Classes of decision analysis

WG1-1

C. Xing, Ghent University, Department of Structural Engineering, Ghent, Belgium
R. Caspeele, Ghent University, Department of Structural Engineering, Ghent, Belgium
D. V. Val, Heriot-Watt University, Institute for Infrastructure and Environment, Edinburgh, United Kingdom

Scope of the factsheet

This factsheet is mainly focused on three difference classes of decision analysis based on prior information, additional information and unknown information, i.e. prior analysis, posterior analysis and pre-posterior analysis respectively. Pre-posterior analysis is considered in more detail and the Value of Information (VoI) is defined.

Abstract

The ultimate task of an engineer consists of developing a consistent decision procedure for the planning, design, construction and use and management of a project. Moreover, the utility over the entire lifetime of the project should be maximized, considering requirements with respect to safety of individuals and the environment as specified in regulations. Due to the fact that the information with respect to design parameters is usually incomplete or uncertain, decisions are made under uncertainty. In order to cope with this, Bayesian statistical decision theory can be used to incorporate objective as well as subjective information (e.g. engineering judgement). In this factsheet, the decision tree is presented and answers are given for questions on how new data can be combined with prior probabilities that have been assigned, and whether it is beneficial or not to collect more information before the final decision is made. Decision making based on prior analysis and posterior analysis is briefly explained. Pre-posterior analysis is considered in more detail and the Value of Information (VoI) is defined.

Basis / theory / methods

Decision tree

The decision process consists of choosing an action a_i out of a set of possible actions $\mathbf{A} = \{a_1, a_2, ..., a_n\}$. The consequence of implementing an action a_i depends on a number of uncertain conditions or events out of the set $\boldsymbol{\theta} = \{\theta_1, ..., \theta_m\}$, which in the context of Structural Health Monitoring (SHM) represent the states of the structure. Moreover, the decision maker may have an option to carry out an experiment/inspection e_j out of the set of possible experiments/inspections $\mathbf{E} = \{e_1, e_2, ..., e_k\}$ in order to obtain additional information on the state of the structure, i.e., $\boldsymbol{\theta}$. Potential outcomes of these experiments/inspections constitute the set $\mathbf{Z} = \{z_1, z_2, ..., z_l\}$ Thus, in general, the decision problem in the context of SHM can be described in the following terms within the framework of Bayesian decision theory (Raiffa and Schlaifer,1961):

$\mathbf{A}$: the set of possible maintenance actions (e.g., do nothing, repair, replace, etc.);

$\boldsymbol{\theta}$: the set of structural states, representing different levels of structural damage, which are usually time-dependent;

$\mathbf{Z}$: the set inspection outcomes, which provide information on the actual structural state;

$\mathbf{E}$: the set of possible inspection actions (e.g., inspection date, type of inspection, location, etc.).

The combination of certain inspection and maintenance actions, inspection outcome and structural state results in a 'value', 'use' or 'utility' $u(e,z,a,\theta)$, which is a numerical (most often monetary) measure that corresponds with the procedure that has been followed. This framework is represented in a 'decision tree' as for example illustrated in Figure 1.

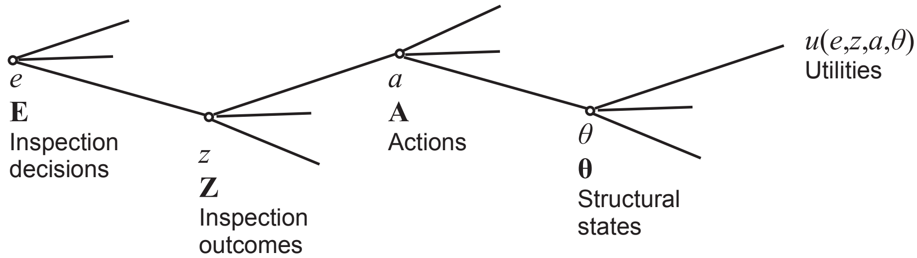

Figure 1. Decision tree

Decisions based on prior information : prior analysis

Prior analysis is referred to a situation when decision is to be made based on previously available, often generic, information. Using this prior information probabilities are assigned to possible structural states/conditions. These assigned probabilities are called prior probabilities and designated as $P'[\theta_j]$. After setting utilities of possible action-state combinations, $u(a_i,\theta_j)$, the expected utilities corresponding to the different actions can be calculated. The decision tree corresponding to this analysis is shown in Figure 2.

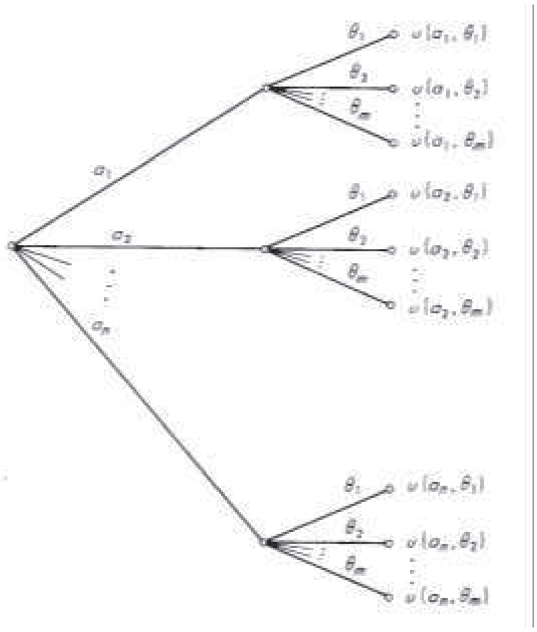

Figure 2. Reduced decision tree

The expected utility of action a_i is given by

$$E'_\theta\left[u(a_i,\theta)\right] = \sum_{j=1}^{m} u(a_i,\theta_j)P'\left[\theta_j\right] \qquad (1)$$

where E'_θ denotes the expectation operation with respect to prior probabilities $P'[\theta]$. Consequently, the decision analysis consists of choosing the action, a^*, which results in the largest expected utility, u^*, i.e.,

$$u^* = \max_a E'_\theta \left[u(a,\theta)\right] \tag{2}$$

or

$$a^* = \arg\max_a E'_\theta \left[u(a,\theta)\right] \tag{3}$$

Decisions based on additional information: posterior analysis

Posterior analysis corresponds to a situation when new information about the structural state becomes available from an inspection but a decision whether to carry out this inspection is not included in the decision process. Using the new information the prior probabilities assigned to the different structural states/conditions can be updated. These updated or posterior probabilities will be denoted as $P''[\theta|z]$. In the case of an experiment/inspection outcome of z_k the posterior probability can found using Bayes' theorem as

$$P''\left[\theta_i \mid z_k\right] = \frac{P\left[z_k \mid \theta_i\right] P'\left[\theta_i\right]}{\sum_{j=1}^{m} P\left[z_k \mid \theta_j\right] P'\left[\theta_j\right]} \tag{4}$$

where $P[z_k|\theta_i]$ is the likelihood function of the experiment outcome z_k. This formula can also be written as

$$\begin{pmatrix} \text{posterior probability} \\ \text{of } \theta_i \text{ based on a} \\ \text{given experiment} \\ \text{outcome} \end{pmatrix} = \begin{pmatrix} \text{normalizing} \\ \text{constant} \end{pmatrix} \begin{pmatrix} \text{likelihood} \\ \text{of experment} \\ \text{outcome based on } \theta_i \end{pmatrix} \begin{pmatrix} \text{prior probability} \\ \text{of } \theta_i \end{pmatrix} \tag{5}$$

The normalizing constant is needed in order to make sure that $0 \le P''[\theta|z] \le 1$. New and old/prior information are incorporated through the product of $P[z_k|\theta_i]$ and $P'[\theta]$. The likelihood function of the experiment outcome can be interpreted as the relative probabilities of the different conditions θ_i considering a given observation z_k. In case the relative probability is larger for θ_i compared to another condition, then $P''[\theta_i|z_k] > P'[\theta_i]$ and vice versa.

Once the posterior probabilities are determined, the decision making procedure is similar to that explained in the previous section except that the expectation operation is now with respect to the posterior probabilities $P''[\theta|z]$, i.e.,

$$u^* = \max_a E''_{\theta|z} \left[u(a,\theta)\right] \tag{6}$$

or

$$a^* = \arg\max_a E''_{\theta|z} \left[u(a,\theta)\right] \tag{7}$$

Decisions based on "unknown information": pre-posterior analysis

Pre-posterior analysis involves both a decision on inspection, e, and a decision on action, a, and the corresponding decision process is presented by the decision tree in Figure 1. There are two forms of pre-posterior analysis: extensive form and normal form. In the extensive form the analysis is carried out backward, i.e., from the right end of the decision tree to its starting point on the left hand side. Initially, it is assumed that the selected experiment/inspection and its outcome, i.e., (e,z), are known. Based on the 'known' outcome, the posterior probabilities $P''[\theta|z]$ can be estimated using Eq. (4). Then, for the considered (e,z) the maximum utility can be found as it has been done in the previous section

$$u^*(e,z) = \max_a E''_{\theta|z}\left[u(e,z,a,\theta)\right] \tag{8}$$

The difference is that the experiment/inspection decision, e, is yet to be done and its outcome, z, is a random variable. To resolve this problem $u^*(e,z)$ needs to be calculated for each possible combination (e,z) and then expected values of the utility should be found for each possible experiment/inspection action. For the latter, the probabilities of experiment outcomes z for a given e, $P[z|e]$, need to be defined. For an experiment e_i the expected utility then can be obtained as

$$E_{z|e}\left[u^*(e_i,z)\right] = \sum_{j=1}^{l} u^*(e_i,z_j)P\left[z_j \mid e_i\right] \tag{9}$$

The experiment e^* that leads to the maximum expected utility u^* can then be selected

$$u^* = \max_e E_{z|e}\left[\max_a E''_{\theta|z}\left[u(e,z,a,\theta)\right]\right] \tag{10}$$

or

$$e^* = \arg\max_e E_{z|e}\left[\arg\max_a E''_{\theta|z}\left[u(e,z,a,\theta)\right]\right] \tag{11}$$

The normal form of analysis progresses forward from the start of the decision tree to its end on the right hand side. The analysis starts with the determination of a decision rule, d, which assigns the optimal action a to each possible outcome z of every experiment/inspection action in **E,** i.e., $a=d(e,z)$. The expected value of utility is then to be found for each combination (e,d). For that the same probabilities that have been used in the posterior analysis (Section 3), i.e., prior probabilities $P'[\theta]$ and likelihood functions $P[z|\theta]$, need to be defined. The only difference is that the likelihood functions now depend on a particular experiment/inspection action and therefore will be denoted as $P[z|\theta,e]$. The expected utility for a combination (e,d) can then be expressed as

$$u(e,d) = E'_\theta\left[E_{z|\theta,e}\left[u(e,z,d,\theta)\right]\right] \tag{12}$$

The optimal combination (e,d) is the one that leads to the maximum expected utility

$$u^* = \max_e \max_d E'_\theta\left[E_{z|\theta,e}\left[u(e,z,d,\theta)\right]\right] \tag{13}$$

The extensive and normal forms of analysis should lead to the same result (Raiffa and Schlaifer, 1961). The normal form of analysis may be convenient for risk/reliability based inspection and maintenance planning as demonstrated, e.g., in (Faber, 1997). In the context of VoI for SHM it may be more convenient to use the extensive formulation.

Value of information (VoI)

One of the main applications of pre-posterior analysis is the evaluation of worth of information. In the context of the extensive form of analysis for each (e,z) the maximum expected utility $u^*(e,z)$ is calculated by Eq. (8). The difference between this utility and the maximum utility obtained by prior analysis, Eq. (2), represents the value of the information z (Raiffa and Schlaifer, 1961). In general, the expected VoI can be found as the difference between the maximum utility obtained in pre-posterior analysis, Eq. (10) or Eq. (13), which will be denoted as u^*_1, and the maximum utility obtained using only prior information, Eq. (2), which will be denoted as u^*_0

$$VoI = u_1^* - u_0^* \tag{14}$$

Since u^*_1 takes into account uncertainty associated with experiments/inspections (i.e., sample data), which prevents perfect identification of the true state/condition θ of a structure, the VoI given by Eq.

(14) is also called the Expected Value of Sample Information (EVSI). The VoI calculated after a particular experiment has been performed and its outcome is known is called the Conditional Value of Sample Information (CVSI). If to assume that an experiment yields exact or perfect information that enables to determine the true state/condition of the structure, the VoI before and after the experiments are called Expected Value of Perfect Information (EVPI) and Conditional Value of Perfect Information (CVPI), respectively. They provide upper bounds for the EVSI and CVSI. The VoI analysis is valid when the utility of any (e,z,a,θ) can be presented as the sum of the sampling utility $u_s(e,z)$ (i.e., utility associated with experiments/inspection and obtained data) and the terminal utility $u_t(a,\theta)$ (i.e., utility associated with terminal action and state/condition of the structure) (Raiffa and Schlaifer, 1961)

$$u(e,z,a,\theta) = u_s(e,z) + u_t(a,\theta) \tag{15}$$

In the context of the VoI in Structural Health Monitoring (SHM), the term ‘utility’ is often replaced by the life-cycle benefit, B (e.g., Faber and Thöns 2014). In this case Eq. (14) is expressed as

$$VoI = B_1 - B_0 \tag{16}$$

where B_0 is the maximum expected life-cycle benefit without SHM estimated by a prior analysis and B_1 the maximum expected life-cycle benefit with SHM estimated by a pre-posterior analysis.

Application areas

The main applications of Bayesian decision analysis are for optimizing the collection of information which leads to a better decision. In the civil engineering field, it is the basis for the computation of the VoI for optimizing inspections and structural health monitoring in deteriorating structures, (Faber and Thöns, 2014), (Straub and Faber, 2006), (Barone and Frangopol, 2013). Similar application were made in the field of transportation infrastructure management (Samer, 1993), geotechnical engineering (Zhang et al., 2009) and in the field of natural hazards (Bensi et al., 2011), (Garrè and Friis-Hansen, 2013). It is and has been applied in many other fields of engineering and science as well, including oil exploration (Demirmen, 1996), and environmental health risk management (Yokota and Thompson, 2004).

Critical appraisal

The posterior and pre-posterior analysis take into account the uncertainties during the decision making process in the framework of Bayesian statistical decision theory, thus enable a well-considered and structured way for making optimal decisions under uncertainty. However, it requires significant computational efforts and statistical modelling which sometimes can be cumbersome in case one wants to apply this methodology to practical engineering applications.

Leading research communities / leading application sectors

DTU Civil Engineering, Technical University of Denmark, Copenhagen, Denmark
Engineering Risk Analysis Group, Technische Universität München
Engineering Research Center for Advanced Technology for Large Structural Systems, Lehigh Univ.

Contact information

Robby Caspeele, Cheng Xing.
Ghent University, Department of Structural Engineering,
Technologiepark-Zwijnaarde 904, 9052 Gent, Belgium.

Dimitri Val,
Heriot-Watt University, Institute for Infrastructure & Environment,
EH14 4AS, Edinburgh, United Kingdom

References

Barone, G. and Frangopol, D.M. (2013). Hazard-based optimum lifetime inspection and repair planning for deteriorating structures. Journal of Structural Engineering, 139(12): 04013017.

Bensi, M.T, Der Kiureghian, A. and Straub D. (2011). A Bayesian network methodology for infrastructure seismic risk assessment and decision support. Report PEER 2011/2, Pacific Earthquake Engineering Research Center.

Demirmen F. (1996). Use of "Value of Information" Concept in Justification and Ranking of Subsurface Appraisal. SPE Annual Technical Conference and Exhibition.

Faber, M.H. (1997). Risk based structural maintenance planning. In: Probabilistic Methods for Structural Design, Ed. C. Guedes Soares, Kluwer Academic Press, Netherlands, 377-402.

Faber, M.H. and Thöns, S. (2014). On the value of structural health monitoring. Safety, Reliability and Risk Analysis: Beyond the Horizon: proceedings of ESREL 2013, Amsterdam, 2535-2544.

Garrè, L. and Friis-Hansen P. (2013). Using Bayesian Networks and Value of Information for risk-based adaptation to climate change: an application of the DNV-ADAPT framework. 11th International Conference on Structural Safety & Reliability, ICOSSAR 2013. Columbia University, New York.

Madanat, S. (1993). Optimal infrastructure management decisions under uncertainty. Transportation Research Part C: Emerging Technologies. 1:77-88.

Raiffa, H. and Schlaifer, R. (1961). Applied statistical decision theory, Boston (Mass.), Harvard university press.

Straub, D. (2004). Generic Approaches to Risk Based Inspection Planning for Steel Structures. ETH Zürich, Institute of Structural Engineering.

Straub, D. and Faber, M.H. (2006). Computational aspects of risk-based inspection planning. Computer-Aided Civil and Infrastructure Engineering, 21:179-192.

Zhang, J., Zhang, L. and Tang, W.H. (2009). Bayesian framework for characterizing geotechnical model uncertainty. J Geotech Geoenviron.135:932-40.

Yokota, F. and Thompson K.M. (2004). Value of information analysis in environmental health risk management decisions: Past, present, and future. Risk Anal. 24:635-50.