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EXPLORING TARGETED ENERGY TRANSFER FOR VIBRATION DAMPING IN NONLINEAR PIEZOELECTRIC SHUNTS

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Abstract. Recently, shunted piezoelectric (PE) patches have been heavily investigated as passive dynamic vibration absorbers (DVAs) to reduce vibrations in mechanical structures. These piezoelectric patches serve as an alternative to mechanical DVAs, which require physical space both for the DVA itself and its stroke, while a piezoelectric DVA only requires space for an electronic circuit. Typically, the patch is shunted with a resistor and an inductor. As attaching the patch and the shunt on a mechanical structure is equivalent to mechanically attaching a spring, mass and damper, mechanical DVA tuning rules are applied to dimensionalize the circuit. This equivalence between mechanical and piezoelectric DVAs is utilized in the paper to apply the advances of the more mature mechanical DVA research field. The effect of nonlinear stiffness in mechanical DVAs has been thoroughly investigated. A typical nonlinear phenomenon is called targeted energy transfer (TET), where the vibration energy is suddenly transferred from the structure to the DVA. In this paper, a nonlinear shunt is proposed, equivalent to a nonlinear mechanical DVA. Then, by applying the nonlinear mechanical DVAs tuning rules, the TET phenomenon is also found in PE patch with a nonlinear shunt.

1 INTRODUCTION

Excessive vibration in mechanical structures can lead to material fatigue or even structural failure. These vibrations can also have an effect on humans, they can experience discomfort, noise pollutions or develop health problems. In *passive vibration absorption*, excessive vibrations in a *main system* are mitigated by adding several passive absorbing elements. As opposed to *active absorption*, passive absorption requires no power source and no sensor, making it an elegant and robust solution to vibration reduction in engineering structures. The elastic structure of fig.1a represents the main structure to be protected such as a building, a metal structure, a turbine vane, an airplane wing, car bodywork, ... A passive element is a correctly dimensioned structure, locally added to the main system, that absorbs and/or counter-acts vibrations of the main system. It is often called a *dynamic vibration absorber* (DVA). The most commonly used DVA is the mechanical type, where a correctly tuned mass-spring-damper system is added to the main structure, see fig.1b. More recently, the DVA of the piezoelectric (PE) type has been given a lot of attention in research. It was first proposed by Hagood and Von Flotow in 1991 [1], where a piezoelectric patch, attached on a beam, was shunted with a resistance and an inductance, as shown in fig.1c. The deformation of this patch induces a voltage in the shunt, caused by the direct piezoelectric effect, causing a charge flow in the shunt. The electric energy is then dissipated in the shunt. While the mechanical DVA both needs space for the mass-spring-damper and its stroke, the PE DVA only needs space for its shunt and does not vibrate itself. In this sense the PE DVA is a *solid state* vibration absorber. Many different type of shunt circuits have been proposed, an overview can be found in [2]. This paper focusses on the piezoelectric DVA, more specifically, on nonlinear shunts, as an analogy for the nonlinear mechanical DVA's.

The main structure is assumed to be linear, and can be decomposed into *linear normal modes* (LNM). Information of the LNM is then used to tune a linear vibration absorber. One of the first, and still very popular tuning method was proposed by Den Hartog [3], where a single vibration mode is damped with an analytic frequency response function optimization. These linear, single mode absorbing DVAs are still used intensively today in both research and industry. Adding a PE DVA with a resistor-inductor (RL) circuit is equivalent to adding a spring-mass-damper, with the inductor being the mass, the resistor being the damper and the an material capacitance being the spring. This was first shown in [4], where the Den Hartog tuning was applied to obtain RL values.

However, these linear DVAs have a few downsides. When properties of the main structure or DVA change (eigenfrequencies, modeshape, ..), for example by age, heavy or sustained loads or by structural modifications, the performance of the DVA might deteriorate seriously. This deterioration is called *detuning* of the absorber. Another downside is the inability to absorb more than one mode with a single DVA. To tackle these downsides, nonlinear vibration absorbers have been proposed, having a nonlinearizable stiffness. In [5] it was shown that DVA with cubic or non-smooth stiffness is more robust against detuning and that a single DVA can absorb multiple eigenmodes.

When the DVA is nonlinear, the main system is still assumed to be linear, while the compound system (main structure + DVA) is now nonlinear. The dynamics of the nonlinear compound system are analysed with so called *nonlinear normal modes* (NNM), a nonlinear extension of LNM. A NNM can be defined as a (nonnecessarily synchronous) periodic motion of the conservative system [7]. The most important differences between LNM and NNM for this paper, is that NNM can differ both in frequency and modeshape, depending on the energy in the structure, and that NNMs can suddenly appear or disappear above a certain energy level, so-called mode bifurcations. A LNM will always have the same frequency and mode shape for every energy level of the compound system and they do not appear or disappear depending on this energy level. NNMs have been around since the 1960s as a theoretical curiosity but recently got more attention in nonlinear vibration absorption [6] and nonlinear structural dynamics [7]. Many numerical and analytic methods [5, 8] have been developed to obtain NNM. As every class of nonlinear systems requires a different analysis method, the numeric toolbox NI2D [9] is used to investigate the system and generate the NNM.

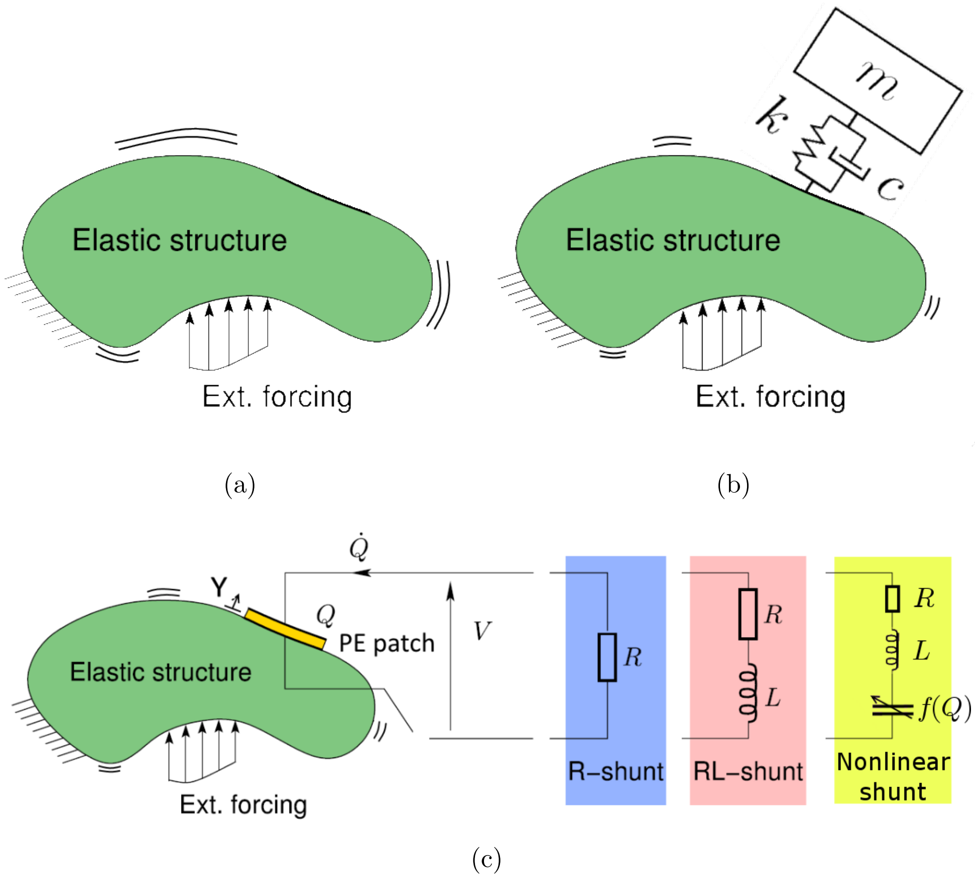

Figure 1: (a) A mechanical elastic structure can experience excessive vibration under forcing or initial conditions. (b) A mechanical DVA, consisting of a mass-spring-damper system can be tuned to mitigate the vibration. (c) A piezoelectric DVA, shunted by an R, RL or the proposed nonlinear circuit. The reduction of vibrations in (b) and (c) is shown here as smaller 'movement' lines. Figures are adapted from [12]

The mechanism of vibration absorption for the considered nonlinear DVAs is called *targeted energy transfer* (TET). It is defined as a sudden, irreversible transfer of vibration energy from the main structure to the DVA [5]. An important property of TET is that this mechanism only occurs when the initial or forcing energy in the main structure is above a certain threshold. This threshold was analytically expressed in [10] where it was shown that a NNM mode bifurcation is responsible for this threshold.

In electric circuits, nonlinearities are often undesirable and avoided, but deliberate nonlinearities have been introduced in for instance variable capacitors (called varicaps) in AM/FM radio tuners, while more recently, in the context of piezoelectric shunts, so-called *synthetic impedances* have been developed, allowing to generate any desired electrical impedance [11]. For now these synthetic impedances have been used to synthesize linear impedances, but can be extended to

nonlinear ones, as the voltage/current relation is programmed in a DSP.
The main contribution of this paper is that TET is possible in a shunted piezoelectric patch if special nonlinear electrical components are added. As an RL shunt is equivalent to adding a linear mechanical DVA, this equivalence is to allow for TET. In section 2, the dynamic model of the effect of the PE patch and shunt (with a nonlinear component) on the main system is constructed. By adding a nonlinear capacitor in series with the inductance and resistor the mechanical-electrical equivalence is extended so that this capacitor is a nonlinear spring connecting the inductance mass with the ground. This nonlinear capacitor is designed to ensure TET in the shunt. In the past, this type of nonlinear, grounded mechanical vibration absorber has been given some attention [6] but most research on TET was investigated ungrounded nonlinear absorbers[5, 10, 13]. In the 3rd section, the NNMs of the compound system are derived, and it is shown that when the shunt contains a 'cubic capacitor', TET indeed does occur if energy in the main system is high enough. In the 4th section, simulations are performed and show the typical properties of TET; as the energy threshold and increased robustness against detuning.

2 SYSTEM ANALYSIS

A continuous mechanical main system is considered with a displacement $\mathbf{Y}(\mathbf{x},t)$, at point $\mathbf{x}$ of the structure at time t. The piezoelectric patch has a charge Q flowing through the shunt and has a voltage V over its electrodes. The main system is assumed to be linear, and either by a mode reduction of the analytic expression [14] , or with a FEM formulation [12], can be decomposed in a discrete number of modes:

$$\mathbf{Y}(\mathbf{x},t) = \sum_{i=1}^{N} \mathbf{\Phi}_i(\mathbf{x}) q_i(t) \tag{1}$$

with $\mathbf{\Phi}_i(\mathbf{x})$ the i-th mode shape and $q_i(t)$ the i-th modal coordinate. The main structure, subjected to both the external forces and piezoelectric force, fig.1c, can be formulated as:

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2 q_i - \underbrace{\chi_i V}_{\text{PE force}} = \underbrace{F_{i,ext}}_{\text{External force}} \quad , \quad i \in [1, N] \tag{2}$$

with ω_i the eigenfrequency of the mode, ζ_i the modal damping and χ_i the modal electromechanical coupling factor (MEMCF)[14].

$$C_p V - Q + \sum_{i=1}^{N} \chi_i q_i = 0 \tag{3}$$

with C_p the equivalent electrical capacitance of the piezoelectric patch. To simplify the analysis, no external forcing is assumed and motion of the mechanical structure is assumed to be dominated by a single mode j, simplifying (2) and (3) to:

$$\begin{aligned} \ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \omega_j^2 q_j - \chi_j V = 0 \\ C_p V - Q + \chi_j q_j = 0 \end{aligned} \tag{4}$$

The proposed shunt contains a resistor R, inductor L and in series, a general electrical component that produced a voltage $V_Z = f(Q)$:

$$V = -L\ddot{Q} - R\dot{Q} - f(Q) \tag{5}$$

This allows (4) and (5) to be rewritten as:

$$\begin{aligned}
\ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \omega_j^2 q_j + \frac{\chi_j^2}{C_p}(q_j - \frac{Q}{\chi_j}) = 0 \\
\chi_j^2 L\frac{\ddot{Q}}{\chi_j} + \chi_j^2 R\frac{\dot{Q}}{\chi_j} + \chi_j f(Q) + \frac{\chi_j^2}{C_p}(\frac{Q}{\chi_j} - q_j) = 0
\end{aligned} \tag{6}$$

or by introducing the variable $\mathcal{Q} = \frac{Q}{\chi_j}$, a charge with unit m:

$$\begin{aligned}
\ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \omega_j^2 q_j + \frac{\chi_j^2}{C_p}(q_j - \mathcal{Q}) = 0 \\
\chi_j^2 L\ddot{\mathcal{Q}} + \chi_j^2 R\dot{\mathcal{Q}} + \chi_j f(\mathcal{Q}) + \frac{\chi_j^2}{C_p}(\mathcal{Q} - q_j) = 0
\end{aligned} \tag{7}$$

which can be interpreted as a two degree of freedom mechanical system, see fig.2. It is as if the modal system is connected with the 'inductance' mass $\chi_j^2 L$ through the linear spring $\frac{\chi_j^2}{C_p}$. The 'inductance' mass itself is also grounded, like the modal system, through a damper, $\chi_j^2 R$. The general impedance $f(\mathcal{Q})$ can be seen as some kind of stiffness connecting the inductance mass to the ground. For instance, if $f(\mathcal{Q}) = \frac{\mathcal{Q}}{C_{lin}}$, actually a linear capacitor, then it is as if a linear spring connects the inductance mass to the ground.

The impedance $f(\mathcal{Q})$ will be designed in order to ensure targeted energy transfer. If $V_z = f(\mathcal{Q}) = \frac{\mathcal{Q}^3}{C_{cub}}$, TET occurs when the system energy is above a threshold coinciding with a NNM mode bifurcation.

3 NONLINEAR NORMAL MODES

Nonlinear normal modes are possibly synchronous periodic solutions of the conservative system. System (7) still has damping, so by setting $R = 0$ and $\zeta_j = 0$ the underlying conservative system reads:

$$\begin{aligned}
\ddot{q}_j + \omega_j^2 q_j + \frac{\chi_j^2}{C_p}(q_j - \mathcal{Q}) = 0 \\
\chi_j^2 L\ddot{\mathcal{Q}} + \frac{\chi_j^4\mathcal{Q}^3}{C_{cub}} + \frac{\chi_j^2}{C_p}(\mathcal{Q} - q_j) = 0
\end{aligned} \tag{8}$$

To construct these nonlinear normal modes, it is assumed that steady state motion of (8) is a synchronous, single frequency vibration:

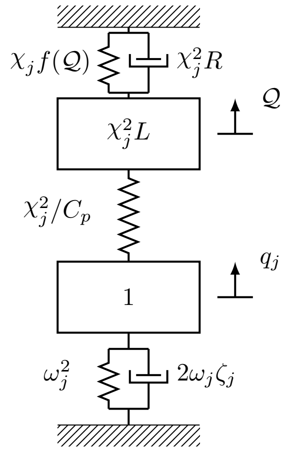

Figure 2: The mechanical equivalent of the structure/patch system

$$\begin{aligned} q_j &= Acos(\omega t) \\ \mathcal{Q} &= Bcos(\omega t) \end{aligned} \tag{9}$$

of which both, A, B and ω are unknown. When replacing (9) in a linear system with n-degree of freedom, n solutions for ω will be found, the eigenfrequencies, and for each ω, eigenvectors or mode shape, which fixes the ratio between A and B. Both the frequencies and eigenvectors will not change depending on the magnitude A or B. In nonlinear system, the 'eigenfrequencies' and the mode shapes depend on the magnitude of A and B.

Replacing (9) in (8) yields the following solutions for A and B:

$$B = \pm \sqrt{\frac{4C_{cub}}{3\chi_j^4} \frac{\chi_j^2 L(\omega_j^2 - \omega^2 + \frac{\chi_j^2}{C_p}) - \frac{\chi_j^2}{C_p}(\omega_j^2 - \omega^2)}{\omega_j^2 - \omega^2 + \frac{\chi_j^2}{C_p}}} \tag{10}$$

$$A = \frac{\frac{\chi_j^2}{C_p} B}{\omega_j^2 - \omega^2 + \frac{\chi_j^2}{C_p}} \tag{11}$$

The potential energy in the conservative system is:

$$E = \frac{1}{2}\omega_j^2 A^2 + \frac{\chi_j^2}{2C_p}(A - B)^2 + \frac{\chi_j^4}{4C_{cub}} B^4 \tag{12}$$

which is the total energy as when the assumed motion (9) is maximal, the speed is zero. In NNMs, it is custom to graphically plot the so-called *frequency-energy plot* (FEP), which reveals the relation (12) and the frequency. Using the numerical values for the nonlinear shunt on tab.1, the FEP is plotted on fig.3 using the toolbox NI2D.

The FEP clearly shows a change of eigenfrequency as the energy increases. In linear systems, the FEP would just consist of horizontal lines. There are 2 main curves, called branches. The first, the $S11+$ branch, is the in-phase nonlinear mode of the system while the second, $S11-$, is the out of phase motion. Both of these branches start horizontally at the linearised frequencies, that is the eigenfrequencies of (8) if it is linearised. As the energy increases, the upper branch $S11-$ bifurcates for $log(E) = -0.817$ into 3 solutions. The branch $S11-$ bifurcates again for $log(E) = -0.174$. It is this last bifurcation and the associated energy that are linked with TET [5, 10, 15]. If the initial energy is just above this $log(E) = -0.174$, the energy in the main system will be transferred to the absorber, where it will be dissipated if there is damping present. Just below this energy level, the energy stays in the main system, and is only slowly mitigated. It is assumed that initially, only the main system has an initial speed. This is called the *impulsive excitation*. The initial (kinetic) energy in the system is then:

$$E = \frac{1}{2}\dot{q}_j^2 = \frac{1}{2}\omega_j^2 A^2 > 10^{-0.174} \Rightarrow \dot{q}_j > 1.15m/s \tag{13}$$

which yields the required initial speed to trigger TET. In the next section, simulations are performed to show this sudden change in behaviour as the initial speed changes and a nonlinear shunt is compared to a linear one.

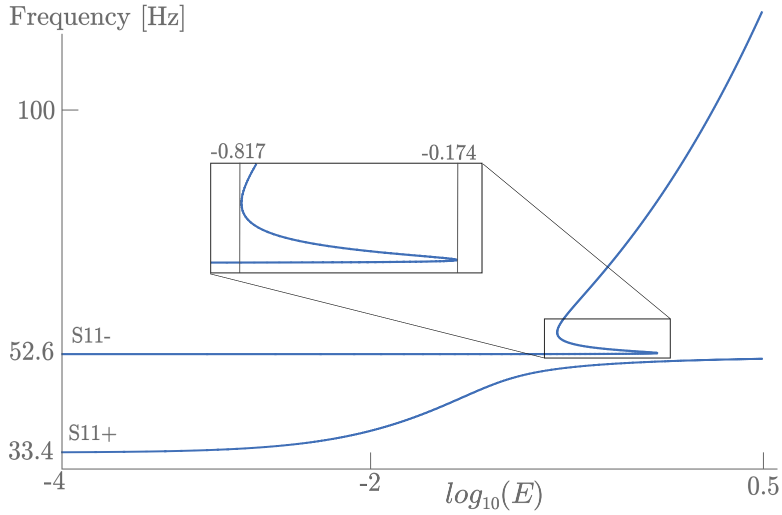

Figure 3: The nonlinear normal modes of system (8), visualised with the frequency-energy plot

4 Simulations

In [14] a cantilever beam was fitted with a piezoelectric patch. The mechanical and piezoelectric parameters from this setup are used to simulate both a nonlinear and linear shunt. The tuning of the linear shunt was performed in [14] see tab.1 for numerical values, with only a

Table 1: Numerical values of mechanical system and shunt. The values for the linear shunt are taken from a mechanical setup from [14]

Quantity	Linear	Nonlinear
ω_j $[\frac{rad}{s}]$	324.5	324.5
χ_j $[\frac{C}{m}]$	0.0045	0.0045
C_p $[F]$	$9.16 \cdot 10^{-9}$	$9.16 \cdot 10^{-9}$
R $[\Omega]$	$58.62 \cdot 10^3$	$58.62 \cdot 10^3$
L $[H]$	1060	2400
C_{cub} $[\frac{C^3}{V}]$	—	$9.16 \cdot 10^{-18}$

resistor and inductor in the shunt circuit. For the nonlinear shunt, the values used to draw the NMM are used.

First, the designed nonlinear shunt is simulated for several initial $\dot{q}_j(0)$, see fig.4. In the previous section, is was determined that $\dot{q}_j(0)$ should be above 1.15 m/s for TET to happen. When the initial speed is below the critical value (here $\dot{q}_j(0) = 0.8$ m/s), the main system vibrations, fig.4a, mitigate very slowly, and the absorber charge, fig.4b, is low compared to the other initial speeds. If the main system is excited with $\dot{q}_j(0) = 1.15$ m/s, the vibrations are reduced very fast, and the absorber charge is persistently larger than before. If the initial speed is then further increased (here $\dot{q}_j(0) = 1.5$ m/s, the vibration reduction is slower, yet still significantly faster than when the initial speed was below 1.15 m/s.

The fact that this vibration absorber don't work below, optimally on, and suboptimally above the energy threshold was already discussed in [5].

Next, the main system's vibration reduction is compared when using the nonlinear shunt and a tuned linear shunt, tuned in [14], on fig.5. First, the main system is excited with the optimal initial speed for the nonlinear shunt, fig.5a. Here, the linear absorber is better than the nonlinear one in the beginning. After 0.29 s however, the linear and nonlinear shunts have reduced the main system vibration to somewhat the same level. A correctly tuned linear absorber is always faster than a correctly tuned absorber of the kind on fig.2 [5, 16]. One of the advantages of the nonlinear shunt is that it is more robust. If the main system is detuned, by reducing eigenfrequency ω_j three times by 0.9, so $\omega_j = 290$, 260 and 232 rad/s, it is observed that linear shunt's performance deteriorates a lot faster than the nonlinear shunt, fig.5a, fig.5b and fig.5c.

5 Conclusion

In this paper, it is shown that the targeted energy transfer (TET) phenomenon, normally associated with mechanical dynamic vibration absorbers (DVA), also occurs in piezoelectric DVAs for a carefully designed shunt. By assuming a single vibration mode in the main system, the electrical charge in the shunt is equivalent to a mechanical displacement. In this mechanical-electrical equivalence, an RL shunt is then seen as a mass spring damper system, attached on the main structure. If a nonlinear electrical element is added in the shunt, the shunt is equivalent to a nonlinear mechanical DVA so the tuning rules to achieve TET in mechanical DVA can be applied to the nonlinear shunt. Some simulations were performed to show the initial energy

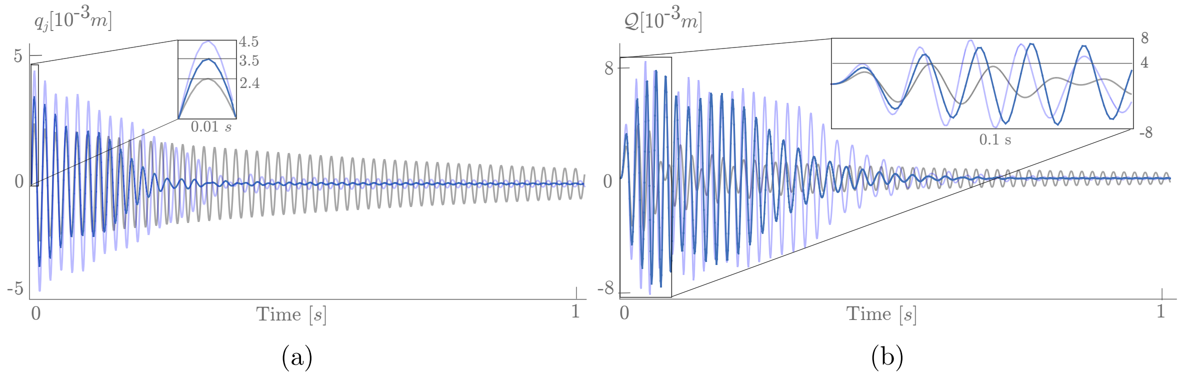

Figure 4: The main systems displacement q_j (a) and charge in the nonlinear shunt$\mathcal{Q}$ (b) for initial main system's speed $\dot{q}_j = 0.8\ m/s$ (grey), $\dot{q}_j = 1.15\ m/s$ (dark blue) and $\dot{q}_j = 1.5\ m/s$ (light blue)

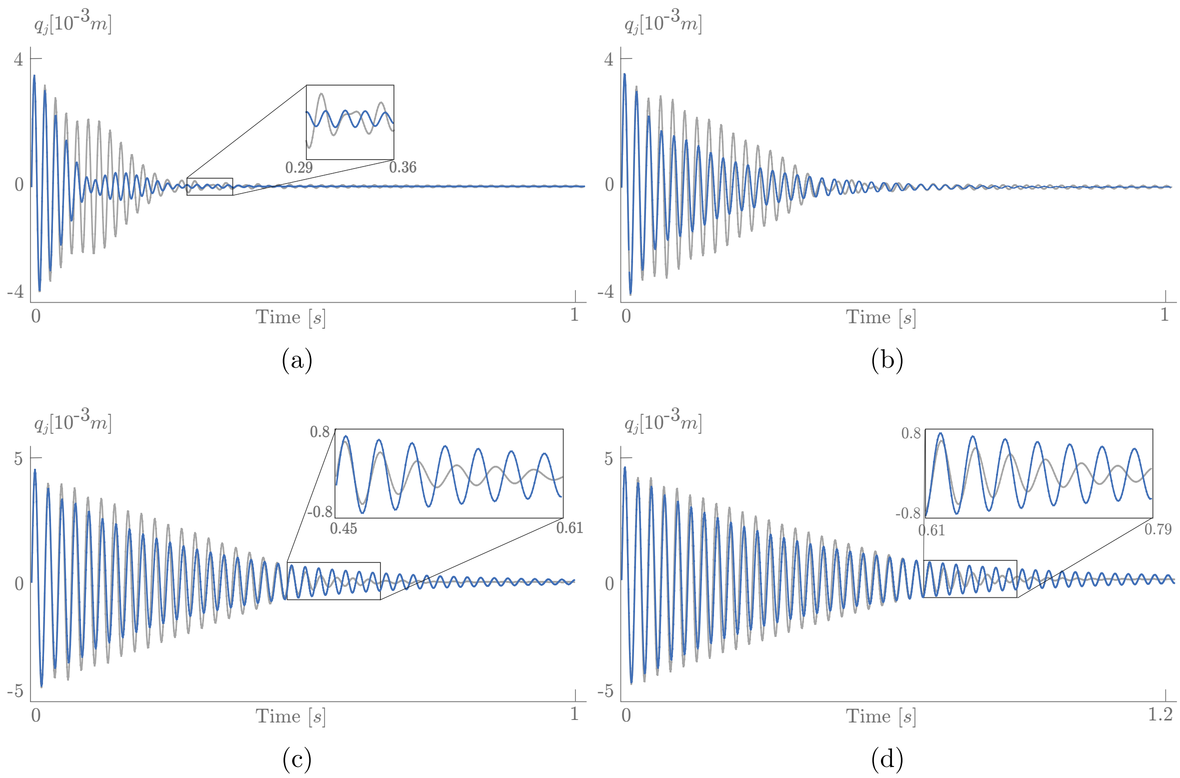

Figure 5: The main systems displacement q_j for initial speed $\dot{q}_j = 1.15$ both when using a linear shunt (blue) and nonlinear shunt (grey) (a), for a detuned eigenfrequency $\omega_j = 290\ rad/s$ (b), $\omega_j = 260\ rad/s$ (c) and $\omega_j = 232\ rad/s$ (d)

dependence on the performance of TET. The nonlinear shunt was also compared to a linearly tuned shunt. While a perfectly tuned linear shunt mitigate vibrations faster than the nonlinear shunt, the nonlinear shunt is a lot more robust if there is detuning.

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