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Estimating Seaway from Ship Motions

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Abstract

We present an autonomous method to estimate the sea state on board of a vessel underway from recorded motion spectra. In principle motion spectra are calculated easily from the seaway spectrum using the response amplitude operators (RAOs). Solutions for the inverse problem, as sought here, are more difficult to obtain and rather sophisticated stochastic algorithms are found in the literature. In this paper a more direct approach is suggested. The algorithm was tested in several experiments performed on small vessels (L=20m). The RAOs were calculated with a commercial strip theory code. While good results were obtained in many cases, deeper investigation was found necessary. The RAO-parameters, such as loading conditions, inertia and the speed dependent damping, need consideration. Furthermore, since the moving ship reacts to the encounter frequency, the question of what part of the seaway spectrum can actually be sensed has been studied.

Keywords

Wave spectrum estimate; ship motions; encounter spectrum

Introduction

Information about sea state is best represented by a power spectrum depending on frequency and direction. It is obviously interesting for meteorological and oceanographic records or even for investigations about climate changes. In nautical practice the watch officer is required to enter the sea state regularly into the ship's log. This is usually done by visual estimation of characteristic height, period and direction. Ship oscillations, in particular roll, are, of course, generated by waves and the avoidance of dangerously high amplitudes is of the utmost importance. Since the reaction of a ship to a particular wave system depends on course and speed, there are decision support systems which rely on the officer's estimate as input. However, the recommendations given by such systems are highly doubtful, as long as the sea state spectrum is reduced to a single point and that depending on the officer's experience.

In many areas wave rider buoys are routinely operated and provide near real-time measurements. The motion of the buoy is sensed by a GPS-receiver and/or a set of accelerometers. These buoys give a full directional spectrum with high reliability. However, with few exceptions, the measurement sites are restricted to coastal areas. Additional independent data are collected via remote sensing satellites. While Synthetic Aperture Radar (SAR) yields peak values of wave height, length and, with some restrictions, direction (Ganguly et al. 2015), only characteristic wave height is obtained from altimetry (Caballero et al. 2014). The orbits of these satellites follow a discrete periodic pattern. As a consequence, data are only available along relatively widely spaced ground tracks with a sampling period of days or weeks. It should be mentioned that ground or ship based radar can also be used to estimate the sea state but, as yet, the results are sometimes ambiguous and not always reliable.

Sea state and ship motions

Quantitative Relations

Some attempts have been undertaken to use the ship itself as a wave buoy or, in other words, to estimate the sea state from measured ship motions (e.g. Iseki 2000, Nielsen 2006). One crucial assumption for all methods is that there is a linear relationship between ship motions and exciting waves. In this case the ship's response to a single harmonic wave is given by

$$A_n(\omega) = h_n(\omega,\alpha)A_{sea}(\omega,\alpha) \qquad (1)$$

Here $A_n(\omega)$ is the complex amplitude of ship motion in the n-th degree of freedom, n=1...3 denotes the linear motions in x-, y- and z-directions, n=4…6 the rotations about the corresponding axes. $A_{sea}(\omega,\alpha)$ denotes the wave amplitude at a given circular frequency ω and a certain direction α. $A_n(\omega)$ and $A_{sea}(\omega,\alpha)$ are related to each other by RAO $h_n(\omega,\alpha)$. The x-axis is oriented along the longitudinal ship axis, the y-axis points to portside and the z-axis upright. Due to the linearity assumption Eq. 1 is easily extended to a linear combination

of waves. Denoting the complex spectrum of seaway by $A_{sea}(\omega,\alpha)$ leads to

$$A_n(\omega) = \int h_n(\omega,\alpha) A_{sea}(\omega,\alpha)\, d\alpha \qquad (2)$$

As stated above, irregular seaway is appropriately described as a random process using statistical properties. Of particular interest is the directional power spectrum of seaway

$$S_{sea}(\omega,\alpha) = < A_{sea}(\omega,\alpha) A^*_{sea}(\omega,\alpha) > \qquad (3)$$

Here <> denotes averaging and $A^*_{sea}(\omega,\alpha)$ the complex conjugate of $A_{sea}(\omega,\alpha)$.The directional power spectrum is related to power and cross spectra of ship motions by means of Eq. 2, which is now illustrated for the roll- and pitch motions. The power spectrum of roll

$$S_{rr}(\omega) = < A_r(\omega) A^*_r(\omega) > \qquad (4)$$

is given by

$$S_{rr}(\omega) = h_r(\omega,\alpha) h^*_r(\omega,\alpha') \int d\alpha \int d\alpha' < A_{sea}(\omega,\alpha)\, A^*_{sea}(\omega,\alpha') > \qquad (5)$$

Assuming that waves from different directions α are statistically independent reduces this equation to

$$S_{rr}(\omega) = \int d\alpha \;\; h_r(\omega,\alpha) h^*_r(\omega,\alpha) S_{sea}(\omega,\alpha) \qquad (6)$$

In a similar way equations for the power spectrum of pitch

$$S_{pp}(\omega) = \int d\alpha \;\; h_p(\omega,\alpha) h^*_p(\omega,\alpha) S_{sea}(\omega,\alpha) \qquad (7)$$

and the cross spectrum of roll and pitch

$$S_{rp}(\omega) = \int d\alpha \;\; h_r(\omega,\alpha) h^*_p(\omega,\alpha) \;\; S_{sea}(\omega,\alpha) \qquad (8)$$

are derived. Eqs. 6-8 and the corresponding equations for the other degrees of freedom form the mathematical basis for estimating the directional spectrum of seaway $S_{sea}(\omega,\alpha)$ from measured motion spectra.

New Approach for Estimating Sea State from Ship Motions

We now want to outline a new approach for estimating the directional spectrum of seaway from measured ship motions. It is assumed that time series of ship motions in several degrees of freedom are sampled and discrete power- and cross-spectra at frequency points ω_j are calculated from these. In our experiments so far pitch and roll motions are measured. It is planned to include other degrees of freedom, in particular the heave motion. While the frequency axis is automatically discretized due to the sampling, the first step of our approach is to discretize the direction into a certain number of sectors. In the examples shown below an angular resolution of 15° is used. For each sector an angular distribution function $f_i(\alpha)$ is defined, the directional spectrum is then expanded into a linear combination of the $f_i(\alpha)$,

$$S_{sea}(\omega_j,\alpha) = \sum_i \alpha_i(\omega_j) \;\; f_i(\alpha) \qquad (9)$$

the coefficients $\alpha_i(\omega_j)$ are real by definition and provide a measure of the wave power in the i-th sector. Inserting the expansion into Eq. 6 results in

$$S_{rr}(\omega_j) = \sum_i \alpha_i(\omega_j) \int d\alpha \;\; h_r(\omega_j,\alpha) h^*_r(\omega_j,\alpha) f_i(\alpha) \qquad (10)$$

Since the RAOs and the distribution functions are known, the integral can be evaluated immediately. This defines the constant

$$c^i_{rr}(\omega_j) = \int d\alpha \;\; h_r(\omega_j,\alpha) h^*_r(\omega_j,\alpha) f_i(\alpha) \qquad (11)$$

Applying this procedure to Eqs. 7 and 8 leads to a linear system of equations for the unknown $\alpha_i(\omega_j)$

$$S_{rr}(\omega_j) = \sum_i c^i_{rr}(\omega_j)\, \alpha_i(\omega_j) \qquad (12)$$

$$S_{pp}(\omega_j) = \sum_i c^i_{pp}(\omega_j)\, \alpha_i(\omega_j) \qquad (13)$$

$$S_{rp}(\omega_j) = \sum_i c^i_{rp}(\omega_j)\, \alpha_i(\omega_j) \qquad (14)$$

Eqs. 12-14 form a system of equations that relates the unknown wave powers $\alpha_i(\omega_j)$ to the measured power- and cross-spectra. It should be noted that, while Eqs.12 and 13 are real by definition, eq. 14 consists of two equations for the real- and imaginary part respectively.

The set of equations is underdetermined in the usual case that more than four test functions are required to approximate $S_{sea}(\omega,\alpha)$ properly. This can be overcome by assuming that the seaway spectrum varies slowly with frequency. Since the spectra are known at discrete frequency points ω_j, a single value of α_i applies to several frequency points ω_j and therefore to several equations from the set of Eqs. 12-14. In the experimental case explained below there are 24 unknown coefficients α_i and four equations for each ω_j. The number of unknowns matches the number of equations if one value of α_i is valid for six frequency points ω_j. To investigate what changes faster with frequency than the seaway spectrum we analyse the well-known equation of motion

$$(-\omega_e^2\,(M + A) + i\omega_e\, B + C)\;\; \hat{\Phi} = F \qquad (15)$$

Here $\hat{\Phi}$ denotes a six dimensional vector of amplitudes for the three motions in x- y- and z-direction and the three rotations about the roll, pitch and yaw axis. The encounter frequency is denoted by ω_e. The remaining coeffi-

cients in the equation are the matrix of solid mass moments M, the added mass caused by hydrodynamic effects A, the damping matrix D and the restoring forces and moments C. Wave forces and moments are combined in the vector F. As an example, the roll coefficients for waves coming from 40° off stern and zero vessel speed are shown in Fig. 1. The data are taken from the model of the Fathom 10 used in the test experiment. The plot indicates that wave forces and moments are the rapidly changing quantity. Consequently, the condition of the set of equations depends on how the ship hull is sampled by the incoming waves.

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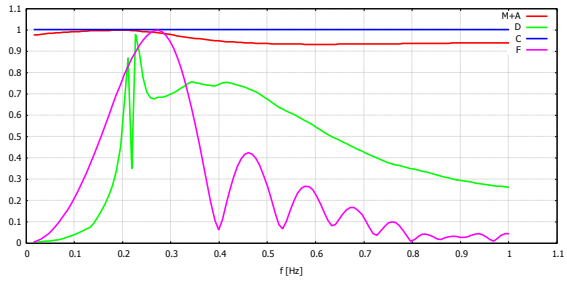

Fig. 1: Coefficients of the equation of motion as a function of the seaway frequency for the Fathom 10 as calculated by the strip theory program OCTOPUS-SEAWAY. The ship is at rest, waves are coming from 40° of stern. The coefficients are normalized to unity.

TEST EXPERIMENT

A test of the described method was performed in an area to the West of Cape Town on 22.Feb.2014. Reference data for wave spectra were obtained from a nearby buoy operated by the Council for Scientific and Industrial Research (CSIR). The experiment was done on the training vessel "Fathom 10" with a waterline length of about 17 m. The craft is a former fishery patrol boat, now owned and operated by the Maritime Department of Cape Peninsula University of Technology (CPUT).

The Response Amplitude Operators (RAOs) were calculated with two alternative software packages for comparison. One is the public domain software PDSTRIP of the Technical University Hamburg-Harburg, the other is the commercially available OCTOPUS-SEAWAY program. Both are based on strip theory (Gerritsma and Beukelman 1971). Calculating the RAOs requires a three dimensional model of the underwater hull form, which was generated from a 3D laser scan performed while the boat was in dry dock. Fig. 2 below shows the result imaged as a solid object. From the high resolution data cross sections spaced by 500 mm were extracted as input to the programs. Further data affecting the RAOs are the loading conditions like displacement, centre of gravity and radii of inertia. These were estimated from the available tables of hydrostatic particulars using the observed fore and aft drafts.

Motion data were recorded using a sensor box placed close to the ship's centre of gravity. Data are sent to a serial interface and/or recorded on a micro SD card. The box has two types of sensors: a 3d accelerometer and two gyros for the pitch- and roll axes. Within this paper, motion spectra are calculated from gyro data. It will be shown in the following text, that the wave estimates can be improved by including linear motions. Linear motions can in principle be obtained by combining gyro and acceleration data. This is left for future work. Data were sampled at a rate of 10 Hz, power and cross spectra were calculated using a 1024 points FFT with a Hanning window and 75% overlap. Exponential averaging over 64 chunks was applied.

Trial runs were performed for headings in a 30° grid. Each run took approximately 15 minutes to get a sufficient number of averages for the power and cross spectra. A low ship's speed of about 7.5 knots was chosen in order to stay close to the wave buoy. Special attention was paid to reduce rudder induced ship motion. In following seas some rudder action was required to keep a steady course. During the experiments a long ocean swell from SW was dominant. The seaway spectrum recorded by the wave buoy showed a peak at a period of about 11 s from a direction of approximately 220°.

RESULTS

Two sample results are shown in Fig. 3. The wave direction is estimated rather well, the spectral peaks are found at the correct position. There are false signals at low frequencies caused by sensor and ship noise. Noise becomes more prominent at low frequencies since the wave slope and therefore the roll and pitch amplitudes are rather small here. This could be overcome by including linear motions (heave) into the equations. Research for estimating linear motions from both the acceleration sensors and the gyros are currently undertaken at Jade-HS. On the right panel spectral components are found for waves mirrored at the centre of the ship. Due to the symmetry of the ship the magnitudes of the response functions are symmetric with respect to the longitudinal ship

Fig. 2: Solid model of the 3D laser scan data.

axis. This symmetry is partially resolved by the cross spectrum which tells if roll and pitch are in or out of phase. But for both results of Fig. 3 roll and pitch are in phase, so there is still an ambiguity between bow and aft. This ambiguity can, in principle, be resolved since the ship has no perfect fore-aft symmetry. In the experiment however, because the ship is very small compared to the wavelength, the geometric difference between bow and aft has only a small effect on the RAOs. To remove this ambiguity completely again leads us to the need of including an additional observable like the heave, which would give another cross spectrum and therefore the second phase information.

Another unknown influence may be caused by the RAO calculations, which are based on several assumptions. For example the uncertainty in the modelling of damping should be noted. Educated guesses had to be used for certain parameters, in particular the radii of inertia had to be estimated.

DISCUSSION

The rather promising results of the previous sections leave the question, to what extent the spectrum of seaway can be measured from a moving vessel. The low frequency limit was discussed in the previous section and can in principle be overcome by including linear motions. In this section the implications of a high frequency limit for estimating seaway are studied. There are two major contributions limiting the motion measurements namely the sensor noise and vibrations of the ship hull generated for instance by engines or propeller induced pressure waves. While the sensor noise is specified by the manufacturer and can be reduced by choosing better sensors, estimates for the hull noise are more difficult to obtain in particular for different ship sizes. As a first example the Fathom 10 is considered. In Fig. 4 the power spectra of the roll and pitch rates are shown for beam waves. Assuming that the marked jump in the roll spectrum is caused by the high frequency decay of the wave spectrum, it seems that the noise floor and therefore the upper frequency limit is reached at approximately f_{max} = 0.6 Hz. For a moving ship this limit is to be read as the encounter frequency f_e which is linked to the seaway frequency f by

$$f_e = f\left(1 - \frac{f}{q}\ v\ \cos\alpha\right) \tag{16}$$

Here v is the ship speed, α the angle of incidence with $\alpha = 0$ meaning following waves and $q = \frac{g}{2\pi}$ with the gravitational acceleration g. For head waves ($\cos\alpha < 0$) the encounter frequency is a monotonic function of f as shown in Fig. 5, thus a unique upper limit for the measurable seaway frequency can be determined for each speed and angle from the maximum encounter frequency f_{max}.

This is generally not the case for following seas. Instead, up to three seaway frequencies f may be mapped to the same encounter frequency f_e as shown in Fig 6. Thus extra knowledge is required to distinguish waves is range I from those in range II on the moving vessel. In the same manner waves in range III have to be distinguished from true head seas. The individual frequency ranges are plotted together in Fig 7 as function of wave angle. The measurable range is clipped to 0.5 Hz in this plot. Head seas are measurable up to approximately 0.33 Hz. Regions I and II of the following block are both in the interesting frequency range, extra assumptions are required to resolve the ambiguity. From knowledge of the seaway spectrum (buoy data) it was clear that region I (Fig. 6) had to be selected for following waves in Fig. 3.

As another example a large container vessel is considered. There is no experimental data available yet, therefore the upper frequency limit f_{max} has to be estimated from basic considerations. It assumed here that the dynamic range for the measurements is approximately the same for all kinds of vessels. The maximum sensitivity of the roll measurements is found at the natural roll frequency. According to the equation of motion, the roll-RAO decreases roughly by 20 dB/decade above the resonance. Thus for a constant dynamic range the ratio of f_{max} and the natural roll frequency should be constant

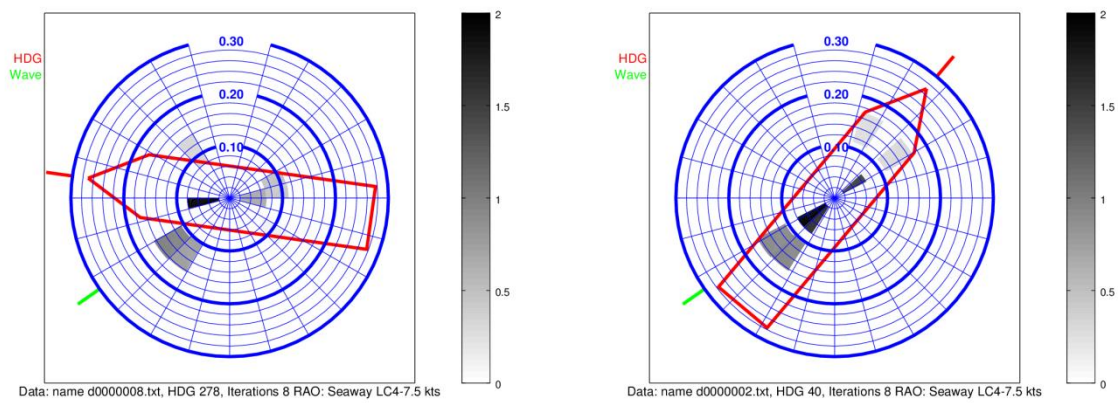

Fig. 3: Estimates of seaway based on gyro data. The green line indicates the wave direction from the wave buoy data. The radial axis contains the frequency in Hertz.

for all types of vessels. From the reasoning for the Fathom 10 a factor of three is assumed in this paper. For the container ship a speed of 22.5 knots and a roll period of 18 seconds are chosen. The frequency plot is shown in Fig. 8.

It is obvious that the range of measurable frequencies is narrowed down, particularly for head and following seas. Beam waves, on the other hand, can be detected to rather high frequencies so that the critical part for nautical decisions seems to be covered. For contributions to oceanographic data bases it remains to be seen how the combination of results from several vessels can yield a complete picture.

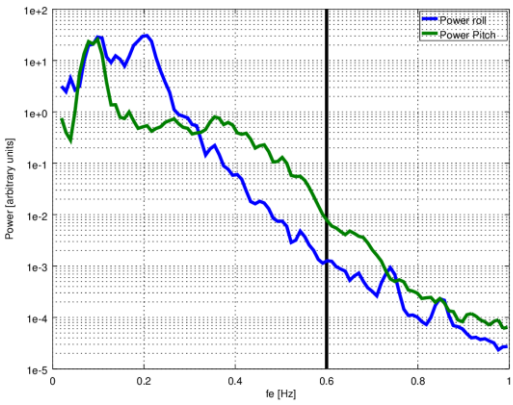

Fig. 4: Power spectra of the angular rates of roll and pitch for beam waves. The peak of the seaway spectrum is visible on the left. The roll resonance shows up very clearly. It is assumed that the marked jump in the spectrum marks the gap between seaway and system noise. This indicates that measurement can be done up to three times the resonance frequency.

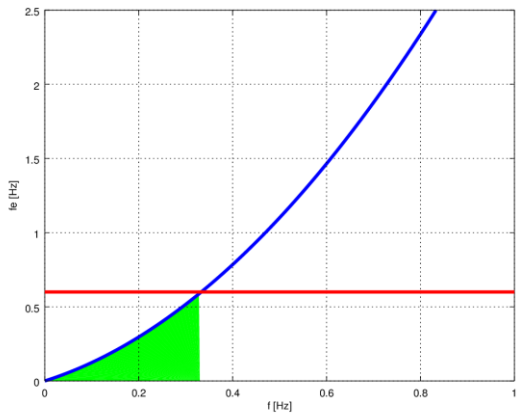

Fig. 5: Encounter frequency f_e as a function of the seaway frequency f for 7.5 knots and head seas using the deep water dispersion relation. For head seas there is a unique relationship between f and f_e, the encounter frequency is higher that the seaway frequency. The assumed limit for the Fathom 10 of $f_e < 0.6$ Hz leads to a measurable seaway frequency of approximately 0.33 Hz.

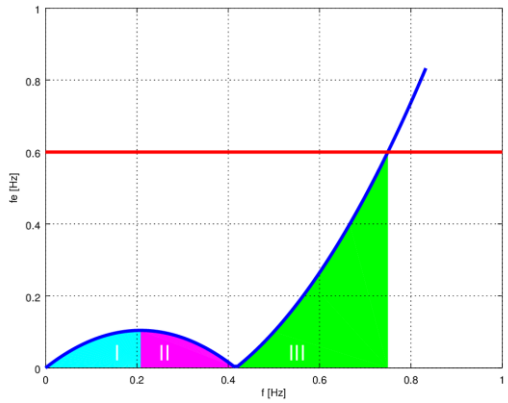

Fig 6: Encounter frequency f_e as a function of the seaway frequency f for 7.5 knots and following seas using the deep water dispersion relation. There are three ranges of seaway frequencies that are mapped onto the same encounter frequency. Waves in range I and II are faster than the ship and appear as following waves. Waves in range III are overtaken by the vessel and appear as head waves.

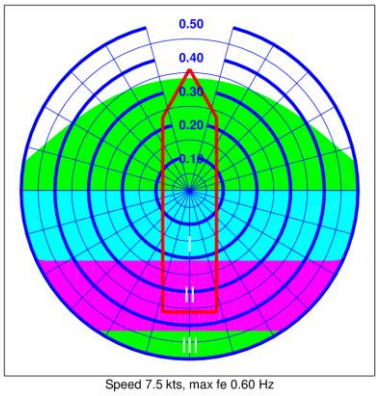

Fig 7: Measurable seaway frequencies for the fathom 10 (7.5 knots). The colors of the frequency ranges correspond to those of figures 5 and 6.

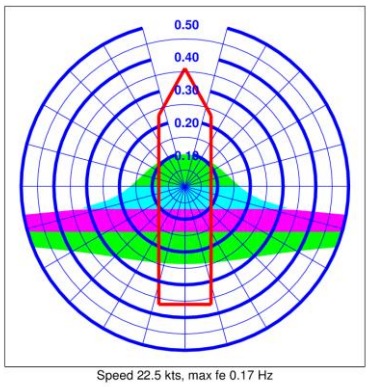

Fig 8: Measurable seaway frequencies for the fictitious container ship.

Conclusions and Outlook

A new method was presented to estimate seaway from ship motions. In a test experiment a low-cost measuring device was used, angular rates of roll and pitch were evaluated. It was shown that a considerable improvement can be expected by including the heave as a linear motion for two reasons: firstly, three independent equations, namely for heave power and real and imaginary part of one cross spectrum with heave, are added to the system. Secondly, the conditioning of the system would be improved by resolving the bow-aft-ambiguity based on the extra phase information. In principle heave can be measured using the accelerometers which are included already in the sensor box. However, an evaluation of these data needs to consider that the sensor can generally not be placed in the centre of gravity and is thus subject to accelerations caused by rotations. A sensor fusion algorithm for the gyro and accelerometer data to give proper heave measurements is currently under development. It was further shown that the range of measurable seaway frequencies is principally restricted. In following seas several seaway frequencies may lead to the same encounter frequency and extra information is required to resolve the ambiguity.

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