Stable and flexible ring-shaped optical lattice generation by use of axially-symmetric polarization elements

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To overcome a trade-off issue between stability and flexibility in generation of ring-shaped optical lattices, we proposed and demonstrated a novel generation method by using axially-symmetric polarization elements. While two optical vortices were coaxially generated, electrically-controlled phase difference between them by an electro-optic modulator enabled a precise rotation of the lattice. Our method has a capability to fulfill both the high-stability and the rapid-rotation.

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When two optical vortices (OVs) with different topological charges (TCs) ℓ_1 and ℓ_2 are coaxially superposed, a ring-shaped optical lattice is generated due to the interference between helical phase fronts with different slopes [1,2]. The lattice pattern has $|\ell_1 - \ell_2|$ lattice sites and can be rotated by changing the initial-phase difference between the superposed vortices. The ring-shaped optical lattice has been applied to the optical manipulations for the ultra-cold atoms and micro-particles [3–6].

For the generation of ring-shaped optical lattices, several methods have been developed so far. Most of the previous methods use spatial light modulators (SLMs) and can be classified into two types according to the configurations of the interferometer, namely the separate-path and common-path interferometers. In the first type, SLMs were used to generate two OVs which respectively travel spatially-separated paths. The vortices are then overlapped to interfere with each other. This configuration has a merit of high-flexibility in the optical design. Especially, it is compatible with external phase/frequency modulators and thereby the rapid rotation of the generated ring-lattice is realizable [2]. In contrast, the second type used an SLM to convert a single plane or Gaussian beam into two co-propagating OVs [6,7]. Since this configuration forms a common-path interferometer, the lattice pattern is almost immune to the external perturbations.

From the practical point of view, both features of the previous methods, namely the rapid-rotation capability and the high-stability, are important. To fulfill both requirements simultaneously, it is desirable to incorporate an external modulator into a common-path ring-lattice generator. However, it is difficult to realize this combination by use of an SLM, because an external modulator cannot give different modulations to two vortices co-propagating from a single SLM.

To overcome this limitation, we developed an alternative method to generate a ring-shaped optical lattice. Axially-symmetric polarization elements are used to generate OVs instead of an SLM. Although two OVs are generated coaxially, their phase difference can be electrically controlled by use of an electro-optic modulator (EOM) because the two vortices have orthogonal polarizations. The present method has a capability to fulfill both the high-stability and the rapid-rotation.

The key devices of this method are two kinds of space-variant polarization elements, an axially-symmetric half-wave plate (AHP) and an axially-symmetric polarizer (ASP), illustrated in Figs. 1(a) and (b). The combination of AHP and ASP allows us to generate a ring-shaped optical lattice. Before describing the principle of the present method, the functions of the respective polarization elements are briefly described by use of the Jones calculus.

The Jones matrix of an AHP whose fast-axis is radially distributed is written as

$$\hat{O}_{\text{AHP}} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}, \tag{1}$$

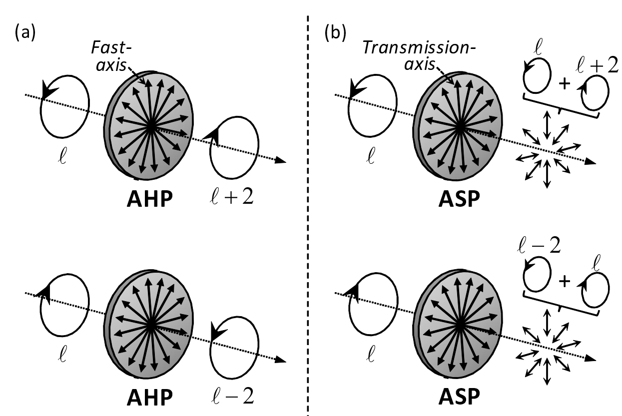

Fig. 1. Schematics of (a) AHP and (b) ASP, and their functions to LCP and RCP vortices.

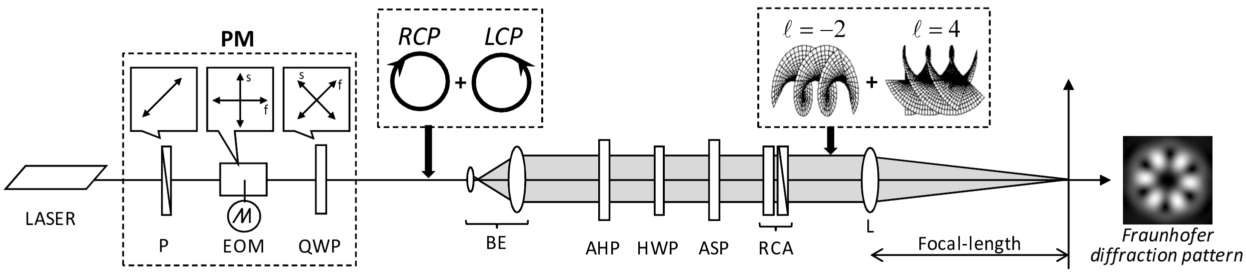

Fig. 2. An example of the ring-shaped optical lattice generator by use of AHP and ASP. This configuration is designed to generate 6-petaled ring lattice.

where θ is the azimuth angle in the beam cross section. Using the Jones vectors $|L\rangle = (1,\ i)^{\mathrm{T}}/\sqrt{2}$ and $|R\rangle = (1,\ -i)^{\mathrm{T}}/\sqrt{2}$ for left and right circularly polarized (LCP and RCP) lights, respectively, we obtain

$$\hat{O}_{\text{AHP}}\ \exp\left[i\ell\theta\right]|L\rangle = \exp\left[i(\ell+2)\theta\right]|R\rangle\,, \tag{2a}$$

$$\hat{O}_{\text{AHP}}\ \exp\left[i\ell\theta\right]|R\rangle = \exp\left[i(\ell-2)\theta\right]|L\rangle\,. \tag{2b}$$

Since the term $\exp\left[i\ell\theta\right]$ represents the spatial phase distribution of an optical vortex with a TC of ℓ, these equations show that the AHP converts a circularly polarized optical vortex into one with a different TC and the opposite polarization-handedness, as has been examined by Biener *et al.* [8] and Marrucci *et al.* [9]. Specifically, the TC ℓ is increased by 2 for the incident LCP vortex whereas ℓ is decreased by 2 for the incident RCP vortex, as shown in Fig. 1(a).

In good contrast, the Jones matrix of an ASP whose transmission-axis is radially distributed is given by

$$\hat{O}_{\text{ASP}} = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}. \tag{3}$$

Any light transmitted through the ASP has a radial polarization, which can be decomposed into LCP and RCP components with same amplitude. Taking into account this fact, we obtain

$$\hat{O}_{\text{ASP}}\, e^{i\ell\theta}|L\rangle = \frac{1}{2}\left[e^{i\ell\theta}|L\rangle + e^{i(\ell+2)\theta}|R\rangle\right], \tag{4a}$$

$$\hat{O}_{\text{ASP}}\, e^{i\ell\theta}|R\rangle = \frac{1}{2}\left[e^{i(\ell-2)\theta}|L\rangle + e^{i\ell\theta}|R\rangle\right]. \tag{4b}$$

Equations (4a) and (4b) show that the ASP converts a circularly polarized optical vortex into the superposition of two OVs possessing different TCs and opposite polarization-handedness [10]. As shown in Fig. 1 (b), when an LCP vortex with a TC of ℓ passes through the ASP, the transmitted light is the superposition of LCP and RCP vortices whose TCs are ℓ and $\ell+2$, respectively. Similarly, a RCP vortex is converted to the superposition of LCP and RCP vortices whose TCs are $\ell-2$ and ℓ, respectively.

We here note that the functions of AHP and ASP are different for LCP and RCP vortices. This fact implies that, when a linearly or elliptically polarized light is transmitted through either device, coaxial OVs with different TCs are generated simultaneously, because the incident light can be treated as the superposition of LCP and RCP components. In addition, the coaxially generation of OVs, with the controlled differences of their TCs, can be realized by combining AHPs and ASPs. On the basis of this approach, we developed a novel, to our knowledge, method to generate the electrically-controllable ring-shaped optical lattice.

Figure 2 illustrates an example of the optical system for generating a ring-shaped optical lattice as interference between two coaxial OVs; this configuration is designed to generate a 6-petaled ring lattice. Laser light first passes through a polarization modulator (PM) consisting of a polarizer P, an EOM, and a quarter-wave plate QWP. The Jones vector of the light emerging from the PM can be written as

$$|E_{\text{PM}}\rangle = |L\rangle/\sqrt{2} + \exp\left(-i\delta\right)|R\rangle/\sqrt{2}, \tag{5}$$

where δ is the retardation of the EOM. This light is linearly polarized and thus can be considered as the superposition of LCP and RCP components with the same amplitudes. The phase difference δ between the LCP and RCP components can be electrically controlled by the EOM. The light from the PM is expanded in its diameter by a beam expander (BE) and successively passes through an AHP, a normal half-wave plate (HWP), an ASP, and a right circular analyzer (RCA). As can be understood from Eqs. (2a) and (2b), the AHP converts the incident LCP and RCP components to the OVs with the TCs of 2 and -2, respectively, and their polarization-handedness of are flipped. Next the HWP flips again the polarization-handedness of the respective components without changing their TCs. The ASP then converts the respective components to the superposition of LCP and RCP OVs. Its Jones vector can be calculated from Eqs. (2a), (2b), (4a), (4b), and (5) to be

$$\begin{aligned} |E_{\text{ASP}}\rangle =& \frac{1}{2\sqrt{2}}\left(e^{i4\theta} + e^{-i(2\theta+\delta)}\right)|R\rangle \\ &+ \frac{1}{2\sqrt{2}}\left(e^{i2\theta} + e^{-i(4\theta+\delta)}\right)|L\rangle\,. \end{aligned} \tag{6}$$

The RCA selects only the RCP components from $|E_{\text{ASP}}\rangle$, which consists of two co-propagating OVs whose complex amplitudes are $e^{i4\theta}$ and $e^{-i(2\theta+\delta)}$. Namely, the TCs of the respective terms are $\ell_1 = 4$ and $\ell_2 = -2$, and the difference between their initial phases can be controlled through the phase term $-\delta$ induced by the EOM. The paired vortices then pass through a convex lens. At the back focal plane of the lens, the respective vortices interfere with each other to generate a 6-petaled ring lattice as a Fraunhofer diffraction pattern.

The azimuth of the generated lattice pattern can be controlled through the phase difference δ between the two vortices, or the retardation of the EOM. For example, the lattice pattern is rotated at a constant speed when δ linearly increases or decreases with time. Although this type of EOM-modulation is not directly achievable because it requires the infinite increase or decrease in δ, an equivalent retardation-modulation can be realized by use of the serrodyne technique [11]. In this technique, a sawtooth signal is applied to the EOM in such a way that its retardation varies between $-\pi$ and π rad. Because of the 2π periodicity of optical waves, the observed beam patterns are the same as those with the infinite increase or decrease in δ.

The feasibility of the present principle was demonstrated by the experiment. A He-Ne laser (wavelength 632.8 nm) is used as a coherent light source. The optical system of Fig. 2 was assembled by use of an AHP and an ASP made of photonic crystals (Photonic Lattice, Inc.). In this assembling, two modifications from Fig. 2 were made to avoid the limitations owing to the experimental apparatus. The first modification is the insertion of a double-diffraction-type imaging system between AHP and ASP. Since the experimental system was composed of bulk optics, the relay imaging system was necessary to circumvent the diffraction effect [13] from the output face of AHP to the input face of ASP. When AHP, HWP, and ASP are closely adhered with one another, the relay imaging system should be unnecessary, because the elements made of photonic crystals are very thin. The second modification is that paired phase modulators (New focus 4002) connected in series were used as an EOM in the PM. The paired modulators are made to work as a single EOM whose phase modulation is doubled to cover $-\pi$ to π rad. For this purpose, the modulators are driven by a common electric amplifier (NF 4055), and their fast axes are set in parallel.

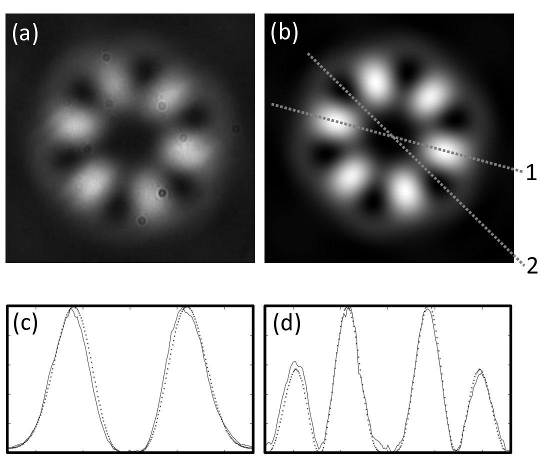

Fig. 3. Observed beam intensity patterns of the ring-lattice. (a) and (b) are two-dimensional beam patterns obtained by the experiment and the numerical simulation, respectively. (c) and (d) are the intensity distributions along the lines 1 and 2, respectively, whose solid and dashed lines indicate the experiment and the simulation results.

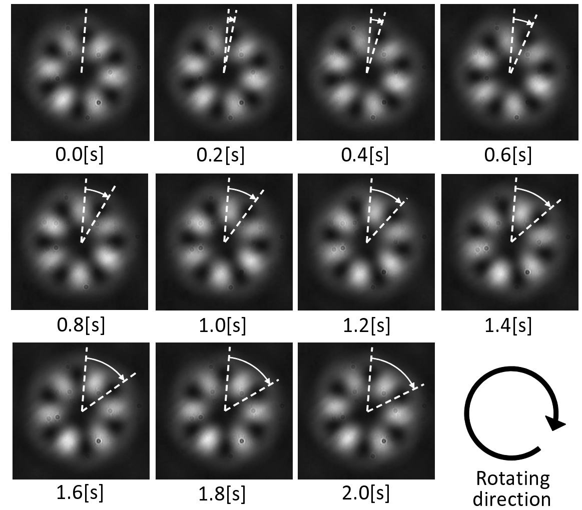

Fig. 4. Rotation of the ring-shaped optical lattice. The initial-phase difference between two vortices is modulated by sawtooth signal with a period of 2 sec.

We first observed a beam pattern without applying an electric signal to the EOM, and compared it with the result by the numerical simulation. Figure 3(a) shows the two-dimensional beam pattern observed at the back focal plane of the lens by use of charge coupled device (210×210 pixels), whereas Fig. 3(b) is the numerically-simulated beam pattern. Figs. 3(c) and (d) are the one-dimensional intensity distribution along the lines 1 and 2 shown in Fig. 3(b), respectively, where solid and dashed lines represent the experiment and the simulation results. A 6-petaled ring-lattice generated by the interference between two OVs with $\ell_1 = 4$ and $\ell_2 = -2$ is clearly seen in Fig. 3(a). The experimentally-obtained pattern shows a good agreement with the pattern obtained by the numerical simulation. The obtained pattern was static and quite stable, proving the high-stability of the present experimental system.

We next observed the change of the beam pattern with the serrodyne retardation-modulation to the EOMs. The sawtooth signal was generated by a programmable oscillator and its period was set to be 2 sec. Figure 4 shows the time variation of the beam pattern. As expected, the ring-lattice continuously rotates with time. Since the lattice has 6-petals, its rotation speed is $(2\pi/6)$ rad / 2sec = $\pi/6$ rad/sec. Furthermore, the beam pattern is preserved and suffers from no distortions.

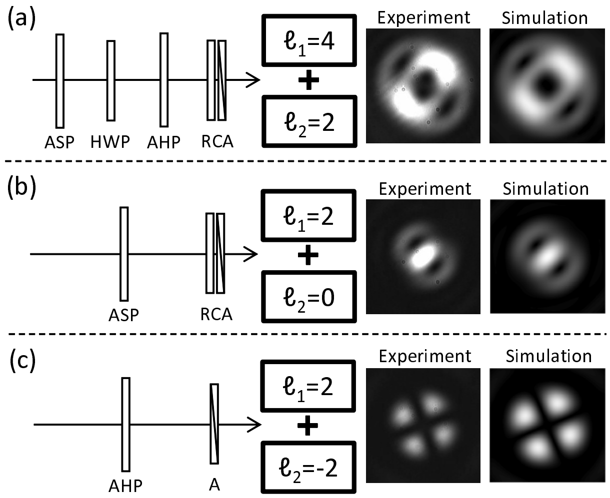

Fig. 5. Examples of other arrangements of AHP and ASP, and the corresponding ring-shaped optical lattices.

We note that the combination of the TC pairs can be changed with the arrangements of AHP, HWP, ASP, and RCA in Fig. 2. As typical examples, we also built the system with three different arrangements of AHP, HWP, and ASP shown in Figs. 5(a), (b), and (c). In any cases, the experimentally observed beam patterns show good agreement with those by the numerical simulation. In addition, we were also able to rotate these patterns with the serrodyne retardation-modulation. It should be noted that we can realize different combinations of ℓ_1 and ℓ_2 by using multiple AHPs and ASPs. The theoretical analysis and its experimental verifications will be presented in elsewhere.

The present method for generating a ring-shaped optical lattice has a feature that two OVs are generated and interfered with each other by the common-path optical system. Accordingly, the generated ring-lattice is almost immune to the environmental perturbations, such as the vibration and the temperature change. In addition, the azimuth of the lattice pattern can be controlled by an external phase modulator EOM. It has been reported that the EOM can be operated up to 1.6 GHz with the serrodyne modulation [12]. Hence the present method has a potential for the rapid rotation of the ring-shaped optical lattice.

In spite of these advantageous features, however, this method has a limitation that the combination of the TCs ℓ_1 and ℓ_2 cannot be electrically controlled. In contrast to the previous methods using SLMs, ℓ_1 and ℓ_2 are fixed to the configuration of the optical system. Nevertheless, we can electrically control the amplitude ratio between two vortex components when we modify the PM in such a way that not only the phase difference but also the amplitude ratio is changed. This ratio-change enables one to transform the shape of ring-lattice between uniform to multi-petaled rings [2]. We also note that the ASP and AHP in the present system do not generate Laguerre-Gaussian beams but point vortices. Since the diffraction characteristics between two constituent waves are different, theoretical analysis needs to be made, taking into account the characteristics of the point vortex.

The performance of the present system may be deteriorated by the imperfection in the imaging between AHP and ASP. While we could not find any apparent defects in the observed ring-lattice, the numbers of AHPs and ASPs should be limited by the imaging quality. However, this limitation will be overcome when the polarization elements are directly and precisely adhered with one another.

In conclusion, we proposed and demonstrated a new method to generate a ring-shaped optical lattice by using axially-symmetric polarization elements. It overcomes a trade-off issue between stability and flexibility in generation. While two OVs were coaxially generated, their phase difference was electrically controlled by use of an EOM, giving a precise rotation of the lattice. Our method has a capability to fulfill both the high-stability and the rapid-rotation, being applicable to the optical manipulation for micro-particles and ultra-cold atoms.

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