



Mathematical model of haptic perception of temperature

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Abstract

The tactile perception of temperature of fabric measured by the thermocouple placed between fabric and a fingertip was analyzed. The experimental data were compared to the mathematical model of heat transfer between body and fabric. A high correlation between mathematical model, which presents a theoretical approach to heat transfer, and the experimental part was found. It proves that the very basic laboratory setup for measurement of heat transfer can be a reliable source of information concerning this phenomenon. It was proven that two time constants, τ and τ_L , and two temperature values, T_0 and ΔT_0 , are the essential parameters to obtain a reliable agreement between the mathematical model and the experimental results.

Keywords

haptic perception of textiles, tactile perception, mathematical modeling of heat transfer

Introduction

Thermal perception of textiles is one of the most important factors that influences our acceptability of textiles and therefore our desire to purchase them. The mechanism of perception of changes of temperature when we have skin contact with objects, especially textiles, is interesting and complex as a result of many factors influencing it. Thermal perception of textiles depends mainly on external conditions in which textiles are touched (air temperature and humidity)^{1–4} as it affects humans and the textiles, but also physical activity of individuals including blood flow in the soft tissue,^{5–7} physical state of the individual (blood pressure and dermatological issues, state of the skin, its roughness, thickness)^{5,8–10} and features of the textiles, including thickness, raw material, and architecture^{8,9} translating directly into their heat capacity and the ease of heat transfer, from the human skin in the direction of textiles in most cases, where the temperature of textiles is lower than the human skin temperature. In this environment objects tend to conduct heat out of the skin when they are touched. The first contact (up to 1 s) determines the thermal tactile perception. However, how fast the temperature exchange stabilizes relates rather to the final comfort perception in this specific textile. This feature is also responsible for maintenance of the specific

microclimate between textiles and the skin. The thermal sensation during the skin–fabric contact is predictable on the basis of a number of mechanisms: the heat transfer in fabrics, the neurophysiological mechanism of thermoreceptors, the heat exchange between the skin and fabric, and the psycho-neurophysiological relationships between subjective perception and thermoreceptors caused by skin temperature changes.^{10–12}

The duration of a thermal stimulus and the rate with which it changes can have a marked effect on perception. With continuous exposure to a thermal stimulus there is a decrease in neural responsiveness, a process referred to as adaptation. The skin adapts to both warm and cold stimuli over time, and for temperatures close to that of the skin the rate at which adaptation

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occurs is rapid, in the order of 60 seconds for changes of $\pm 1^\circ\text{C}$ in skin temperature. It takes much longer for the skin to adapt to more extreme temperatures, and for the forearm complete adaptation occurs within about 25 minutes for temperatures between 28 and 37.5°C .¹³ The time required to respond to a thermal stimulus depends on the intensity of the stimulus and the response required. Temperatures close to the thermal pain thresholds are responded to rapidly because of the possibility of tissue damage, but the response to more moderate temperatures is sluggish when compared to other sensory systems.

The thermal adaptation to an object when touched (equilibrium achievement) will be investigated in this paper as it deals with assessment of the conditions when the exchange of the temperature starts stabilizing. The purpose of this paper is to establish a mathematical model to explain the recorded temperature profiles measured when a fingertip is touching a fabric. It will also be shown that only four parameters are needed to represent completely the observed phenomena: two time constants and two temperature values.

Materials and methods

The methodology of the experimental part is the following:

- Design and production of three woven fabrics: two different linen/cotton woven fabrics and one cotton chenille woven fabric, as presented in Figure 1. These fabrics were characterized by assessment of their physical parameters, which are presented in Table 1.
- Performing an objective assessment of the thermal properties: objective assessment of thermal features of woven fabrics was conducted by measuring so called Q_{MAX} , which is the peak value of heat measured immediately after the heat stored on a pure copper plate of the device travels to a fabric when

the plate touches the surface of this fabric. It accurately reproduces the warm/cool feeling (heat transfer up to 1 s) experienced when a human finger touches an object briefly for the first time. Q_{MAX} was assessed using KES-F7 Precise and Fast Thermal Property-Measuring Instrument Thermo Lab.

- Performing an objective assessment of the temperature change between the skin of the right hand index fingertip of a female, aged 35, and three woven fabrics. It was to reveal what the temperature between textiles and human skin is when textiles are touched (also what the heat transfer from the surface of the skin is in the direction of textiles).

Experimental measurements

The female subject remained seated for 15 min at a room temperature of 21°C and 65% of humidity, as the outdoor air temperature was in the range of $17\text{--}19^\circ\text{C}$. The period of 15 min of acclimatization was enough to adapt to the room temperature. This test was a part of a bigger study. The thermocouple was connected to the voltmeter that indicated the voltage change depending on the change of the detected temperature. A double layer of fabric was placed on top of an insulating polystyrene foam with a thickness of 2 cm. The isolation foam was utilized to block the effect of the temperature of the table on measurement results. Double layers of selected fabrics were placed on the isolation foam to minimize the interference from the measuring system. The thermocouple was placed directly on this double layer and next the test subject placed her washed right hand index fingertip on the thermocouple, just on the thermocouple junction so that the skin temperature variation could be recorded electronically. A measurement of the temperature change between the fabric and human fingertip took place. This is schematically shown in Figure 2. The contact force between the finger and the textile was

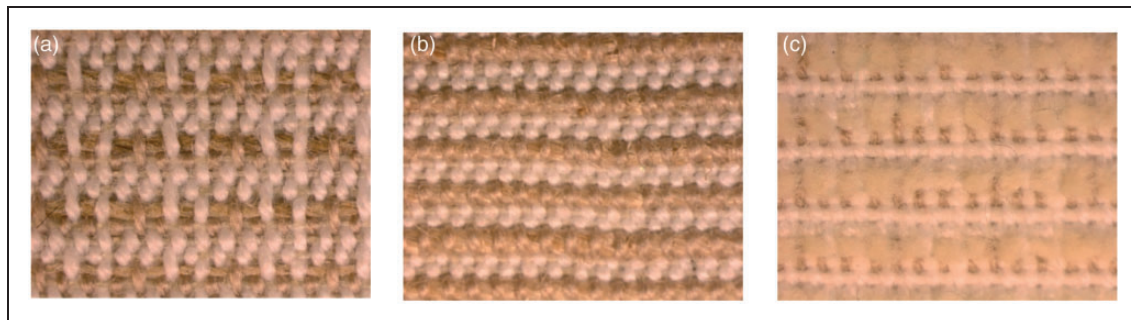


Figure 1. Woven fabrics made of: (a) linen/cotton with lower mass surface, (b) linen/cotton with higher mass surface, and (c) cotton chenille/linen; utilized in the experiment where the temperature between human fingertip and these fabrics was measured.

Table 1. The characteristic of the woven fabrics

	(Figure 1a) Linen 65% Cotton 35%	(Figure 1b) Linen 60% Cotton 40%	(Figure 1c) Linen 30% Cotton 70%
Raw material			
Mass surface of the sample [g/m ²]	354	400	466
Thickness [mm]	0.92	0.75	1.25
Yarn linear density [tex]	Linen ¹ : 240 (weft) Linen ² : 30 (warp and weft) Cotton: 15 (warp)	Cotton: 15 (warp and weft) Linen: 30 (warp and weft)	Cotton ¹ : 15 (warp and weft) Linen: 30 (warp and weft) Chenille yarn – cotton ² : 250 (weft)
Structure type; stitch/weave type and yarns density [dm]	Woven fabric; ½ Z Warp density: 123 linen ² + 248 cotton Weft density: 64 linen ¹ + 127 linen ²	Woven fabric; combined weave; Warp density: 210 cotton + 160 linen; Weft density: 120 cotton + 120 linen	Woven fabric; ½ Z; Warp density: 140 cotton ¹ + 140 linen; Weft density: 180 cotton ¹ + 50 cotton ²
Thermal resistance [(m ² × °C/W)]	0.0164	0.0145	0.0235
Q _{MAX} by KES-F7	0.0860	0.0825	0.0700
Face:	0.1165	0.0950	0.0750
Back:	0.1012	0.0890	0.0725
average			

¹ and ² refer to the type of the yarns utilized as warps and wefts in textiles; ¹ and ² are introduced to demonstrate that two different linen yarns and two different cotton yarns were utilized.

not measured. However, the female subject was trained to perform the tests and how to keep her finger in the same position for a longer period of time so that only very small, negligible, variations would be possible in terms of contact force. It happened that after several seconds of keeping her finger in the same position, the subject made a minor move that affected the test results. In that case, the results were not taken into account. The thermocouple was made from very thin wires of nickel and nickel-chromium. The electromotive force of the thermocouple was $36.8 \mu\text{V}/^\circ\text{C}$. The thermocouple junction was soldered into a spherical shape with a diameter of only 0.5 mm.

Figure 2 shows a schematic view of the experimental assembly. It should be emphasized here that we examined the time dependent behavior of the temperature at the interface between the fingertip and the fabric.

Three sets of experimental data were collected. Each of them represent the temperature measurement when the test subject kept her fingertip on the thermocouple and on the fabric. The data are presented in Figure 3. The cold junction of the thermocouple was kept at room temperature. As a consequence Figure 3 shows

the temperature rise above ambient. Part of this measuring methodology was previously published. These sets of data will support the verification of a mathematical model that can explain the observed phenomenon of interface finger–fabric temperature change.

Experiments were carried out on a double layer of woven linen/cotton fabrics (Δ and \bullet) and on a double layer of woven chenille fabric (\circ). At the very beginning ($t=0$) the thermocouple records the heating at the interface between the fingertip and the fabrics. This transient phenomenon stops after 20 s. After that, even a slight decrease in the temperature is observed (e.g. \bullet) because the thermal energy in a fingertip is not enough to continue heating for a longer time. All curves have similar shapes, although different temperature transients have been found. This is attributable to the fabric itself because different fabrics have different thermal properties. The physiological condition of the test person also has an influence. If the person is in a condition of intensified blood flow, perhaps on account of some previous sport activities, the fingertip has a higher bulk temperature. However, these effects only influence the amplitude of the temperature transients, not the time constants, which explains the different temperature values observed in Figure 2.

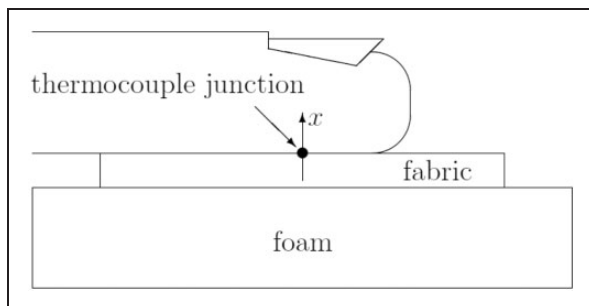


Figure 2. Schematic view of the experimental setup.

Theoretical analysis

The contact temperature between a fingertip and a woven fabric is recorded by means of a thermocouple. The study is limited to the short time transient. Cooling by convection or conduction through objects further away is not taken into account. To simplify the calculation a one-dimensional approach will be used. A schematic view of this approach is presented in Figure 4.

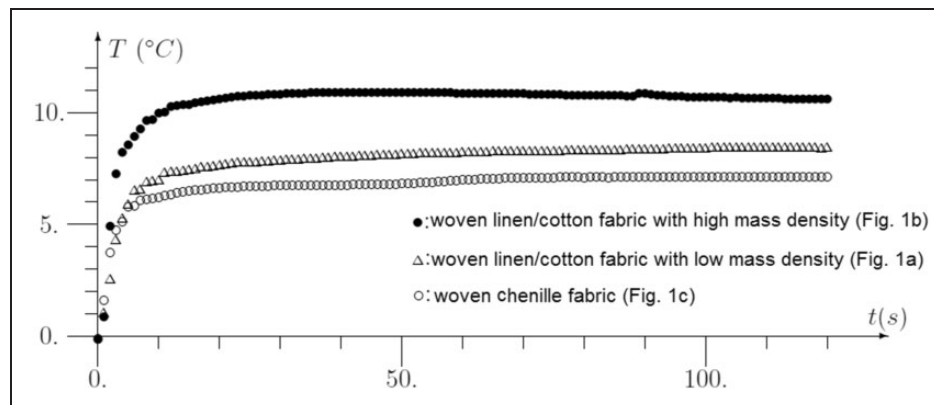


Figure 3. The temperature transients recorded by the thermocouple when a human finger was in a contact with fabrics as demonstrated in Figure 2.

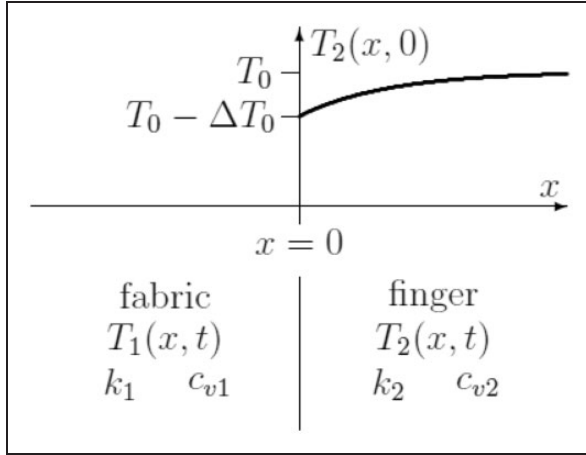


Figure 4. Schematic view of the fingertip–fabric interface. The initial fingertip temperature (6) is also presented.

The x -axis is perpendicular to the fabric–finger interface. Both the temperature distributions $T_1(x, t)$ in the fabric and $T_2(x, t)$ in the fingertip satisfy the diffusion equation

$$k_i \frac{\partial^2 T_i}{\partial x^2} = c_{vi} \frac{\partial T_i}{\partial t} \quad (1)$$

The value $T_i(x, t) = 0$ is set to the ambient temperature. Hence, (1) gives us the temperature rise above ambient. The index $i = 1$ denotes the fabric whereas $i = 2$ denotes the fingertip. k_i is the thermal conductivity and c_{vi} the specific heat per unit volume. At the interface $x = 0$ the temperature remains continuous

$$T_1(0, t) = T_2(0, t) \quad (2)$$

A second boundary condition requires the continuity of the heat flux or $k_1 (\partial T_1 / \partial x) = k_2 (\partial T_2 / \partial x)$ in $x = 0$. However, with this boundary condition the theoretical results were far from being satisfactory. A reliable fitting to the experimental results turned out to be impossible. The reason for it is that the thermal capacity of the thermocouple joint is not negligible. Therefore the aspect of a thermocouple should also be taken into consideration and modeled as an additional thermal capacity C_{th} , which has to be inserted in the second boundary condition

$$k_1 \frac{\partial T_1}{\partial x} = k_2 \frac{\partial T_2}{\partial x} - C_{th} \frac{dT_c}{dt} \quad \text{at } x = 0 \quad (3)$$

where T_c is the thermocouple temperature. Physically, equation (3) describes the heat absorption by the thermocouple when the finger makes contact, k_1 is thermal conductivity of the fabric, and k_2 is thermal

conductivity of the fingertip skin. The boundary condition (2) must then be replaced by

$$T_c(t) = T_1(0, t) = T_2(0, t) \quad (4)$$

expressing the continuity of the temperature.

The initial conditions are special because the finger only makes contact with the fabric for $t > 0$. At $t = 0$ the fabric and the thermocouple are both at ambient temperature

$$T_1(x, 0) = T_c(0) = 0 \quad (5)$$

In order to take into account the fact that the skin surface temperature is always less than the bulk body temperature, one assumes the initial temperature distribution having the following relationship

$$T_2(x, 0) = T_0 - \Delta T_0 e^{-x/L} \quad (6)$$

Equation (6) is also presented in Figure 4. T_0 is the temperature inside the fingertip where the blood perfusion is active. In the skin layers the blood perfusion is reduced and almost unexisting in the skin layer (stratum corneum). Hence the temperature will no longer be controlled by the blood flow resulting in a negative temperature slope toward the skin. The relation (6) is used to model this phenomenon. The length constant L can be interpreted as the distance over which the temperature ΔT_0 decay takes place. For a fingertip, L should be around 1 mm. The initial temperature distribution (6) inside the finger was taken into account in order to set up a model which fit the experimental results.

The easiest way to solve the problem is to convert all equations and boundary conditions into the Laplace domain. Equation (1) for the fabric is then converted to

$$k_1 \frac{d^2 T_1}{dx^2} - s c_{v1} T_1 = 0 \quad (7)$$

and for the fingertip

$$k_2 \frac{d^2 T_2}{dx^2} - s c_{v2} T_2 = -c_{v2} T_2(x, 0) = -c_{v2} [T_0 - \Delta T_0 e^{-x/L}] \quad (8)$$

where s is the Laplace variable, c_{v1} is the specific heat per unit volume of the fabric, and c_{v2} is the specific heat per unit volume of the skin. The initial condition (6) has been taken into account in (8) by introducing the initial finger temperature distribution (6) as a temperature step in the right side element of the equation (8).

The general solutions of (7) and (8) are

$$T_1(x) = A \exp\left[\sqrt{\frac{sc_{v1}}{k_1}}x\right] \quad (9)$$

$$T_2(x) = B \exp\left[-\sqrt{\frac{sc_{v2}}{k_2}}x\right] + \frac{T_0}{s} - \frac{c_{v2}T_0}{sc_{v2} + \frac{k_2}{L}} e^{-\frac{x}{L}} \quad (10)$$

where A and B are two integration constants to be determined by the boundary conditions (3) and (4) at $x=0$.

The boundary condition (4) gives rise to

$$A = B + \frac{T_0}{s} - \frac{c_{v2}\Delta T_0}{sc_{v2} + \frac{k_2}{L}} \quad (11)$$

The boundary condition (3) involves the time derivative of the thermocouple temperature, which, as already pointed out, equals the temperature T_1 at the boundary $x=0$. The boundary condition (3) in the Laplace domain reads

$$k_1 \frac{dT_1}{dx} = k_2 \frac{dT_2}{dx} - sC_{th}T_1 \quad (12)$$

which gives rise to

$$A\sqrt{k_1c_{v1}s} = B\sqrt{k_2c_{v2}s} + \frac{c_{v2}T_0}{sc_{v2} + \frac{k_2}{L^2}} \frac{k_2}{L} - sC_{th}A \quad (13)$$

Using (9) it is clear that the interface temperature or the thermocouple temperature is just given by the constant A. Eliminating B from (11) and (13) gives

$$\begin{aligned} & \left[\sqrt{k_1c_{v1}s} + \sqrt{k_2c_{v2}s} + sC_{th}\right]A \\ &= \sqrt{k_2c_{v2}s} \left[\frac{T_0}{s} + \frac{c_{v2}\Delta T_0}{sc_{v2} - k_2/L^2}\right] + \frac{c_{v2}\Delta T_0}{sc_{v2} - k_2/L^2} \frac{k_2}{L} \end{aligned} \quad (14)$$

We define the following two time constants τ and τ_L

$$\tau = \left[\frac{C_{th}}{\sqrt{k_1c_{v1}} + \sqrt{k_2c_{v2}}}\right]^2 \quad (15)$$

$$\tau_L = \frac{c_{v2}L^2}{k_2} \quad (16)$$

The time constant τ is determined by the thermal capacity of the thermocouple C_{th} , whereas the other time constant τ_L is determined by the length constant L. The temperature is distributed along this length

inside the fingertip. Thus, the expression for A can be simplified to

$$A = \frac{k_2c_{v2}}{C_{th}} \left[\frac{T_0}{s(\sqrt{s} + 1/\sqrt{\tau})} - \frac{\Delta T_0}{\sqrt{s}(\sqrt{s} + 1/\sqrt{\tau})(\sqrt{s} + 1/\sqrt{\tau_L})} \right] \quad (17)$$

Taking into account the inverse Laplace transform of (17)

$$\mathcal{L}^{-1} \frac{1}{\sqrt{s}(a + \sqrt{s})} = e^{a^2t} \operatorname{erfc}(a\sqrt{t}) \quad (18)$$

The last term of (17) can be handled by expansion into partial fractions. The inverse Laplace transform is then also obtained by using (18). The inverse Laplace transform of (17) is found to be

$$\begin{aligned} \mathcal{L}^{-1}A = T_c(t) &= T_0 \frac{\sqrt{k_2c_{v2}}}{\sqrt{k_1c_{v1}} + \sqrt{k_2c_{v2}}} \\ &\times \left[1 - \left(1 - \frac{\sqrt{\tau_L}}{\sqrt{\tau_L} - \sqrt{\tau}} \frac{\Delta T_0}{T_0} \right) e^{\frac{t}{\tau}} \operatorname{erfc}\left(\sqrt{\frac{t}{\tau}}\right) \right. \\ &\left. - \frac{\sqrt{\tau_L}}{\sqrt{\tau_L} - \sqrt{\tau}} \frac{\Delta T_0}{T_0} e^{\frac{t}{\tau}} \operatorname{erfc}\left(\sqrt{\frac{t}{\tau_L}}\right) \right] \end{aligned} \quad (19)$$

where erfc denotes the complementary error function.

Experimental verification

In Figure 5 the comparison between experimental results and theoretical results have been plotted and the time scale has been limited to 20 s. As already pointed out in the experimental measurements section, a decay of the temperature is observed after some time because the thermal field is no longer limited to the fingertip and the fabric. Heating of the underlying insulating layer and convective heat transfer to the ambient air cannot be neglected for longer times. Hence our model, and consequently the fitting, has to be limited to 20 s.

First of all the results obtained with woven linen/cotton fabric (●) will be analyzed in detail. Then the two other materials will be discussed.

The continuous curve is calculated according to

$$\begin{aligned} T &= 13.043 \left[1 - 0.85416e^{t/\tau} \operatorname{erfc}\left(\sqrt{t/\tau}\right) \right. \\ &\left. - 0.14583e^{t/\tau_L} \operatorname{erfc}\left(\sqrt{t/\tau_L}\right) \right] \end{aligned} \quad (20)$$

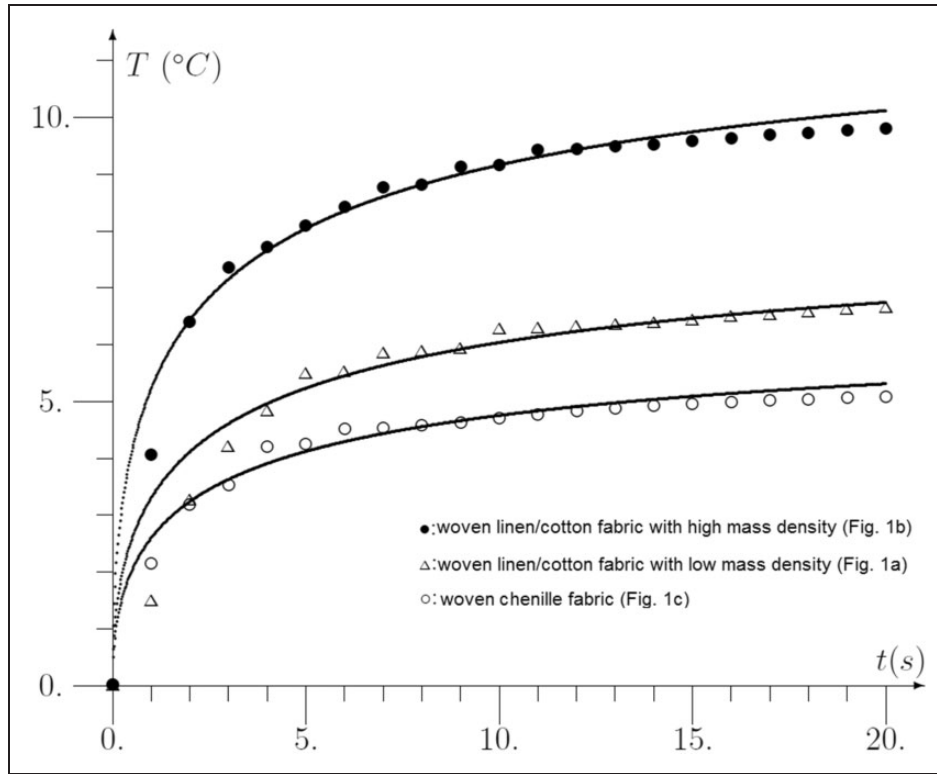


Figure 5. Comparison between experimental data (●), (Δ), and (○) obtained on the base of temperature measurements between the fabric (Figure 1a, b, and c) and the finger and theoretical results (black lines corresponding to the theoretical analysis).

where the values $\tau = 3$ s, $\tau_L = 10$ s, $T_0 = 13.043^\circ\text{C}$, and $\Delta T_0 = 0.86^\circ\text{C}$ were utilized. The fitting of mathematical model–curve to the experimental data (●) is good and has been limited to the first 20 seconds of the measurements. For longer times the foam below the textile becomes heated and this aspect was not included in the model. Moreover, after some time, the one-dimensional model outlined in the foregoing section is no longer valid because the lateral heat spreading can no longer be neglected. A similar approach was presented in other studies, where the temperature response of paper subjected to laser heating was investigated.¹⁵

A first verification of the model can be made by evaluating the value of the temperature T_0 . Taking into account that $k_2 > k_1$ and $c_{v2} > c_{v1}$ we obtain the value $T_0 = 13.043^\circ\text{C}$. With a room temperature of 20°C , one obtains a fingertip temperature of 33.043°C , which is a realistic value recalled in the literature.¹⁶

From the time constant $\tau = 3$ s one is able to find the thermal capacity of the thermocouple joint

$$C_{th} = \left(\sqrt{k_1 c_{v1}} + \sqrt{k_2 c_{v2}} \right) \sqrt{\tau} \cong \sqrt{k_2 c_{v2} \tau} = 4.47210^3 \frac{\text{J}}{\text{m}^2 \text{K}} \quad (21)$$

where the values $c_{v2} = 4 \text{ MJ/m}^3 \text{K}$, $k_2 = 0.5 \text{ W/mK}$ have been used.¹⁷ The approximation takes into account the fact that thermal capacity of the textile (c_{v1}) is lower than the corresponding value of a fingertip (c_{v2}). The value in equation (21) is a thermal capacity per unit interface area. In the experiments the thermocouple junction was spherical with a diameter of 0.5 mm. At the interface it occupies an area given by $\pi 0.5^2 / 4 \text{ mm}^2 = 0.1963 \cdot 10^{-6} \text{ m}^2$ so that the thermal capacity expressed in J/K is

$$C_{th} = 4.47210^3 \times 0.196310^{-6} = 0.87810^{-3} \frac{\text{J}}{\text{K}} \quad (22)$$

In order to verify whether this value is realistic and correct, the thermal capacity will be calculated in a completely different way. By definition a thermal capacity is the specific heat per unit volume (c_v) times the volume of a sphere ($4\pi 0.25^3 \cdot 10^{-3} \text{ m}^3$). Hence we obtain

$$C_{th} = 4.10^6 4\pi 0.25^3 10^{-9} = 0.78510^{-3} \frac{\text{J}}{\text{K}} \quad (23)$$

Both results in equations (22) and (23) are relatively consistent. One utilized $c_v = 4.10^6$, which is the value

for Ni for the thermocouple junction.¹⁸ This agreement is a second validation of the mathematical model.

A third validation can be performed when fitting the value for the time constant $\tau_L = 10$ s. One can find the characteristic length L based on (16)

$$L = \sqrt{\frac{k_2 \tau_L}{c_{v2}}} = 1.11 \text{ mm} \quad (24)$$

which is also a very acceptable value because it is almost equal to the thickness of the skin of a fingertip. In the skin layers, the blood perfusion is limited compared to the soft tissues deep inside of the finger, where the thermoregulation keeps the temperature constant by regulating the blood flow. As a consequence, there will be always a temperature decay toward the skin surface with a length constant comparable to the skin thickness as modeled by the equation (6).

A fourth verification of the model relies on assessment of the quotient of ΔT_0 and T_0 and the numerical coefficient 0.14583 in (20). If one compares (19) and (20), one obtains

$$\frac{\Delta T_0}{T_0} = \frac{\sqrt{\tau_L} - \sqrt{\tau}}{\sqrt{\tau_L}} 0.14583 = \frac{\sqrt{10} - \sqrt{3}}{\sqrt{10}} 0.14583 = 0.0659 \quad (25)$$

which gives the value $\Delta T_0 = 0.86^\circ\text{C}$.

In order to verify whether it is an acceptable value, ΔT_0 will be also calculated independently using the thermal equivalent network presented in Figure 6. The skin has a thermal resistance (per unit area) given by

$$\frac{t_s}{k} = \frac{0.001}{0.127} = 0.00784 \frac{\text{Km}^2}{\text{W}} \quad (26)$$

where $t_s = 1$ mm denotes the skin thickness and $k = 0.127$ W/mK denotes the thermal conductivity of the epidermic layer.¹⁹ Here ΔT_0 is the temperature drop across the skin layer, as shown in Figure 5.

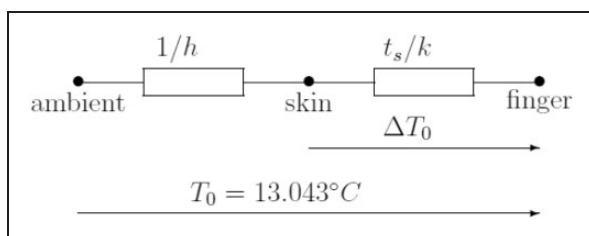


Figure 6. Thermal network model for fingertip in ambient air.

Heat is removed from the skin surface by convection and radiation to the ambient. One may utilize a typical value for the heat transfer coefficient $h = 10$ W/m²K for natural convection cooling.²⁰ Thus, the equivalent thermal resistance is

$$\frac{1}{h} = \frac{1}{10} = 0.1 \frac{\text{Km}^2}{\text{W}} \quad (27)$$

The entire temperature drop exists between the internal finger tissues and the ambient (see Figure 2). Based on the equivalent network, the temperature drop ΔT_0 is then evaluated as

$$\Delta T_0 = T_0 \frac{\frac{t_s}{k}}{\frac{1}{h} + \frac{t_s}{k}} = 13.043 \frac{0.00784}{0.1 + 0.00784} = 0.948 \quad (28)$$

which is quite close to the value 0.86 obtained from (25).

The fact that we obtained very acceptable values for the thermal capacitance of the thermocouple, the length constant L , and the temperature drop ΔT_0 proves that our model is realistic and adequate. On the other hand it must be emphasized here that the measuring device, namely, the thermocouple, should not be neglected even if its diameter is just 0.5 mm.

The two other curves could also be fitted quite well, as shown in Figure 5. For the woven linen/cotton fabric with low mass density (Δ) the best fitting was obtained for $\tau = 4$ s, $\tau_L = 10$ s, $T_0 = 8.96^\circ\text{C}$ and $\Delta T_0 = 0.498^\circ\text{C}$, which gives rise to a value $\Delta T_0/T_0 = 0.0556$, which is close to the value (25) obtained for woven linen/cotton with high density (\bullet). The value of τ_L is the same, which is quite understandable because, as a result of (16), the time constant τ_L is determined only by the thermal conductivity k_2 , the specific heat c_{v2} , and the length constant L of the fingertip only. The other time constant τ changed because it depends on the thermal parameters of both the fingertip and the fabric as in (15). For the woven chenille fabric, the best fitting was obtained for the set: $\tau = 4$ s, $\tau_L = 10$ s, $T_0 = 7.06^\circ\text{C}$, and $\Delta T_0 = 0.39^\circ\text{C}$ or $\Delta T_0/T_0 = 0.056$. One can conclude that the time constants do not vary much if a different fabric is used. Also the ratio $\Delta T_0/T_0$ remains around the value 0.6, so that the calculation in relation to Figure 6 will provide similar results for the thermal conductivity of the finger. The main difference between the fabrics is observed in the parameter T_0 . Referring to Figure 6, this can be explained by variations in the heat transfer coefficient h and or course the initial fingertip temperature.

It must be emphasized that experiments have been carried on several other fabrics as well; made of different raw materials including angora wool and polyester

with Outlast finishing. All the transient temperature curves present similar shapes.

Discussion

It was mentioned in the theoretical analysis that the boundary condition or $k_1 (\partial T_1/\partial x) = k_2 (\partial T_2/\partial x)$ in $x=0$ was totally unsatisfactory. It was observed that this boundary condition had to be replaced by (3) in order to include the influence of the thermal capacity C_{th} of the thermocouple. Without C_{th} no reasonable fitting with the experimental curves could be obtained.

If one neglects the thermal capacity $C_{th}=0$ or the time constant $\tau=0$ according to (15), and if $\tau=0$, the final solution (19) is changed into

$$T_j(t) = T_0 \frac{\sqrt{k_2 c_{v2}}}{\sqrt{k_1 c_{v1}} + \sqrt{k_2 c_{v2}}} \left[1 - \frac{\Delta T_0}{T_0} e^{\pm t/\tau} \operatorname{erfc}\left(\sqrt{\frac{t}{\tau_L}}\right) \right] \quad (29)$$

where $T_j(t)$ denotes the time dependent junction temperature between the fingertip and the fabric. At the initial moment $t=0$, the expression (29) is

$$T_j(0) = T_0 \frac{\sqrt{k_2 c_{v2}}}{\sqrt{k_1 c_{v1}} + \sqrt{k_2 c_{v2}}} \left[1 - \frac{\Delta T_0}{T_0} \right] \quad (30)$$

which means that the interface temperature T_j makes a sudden jump from $T_j=0$ to the value (30). This phenomenon is known as the interface contact problem and occurs when two solid bodies at different temperatures make contact with each other. Taking into account that $\Delta T_0 \ll T_0$, it simply means that the temperature jumps from zero to almost its final value. This fact was not observed experimentally. All experiments clearly showed curves starting with $T=0$ at the initial time $t=0$. From mathematical point of view, this discontinuity in a form of temperature jump, can be avoided by taking the limit of (19) for $\tau \rightarrow 0$. Further development of this matter is a laborious evaluation and it is beyond the scope of this paper.

Conclusion

The paper presents a thermal model, which was elaborated for the contact temperature between a fingertip and a fabric. The model reflects a dynamic behavior of both these bodies. The model was verified by comparing the theoretical data with experimental measurements by inserting a thermocouple junction between the fabric and the fingertip. It was found that, for an acceptable fitting, it was necessary to take the thermal capacity of the thermocouple into account. Moreover it was mandatory to include the initial temperature

distribution inside the fingertip in order to obtain an acceptable fitting.

The modeling could be restricted to four parameters: two time constants, τ and τ_L , and two temperature values, T_0 and ΔT_0 . From the values of these four parameters, obtained through curve fitting, one was able to estimate the thermal capacity of the thermocouple, the finger skin thickness, and the fingertip skin temperature. These three values were in agreement with data obtained from published information, which is the additional proof for the reliability of the model proposed in this paper. It has also been proved that neglecting the thermal capacity of the thermocouple did not provide a feasible model.

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