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Value of Flexible Resources, Virtual Bidding, and Self-Scheduling in Two-Settlement Electricity Markets With Wind Generation – Part II

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***Abstract*—In Part II of this paper, we present formulations for three two-settlement market models: baseline cost-minimization ($Stoch$-Opt); and two sequential market models in which an independent system operator (ISO) runs real-time (RT) balancing markets after making day-ahead (DA) generating unit commitment decisions based upon deterministic wind forecasts, while virtual bidders arbitrage the two markets (Seq and Seq-SS). The latter two models differ in terms of whether some slow-start generators can self-schedule in the DA market while anticipating probabilities of RT prices. Models in Seq and Seq-SS build on components of the two-settlement equilibrium model ($Stoch$-MP) defined in Part I of this paper [1]. We then provide numerical results for all four models. A simple single-node case illustrates the economic impacts of flexibility, virtual bidding, and self-schedules, and is followed by a larger case study based on the 24-node IEEE reliability test system. Their results confirm that flexible resources, including fast-start generators and demand response, can reduce expected costs in a sequential two-settlement market. In addition, virtual bidders can also improve the functioning of sequential markets. In some circumstances, virtual bidders (together with self-scheduling by slow-start generators) enable deterministic ISO DA markets to obtain the least (expected) cost unit commitments.**

Index Terms– Operational flexibility, wind uncertainty, equilibrium, day-ahead, real-time, demand response, virtual bidding.

I. Introduction

In the first of this two-paper series [1], we presented the assumptions and formulation of an equilibrium model for a two-settlement electricity market ($Stoch$-MP). In this model, slow-start resources make commitment and tentative energy decisions day-ahead (DA), and then real-time (RT) markets resolve imbalances arising from inaccurate wind forecasts by adjusting schedules of slow-start resources and committing fast-start resources. All resources are expected profit maximizers who possess no market power and correctly anticipate the probability distribution of RT prices when making DA commitments. In this, the second paper of the series, we first provide the formulations of $Stoch$-Opt, Seq and Seq-SS. $Stoch$-Opt is a baseline model in which the expected costs in the two-settlement market are minimized. We demonstrate that this model is equivalent to $Stoch$-MP, the two-settlement equilibrium, if players correctly anticipate the probability distribution of RT prices and behave competitively, which implies that a two-settlement market equilibrium is efficient under our assumptions. Meanwhile, Seq and Seq-SS instead reflect the present design in US markets, in which an independent system operator (ISO) schedules DA generation against a deterministic wind forecast, and then runs RT imbalance markets. In particular, PJM and CAISO use a sequential model that includes virtual bidding (Seq). In addition, US markets also allow generators to submit self-schedules if they prefer (Seq-SS), for example by offering their entire capacity at a price equal to the bid floor (-150 \$/MWh in California). Financial players arbitrage the DA and RT markets by buying in one and selling in the other, resulting in DA prices equaling expected RT prices. Seq assumes the ISO schedules all generation and, in one variant, allows virtual bidding (VB) between DA and RT markets. Meanwhile, Seq-SS allows some slow-start generators to self-schedule in the DA market while anticipating the probability distribution of RT prices. Then we apply the four models to two case studies, including an application based on the 24-node IEEE reliability test system (RTS) [2]. The case studies are used to quantify the economic value of adding flexible (fast-start) resources to the system to manage wind variability. They also illustrate how VB together with rational self-scheduling by some slow-start generators has the potential to yield the same efficient outcomes in deterministic DA markets (Seq-SS) as the optimal stochastic model ($Stoch$-Opt), without the ISO having to solve stochastic DA unit commitment problems.

In the next section, we provide the formulations of $Stoch$-Opt, Seq and Seq-SS, based upon the assumptions presented in the companion paper [1], where definitions of the notation used throughout this paper can be found. In Section III, we present the two case studies, including a simple three thermal unit and one wind plant example to illustrate the basic results and an case study based upon the 24-node IEEE RTS. Section IV presents conclusions. The appendix describes the equilibrium conditions for the two-settlement equilibrium ($Stoch$-MP), consisting of each market party's first-order Karush-Kuhn-Tucker (KKT) conditions combined with clearing conditions for each market; these turn out to the be same as the first-order KKT conditions of $Stoch$-Opt, proving their equivalence. The appendix also summarizes differences between the equilibrium conditions of the sequential market of Seq and the equilibrium conditions of $Stoch$-MP.

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II. Market-clearing models

In the following presentations of $Stoch$-Opt, Seq and Seq-SS, note that all references to (1) are to the equations of $Stoch$-

MP, presented in [1].

A. *Stoch-Opt*: Stochastic Optimization Model, Which Minimizes the Total Expected System Cost

As illustrated in Fig. 3 of the companion paper [1], this model represents a single linear optimization problem in which the grid operator chooses all schedules in order to minimize the total expected cost across all players in both DA and RT markets. This problem is given by (2) below:

$$\begin{aligned}
&\underset{\Xi^{\mathrm{Stoch-Opt}}}{\text{Minimize}} \sum_{i\in(\mathcal{S}\cup\mathcal{F}),t} \left(c_{i,t}^{\mathrm{DA}} + C_i\ p_{i,t}^{\mathrm{DA}}\right)\\
&+ \sum_{i\in(\mathcal{S}\cup\mathcal{F}),t,s} \phi_s\ C_i\ p_{i,t,s}^{\mathrm{RT}} + \sum_{i\in\mathcal{F},t,s} \phi_s\ c_{i,t,s}^{\mathrm{RT}}\\
&+ \sum_{d\in(\mathcal{SDR}\cup\mathcal{FDR}),k,t} \left(C_{d,k,t}^{\downarrow}\ d_{d,k,t}^{\mathrm{DA}\downarrow} - C_{d,k,t}^{\uparrow}\ d_{d,k,t}^{\mathrm{DA}\uparrow}\right)\\
&+ \sum_{d\in\mathcal{FDR},k,t,s} \phi_s \left(C_{d,k,t}^{\downarrow}\ d_{d,k,t,s}^{\mathrm{RT}\downarrow} - C_{d,k,t}^{\uparrow}\ d_{d,k,t,s}^{\mathrm{RT}\uparrow}\right) \qquad \text{(2a)}
\end{aligned}$$

subject to

$$\begin{aligned}
&\text{(1ab)-(1ah)} \quad \forall i \in \mathcal{S} && \text{(2b)}\\
&\text{(1bb)-(1bh)} \quad \forall i \in \mathcal{F} && \text{(2c)}\\
&\text{(1cb)-(1cd)} \quad \forall i \in \mathcal{W} && \text{(2d)}\\
&\text{(1db)-(1dd)} \quad \forall d \in \mathcal{SDR} && \text{(2e)}\\
&\text{(1eb)-(1ed)} \quad \forall d \in \mathcal{FDR} && \text{(2f)}\\
&\text{(1fb)} \quad \forall f && \text{(2g)}\\
&\text{(1gb)-(1gg), (1ha), (1hb).} && \text{(2h)}
\end{aligned}$$

The set of decision variables of problem (2), i.e., $\Xi^{\mathrm{Stoch-Opt}}$, contain all variables included in problems (1a)-(1h) of *Stoch-MP*. The set of constraints (2b)-(2h) include the same constraints as considered by the players in *Stoch-MP*, as well as market clearing.

Note that *Stoch-Opt* is *equivalent* to *Stoch-MP* because the KKT conditions of *Stoch-Opt* are identical to the complementarity equilibrium problem for *Stoch-MP*, as described in Appendix. This proves that the multi-player stochastic two-settlement equilibrium model (*Stoch-MP*) describes a market that results in an economically efficient (least expected system cost) solution under the assumptions we made. It also turns out that in *Stoch-MP* and *Stoch-Opt*, the DA market-clearing price is *equal* to the expected RT price since the DA and RT markets are arbitraged by the generators and fast demand response (DR) providers; therefore, including separate VB has no impact on market-clearing outcomes in those models.

We need to point out, however, that modeling simplifications as well as computational and data limitations mean that any real-world implementation of *Stoch-Opt* as a stochastic unit commitment (e.g., [3]-[5]) would not actually achieve the minimum expected cost solution [6]. For instance, the curse of dimensionality means that it is not possible to consider more than just a few of the possible actual random wind and load realizations, nor the full multistage nature of the problem (24 DA decision intervals, together with 288 daily 5-minute RT intervals, not to mention the impacts on decisions in later days). Thus, the solution to *Stoch-Opt* (and thus equilibrium model in *Stoch-MP*) should be, strictly speaking, viewed not as actual social optima but more narrowly as a lower bound to the costs that would be reported by the market models of *Seq* and *Seq-SS*, assuming the same set of wind scenarios and probabilities.

We also note that in the presence of market distortions, such as feed-in tariffs, average cost-based transmission prices, or market power, the equilibrium problem (*Stoch-MP*) might not achieve the social cost minimum, nor might it have an equivalent single optimization problem (analogous to *Stoch-Opt*). Gabriel et al. [7] discuss general conditions under which an economic equilibrium problem posed as a complementarity problem has an equivalent single optimization problem. Incorporating such distortions, which arise from policies and practical operating conditions, into *Stoch-MP*, *Seq* and *Seq-SS* is an important research agenda. In particular, such models can be used to assess the efficiency impacts of those distortions, and the benefits of market design changes to minimize them.

B. *Seq*: Sequential Two-Settlement Equilibrium Model

As illustrated in Fig. 4 of the companion paper [1], this equilibrium model contains three kinds of optimization problems: a deterministic DA market-clearing problem, a RT market-clearing problem for each scenario, and a profit-maximization problem for each virtual bidder. The deterministic DA market-clearing problem is given by (3a), whose objective is the minimization of the cost of day-ahead generation and DR schedules within that market subject to the deterministic wind power forecast:

$$\begin{aligned}
&\underset{\Xi^{\mathrm{Seq,DA}}}{\text{Minimize}} \sum_{i\in(\mathcal{S}\cup\mathcal{F}),t} \left(c_{i,t}^{\mathrm{DA}} + C_i\ p_{i,t}^{\mathrm{DA}}\right)\\
&+ \sum_{d\in(\mathcal{SDR}\cup\mathcal{FDR}),k,t} \left(C_{d,k,t}^{\downarrow}\ d_{d,k,t}^{\mathrm{DA}\downarrow} - C_{d,k,t}^{\uparrow}\ d_{d,k,t}^{\mathrm{DA}\uparrow}\right) \qquad \text{(3aa)}
\end{aligned}$$

subject to

$$\begin{aligned}
&\text{(1ab), (1ad), (1af)-(1ah)} \quad \forall i \in \mathcal{S} && \text{(3ab)}\\
&\text{(1bb)} \quad \forall i \in \mathcal{F} && \text{(3ac)}\\
&\text{(1cb), (1cd)} \quad \forall i \in \mathcal{W} && \text{(3ad)}\\
&\text{(1db)-(1dd)} \quad \forall d \in \mathcal{SDR} && \text{(3ae)}\\
&\text{(1eb)} \quad \forall d \in \mathcal{FDR} && \text{(3af)}\\
&\text{(1gb), (1gd), (1gf), (1ha).} && \text{(3ag)}
\end{aligned}$$

The optimization variable set of problem (3a), which is $\Xi^{\mathrm{Seq,DA}}$, contains all DA variables included in problems (1a)-(1h) of *Stoch-MP*, except for the arbitrage quantity $v_{f,t}^{\mathrm{DA}}$, which is treated as exogenous (fixed) in the operator's DA market-clearing problem. That is, virtual bidders are assumed in their model to self-schedule the amount of power they buy (sell) in the DA market, and then sell (buy) back in the RT market.

The RT market-clearing problem for wind generation scenario s is given by (3b), whose objective is to minimize the probability-weighted system cost in the RT market under the scenarios considered:

$$\left\{ \underset{\Xi^{\mathrm{Seq,RT}}}{\text{Minimize}} \sum_{i \in (\mathcal{S} \cup \mathcal{F}),t} \phi_s \; C_i \; p^{\mathrm{RT}}_{i,t,s} + \sum_{i \in \mathcal{F},t} \phi_s \; c^{\mathrm{RT}}_{i,t,s} \right.$$

$$+ \sum_{d \in \mathcal{FDR},k,t} \phi_s \; \left(C^{\downarrow}_{d,k,t} \; d^{\mathrm{RT}\downarrow}_{d,k,t,s} - C^{\uparrow}_{d,k,t} \; d^{\mathrm{RT}\uparrow}_{d,k,t,s} \right) \quad \text{(3ba)}$$

subject to

$$\text{(1ac), (1ae)} \quad \forall i \in \mathcal{S} \quad \text{(3bb)}$$

$$\text{(1bc)-(1bh)} \quad \forall i \in \mathcal{F} \quad \text{(3bc)}$$

$$\text{(1cc)} \quad \forall i \in \mathcal{W} \quad \text{(3bd)}$$

$$\text{(1ec)-(1ed)} \quad \forall d \in \mathcal{FDR} \quad \text{(3be)}$$

$$\left. \text{(1gc), (1ge), (1gg), (1hb)} \quad \right\} \; \forall s. \quad \text{(3bf)}$$

The optimization variable set of problem (3b), i.e., $\Xi^{\mathrm{Seq,RT}}$, contains all RT variables included in problems (1a)-(1h) of *Stoch-MP*, except for the arbitraged quantity $v^{\mathrm{RT}}_{f,t}$, which is exogenous. One important observation is that all DA market-clearing outcomes, i.e., decision variables included in set $\Xi^{\mathrm{Seq,DA}}$, take on fixed values within (3b). Note that given the values of the DA variables, the RT model can be decomposed into separate RT problems for each wind realization. Finally, the profit-maximization problem of each virtual bidder f to be included in *Seq* is identical to (1f) in *Stoch-MP* in [1]. The solution of *Seq* can be obtained by simultaneously solving the KKT conditions corresponding to problems (1f), (3a) and (3b), as illustrated in Fig. 1. The resulting model is a mixed linear complementarity problem (MLCP) which we solve using PATH.

One important observation is that *Seq* is *not* necessarily equivalent to *Stoch-MP* and *Stoch-Opt* because *Seq* yields a different set of KKT conditions than those of *Stoch-MP* and *Stoch-Opt*. Further description can be found in Appendix. Since *Stoch-MP* and *Stoch-Opt* yield the expected cost minimization solution, this means that the solution of *Seq* might be inefficient, as we illustrate in our application later. Another important observation is that unlike *Stoch-MP* and *Stoch-Opt*, the DA market-clearing price obtained in *Seq* is *not* necessarily equal to the expected RT price, unless there are arbitragers. Therefore, the virtual bidders (and the self-scheduled slow-start generators that we introduce in *Seq-SS*) can potentially help fix this price distortion. In this paper, we numerically show how VB alone or a combination of VB and self-scheduling by some slow-start generators can eliminate market distortions due to deterministic DA scheduling in some circumstances. This implies that the grid operator does not necessarily need to do stochastic unit commitment (as in *Stoch-Opt*) for the optimal solution to occur, and arbitrage and some self-scheduling together with deterministic DA scheduling by ISOs could accomplish this.

C. *Seq-SS*: Extended *Seq* with Self-Scheduling Slow-Start Generators

As Fig. 5 of the companion paper [1] shows, this equilibrium model is a collection of four kinds of optimization

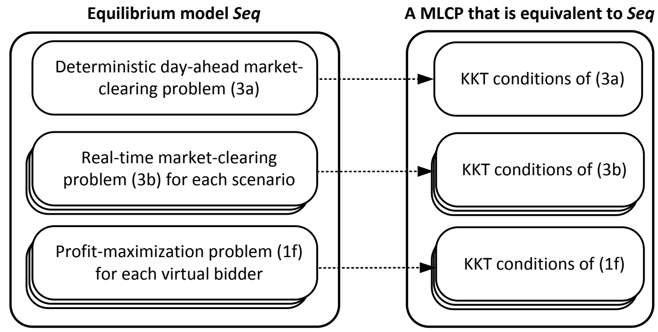

Fig. 1. Model *Seq* recast as a mixed linear complementarity problem (MLCP).

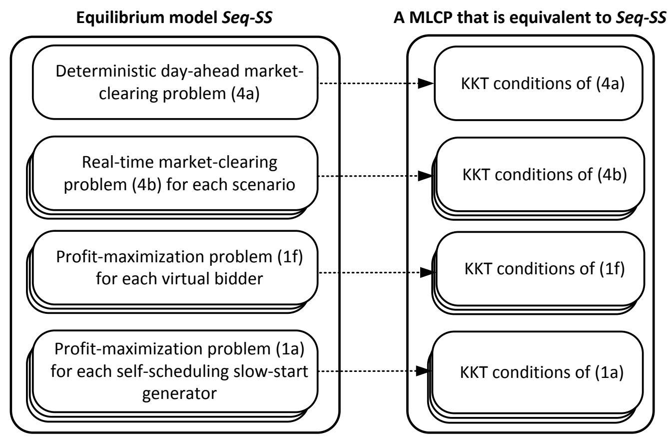

Fig. 2. Model *Seq-SS* recast as a MLCP.

problems. These include a deterministic DA market-clearing problem, a RT market-clearing problem for each scenario, a profit-maximization problem for each virtual bidder, and an expected profit-maximization problem for each self-scheduling slow-start generator $i \in \mathcal{SS}$. The deterministic DA market-clearing problem is similar to (3a) in *Seq*, except that the ISO respects the self-scheduling decisions of certain slow-start generators. This market-clearing problem is given by (4a) below, and its objective is the system cost minimization in the DA market subject to the deterministic wind power forecast:

$$\underset{\Xi^{\mathrm{Seq-SS,DA}}}{\text{Minimize}} \sum_{\substack{i \in (\mathcal{S} \cup \mathcal{F}),t \\ i \notin \mathcal{SS}}} \left(c^{\mathrm{DA}}_{i,t} + C_i \; p^{\mathrm{DA}}_{i,t} \right)$$

$$+ \sum_{d \in (\mathcal{SDR} \cup \mathcal{FDR}),k,t} \left(C^{\downarrow}_{d,k,t} \; d^{\mathrm{DA}\downarrow}_{d,k,t} - C^{\uparrow}_{d,k,t} \; d^{\mathrm{DA}\uparrow}_{d,k,t} \right) \quad \text{(4aa)}$$

subject to

$$\text{(1ab), (1ad), (1af)-(1ah)} \quad \forall i \in \mathcal{S}, i \notin \mathcal{SS} \quad \text{(4ab)}$$

$$\text{(1bb)} \quad \forall i \in \mathcal{F} \quad \text{(4ac)}$$

$$\text{(1cb), (1cd)} \quad \forall i \in \mathcal{W} \quad \text{(4ad)}$$

$$\text{(1db)-(1dd)} \quad \forall d \in \mathcal{SDR} \quad \text{(4ae)}$$

$$\text{(1eb)} \quad \forall d \in \mathcal{FDR} \quad \text{(4af)}$$

$$\text{(1gb), (1gd), (1gf), (1ha).} \quad \text{(4ag)}$$

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The optimization variable set of problem (4a), which is $\Xi^{\mathrm{Seq-SS,DA}}$, contains all DA variables included in problems (1a)-(1h) of *Stoch-MP*, except for $u^{\mathrm{DA}}_{i\in\mathcal{SS},t}$, $c^{\mathrm{DA}}_{i\in\mathcal{SS},t}$, $p^{\mathrm{DA}}_{i\in\mathcal{SS},t}$ and $v^{\mathrm{DA}}_{f,t}$. Note that the power dispatch decisions of self-scheduling slow-start generators, i.e., $p^{\mathrm{DA}}_{i\in\mathcal{SS},t}$, and the arbitrage quantity, i.e., $v^{\mathrm{DA}}_{f,t}$, appear in power balance constraints (1ha) within (4ag), but they are treated by the ISO as exogenous (fixed) parameters.

Similar to problem (3b) in *Seq*, the RT market-clearing problem in *Seq-SS* for wind generation scenario s is given by (4b), whose objective is to minimize the probability-weighted system cost in the RT market under the scenarios considered:

$$\left\{\begin{aligned}&\underset{\Xi^{\mathrm{Seq-SS,RT}}}{\text{Minimize}} \sum_{\substack{i\in(\mathcal{S}\cup\mathcal{F}),t\\ i\notin\mathcal{SS}}} \phi_s\, C_i\, p^{\mathrm{RT}}_{i,t,s} + \sum_{i\in\mathcal{F},t} \phi_s\, c^{\mathrm{RT}}_{i,t,s}\\ &+ \sum_{d\in\mathcal{FDR},k,t} \phi_s \left(C^{\downarrow}_{d,k,t}\, d^{\mathrm{RT}\downarrow}_{d,k,t,s} - C^{\uparrow}_{d,k,t}\, d^{\mathrm{RT}\uparrow}_{d,k,t,s}\right) \qquad \text{(4ba)}\\ &\text{subject to}\\ &\text{(1ac), (1ae)} \quad \forall i\in\mathcal{S}, i\notin\mathcal{SS} \qquad \text{(4bb)}\\ &\text{(1bc)-(1bh)} \quad \forall i\in\mathcal{F} \qquad \text{(4bc)}\\ &\text{(1cc)} \quad \forall i\in\mathcal{W} \qquad \text{(4bd)}\\ &\text{(1ec)-(1ed)} \quad \forall d\in\mathcal{FDR} \qquad \text{(4be)}\\ &\text{(1gc), (1ge), (1gg), (1hb)} \end{aligned}\right\} \forall s. \qquad \text{(4bf)}$$

The optimization variable set of problem (4b), i.e., $\Xi^{\mathrm{Seq-SS,RT}}$, contains all RT variables included in problems (1a)-(1h) of *Stoch-MP*, except for $p^{\mathrm{RT}}_{i\in\mathcal{SS},t,s}$ and $v^{\mathrm{RT}}_{f,t}$, which are exogenous. Similar to *Seq*, all DA market-clearing outcomes, i.e., decision variables included in set $\Xi^{\mathrm{Seq-SS,DA}}$, are fixed values within (4b). In addition, similar to *Seq*, the RT model can be decomposed into separate RT problems for each wind realization given the values of the DA variables.

The profit-maximization problem of each virtual bidder f to be included in *Seq-SS* is identical to (1f) in *Stoch-MP*. Finally, the expected profit-maximization problem for each self-scheduling slow-start generator is identical to problem (1a) in *Stoch-MP*, but only for each $i\in\mathcal{SS}$.

We obtain the solution of *Seq-SS* by simultaneously solving the KKT conditions corresponding to problems (1a) specifically for $\forall i\in\mathcal{SS}$, (1f), (4a) and (4b), as illustrated in in Fig. 2. The resulting model is a MLCP.

III. Numerical Results

This section provides numerical results for a simple illustrative example and a larger case study based on the 24-node IEEE RTS [2]. The two systems are used to highlight the relationships among the four models; in particular, that the stochastic equilibrium market (*Stoch-MP*) yields the cost minimizing solution of *Stoch-Opt*, which might be viewed as an ISO that runs a stochastic unit commitment model. Meanwhile, *Seq* (in which the ISO runs a deterministic DA market model, followed by RT markets) is shown to be, in general, inefficient, although virtual bidders and a few self-scheduled slow-start generators can eliminate that inefficiency (*Seq-SS*). In addition, the RTS-based case study shows how the models can be used to calculate the expected economic value of adding flexible resources to the system that can manage unforecast wind variations.

TABLE I
Illustrative Example: Technical characteristics of dispatchable generators

Unit	Type	$\underline{P}_i$ [MW]	$\overline{P}_i$ [MW]	R_i^{D} [MW/h]	R_i^{U} [MW/h]	C_i [$/MWh]	C_i^{SU} [$]
G1	Slow-start	1000	1000	1000	1000	40	15,000
G2	Slow-start	0	1000	1000	1000	60	10,000
G3	Fast-start	0	500	500	500	120	1000

TABLE II
Illustrative Example: Results

		Stoch-MP and *Stoch-Opt* (with/without VB)	*Seq* (without VB)	*Seq* (with VB)	*Seq-SS* (with VB)
DA schedule [MW]	G1	500 (0.5)*	750 (0.75)	1000 (1.0)	500 (0.5)
	G2	250 (0.5)	0 (0)	0 (0)	0 (0.5)
	G3	0 (0)	0 (0)	0 (0)	0 (0)
	WP	250	250	250	250
	VB	-	-	-250	+250
RT operation under scenario s_1 [MW]	G1	0	0	0	0
	G2	+250	0	0	+500
	G3	0 (0)	250 (0.5)	0 (0)	0 (0)
	WP	-250	-250	-250	-250
	VB	-	-	+250	-250
RT operation under scenario s_2 [MW]	G1	0	0	0	0
	G2	-250	0	0	0
	G3	0 (0)	0 (0)	0 (0)	0 (0)
	WP	+250	0	-250	+250
	VB	-	-	+250	-250
Expected wind curtailment [MW]		0	125	250	0
Prices [$/MWh]	DA	55	55	[55,61]	55
	RT (s_1)	[60,110]†	122	[110,122]	[55,110]
	RT (s_2)	[0,50]	0	[-12,0]	[0,55]
Total expected system cost [$]		47,500	56,500	55,000	47,500

* The value inside parentheses indicates the (relaxed) commitment status.
† The square brackets indicate the range of possible prices consistent with the solution.

In all case studies, the main criterion for comparing the performance of the market models is the total expected system cost, i.e., system cost in DA plus the probability-weighted system cost in RT. The total expected system cost in each model is calculated using the optimal or equilibrium solution achieved, and its formulation is identical to objective function (2a) in *Stoch-Opt*.

A. Simple Illustration of Relationships Among the Models

A single-hour two-settlement market is considered in this illustrative example. The market includes two slow-start generators (G1 and G2), a fast-start generator (G3), a wind generator (WP), a virtual bidder, and a single load, all located at a single node. For simplicity, demand-side flexible resources are not considered; however, they are modeled in the RTS-based case study of the next subsection. Table I gives the technical characteristics of the dispatchable generators. The initial commitment status of all dispatchable generators is set to zero, so that start-up costs need to be incurred if a generator is to be dispatched. The wind power forecast in DA market is 250 MW, while its actual RT realization is uncertain. This uncertainty is modeled via two equiprobable scenarios (s_1 and s_2), in which the wind production is 0 MW and 500 MW, respectively. Finally, the system load is 1000 MW.

Table II provides the results for the four models. The values within parentheses give the (relaxed) commitment status of dispatchable generators in the DA market and the adjusted commitment for fast-start generator in RT market. As we mentioned above, VB has no impact on market-clearing outcomes in $Stoch$-MP and $Stoch$-Opt (column 2 of Table II) since DA and RT markets are arbitraged. However, VB alters the outcomes of sequential two-settlement market in Seq and Seq-SS (columns 3 to 5). Note that despite the large wind uncertainty, there is sufficient flexible capacity to ensure that there is never any unserved load, although wind is curtailed in the suboptimal (Seq) solutions.

In $Stoch$-MP and $Stoch$-Opt (column 2 of Table II), which provide the ideal (expected cost-minimizing) solution, the slow-start generation sources G1 and G2 as well as WP supply the load while the fast-start generator G3 is off. All generators know the probability distribution of RT prices. This allows G2 to efficiently manage the wind forecast error in RT by committing half of its capacity.[1] This slow-start generation fully compensates for the 250 MW wind power shortfall (relative to the forecast) under scenario s_1, while reducing its own production under scenario s_2 due to 250 MW of excess wind power. The expected wind power curtailment is zero, and the costly fast-start generator G3 is never called upon.

Turning to the prices in the efficient solutions ($Stoch$-MP and $Stoch$-Opt), we see that the DA price is unique, and equals the marginal cost of generator G1 covering its production and start-up costs. Unlike the DA price, the RT prices under both scenarios are non-unique due to degeneracy. The RT price under scenario s_1 lies between \$60/MWh and \$110/MWh. The lower price bound, i.e., \$60/MWh, is indeed the G2's offer price (production cost) in case a marginal decrease in production level is needed. Similarly, the higher price bound should be determined based on the cost incurred by a marginal increase in production level, which is \$122/MWh (G3's offer price covering its production and start-up costs). However, this bound cannot exceed \$110/MWh. The reason, as discussed in the companion paper [1], is that in $Stoch$-MP and $Stoch$-Opt (and also in Seq with VB, and Seq-SS) the expected RT price should be equal to the DA price. Since DA price is \$55/MWh, and the lowest RT price under scenario s_2 is 0, the maximum possible RT price under scenario s_1 is \$110/MWh. Likewise, the RT price under scenario s_2 varies between 0 and \$50/MWh. This price multiplicity implies that a choice of RT price for one scenario will determine the price uniquely for the other. An example of feasible selection of RT prices is \$80/MWh for scenario s_1 and \$30/MWh for s_2, since their expected value equals \$55/MWh. The appropriate selection of prices in cases with such a degeneracy condition is outside the scope of this paper. However, the interested reader is referred to [8] and [9].[2]

In Seq without VB (column 3 of Table II), the DA market is first cleared against a deterministic net load forecast, which results in scheduling the least-cost generators, i.e., WP and G1. Then, in the sequential market-clearing process, the DA schedules are fixed boundary conditions for the RT market-clearing problem. Thus, the slow-start generator G2 is unavailable in the RT market, since it was not committed in DA market. Therefore, unlike $Stoch$-MP and $Stoch$-Opt, the wind power deficit under scenario s_1 is made up by starting up the most expensive generator, i.e., the fast-start unit G3. In addition, the excess wind power under scenario s_2 is curtailed since G1, once committed, does not have the capability to be turned down. In this case, the DA and the expected RT prices are unique but not identical. The total expected system cost increases in this case to \$56,500, which is 18.9% higher than that in $Stoch$-MP and $Stoch$-Opt. The reason for this increase is that all players, especially slow-start generator G2, are naive in the sense that they are dispatched deterministically in DA based on the forecast in that market without consideration of the value of flexibility in the RT market. However, the two solutions discussed next show how the participation of some market players who anticipate the probability distribution of RT prices can induce more efficient market-clearing solutions, even though the DA and RT markets are still cleared sequentially.

In the first of these two solutions (column 4 of Table II), the virtual bidder participates in the sequential DA-RT markets of Seq. In equilibrium, the bidder buys 250 MW in DA and sells it back in RT. This arbitrage action has the beneficial effect of avoiding use of the expensive fast-start generator G3 in RT under scenario s_1. However, compared to Seq without VB, the arbitrage action causes wind power to be curtailed and price to fall to zero in the RT market under scenario s_2. Overall, VB decreases the expected system cost from \$56,500 (Seq without VB) to \$55,000 (Seq with VB), but the latter cost still exceeds the ideal solution (\$47,500 in $Stoch$-MP and $Stoch$-Opt). Thus, VB has improved market efficiency, but the resulting solution is still more costly than optimal. Note that in this case, just like Seq without VB, the slow-start generator G2 is not started up in DA and therefore is unavailable to the RT market.

Exploring the prices in Seq further, we see that the markets have multiple price equilibria, but subject to the constraint that DA and expected RT price differences being arbitraged to zero. An example of feasible market prices in this case is \$55/MWh (DA), \$110/MWh (RT under scenario s_1) and 0 (RT under scenario s_2). We also see that if the markets allow for negative RT prices (which is the case in US markets, where

[1] This might be viewed as an approximation of a large system in which there are multiple generators of each type, and the fraction committed represents the proportion of those generators that are scheduled DA to be on-line. The generator's decision to commit only a proportion of the capacity represents a compromise between saving commitment costs and the reduced flexibility that results from having only half of the ramping capability available to cope with net load uncertainty. This diminished rampability is appropriately reflected in the constraints in (1a).

[2] In [10], it is pointed out that such degeneracy can usually be gotten rid of if a small amount of price-responsive demand is represented as a continuous demand function.

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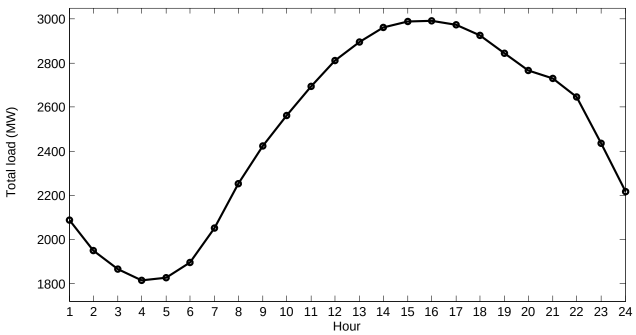

Fig. 3. IEEE RTS case study: Total load in different hours

price floors are negative), then it is possible for equilibrium RT prices in s_2 be negative.

In Seq-SS, which is the second of the two solutions with more participants in the successive markets (last column of Table II), one of the slow-start generators (G2) is allowed to self-schedule. A combination of VB and self-scheduling by G2 results in the most efficient solution (yielding the same expected system cost as for $Stoch$-MP and $Stoch$-Opt), even though the ISO clears the DA market without explicitly considering the need for flexibility in the RT market. In $Stoch$-MP, $Stoch$-Opt and Seq-SS, generator G2 is committed in DA, generator G3 is off, and neither load nor wind power is curtailed.

One question is: how might the self-scheduling of G2 come about? The answer is that G2 would find it profitable, given the prices that would otherwise occur. For instance, consider the prices observed in the solutions of Seq without VB: if the profit-maximizing owner of G2 anticipated the RT prices resulting in that solution, it would find it profitable to self-commit in the DA market, even if it does not sell any DA power. Such self-commitment would make G2 available in scenario s_1 to generate under the high prices that prevail under that net load realization. If G2 is price-taking, it would then expand its DA commitment to half of its capacity, at which point additional commitment would no longer have a positive profit. Similarly, if the DA price in Seq (with VB) is over \$55/MWh, G2 would profitably increase its self-commitment, which would then drive down price to \$55/MWh (when self-commitment reaches 0.5), at which point it would commit no additional capacity.[3]

These same relationships among the models are shown in the next set of case studies, and in addition we show how the model can quantify the economic value of introducing additional flexible resources.

[3]The presence of transmission constraints, which result in unequal LMPs at different locations whose differences will depend on which constraints bind, will make characterization of probability distributions of RT LMPs more challenging for self-scheduling generators. However, we do note that variations among LMPs within a subregion of a market (such as the Bay Area of California) are much less than variations over time (offpeak vs peak) [11], and that power marketers do carefully analyze and characterize those distributions.

TABLE III
IEEE RTS CASE STUDY: TECHNICAL CHARACTERISTICS OF DISPATCHABLE GENERATORS

Unit (i)	$\underline{P}_i$ [MW]	$\overline{P}_i$ [MW]	R_i^{D} [MW/h]	R_i^{U} [MW/h]	C_i [\$/MWh]	C_i^{SU} [\$]	Initial dispatch [MW] (initial commitment)
G1	10	40	10	10	11.09	17,462	40 (1)
G2	12	152	30	30	16.60	13,207	0 (0)
G3	10	40	10	10	11.09	17,462	40 (1)
G4	12	152	30	30	16.60	13,207	0 (0)
G5	75	300	105	105	18.52	22,313	0 (0)
G6	100	591	130	130	19.10	28,272	0 (0)
G7	0	60	60	60	22.41	10,721	0 (0)
G8	80	155	100	100	14.08	21,450	100 (1)
G9	80	155	100	100	14.08	21,450	0 (0)
G10	400	400	400	400	10.17	90,000	400 (1)
G11	400	400	400	400	10.17	90,000	400 (1)
G12	40	300	150	150	17.80	10,000	0 (0)
G13	160	310	200	200	14.08	42,900	200 (1)
G14	220	350	40	40	10.46	33,921	350 (1)

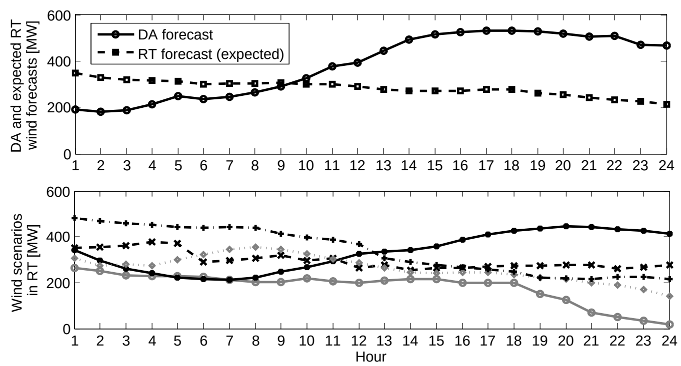

Fig. 4. IEEE RTS case study: DA and expected RT wind power forecasts (upper plot), and five scenarios in RT (lower plot)

B. IEEE RTS Case Studies

In this section, we analyze the value of various flexible resources, i.e., fast-start peaking units, slow and fast DR providers, and also VB using several cases based on the 24-node IEEE reliability test system [2].

We consider a daily time horizon (24 hours). The original system in [2] contains 14 dispatchable generators (G1 to G14), 17 loads (D1 to D17) and 34 transmission lines. The data for dispatchable generators are provided in Table III. The total installed capacity of dispatchable generators is 3405 MW. Which generators belong to which category (slow-start or fast-start) is defined below, and depends upon the specific case considered. The aggregate load profile is depicted in Fig. 3. The shape of this load profile is consistent with the load in PJM on June 25, 2015 [12]. The total load in peak hour 16 is equal to that assumed in [2] raised by 5%., i.e., 2,992.5 MW. The total load is distributed among different nodes as in [2]. The costs of load shedding for all loads are assumed to be identical, i.e., \$300/MWh.

In addition to dispatchable generators in the original paper [2], we add a wind farm at node 15, whose RT production uncertainty is represented by five equiprobable scenarios. The upper plot of Fig. 4 depicts the DA and the expected RT forecasts of wind power production, while the lower plot illustrates the five wind power scenarios in RT. During hours 1 to 9, the wind power forecast in DA is comparatively lower than the expected one in RT, while it is higher during the rest of hours. The wind power penetration, i.e., the level of expected wind power divided by total load, is 15.4% based on the DA forecast, while it is 11.4% based on the expected RT realizations. Thus, the expected wind forecast error is 4% as a fraction of total load. Introducing this "bias" into the wind forecasts makes the analysis more interesting by illustrating the impact of systematic ISO forecast error, and the economic value of flexibility and VB.

This DA forecast bias does not affect the solution of stochastic models, i.e., $Stoch\text{-}MP$ and $Stoch\text{-}Opt$, because the correct distribution of RT realizations is considered at the DA stage. However, such a bias can affect the solutions from sequential models (Seq and $Seq\text{-}SS$) since the ISO considers only the (biased) DA forecast when it clears the DA market, although virtual bidders and some generators recognize the full range of possible RT outcomes when developing their offers. Figs. 6 and 7 in the companion paper [1] illustrate the decision sequence in different models and indicate when each information is revealed. We acknowledge that the value of flexibility and VB to the market could be less in case the ISO forecast is instead unbiased, i.e., DA forecasts and expected RT wind are identical.

We augment the original RTS by adding DR providers. They are defined as loads D5, D8, D9, D13 and D16. The first three provide slow DR, while D13 and D16 are fast DR providers. We define two tranches (or "blocks") of adjustments for each DR provider. Each block for each DR provider represents an adjustment of 1% of its hourly load. The DR bidding curves of DR providers in DA and RT are schematically depicted in Fig. 5. We consider identical bid prices for upward DR in the DA market (provided by all slow and fast DR providers) and in RT (provided by only fast DR providers). More sophisticated assumptions are possible (e.g., feasible adjustments in RT might be less than DA). Each slow/fast DR provider bids the first and second upward DR blocks at prices of \$15/MWh and \$10/MWh, respectively. These represent the willingness to pay for additional power consumption. Meanwhile, each slow/fast DR provider offers the first and second downward DR blocks in DA market at prices \$60/MWh and \$70/MWh, respectively. However, each fast DR provider offers the same blocks but in RT at comparatively higher price, i.e., \$80/MWh and \$100/MWh. These prices represent the willingness to accept compensation in return for reducing load.

We consider the following four cases:

- Case 1: All dispatchable generators (G1 to G14) are slow-start, and DR resources are unavailable.
- Case 2: This case is similar to Case 1, but G7 is a fast-start dispatchable generator.
- Case 3: This case is the same as Case 2, but G5 is also a fast-start dispatchable generator.
- Case 4: This case is identical to Case 3, except that DR resources are also available.

Case 1 refers to a highly inflexible market without any flexible resource. Compared to Case 1, Case 2 is comparatively flexible, since we consider generator G7 with relatively small capacity (60 MW) as a fast-start unit. The market becomes more flexible in Case 3 in which the relatively large generator G5 with the capacity of 300 MW is considered as a fast-start unit too. Case 4 is the most flexible case, in which both slow and fast DR resources are also available.

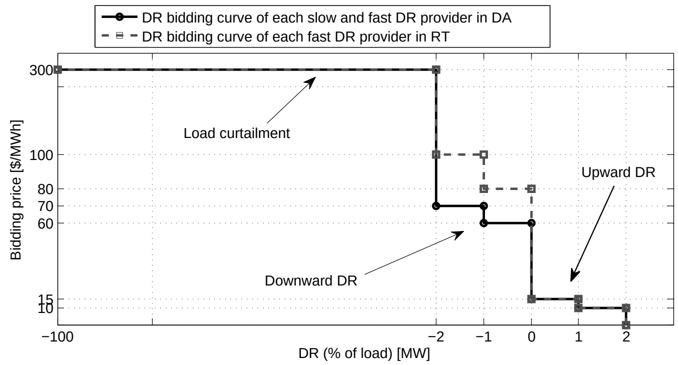

Fig. 5. IEEE RTS case study: DR bidding curves of DR providers in DA and RT

TABLE IV
IEEE RTS CASE STUDY: RESULTS OBTAINED FOR $Stoch\text{-}MP$, $Stoch\text{-}Opt$ AND Seq WITHOUT VIRTUAL BIDDING

Case	Outcome	$Stoch\text{-}MP$ and $Stoch\text{-}Opt$	Seq
Case 1	Total expected system cost [\$]	740,091	1,487,869
	Total expected curtailed load [MW]	0	2,712.3
	Average DA price [\$/MWh]	19.74	20.12
	Average expected RT price [\$/MWh]	19.74	178.72
Case 2	Total expected system cost [\$]	739,249	1,291,546
	Total expected curtailed load [MW]	0	1,965.8
	Average DA price [\$/MWh]	19.78	20.12
	Average expected RT price [\$/MWh]	19.78	160.29
Case 3	Total expected system cost [\$]	737,791	762,277
	Total expected curtailed load [MW]	0	58.1
	Average DA price [\$/MWh]	19.78	20.12
	Average expected RT price [\$/MWh]	19.78	29.81
Case 4	Total expected system cost [\$]	685,143	748,049
	Total expected curtailed load [MW]	0	0
	Average DA price [\$/MWh]	19.87	20.05
	Average expected RT price [\$/MWh]	19.87	22.84

We start by first excluding the virtual bidders, and solve $Stoch\text{-}MP$, $Stoch\text{-}Opt$ and Seq for all four cases. The results obtained are presented in Table IV. Note that the "average DA price" reported in Table IV is obtained averaging all nodal DA prices across all hours.

Similarly, the "average expected RT price" is achieved based on the expected RT prices at all nodes across all hours. As expected, $Stoch\text{-}MP$ and $Stoch\text{-}Opt$ (stochastic equilibrium and expected cost minimization/ISO stochastic optimization) yield the same market outcomes, while those of Seq (sequential ISO-based market clearing using deterministic unit commitment DA) are different and less efficient. Also as expected, the total expected system cost decreases as the flexible resources increase.[4] The cost savings can be considered the economic value of additional flexibility.

Surprisingly, the value of flexibility can be vastly different

[4]Note that cost cannot increase because making a resource flexible by allowing start-ups in RT increases the size of the feasible region.

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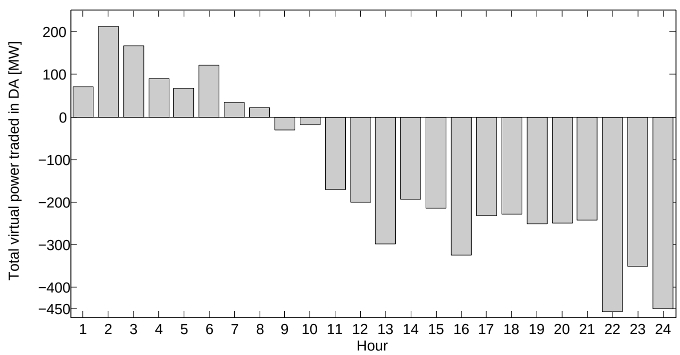

Fig. 6. IEEE RTS case study: Total virtual power traded in DA across different hours in Case 1

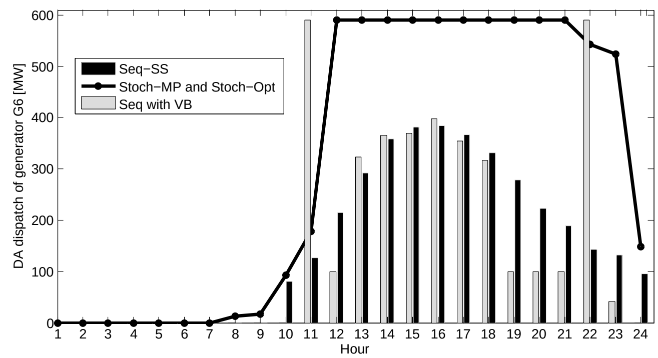

Fig. 7. IEEE RTS case study: DA schedule of generator G6 (without enforcing ramping constraints in DA) in $Stoch\text{-}MP$, $Stoch\text{-}Opt$, Seq with VB, and $Seq\text{-}SS$ with VB while generator G6 self-schedules

depending on the market design. In particular, the system cost reduction is larger in inefficient Seq than in the cost-optimal models. For example, a comparison of Cases 1 and 2 implies that making G7 a fast-start generator reduces costs slightly in $Stoch\text{-}MP$ and $Stoch\text{-}Opt$ ($842), which corresponds to about $22.61/MWh for each MWh of expected production from G7. Meanwhile, the decrease resulting from making G7 flexible in Seq is orders of magnitude larger ($196,323). This reason for this huge cost reduction in the latter case is that the inefficient model yields a costly loss of load. Its expected curtailed load across the daily horizon is reduced from 2,712.3 MW in Case 1 to 1,965.8 in Case 2. This change is even greater in Case 3 in which G5 is also considered as a fast-start unit.

Examining the prices, we see that, as expected, the DA and the expected RT prices are identical in $Stoch\text{-}MP$ and $Stoch\text{-}Opt$ across all cases. However, DA and RT prices are not arbitraged in Seq, with the average expected RT price being much higher than that in DA due to RT load curtailments. This DA-RT price gap reduces in Seq as more flexible resources become available. For example, in the most flexible case, i.e., Case 4, load curtailments are eliminated and the price gap is only $2.79/MWh. However, the expected system cost of Seq still exceeds that in $Stoch\text{-}MP$ and $Stoch\text{-}Opt$ due to less efficient utilization of generators and DR providers in Seq.

We then introduce virtual bidders at every node to Seq for all four cases. Note that VB would not change the results of $Stoch\text{-}MP$ and $Stoch\text{-}Opt$. As discussed in the companion paper [1], we assume that all virtual bidders in DA have perfect knowledge of the distribution of RT prices. Remarkably, such virtual bidders are sufficient to make Seq's results converge to the expected cost minimization outcomes of $Stoch\text{-}MP$ and $Stoch\text{-}Opt$, i.e., the third column of Table IV. This means that in these particular cases, virtual bidders by themselves lead the deterministic day-ahead market to choose the most efficient unit commitment. In contrast, in the simple illustration of the previous subsection, self-scheduling by a profit-maximizing slow-start generator was also needed. We consider Case 1 to explore why this occurred. In Seq without VB, there are out-of-merit dispatches in DA due to ramping constraints of dispatchable generators. For example, those constraints force the ISO to commit generator G9 despite its comparatively high start-up cost, while generators G2 and G4 with lower start-up costs are kept off. But in the same model with VB, virtual trading alters the net DA load in each hour and consequently changes the DA dispatch so that G9 is constrained off, whereas G2 and G4 are dispatched. The total virtual power traded in DA market across different hours is illustrated in Fig. 6. Virtual bidders sell energy in DA during hours 1 to 9, when the DA wind forecast is lower than the expected RT forecast (upper plot of Fig. 4), and buy back the power in RT. Then in the rest of the hours, VBs buy power in the DA market and sell back in RT to compensate for DA overforecasts of wind output. In these cases, $Seq\text{-}SS$, with the inclusion of self-scheduling slow-start generators, is unneeded to achieve the efficient solution; VB suffices.

To analyze a case with less out-of-merit order dispatch in DA market, we relax the ramping constraints of dispatchable generators in DA (while still enforcing them in RT) and solve Case 1 considering the three different models. Thus, the DA model in Seq is inaccurate not only because it ignores the need for flexibility in RT but also because it omits important generator limitations. The results obtained are presented in Table V. The total expected system cost in $Stoch\text{-}MP$ and $Stoch\text{-}Opt$ is identical to that in Table IV with ramping constraints in DA. However, the DA schedules can be different, which indicates the optimal schedule is not unique. But unlike $Stoch\text{-}MP$ and $Stoch\text{-}Opt$, the DA ramping constraints were active in Seq (without VB), because the expected system cost is increased from $1.48 million (Table IV) to $1.96 million (Table V). The reason for this increase is that the DA dispatches without ramping constraints in the sequential two-settlement system are more inefficient, in the sense that they increase the magnitude of infeasibility in RT that the operator needs to try to correct, resulting in greater RT load curtailment. However, the inclusion of VB in Seq reduces expected system costs significantly, but now they remain slightly higher than costs in $Stoch\text{-}MP$ and $Stoch\text{-}Opt$. This confirms the conclusion of the simple example (Section III.A) that VBs alone may not lead a deterministic DA market to choose the most efficient resource commitment. A comparison of the schedules of $Stoch\text{-}MP$, $Stoch\text{-}Opt$ and Seq (with VB) indicates that the DA position of some slow-start generators, especially G6 (see Fig. 7), in Seq is still inefficient. According

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TABLE V
IEEE RTS CASE STUDY: RESULTS IN CASE 1 WITHOUT ENFORCING RAMPING CONSTRAINTS IN DA

Outcome	*Stoch-MP* and *Stoch-Opt* (with/without VB)	*Seq* (without VB)	*Seq* (with VB)	*Seq-SS* (with VB; generator G6 self-schedules)
Total expected system cost [$]	740,091	1,962,780	751,579	740,519
Total expected curtailed load [MW]	0	4,406.1	10.1	0

TABLE VI
IEEE RTS CASE STUDY WITH INCREASED WIND PENETRATION: RESULTS IN CASE 1 WITHOUT ENFORCING RAMPING CONSTRAINTS IN DA

Wind power penetration* based on DA and expected RT forecasts	Outcome	*Stoch-MP* and *Stoch-Opt* (with/without VB)	*Seq* (without VB)	*Seq* (with VB)	*Seq-SS* (with VB; generator G6 self-schedules)
30.8% (DA), 22.8% (RT)	Total expected system cost [$]	623,947	2,359,824	644,046	624,845
	Total expected curtailed load [MW]	0	6,260.8	9.2	0

* The wind power penetration refers to the level of (expected) wind power forecast divided by total load.

to Fig. 7, generator G6 in *Stoch-MP* and *Stoch-Opt* (ideal solution) is fully dispatched in hours 12 to 21, and its expected profit is $2139. However, in *Seq* with VB, its hourly DA schedule is often lower than that in the ideal solution. In addition, G6 incurs significant start-up costs in hours 11 and 22. This naive DA schedule of G6 considerably reduces its expected profit to -$18,000. If G6 is aware of this expected loss, it would prefer to self-schedule, as in *Seq-SS*, because it could eliminate its entire expected loss (earning zero profit in *Seq-SS*), while lowering overall system costs. However, its earnings are still lower than what it would earn in the fully efficient market ($2139 in *Stoch-MP* and *Stoch-Opt*), where market efficiency is also better. This example demonstrates that virtual bidders plus self-scheduling by one slow-start generator can help improve market efficiency when the ISO uses a deterministic DA scheduler, but is not sufficient to achieve full optimality.

C. IEEE RTS Case Study with Increased Wind Penetration

In this section, we analyze the impact of increased wind power penetration on performance of the four models. This is motivated by European power systems with high wind penetration, such as Denmark and Germany. Such systems have been the focus of other work on market-clearing mechanisms. For instance, [13] addresses the limitations of the current sequential market-clearing system in Germany in terms of coping with high variable renewable penetration. Likewise, [14] examines the role of support payments to flexible resources in changing the DA dispatch in Germany in order to back up variable renewable output.

We build upon the last version of Case 1 from Section III.B, in which we relaxed the ramping constraints of dispatchable generators in DA while enforcing them in RT (Table V). In particular, we consider a high wind case (Table VI). This case has double the amount of DA wind forecast (from 15.4% in Section III.B to 30.8%) as well as double the wind in each RT scenario (wind power penetration based on expected RT output is 22.8%, rather than the 11.4% in Section III.B). Accordingly, the DA wind forecast bias is 8% of the total load while it was 4% in Table V of Section III.B.

Note that the hourly patterns of wind power scenarios are identical to that in Fig. 4 of Section III.B, but their magnitude is scaled up in proportion to the overall increase in wind MWh. The capacity of transmission lines is raised by 15% to facilitate higher wind integration.

The results obtained are given in the second row of Table VI, whose structure is identical to that of Table V in Section III.B. In this case with increased wind penetration (and increased forecast error), the total expected system cost in *Seq* (with VB) and *Seq-SS* is 3.22% and 0.13% higher than that in the ideal solution (*Stoch-MP* and *Stoch-Opt*), respectively. In comparison, those two values in Section III.B (Table V) were roughly half as large (1.55% and 0.06%, respectively). However, these inefficiencies are much less than what occurs in the absence of VB and self-scheduling, in which case the costs are several times larger than the efficient cost level. This example demonstrates that VB and self-scheduling can still greatly enhance market efficiency when the ISO uses a deterministic DA scheduler, but that the remaining inefficiency can increase with the amount of wind penetration.

One interesting observation is that the results given in Table VI, except for *Seq* (without VB), are unchanged across the two cases with the same RT wind realizations but with a lower degree of bias in the DA forecast (either 4% and 0% overforecast, instead of the value 8% in Table VI, i.e, the forecast DA wind penetration in these two cases is 26.8% and 22.8%, respectively). Note that the case with 0% DA forecast bias still has some bias in some hours, since the daily pattern of DA and expected RT forecasts is different. The reason for achieving the same results (except for *Seq* without VB) in this specific example is that the perfect VB adapts itself to this bias and eliminates it.

D. Computational Performance

The linear optimization problem in *Stoch-Opt* and the sequential linear optimization problems in *Seq* (without VB) are solved using CPLEX under GAMS. In addition, the MLCPs for models *Stoch-MP*, *Seq* (with VB) and *Seq-SS* are solved using PATH under GAMS. We solve all problems on an Intel(R) Xeon(R) E5-1650 with 12 processors clocking at 3.50 GHz and 32 GB of RAM. The maximum number of variables occurs in model *Seq-SS*, which is 27,456 in the cases reported in Sections III.B and III.C. The CPU time for linear problems in Sections III.B and III.C is a few seconds, while it is about 4-7 minutes for the MLCPs.

IV. Conclusions

Our four models and their computational applications show that deterministic day-ahead scheduling by ISOs can result in large inefficiencies that, however, can be largely or entirely overcome by financial arbitragers (virtual bidding) together with some self-scheduling by large slow-start conventional generators.

Therefore, it is possible that a subset of market parties acting on high quality stochastic information can help the market achieve the same efficiencies as central stochastic clearing by the ISO. Hence, in spite of the theoretical appeal of having an ISO use stochastic unit commitment to clear the market, the practical and political difficulties of that approach together with the efficiency of the $Seq\text{-}SS$ results suggest that ISOs should not rush to embrace the central stochastic model. Rather, they should instead carefully consider whether self-scheduling and virtual bidding in fact already allow markets to realize most of the potential efficiencies of stochastic scheduling.

However, the above conclusions presume that the virtual bidders and self-schedulers can anticipate the probability distribution of real-time prices based on experience and analysis. To the degree that price formation (rules for price calculation and settlement) and market conditions (such as line outages) are transparent in markets, and to the extent that market conditions are stable, this assumption is more likely to be valid. Highly complex market software and poorly communicated or highly unstable market conditions will mean that prices and their distributions will be difficult for market participants to forecast. This will lessen or even eliminate the ability of those participants to correct the inefficiencies of deterministic market-clearing algorithms used by ISOs.

To provide a more robust estimate of the value of flexible resources, virtual bidding and self-scheduling, the comparison of the market models in this paper can be extended to include an "out-of-sample" simulation, considering many samples of possible wind realizations. These samples are not necessarily identical to those wind scenarios considered at the day-ahead stage. Such out-of-sample analysis would assess the impact of a fundamental limitation of stochastic programming: that only a finite set of scenarios can be considered in the optimization [6].

A further extension of interest would be to consider "imperfect" virtual bidders, whose knowledge on probability distribution of real-time prices are not perfect. We hypothesize that imperfect virtual bidding can still improve market solutions, as long as the bidders' distributions are not too far from actual distributions; on the other hand, large errors in those distributions might worsen market outcomes. Moreover, in line with [15] and [16], it could be of interest to analyze the impacts of virtual bidding on market power mitigation in oligopolistic electricity markets.

Appendix

The KKT conditions associated with $Stoch\text{-}MP$ in the companion paper [1] include three condition sets: i) all constraints within problems (1a)-(1g) and balance conditions (1h), ii) complementarity conditions corresponding to inequality constraints, and iii) conditions obtained from differentiating the Lagrangian of each optimization problem (1a)-(1g) with respect to its primal variables. An example of the members of the third condition set is given by (5) below, which is derived from differentiating the Lagrangian of problem (1a) with respect to DA schedule of slow-start generator $i \in \mathcal{S}$, i.e., $p_{i,t}^{\mathrm{DA}}$:

$$\begin{aligned}
&C_i - \lambda^{\mathrm{DA}}_{(n:i\in\Psi_n),t} + \overline{\mu}_{i,t} - \underline{\mu}_{i,t} + \mu^{\mathrm{U}}_{i,t} - \mu^{\mathrm{U}}_{i,(t+1)} \\
&- \mu^{\mathrm{D}}_{i,t} + \mu^{\mathrm{D}}_{i,(t+1)} + \sum_s \Big[\overline{\rho}_{i,t,s} - \underline{\rho}_{i,t,s} + \rho^{\mathrm{U}}_{i,t,s} \\
&-\rho^{\mathrm{U}}_{i,(t+1),s} - \rho^{\mathrm{D}}_{i,t,s} + \rho^{\mathrm{D}}_{i,(t+1),s}\Big] = 0 \quad \forall i \in \mathcal{S}, t. \qquad (5)
\end{aligned}$$

The KKT conditions associated with the single optimization problem in $Stoch\text{-}Opt$ are identical to those conditions in $Stoch\text{-}MP$. This proves that $Stoch\text{-}MP$ and $Stoch\text{-}Opt$ are equivalent. These two models provide benchmarks for evaluating other models, since they determine the efficient DA commitment and energy schedules that minimize expected system cost. However, Seq differs from the first two models, since the ISO uses a short-sighted deterministic unit commitment model in DA market. The KKT conditions associated with Seq are *not* identical to those of $Stoch\text{-}MP$ and $Stoch\text{-}Opt$. For example, the condition derived from differentiating the Lagrangian of problem (3a) with respect to $p_{i,t}^{\mathrm{DA}}$ is given by (6), which is different than (5):

$$\begin{aligned}
&C_i - \lambda^{\mathrm{DA}}_{(n:i\in\Psi_n),t} + \overline{\mu}_{i,t} - \underline{\mu}_{i,t} + \mu^{\mathrm{U}}_{i,t} - \mu^{\mathrm{U}}_{i,(t+1)} \\
&- \mu^{\mathrm{D}}_{i,t} + \mu^{\mathrm{D}}_{i,(t+1)} = 0 \quad \forall i \in \mathcal{S}, t. \qquad (6)
\end{aligned}$$

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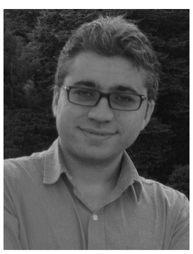

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