand variability per period			Demand variability						Demand variability					
			Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6
	Hired employees	4 empl	374.39	404.05	426.91	438.74	443.10	445.41	381.77	410.24	432.75	444.58	448.98	451.32
		5 empl	360.15	392.61	415.23	426.94	431.30	433.40	367.93	398.58	420.40	432.22	436.51	438.65
		6 empl	358.66	392.38	414.04	426.02	430.33	432.64	364.17	396.24	417.24	428.50	432.80	435.00
		7 empl	366.73	400.47	422.11	433.17	437.65	439.78	369.89	401.97	422.21	432.99	437.13	439.40
		8 empl	386.15	417.71	438.00	448.24	452.42	454.79	384.11	414.34	433.52	443.11	447.19	449.40
		9 empl	409.56	440.20	458.91	469.14	473.19	475.38	403.12	432.22	449.92	459.75	463.53	465.64
		10 empl	442.52	470.66	488.17	497.21	500.74	503.00	428.74	455.45	471.92	480.61	484.16	486.23

5.5 Conclusions

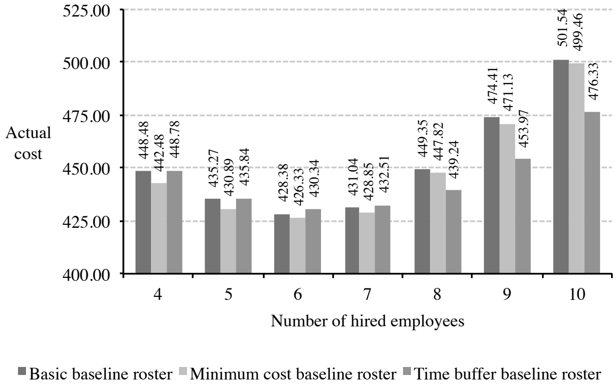
In this chapter, we discuss the impact of overtime as a time buffer strategy on the robustness of a personnel roster. In the personnel planning process, a decision on the overtime budget is typically taken in the staffing phase as this decision is interconnected with the hiring policy in an organisation and overtime may reduce the required number of employees. Overtime is defined as the extension of the daily working time and/or of the total working time over the planning period. In the personnel planning process, decisions taken in the higher-level hierarchical phases impact lower-level phases. The overtime budget has undoubtedly an impact on the operational allocation phase where operational variability arises and overtime may offer some flexibility to solve schedule disruptions in a reactive way. In this perspective, we explore the trade-off between the number of hired employees and the overtime budget. Additionally, we investigate the impact of the overtime policy, which determines how overtime is used in the personnel planning process. Overtime may be introduced in the integrated staffing and scheduling phase to construct the baseline personnel roster as a proactive scheduling strategy and in the operational allocation phase as a reactive allocation strategy to balance supply and demand. We utilise a three-step methodology to assess three types of personnel rosters, which differ in the availability of overtime and the definition of specific staffing requirements for overtime duties as a capacity buffer to hedge against uncertainty.

The results of the computational experiments show that the introduction of overtime reduces both the planned and actual cost. The degree in which overtime should be included in the baseline personnel roster depends on the number of hired employees. A low number of hired employees requires that more overtime is proactively scheduled to reduce the planned understaffing while a larger number of hired employees benefits from a combination of scheduled and unscheduled overtime. The definition of additional staffing requirements as a capacity buffer is most useful when the workforce size is higher than the minimum required workforce size. We investigated the size of this workforce buffer on top of the minimum number of required employees. Moreover, the additional staffing requirements are only relevant when the demand variability is sufficiently high.

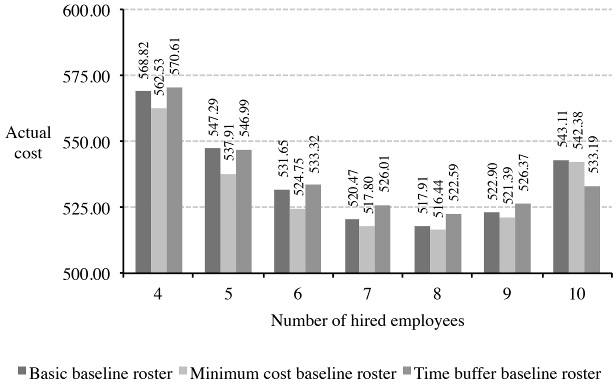
5.A Appendix - Supporting figures and tables



(a) TCC=0.30



(b) TCC=0.40



(c) TCC=0.50

Figure 5.A.1 The impact of the workforce size on the actual cost for different TCC-values

6

General conclusions and recommendations for future research

6.1 Conclusions

In this book, we investigated the impact of uncertainty on the service level and associated personnel cost and satisfaction in the personnel planning process. In order to provide managerial guidelines that are applicable in many problem areas, we focused on a general personnel shift scheduling problem with uncertain personnel demand and employee availability. This problem assigns employees to a duty, i.e. an early, late or night shift, or to a day off while considering the staffing requirements per duty and the time-related constraints for each employee.

In Chapter 1, we claimed that uncertainty significantly deteriorates the service level and personnel cost and satisfaction in the short-term. In this respect, we proposed to improve the robustness of the personnel shift roster. In the academic literature, robustness is defined based on roster stability (Dück et al., 2012) and flexibility (Ionescu and Kliewer, 2011). Hence, a roster should be able to absorb disruptions and provide those adjustment possibilities such that the personnel planner can efficiently and effectively recover the roster. This roster stability and flexibility requires the application of proactive strategies in the strategic staffing phase and the tactical scheduling phase. Moreover, reactive strategies need to enable the full potential of these proactive strategies in the operational allocation phase. As such, we formulated and investigated different proactive and reactive strategies throughout this book.

In order to assess the robustness of these strategies, we established a general framework in Chapter 2. This framework imitates the personnel planning process and comprises a three-step methodology consisting of roster construction, day-by-day simulation and adjustments, and evaluation. In the first step, the (integrated strategic staffing and) tactical scheduling phase is considered and can be enhanced by the application of a proactive strategy. The resulting baseline personnel shift roster is subjected to a day-by-day simulation and adjustment in the operational allocation phase. Each day, the personnel demand and employee availability are simulated to imitate the variability in the short-term operational allocation phase. This may result in disruptions that need to be recovered with a reactive strategy. Given the performance of the baseline personnel shift roster in the first and second step, the robustness can be determined through an evaluation of its planned and actual performance in the third step.

Table 6.1 provides an overview of the studies for which this three-step methodology was utilised to test the robustness corresponding to the formulated proactive and reactive strategies.

Proactive strategies

We focused on the construction of *stable* baseline personnel shift rosters in Chapters 2 and 3. In Chapter 2, we formulated different reserve duty scheduling strategies. These strategies aim to assign reserve duties to employees such that a larger-than-expected demand can be absorbed in the operational allocation phase. In contrast, we explicitly considered the possibility of both a larger-than-expected and smaller-than-expected demand in Chapter 3 to avoid understaffing and overstaffing in the operational allocation phase. As such, we aimed to improve the stability of the baseline personnel shift roster through the application of uncertainty set characterisation strategies, which enable a deterministic formulation of a stochastic problem.

Roster *flexibility* was explicitly considered in Chapter 4 by expanding our problem definition to include the strategic staffing phase. In this phase, we defined a heterogenous workforce characterised by the number and type of skills the employees possess. We investigated the impact of different degrees of cross-training in the personnel planning process by defining varying skill possession settings for the workforce. Moreover, we proactively improved the flexibility of the baseline personnel shift roster in the tactical scheduling phase by considering employee substitutability strategies.

In Chapter 4, we established that the strategic staffing phase can have an important impact on the personnel planning process. In this respect, we integrated the strategic staffing phase and the tactical scheduling phase in Chapter 5. In this chapter, we aimed to improve the roster *stability and flexibility* by applying time-based strategies for varying homogenous workforce sizes. These strategies entail the assignment of overtime during complete shifts and shift extensions, and the introduction of capacity buffers. Additionally, we investigated the trade-off between the workforce size and overtime.

Irrespective of the applied proactive strategy, we have shown that the formulated strategies enable us to provide a baseline personnel shift roster that is more robust than the minimum cost scheduling strategy. This strategy assumes a deterministic operating environment and does not appropriately reflect the operating environment in the short-term operational allocation phase.

However, the proactive strategies should be applied with care. First, it is important that the demand profile and more specifically, the profile of the demand uncertainty is considered in the definition of the proactive strategy. In this book, the profile of the demand and the demand uncertainty are closely related given the assumption that a Poisson distribution characterises the demand uncertainty. As such, shifts with large staffing requirements represent a large variability and therefore require more consideration of proactive strategies than other shifts. Second, the cost of robustness needs to be monitored such that not too much planned cost is incurred for extra robustness. In this respect, the trade-off between cost and robustness is an important consideration in the tactical scheduling phase. Third, the choice between proactive strategies promoting stability and/or flexibility depends on the problem setting. Stability and flexibility strategies can only be applied when the total demand, the time-related constraints, the personnel mix and workforce size provide sufficient opportunity to introduce proactive strategies in the baseline personnel shift roster. Stability strategies are especially valuable if the inherent flexibility corresponding to the problem setting is small while flexibility strategies require a certain level of implicitly available flexibility in the baseline personnel shift roster.

Reactive strategies

Throughout Chapters 2-5, we utilised reassignments and cancellations to recover the baseline personnel shift rosters when disruptions occur in the operational allocation phase. In Chapter 2, we also considered the conversion of reserve duties into working duties. These conversions may pertain to the same shift or to differing shifts.

Moreover, we considered different levels of operational flexibility in Chapters 2 and 3, i.e. a low, medium and high level of operational flexibility. The low level corresponds to a fixed reactive mechanism in which no reassignments are allowed except the conversion of a reserve duty to a working duty or an overstaffed duty to a regular duty during the same shift. The medium level represents an adjustable mechanism that allows reassignments of working duties while a high level also allows employees with a day off to be called in to work a duty. In Chapter 4, we also investigated different levels of flexibility by allowing specific reassignments, i.e. between-skill substitutions, within-skill substitutions, day-off-to-work substitutions or all substitutions. We extended the reassignments in the operational allocation phase by allowing the allocation of unscheduled overtime in Chapter 5. However, the flexibility to allocate this overtime depends on the degree to which overtime was proactively scheduled in the integrated strategic staffing and tactical scheduling phase. As such, we investigated the trade-off between proactively scheduling overtime to obtain a stable roster and reactively allocating overtime as a flexible mechanism to react to disruptions.

The reactive strategies and the flexibility they represent have an important impact. As the operational flexibility increases, it becomes more difficult for the proactive strategy to discern itself from the minimum cost scheduling strategy.
Moreover, a strategy that proactively improves the stability of the baseline personnel shift roster is most valuable in combination with a reactive strategy that is focused on roster stability rather than flexibility. Similarly, proactive flexibility needs to be converted into reactive flexibility such that the necessary adjustment possibilities can be executed.

Thus, it is very important that the proactive and reactive strategies accommodate one another to enable their full potential and a maximal impact on the robustness of the personnel roster. Given that the decision freedom is smallest in the operational allocation phase, the personnel planner should define proactive strategies based on the type of available and preferred reactive recovery options. In case no adjustments are allowed for example, the proactive strategies should focus on the creation of capacity buffers through reserve duty scheduling and/or uncertainty set characterisation strategies. As more adjustments are allowed and acceptable, the personnel planner may opt to reduce capacity buffers and focus more on strategies that increase the flexibility of personnel rosters. The time buffer strategies can be utilised to combine stability and flexibility. If flexibility is the main focus in the operational allocation phase, employee substitutability is the fitting strategy. Irrespective of the multi-skilled nature of employees, substitutability can be improved by considering within-skill and day-off-to-work substitutions.
Once the type of proactive strategy has been chosen, the specific definition of this strategy can be based on the need for stability and/or flexibility in the operational allocation phase, the inherent stability or flexibility in the baseline personnel shift roster and the demand profile/profile of demand uncertainty.

To conclude, the contribution of this book is fourfold. First, we established a general three-step methodology to assess the robustness of a personnel shift roster in a general personnel planning process. Second, we proposed a variety of proactive and reactive strategies to improve the robustness of personnel shift rosters and provided the corresponding mathematical formulations. Third, we evaluated the cost efficiency of the proactive and reactive strategies and focused on the associated cost of robustness such that the resulting baseline personnel shift roster performs well for many realistic operational scenarios. Fourth, we have continuously provided managerial guidelines about the relevance of proactive and reactive strategies in specific problem settings.

6.2 Future research

In this book, we have provided a number of studies that may be expanded and adapted in future research.

First, dedicated algorithms should be developed to construct baseline personnel shift rosters in the tactical scheduling phase. In this book, we provided a number of general algorithms that worked well given the considered problem size. However, these algorithms should be adapted such that larger real-life problems with additional case-specific time-related constraints can be efficiently solved.

Second, the assumptions underlying the simulation procedure may be adapted to investigate their impact on the formulated proactive and reactive strategies. In this respect, dependencies between different demand periods or shifts can be considered such that personnel demand that was not satisfied during a specific demand period may be either lost or transferred to a later period. Queueing analysis and Markov chains can be utilised to investigate the interdependency of the demand between different demand periods. Moreover, the simulation of the uncertainty of demand can be adapted. The impact of other realistic demand uncertainty scenarios or other probability distributions can be investigated. Similarly, the simulation of the uncertainty of capacity can be adjusted such that both the unavailability and the length of the unavailability are simulated.

Third, the presented strategies can be combined in different ways. This is an interesting research path because of the residual planned overstaffing in the constructed baseline personnel shift rosters. This overstaffing has not been positioned according to one of the formulated strategies and therefore provides an opportunity to obtain additional improvements. The residual overstaffing after the application of uncertainty set characterisation strategies can, for example, be positioned according to reserve duty scheduling or employee substitutability strategies. Moreover, this overstaffing can be avoided by hiring a lower number of employees. An important consideration in the combination of proactive strategies is the complementarity of these strategies. In this respect, the choice between proactive strategies that focus on stability, flexibility or both should be refined based on specific problem settings. A combination of stability and flexibility strategies will have a significant impact on the way disruptions should be resolved in the operational allocation phase. In this phase, an interesting trade-off will arise between stability- and flexibility-enhancing reactive strategies. In this respect, it is interesting to value this trade-off and to investigate the impact on the actual performance.

Fourth, the knowledge underlying the news vendor problem can be utilised as a proactive strategy to define the stochastic staffing requirements in the tactical scheduling phase. The robustness of personnel rosters obtained with these staffing requirements should be compared with the robustness of rosters obtained with the expected staffing requirements.

Finally, the impact of the proactive strategies can be investigated with a longer reactive planning horizon to execute adjustments after the occurrence of disruptions. In this case, an intermediate phase between the tactical scheduling phase and the operational allocation phase is considered. In this phase, the personnel roster is reconsidered for multiple days while simultaneously ensuring that appropriate proactive mechanisms remain available for the future.

Table 6.1 A general overview of the research studies

			Chapter 2	Chapter 3	Chapter 4	Chapter 5
Personnel characteristics			**Homogenous** workforce with a **single skill**	**Homogenous** workforce with a **single skill**	**Heterogenous** workforce with **multiple skills**	**Homogenous** workforce with a **single skill**
Research objective			Improve the **robustness** of personnel shift rosters by defining proactive strategies that focus on **roster stability.** A stable roster is able to (partially) absorb disruptions and needs a small number of adjustments.	Improve the **robustness** of personnel shift rosters by defining proactive strategies that focus on **roster stability.** A stable personnel roster is able to (partially) absorb disruptions and needs a small number of adjustments.	Improve the **robustness** of personnel shift rosters by defining proactive strategies that focus on **roster flexibility.** A flexible roster entails a sufficient number of adjustment possibilities to efficiently recover from disruptions.	Improve the **robustness** of personnel shift rosters by defining proactive strategies that focus on **roster stability and flexibility.** A stable personnel roster is able to (partially) absorb disruptions and needs a small number of adjustments. A flexible roster entails a sufficient number of adjustment possibilities to efficiently recover from disruptions.
Problem definition	Proactive robustness strategies		**Reserve duty scheduling strategies**: Definition of the time point (day, shift) when to add a **surplus number of employees**. A reserve duty is defined as an on-call assignment on top of the working assignments that are scheduled in accordance with the minimum staffing requirements.	**Uncertainty set characterisation strategies**: Definition of the time point (day, shift) when to anticipate smaller-than-expected and larger-than-expected staffing requirements. The anticipation of smaller-than-expected staffing requirements leads to a **smaller number of scheduled employees** (requirements diminution) while larger-than-expected staffing requirements result in a **surplus number of employees** (requirements enlargement).	**Employee substitutability strategies**: Definition of the time point (day, shift) and skill category to schedule **additional substitution possibilities**. A substitution is defined as the event where an employee takes over the assignment of another employee	**Capacity buffer strategies:** Definition of the time point (day, demand period) to schedule additional employees. These additional employees may be assigned in regular time or in overtime. **Hiring strategies:** Definition of the number of employees that need to be hired. **Overtime strategies:** Definition of the allowance to schedule overtime.
	Model formulation	Model type	**Deterministic model** including strategies to introduce reserve duties.	**Deterministic model** including strategies that characterise the uncertainty set, which introduces the stochasticity underlying this model	**Deterministic model** including strategies to introduce substitution possibilities.	**Deterministic model** including strategies to introduce additional employees and to assign overtime.
		Objective	Reserve duty cost optimisation	Uncertainty set optimisation	Employee substitutability value optimisation	Personnel and overtime cost minimisation
		Constraints	Reserve duty staffing requirements Reserve duty time-related constraints	Deviation measure constraints Uncertainty budget constraints	Employee substitutability measuring constraints	Extra staffing requirements Hiring constraint Overtime time-related constraints
Solution methodology			Dedicated **branch-and-price**	Dedicated **column generation**	Dedicated **two-phase preemptive programming** approach: Truncated **branch-and-bound** (Gurobi) Problem-specific **branch-and-price** with stopping criterion	Branch-and-bound (Gurobi)
Experimental design	Test instances		**Artificial** setting	**Artificial** setting	**Artificial** setting **Real-life** setting	**Artificial** setting
	Experiments		Impact of reserve duty scheduling strategies	Impact of the definition of uncertainty sets Impact of strategies underlying the uncertainty sets	Impact of group employee substitutability strategies Impact of individual employee substitutability strategies Impact of cross-training of employees	Impact of hiring and overtime strategies Impact of capacity buffers Impact of demand variability
	Managerial guidelines	Cost of robustness	The cost of robustness needs to allow a rise in the wage and the preference penalty costs such that reserve duties can be introduced	The cost of robustness needs to allow a rise in the wage and the preference penalty costs to increase the number of working assignments such that an appropriate combination of uncertainty budget and deviation measures can be utilised	The cost of robustness should allow additional substitution possibilities with a different position but needs to remain limited depending on the implicitly available flexibility (i.e. the employee cross-training).	The cost of robustness should enable the organisation to hire additional employees such that capacity buffers can be properly positioned. However, the number of additional employees cannot be too large.
		Proactive strategies	The structure and height of the working duty staffing requirements should be carefully considered in the definition of reserve duty scheduling strategies	The size of the deviation measures should be limited to enable an equal distribution of smaller requirements diminutions and enlargements over a larger number of days and shifts	The counting strategies should be defined based on structure of the staffing requirements and the implicitly available flexibility	In contrast to a high number of hired employees, a low number requires that overtime is mostly utilised proactively such that the planned understaffing remains limited
			A combination of reserve duty staffing requirements and time-related constraints offers the best results	The available uncertainty budget should depend on the size of the deviation measures such that an appropriate level of uncertainty is proactively anticipated	The value strategies and corresponding benefit value need to be defined based on the structure of the staffing requirements to ensure an appropriate position of the substitution possibilities	There needs to be a balance between the budget for hiring employees and the budget to assign overtime such that a sufficient number of employees can be assigned an appropriate level of overtime
				The utility of non-standard uncertainty budget allocation strategies depends on the liberty to diminish or enlarge the staffing requirements, i.e. the magnitude of the uncertainty budget and deviation measures	The utilisation of the specific substitution types should depend on and exploit the degree of cross-training of employees	A higher demand variability benefits more from a capacity buffer than a smaller demand variability
		Reactive strategies	A larger reactive flexibility requires a smaller number of reserve duties and enables a better actual performance. However, it also reduces the differences between reserve duty and minimum cost scheduling strategies	A larger reactive flexibility requires a smaller uncertainty anticipation and enables a better actual performance. However, it also reduces the differences between robust and minimum cost scheduling strategies	Individual employee substitutability should be facilitated to perform better than group employee substitutability in the operational phase	A higher number of hired employees benefits from the reactive allocation of overtime in the operational allocation phase

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