

Spiral wave chimeras in locally coupled oscillator systems

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The recently discovered chimera state involves the coexistence of synchronized and desynchronized states for a group of identical oscillators. In this work, we show the existence of (inwardly) rotating spiral wave chimeras in the three-component reaction-diffusion systems where each element is *locally* coupled by diffusion. A transition from spiral waves with the smooth core to spiral wave chimeras is found as we change the local dynamics of the system or as we gradually increase the diffusion coefficient of the activator. Our findings on the spiral wave chimera in the reaction-diffusion systems suggest that spiral chimera states may be found in chemical and biological systems that can be modeled by a large population of oscillators indirectly coupled via a diffusive environment.

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Introduction. Collective behavior, which occurs commonly in physical, chemical, and biological systems, has been a subject of continued interest in nonlinear science over the past several decades [1–4]. In neural and biological systems, a typical collective behavior observed is the coherent motion of oscillators. This phenomenon of synchronization has been widely regarded as having important implications to the function and performance of those systems [5,6]. For instance, asynchronous contraction of the heart may be triggered by multiple electrical spiral waves, which eventually leads to heart dysfunction [7].

Recently, much attention has been paid to a particular hybrid state in which an ensemble of identical oscillators with the same coupling scheme spontaneously degenerates to one group with synchronization, and another group with desynchronization [8–20]. This fascinating counterintuitive state was first discovered by Kuramoto and co-workers [8] and later called “chimera state” [9]. The existence of chimera states has been experimentally confirmed in diverse systems such as coupled maps [10], chemical oscillators [11,12], and mechanical pendulums [13]. In two-dimensional systems, chimera states usually take the form of spiral waves [17–23]. These so-called spiral wave chimeras exhibit phase-locked oscillators in the spiral arm but a phase-randomized spiral core [17]. An analytical solution for a spiral wave chimera was further demonstrated by using a Kuramoto-type phase equation with nonlocal coupling [19]. Recently, the spiral wave chimera state has also been reported experimentally in chemical oscillators [12] and numerically in complex and chaotic oscillators [20] where nonlocal coupling is introduced.

It was initially believed that the chimera state arises from nonlocal coupling, i.e., each oscillator in the system is affected *instantaneously* by a group of oscillators within certain interacted range and its coupling strength decreases as a function of the distance. Therefore, nonlocal coupling is intermediate between the cases of local and global coupling. However, some recent works show that the nonlocality conditions for occurrence of the chimera state can be further relaxed [14,15]. For example, a generalized chimera state called “amplitude

mediated chimera state” was observed in a population of oscillators in the case of global coupling [14]. Very recently, chimera states in network with purely local coupling are also reported [16].

It is worth pointing out, on one hand, that in the seminal work of Kuramoto *et al.*, a key assumption to observe spiral wave chimeras is that the third component of reaction diffusion (RD) changes so fast that it can be eliminated adiabatically [17]. In this way, the three-component RD system is essentially reduced to an effective two-component one with an extra nonlocal term. On the other hand, to observe chimera state in experiments, the realization of nonlocal coupling strongly relies on a computer [10–13]. Therefore, it remains an open question whether spiral wave chimeras exist in systems where the time scale of each component in the system is comparable and the coupling will be mediated by a natural law such as diffusion. Furthermore, classical spiral waves can in principle rotate outwardly or inwardly [24], whereas all the spiral wave chimeras reported previously are outward. There is still very little information on the existence of inwardly rotating spiral wave chimeras.

In this work, we report the existence of spiral wave chimeras in a three-component RD system which involves one activator and two inhibitors. By only *locally* coupling one of the inhibitors, spiral wave chimeras are found for the nondiffusing components; the other diffusible component shows the coherent spiral structure. Moreover, the observed spiral wave chimeras rotate inwardly, i.e., a coherent wave propagates towards the phase-randomized center. We also report a continuous transition from a spiral wave with a smooth core to spiral wave chimeras. Our findings in RD systems show that the occurrence of the chimera states does not require nonlocal or global coupling and therefore provide key hints to explore the chimera state in realistic chemical and biological systems that can be modeled by a population of oscillators indirectly communicated through a diffusive environment.

Reaction-diffusion model. Our starting point is a three-component RD system in two spatial dimensions [25–27]:

$$\begin{aligned}\partial_t X &= \phi(aX - \alpha X^3 - bY - cZ) + D_X \nabla^2 X, \\ \partial_t Y &= \phi \epsilon_1 (X - Y), \\ \tau \partial_t Z &= \phi \epsilon_2 (X - Z) + D_Z \nabla^2 Z.\end{aligned}\tag{1}$$

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These equations describe the evolution of concentrations of chemical reactants X , Y , and Z , where X is an activator and Y and Z are inhibitors. D_X and D_Z denote the diffusion coefficients of chemical species of X and Z , respectively. The dimensionless parameter τ represents the characteristic time scale of the system variable Z , and will be assigned a finite nonzero value. In principle, the parameter ϕ can be absorbed to a , b , c , α , ϵ_1 , and ϵ_2 which represent other parameters, but we still write it explicitly here to keep the same form as in Ref. [25]. This three-component RD system actually is an extension of the two-component FitzHugh-Nagumo model by coupling to a third variable Z , which was proposed to study pattern formation in the Belousov-Zhabotinsky (BZ) reaction dispersed in water droplets of a water-in-oil aerosol OT microemulsion system [25] and to model spot dynamics in gas discharges [28].

This study will differ from previous works on pattern formation in RD systems in two aspects. The first modification is related to the ratio of the diffusion coefficients $\delta = D_X/D_Z$. Traditionally $\delta \geq 1$ or $\delta \rightarrow \infty$, but in our study we take $\delta \ll 1$ or $\delta = 0$. In the case of $\delta = 0$, we can consider Eqs. (1) from a dynamical point as a model that describes a large number of oscillators that communicate with each other via a diffusive environment. Such kind of models may be related to various systems such as chemical oscillators [29], genetically engineered bacteria [30], yeast cells [31], and social amoeba *Dictyostelium discoideum* [32]. The second important aspect is the time scale τ . In this work, we omit the key assumption of the fast change of the third variable Z (i.e., $\tau \rightarrow 0$) that was made previously [17,26].

Order parameter. To analyze the state of the coupled oscillators, we introduce the oscillator phase θ by $\tan \theta(\vec{r}) = Y(\vec{r})/X(\vec{r})$. We define the phase in the X - Y plane as we find only in this plane a phase portrait demonstrating a limit-cycle-like trajectory with defined rotation around the origin even at the core. Since several observed states will be incoherent at the level of the simulation grid, we consider the domain as a discrete set of oscillators at positions $x = I\Delta x$, $y = J\Delta y$ with phase $\theta_{I,J} = \arctan(Y_{I,J}/X_{I,J})$.

To quantitatively study the size of the region with incoherent oscillations, we define the time averaged order parameter

$$\langle \sigma_{I,J} \rangle = \frac{1}{(2m+1)} \left\langle \left| \sum_{[I,J]} \exp[i\theta_{I,J}(t)] \right| \right\rangle, \quad (2)$$

where $\theta_{I,J}$ is the oscillator phase as defined above. The notation $[I,J]$ denotes the set of nearest neighbors including itself and $1/(2m+1)$ is a normalization factor where m denotes the number of oscillators with the nearest coupling along a given spatial dimension. In the present case, we set $m = 2$ since we know the diffusion term in Eqs. (1) was computed using a five-point discrete Laplacian. Finally, the time average $\langle \dots \rangle$ is computed over the interval time $\Delta T = 20\,000$ in our simulations. The radius of the spiral wave chimera core is approximately computed according to $R = (d_x + d_y)/4$ where d_x (d_y) denotes maximal distance between the grid points along the x (y) line when $\langle \sigma_{I,J} \rangle \leq 0.9$.

Results. Figure 1 demonstrates the existence of a spiral wave chimera state in the locally coupled RD system (1) with inhibitor diffusion only ($\delta = 0$). The snapshot of variable X

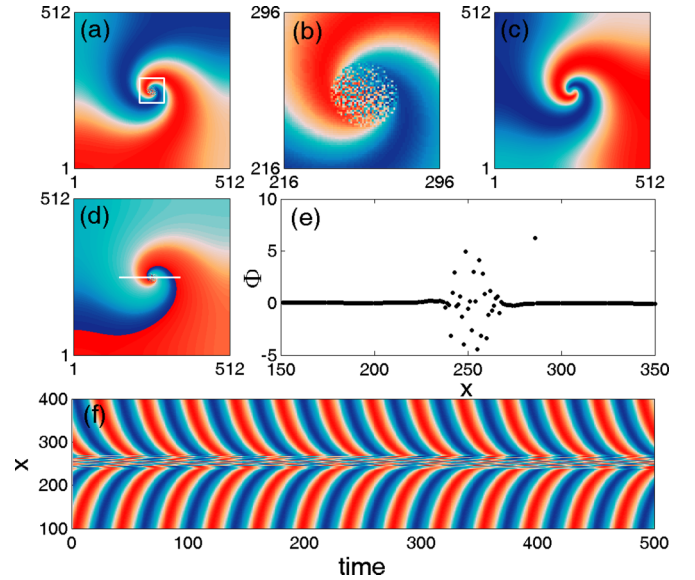


FIG. 1. Spiral wave chimeras in a three-component RD system (1) with $a = 1.50$ and $\epsilon_2 = 0.15$. (a) is a snapshot of the variable X ; (b) is the magnification the square region in (a); (c) represents the variable of Z ; (d) is a snapshot of phase defined $\tan \theta(\vec{r}) = Y(\vec{r})/X(\vec{r})$; and (e) is a cross section along the line in (d) by defining $\Phi = \theta_{I+1,J} - \theta_{I,J}$. It is noted that the spiral core is incoherent for the variable X , while it is coherent for the variable Z . (f) is the spatiotemporal pattern of X along the middle horizontal line of (a) showing wave propagation towards the spiral chimera core. In panels (a)–(d), the wave pattern rotates clockwise.

and its magnification around the core are shown in Figs. 1(a) and 1(b), respectively, at time $t = 10^5$ after the initiation of the spiral wave. The X variable around the spiral core is discontinuous, while it is smooth far from the core, which is similar to previously reported spiral wave chimeras in nonlocal systems. This feature is also true for the other nondiffusing variable Y (figure not shown).

However, in contrast to X and Y , for the diffusing inhibitor Z , the whole spiral pattern is relatively smooth even in the region close to the spiral core as shown in Fig. 1(c). This property of the system is different from previously reported nonlocally coupled oscillator systems where all of the variables describing the oscillator demonstrate a similar chimera character.

The corresponding phase distribution of $\theta_{I,J}$ and the dependency of $\Phi_{I,J} = \theta_{I,J} - \theta_{I+1,J}$ on the position along a horizontal cross section through the center of the medium ($J = N_y/2$) is illustrated in Figs. 1(d) and 1(e), respectively. Close to the spiral center, the quantities $\Phi_{I,J}$ are nonzero, which means that the phases of the oscillators around the spiral core are discontinuous or incoherent. However, far from the core region, the phases become continuous or coherent as indicated by the vanishing of $\Phi_{I,J}$. This behavior is the defining property of a chimera state for a group of identical oscillators: some oscillate in a coherent way while a localized subgroup oscillates incoherently [8].

It is worth pointing out that the spiral wave chimera shown in Fig. 1 rotates inwardly, i.e., clockwise. In other words, one sees the coherent waves propagate toward the

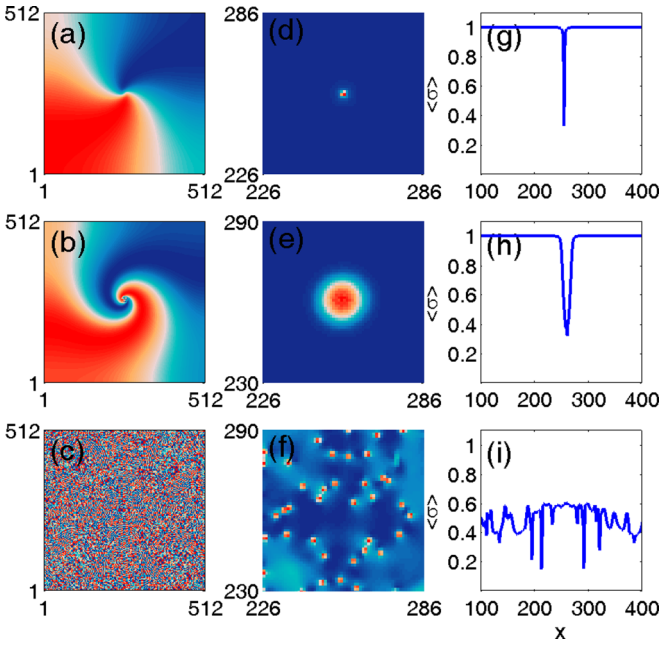


FIG. 2. Transition of chimera spiral waves as a function of a . In the case of small $a = 0.90$, a coherent spiral wave is observed (a) and it is gradually changed to a spiral wave chimera (b) as we increase the system parameter $a = 1.40$. Further increasing a led to the formation of completely disordered state (c) where $a = 1.70$. We here also show the corresponding time averaged order parameter σ [refer to (d)–(f)] and the order parameter but along the center of medium (g)–(i). Except for a , other parameters are taken as in Fig. 1.

phase-randomized core. This can also be seen from the spatiotemporal pattern for X variable along a horizontal cross section through the center of the medium ($J = N_y/2$) shown in Fig. 1(f). Inwardly rotating spiral waves with coherent cores, also termed antispiral waves, were reported 15 years ago [24]. However, here we observe inwardly rotating spiral wave chimeras in a RD system.

To get more insight into the formation and robustness of the spiral wave chimera, we investigate its behavior under a sweep of the model parameter a . From [25], we recall that the system (1) changes from a stationary state to an oscillatory state via a Hopf bifurcation when the value of the parameter a is increased. For $\epsilon_2 = 0.15$, we find the Hopf bifurcation point for $a_c \simeq 0.84$. For $a \leq 1$ but beyond the Hopf bifurcation point, we observe inwardly rotating spiral waves with a smooth core [see Fig. 2(a)]. From $a = 1.1$ onwards, spiral chimera states are found as shown in Fig. 2(b). The incoherent region of the chimera state monotonically grows with increasing a , until the incoherent state fully occupies the domain. This completely incoherent state is already reached in Fig. 2(c) for $a = 1.7$.

The time-averaged order parameter for different values of a is shown in Figs. 2(d)–2(f). In the incoherent chimera core, $\langle \sigma_{I,J} \rangle$ is smaller than 1, whereas in the outer region $\langle \sigma_{I,J} \rangle \approx 1$ due to the continuity of phase. Therefore, in the case of spiral waves, the region with $\langle \sigma_{I,J} \rangle \approx 1$ is extremely small [see Fig. 2(d)]. On the contrary, for the turbulent state, $\langle \sigma_{I,J} \rangle \ll 1$ everywhere. These statements can also be seen clearly from Figs. 2(g)–2(i), which shows the order parameter $\langle \sigma_{I,J} \rangle$ along the line $J = N_y/2$.

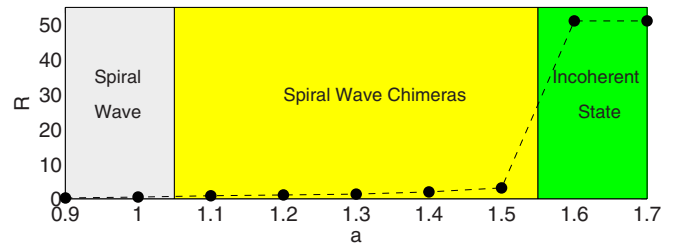


FIG. 3. Radius of the incoherent chimera core as a function of model parameter a , for $\epsilon_2 = 0.15$.

Figure 3 displays the chimera core radius as a function of the parameter a . When $0.90 \leq a \leq 1.0$, the radius $R \approx \Delta x$ and thus in such a case, a classical spiral wave emerges. However, when a lies between 1.1 and 1.5, the radius of the incoherent spiral core is clearly finite and it is much larger than the order of Δx , corresponding to the spiral wave chimera state. From this figure, we find that there is a continuous transition from spiral waves to spiral wave chimeras and a sudden transition from spiral wave chimeras to a completely turbulent state controlled by the parameter a .

To give a global picture about how the local dynamics of the system affects the behavior of the spiral wave chimeras, we further investigated the occurrence of the chimera state in a wide range of the a - ϵ_2 parameter space. The results are summarized in Fig. 4. In this figure, full circles represent the stable spiral wave chimeras and the full triangles and squares denote the spiral waves and incoherent state. The cross means that oscillations are not sustained. In the parameter space, the separation line between spiral and spiral wave chimera is almost vertical, which means that a plays the key role in

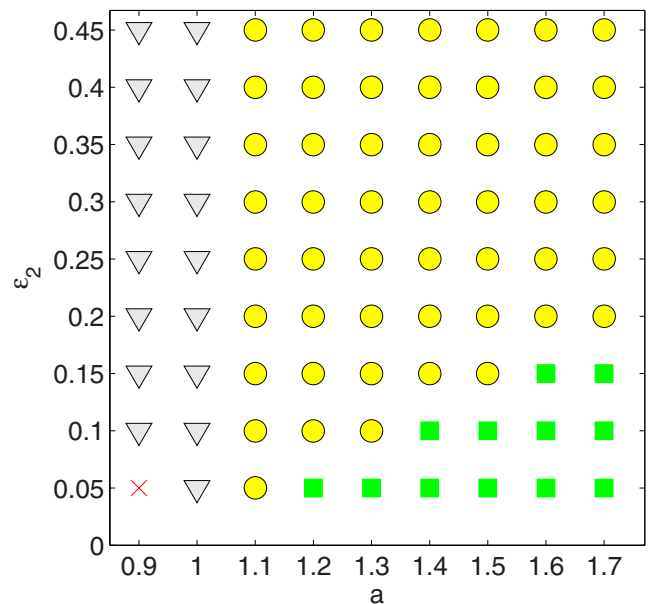


FIG. 4. Phase diagram for the spiral wave chimera state in $a - \epsilon_2$ parameter space showing the occurrence of spiral waves (triangles), fully incoherent state (squares), spiral wave chimera states (full circles), and a stable resting state (cross). All the patterns were observed at $t = 10^5$ after the initiation of the spiral wave.

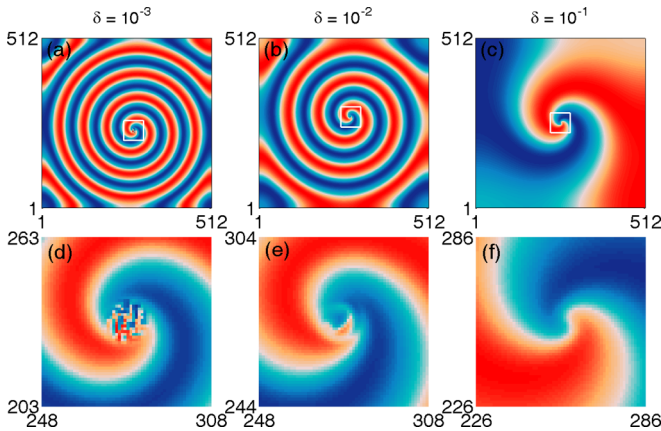


FIG. 5. The impact of finite δ on spiral wave chimera. (a) $\delta = 10^{-3}$, spiral wave chimeras; (b) $\delta = 10^{-2}$, spiral wave with split core; (c) $\delta = 10^{-1}$, spiral wave with smooth core. (d)–(f) are the corresponding core regions as indicated by the white square region in (a)–(c). $\epsilon_2 = 0.45$, $a = 1.70$. The time step used here is $\Delta t = 0.008$.

determining such state, while ϵ_2 has a more important role in determining whether spiral chimeras or turbulent states appear.

Spiral wave chimeras with finite activator diffusion. Until now, we have considered RD systems where only the diffusion of the inhibitor Z is present, i.e., $\delta = 0$. A natural question is: how will this kind of spiral wave chimera vary when we increase δ from zero to a finite value? Typical results are presented in Fig. 5. For $\epsilon_2 = 0.45$, spiral wave chimeras survive as we increase to $\delta = 10^{-3}$ [see Fig. 5(a) and magnification of the core region 5(d)]. The core region becomes smooth but somehow splits as we increase to $\delta = 10^{-2}$ [see Figs. 5(b) and 5(e)]. Further increasing $\delta = 10^{-1}$ finally leads to the appearance of the spiral wave with smooth core as illustrated in Figs. 5(c) and 5(f). These results are understandable as the diffusion coupling of the activator has a smoothing effect.

Discussion. With a three-component RD model, we have shown that spiral wave chimeras are possible in a spatially extended system of locally coupled oscillators. This result is a useful and nontrivial extension to the previous studies of chimera states. Prior to our work, spiral wave chimeras were only observed in nonlocally coupled systems. Very recently, chimera states in networks of purely local coupling were reported. However, this author did not consider spiral chimera states. For the systems with nonlocal coupling, the chimera feature appeared simultaneously for each state variable. In our findings, however, only the nondiffusive components of the system show chimeras, while diffusing variables are found in the fully coherent state. Moreover, the observed spiral wave chimera rotates inwardly, which has not been reported to our knowledge. Our results are not limited to the particular RD system (1), when it is linearly coupled to the third variable that diffuses. For example, we also find spiral wave chimeras in a system of Stuart-Landau oscillators coupled to a diffusive environment (results not shown). In view of those points, the finding of the spiral chimera state in local (diffusive) coupling extends our knowledge and improves our understanding of the existence of the chimera state in coupled oscillator systems.

Qualitatively, the origin of the chimera core may be understood in the following manner. The system may be seen as a population of oscillators (described by X and Y) subjected to spatiotemporal forcing of Z , which is relatively smooth but inhomogeneous. Since the amplitude of Z is sufficiently large far from the core (no figures shown), oscillators in those regions can be phase locked by the forcing of Z , while inside the core region, its oscillation amplitude of Z is too weak and thus is insufficient to keep the phases together. As a result, one observes coherent motion of oscillators far from the core and incoherent motion of oscillators inside the core. This explanation is phenomenological and simplified, and a more accurate and quantitative interpretation should be explored further in the future.

There may be various reasons for not observing chimeras in natural experimental settings before. From our study we note two necessary conditions: first, the local dynamics of each element needs to be oscillatory. Secondly, one requires a nearly vanishing spatial coupling of the observed variable.

Finally, we note the essential property of system (1) for $\delta = 0$ used in our study is that each oscillator (described by X and Y) is coupled indirectly via a nonuniform dynamical environment, which is realized by the variable Z from a dynamics viewpoint. The systems with such property may denote a broad class of the systems such as chemical oscillators BZ particles immersed in catalyst-free solutions [29], genetically engineered bacteria [30], yeast cells [31], and social amoeba *Dictyostelium discoideum* [32]. Therefore, we expect that chimera states are highly possible in biological or chemical systems which can be modeled by RD equations like Eq. (1). Furthermore, although we take the form of local coupling in the Z variable, in the limit case of $\delta = 0$, such kind of local coupling of the inhibitor Z may give rise to nonlocal effects in the absence of diffusion of the activator X . However, these nonlocal effects, which largely differ from the traditional nonlocal coupling (where each oscillator in the system will be affected *instantaneously* by a group of oscillators within certain interacted range), are not only spatial but also temporal, because of the finite τ . The systems with the local coupling that nevertheless demonstrate spatiotemporally nonlocal effects have been largely overlooked in past decades. However, such systems may be very common in biological systems and thus deserve further investigation.

Conclusion. In summary, we have shown that inwardly rotating spiral wave chimeras do exist in spatially extended oscillatory media where only nearest-nearest interaction between the elements is present. In such a system, the nondiffusing components appear as a spiral wave chimera, while the diffusing variables show a coherent spiral wave structure.

A continuous transition from coherent spiral waves to spiral wave chimeras is observed as we increase the model parameter a that controls the Hopf bifurcation. A phase diagram for spiral wave chimeras is identified in the wide parameter space. We further discussed the smooth effects on the spiral core by the diffusion of the activator X . Our results on the spiral wave chimera in locally coupled oscillator systems improves our understanding of the chimera state and provides indications that the chimera state can be found in biological and chemical systems where each oscillating element communicates via a diffusive dynamical environment.

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