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Pucher, Isler and form finding of shells

In 1862 Airy found that the in-plane stresses in an elastic plate could be described by the curvature of a surface. This surface he called the stress function. In two papers from 1934 and 1937 Pucher showed that for a membrane shell subjected to only vertical load, the stress function could be found directly from the equation of equilibrium if the shell geometry and the load were known. Also he pointed out that the shell surface and the stress surface were equal, meaning that if one of the surfaces is the stress surface then the other is the shell surface for the considered load.

In this presentation Pucher's findings is taken a step further and used for force driven form finding of shells. It is exemplified with two very significant shell surfaces with square plans.

The first step is to find the stress surface, using the fact that it represents the horizontal equilibrium of forces in any point of the shell surface. This means that we can consider the plan projection of the shell as an elastic plate and determine the stresses by applying the anticipated reactions from the foundation as loads.

For a shell with a square plan, subjected to a uniform load and only supported at the corners, the stress surface S is shown in (Figure 1).

The second step is then to determine the shell surface using the equation of vertical equilibrium in a membrane shell. In Pucher's notation:

$$\frac{\partial^2 S}{\partial y^2} \frac{\partial^2 F}{\partial x^2} - 2 \frac{\partial^2 S}{\partial x \partial y} \frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 S}{\partial x^2} \frac{\partial^2 F}{\partial y^2} = -q_z$$

Here S is the stress surface, F the shell surface and q the area load. It is seen that the equation is completely symmetrical in S and F . For the considered case the edges are free except at the supports and the shell surface is shown in (Figure 2).

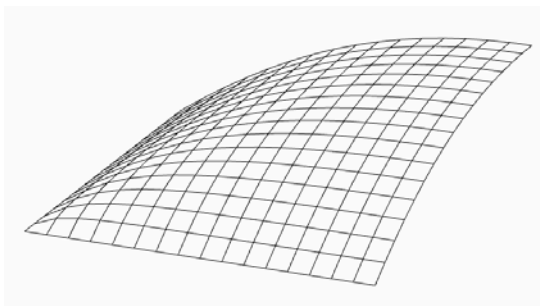


Figure 1. $\frac{1}{4}$ of surface S , the stress surface for a square plate subjected to a diagonal load in each corner.

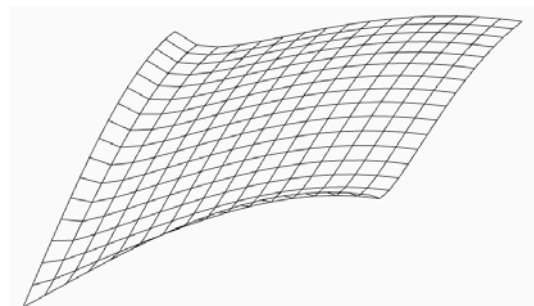


Figure 2. $\frac{1}{4}$ of surface F , the equilibrium surface for a square shell supported at the corners and subjected to uniform vertical load.

These surfaces have been found using the methods of finite difference, calculated by dynamic relaxation and programmed in Processing.

Since the stresses are determined by elasticity the resulting shell forms can be considered as guidelines for structurally optimal shell forms.

The Swiss engineer Heinz Isler designed and built many reinforced concrete shells. He used three form finding methods: rubber membrane under pressure, wet hanging fabric and flow form.

The first method lead to his “bubble” shells that reminds very much of surface S, see Figure 3.

The second method lead to several different designs which all reminds of surface F, see Figure 4.

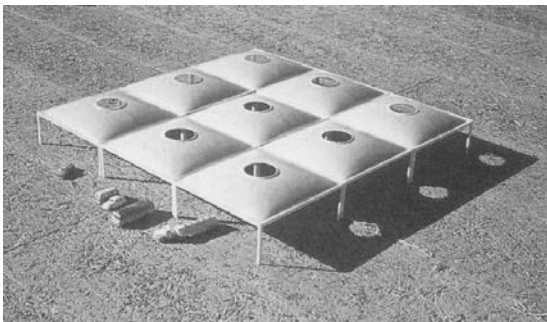


Figure 3. “Bubble” shells designed by Heinz Isler.

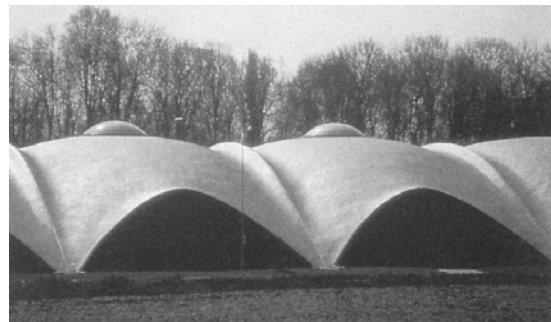


Figure 4. Tennis halls designed by Heinz Isler.

It is interesting that the negative curvature along the edges of the hanging fabric forms is caused by equilibrium and not a specific property of fabric models. Also it is interesting that Isler’s “bubble” shells have positive curvature all over the surface as S, because the surface of a rubber membrane under pressure has negative curvature near the vertices.

These and other results will be discussed at the presentation, which also is a follow up on a presentation at the Nordic meeting in Trondheim 2007.

References:

Pucher, A. (1934): *Ueber den Spannungszustand in gekrümmten Flächen*. Beton und Eisen. 33. Jahrgang, Heft 19, 298- 304.

Pucher, A. (1937): *Die Berechnung von doppelt gekrümmten Schalen mittels Differenzgleichungen*. Der Bauingenieur. 18. (1937) Heft 9/10, 118-123.

Ramm, E., & Schunck, E. (1986). *Heinz Isler: Schalen: Katalog zur Ausstellung*. Stuttgart: Karl Krämer.

Almegaard, H. (2007): *Skallers statiske virkemåde*. Nordisk Tekniklærermøte 2007. Trondheim 2007.