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# Effect of thermal cycling on martensitic transformation and mechanical strengthening of stainless steels – A phase-field study

Hemantha Kumar Yeddu<sup>a,\*</sup>, Brian A. Shaw<sup>a</sup>, Marcel A.J. Somers<sup>b</sup>

<sup>a</sup>*School of Mechanical and Systems Engineering, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom.*

<sup>b</sup>*Department of Mechanical Engineering, Technical University of Denmark (DTU), 2800 Kgs. Lyngby, Denmark.*

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## Abstract

A 3D elastoplastic phase-field model is used to study the effect of thermal cycling on martensitic transformation as well as on mechanical strengthening of both austenite and martensite in stainless steel. The results show that with an increasing number of thermal cycles, martensite becomes more stable. Increase in strain, plastic strain and strain hardening lead to strengthening of austenite.

*Keywords:* phase-field model, martensitic transformation, reversion, microstructure, thermal cycling, steels

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## 1. Introduction

Phase transformations play an important role in enhancing the mechanical properties of stainless steels. The solid state phase transformation of austenite to martensite, known as martensitic transformation, occurs during

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\*Corresponding Author. E-mail: hemanth.yeddu@ncl.ac.uk, hemu23@gmail.com

quenching and imparts significant strength to steels. Reverse phase transformation of martensite to austenite occurs during intercritical annealing and has been reported to improve the yield strength, by grain refinement, and the ductility of steels [1–3].

Reversion of martensite can occur either by a shear mechanism or a diffusion-controlled mechanism. During reversion by a shear mechanism, dislocations from martensite are inherited into reversed austenite and thereby increase the ductility of steels [1, 4]. Moreover, reversion of martensite leads to grain refinement [1, 2, 5, 6] and grain boundary strengthening [7], which are reported to be effective strengthening mechanisms. Grain refinement can lead to reduction of  $M_s$  temperature [8], increased retained austenite [9, 10] and dislocation density [11].

Owing to the importance of martensite formation, reverse phase transformation and grain refinement in enhancing the mechanical properties of steels, several thermo-mechanical processing methods have been developed [1, 6, 12]. Thermal cycling, i.e. repeated quenching and subsequent heating, has proved to be an effective way of grain refinement and strengthening of steels [10]. Durlu reported an increase in dislocation density and strength after thermal cycling of Fe-Ni-C single crystals [13]. Alaei et al. have recently showed that dislocations are inherited from martensite to reversed austenite and that the dislocation density as well as the yield strength increase with increasing number of thermal cycles in an Fe-Ni-C TRIP steel [14]. Although the experimental studies showed that thermal cycling leads to strengthening of steels [10, 13, 14], it is essential to study the role of austenite and martensite in mechanical strengthening due to thermal cycling.

Several constitutive and phenomenological models have been proposed to study martensite formation and the relation between phase transformation and plasticity [15–17]. The phase-field approach [18, 19] has been successfully applied to study martensitic transformation and other solid state phase transformations [20–27] as well as the reversion of martensite to austenite by a shear mechanism [26, 28, 29]. In the present work, the effect of thermal cycling on martensite formation and reversion of martensite by a shear mechanism as well as on mechanical strengthening of stainless steel is studied, by using a 3D elastoplastic phase-field model [20, 28].

## 2. Phase-field model

The phase-field equation governing the microstructure evolution is given by:

$$\frac{\partial \eta_p}{\partial t} = - \sum_{q=1}^{q=v} L_{pq} \frac{\delta G}{\delta \eta_q} \quad (1)$$

where  $\eta_q$  is the phase field variable that tracks the evolution of martensite,  $v$  is the total number of martensite variants and  $L_{pq}$  is a matrix of kinetic parameters. Martensite variants (laths), which form in 24 different crystallographic orientations according to the Kurdjumov-Sachs (K-S) orientation relationship (OR), can be grouped into three basic variants known as Bain variants [30–32]. In order to simulate 24 different martensite variants, the model needs to consider 24 phase-field variables, which increases the computational complexity of the model. Therefore in the present work three phase-field variables ( $\eta_1, \eta_2, \eta_3$ ) that correspond to the three Bain variants, which form the basis for the K-S OR, are considered [24].

The Gibbs energy of a system undergoing athermal martensitic transformation can be expressed as:

$$G = \int_V (G_v^{chem} + G_v^{grad} + G_v^{el}) dV \quad (2)$$

where  $G_v^{chem}$  corresponds to the chemical part of the Gibbs energy density,  $G_v^{grad}$  is the gradient energy term,  $G_v^{el}$  is the elastic strain energy density.

$G_v^{chem}$  is expressed as a Landau-type polynomial [20, 21]:

$$G_v^{chem}(\eta_1, \eta_2, \eta_3) = \frac{1}{V_m} \left[ \frac{1}{2} A (\eta_1^2 + \eta_2^2 + \eta_3^2) - \frac{1}{3} B (\eta_1^3 + \eta_2^3 + \eta_3^3) + \frac{1}{4} C (\eta_1^2 + \eta_2^2 + \eta_3^2)^2 \right] \quad (3)$$

where  $V_m$  is the molar volume and the coefficients  $A, B, C$  are expressed in terms of Gibbs energy barrier and the driving force [20].

$G_v^{grad}$  is expressed as [20, 21]:

$$G_v^{grad} = \frac{1}{2} \sum_{p=1}^{p=3} \beta_{ij}(p) \frac{\partial \eta_p}{\partial r_i} \frac{\partial \eta_p}{\partial r_j} \quad (4)$$

where  $\mathbf{r}(\mathbf{x}, \mathbf{y}, \mathbf{z})$  is the position vector expressed in Cartesian coordinates.  $\beta_{ij}$  is the gradient coefficient matrix expressed in terms of the interfacial energy, molar volume and the Gibbs energy barrier.

$G_v^{el}$  can be expressed as:

$$G_v^{el} = \int_{\epsilon_{ij}^0(\mathbf{r})}^{\epsilon_{ij}(\mathbf{r})} c_{ijkl} \left( \epsilon_{kl}(\mathbf{r}) - \epsilon_{kl}^0(\mathbf{r}) - \epsilon_{kl}^{pl}(\mathbf{r}) \right) d\epsilon_{ij}(\mathbf{r}) \quad (5)$$

where  $c_{ijkl}$  is the tensor of elastic constants,  $\epsilon_{ij}(\mathbf{r})$  is the total strain,  $\epsilon_{kl}^{pl}(\mathbf{r})$  is the plastic strain and  $\epsilon_{ij}^0(\mathbf{r})$  is the stress-free transformation strain expressed in terms of  $\eta_q$  and Bain strains ( $\epsilon_{ij}^{00}$ ).

The material undergoes plastic deformation when the internal stress exceeds the yield limit. The evolution of plastic strain  $\epsilon_{ij}^{pl}(\mathbf{r})$  is governed by [20, 23]:

$$\frac{\partial \epsilon_{ij}^{pl}(\mathbf{r})}{\partial t} = -k_{ijkl} \frac{\delta G_v^{shear}}{\delta \epsilon_{kl}^{pl}(\mathbf{r})} \quad (6)$$

where  $G_v^{shear}$  is the shear energy density and  $k_{ijkl}$  is the plastic kinetic coefficient.

Linear isotropic strain hardening is considered by using the following expression [33]:

$$\sigma_y = \sigma_y^0 + H \epsilon^{pl}(\mathbf{r}) \quad (7)$$

where  $\sigma_y$  is yield stress of the material that depends on plastic strain,  $\sigma_y^0$  is initial yield stress,  $H$  is hardening modulus and  $\epsilon^{pl}(\mathbf{r})$  is von Mises equivalent plastic strain.

Finally the total strain is calculated by solving the mechanical equilibrium equation:

$$c_{ijkl} \left( \frac{\partial \epsilon_{kl}}{\partial r_j} - \sum_{p=1}^{p=v} \epsilon_{kl}^{00}(p) \frac{\partial \eta_p(\mathbf{r})}{\partial r_j} - \frac{\partial \epsilon_{kl}^{pl}(\mathbf{r})}{\partial r_j} \right) = 0 \quad (8)$$

The following input simulation data corresponding to stainless steels with a composition of Fe-17 wt %Cr-7 wt %Ni are acquired from different sources, such as CALPHAD, ab initio calculations and experiments [25, 28]:  $A = 1188$  J/mol,  $B = 3564$  J/mol,  $C = 2376$  J/mol,  $\beta = 0.1061 \times 10^{-10}$  J/m; Bain strains are  $\epsilon_1 = 0.1316$ ,  $\epsilon_3 = -0.1998$ ; elastic constants of austenite are  $C_{11} = 209$  GPa,  $C_{12} = 133$  GPa and  $C_{44} = 121$  GPa; elastic constants of martensite are  $C_{11} = 248$  GPa,  $C_{12} = 110$  GPa and  $C_{44} = 120$  GPa;  $\sigma_y^0$  (austenite) = 500 MPa,  $\sigma_y^0$  (martensite) = 800 MPa,  $H = 738$  MPa,  $k = 0.2$   $GPa^{-1}s^{-1}$ , driving force =  $-3600$  and  $+150$  J/mol at  $T = 263$  K and  $975$  K,

respectively.

A single crystal of austenite of 1  $\mu m$  grain size is subjected to several thermal cycles with temperature varying between 263 K and 975 K as shown in Fig. 1a. The same process is repeated under two different conditions, viz. (a) with and (b) without strain hardening in order to study its effect on the transformation. A pre-existing martensite embryo is considered in the center of the grain, in order to maintain the symmetry in the cubic grain. Dirichlet (clamped) boundary conditions are considered. Simulations are performed on a 50 x 50 x 50 mesh by using FemLego software [34]. Due to the lack of available experimental data on the kinetics of lath martensite,  $L_{pq}$  in Eq. (1) is considered to be unity and the microstructure evolution is discussed in terms of dimensionless time,  $t^*$ .

### 3. Results and discussion

Fig. 1b (side view) and 1c-f (top view) shows the microstructures obtained during different stages of thermal cycling shown in Fig. 1a. Martensite laths (variants), plotted in red, blue and green, are formed during quenching (Fig. 1b). The regions where martensite has formed during quenching are marked by a boundary on the  $(111)^\gamma$  plane (Fig. 1c and e) and these boundaries are superimposed on the microstructures obtained after the corresponding thermal cycles (Fig. 1d and f), i.e. microstructure boundary in Fig. 1c is superimposed on Fig. 1d and the boundary in Fig. 1e is superimposed on Fig. 1f. The regions in white that lie inside the boundary in Fig. 1d and f correspond to reversed austenite, whereas the regions in white that lie outside the boundary correspond to retained austenite. Fig.

1d shows that martensite units revert to austenite (circles, rectangles and arrows) during heating. Some martensite units (compare Fig. 1c and d) completely revert (rectangles), whereas some partially revert to austenite (circles). Reversion initiates at lath boundaries, where a low nucleation energy is sufficient to overcome the transformation barrier (Fig. 1d and f) [28]. Reversion proceeds by interface migration, which is in good agreement with Durlu's experimental study on Fe-Ni-C alloy [13]. Martensite units that are surrounded by retained austenite (arrows in Fig. 1d and f) offer less resistance to reversion and hence revert more compared to the martensite units that are surrounded by other martensite units (big variant in red at the center). Reversion of martensite is prominent during early stages of thermal cycling (compare Fig. 1c and d), whereas less volume fraction of martensite reverts to austenite during the later stages of thermal cycling (compare Fig. 1e and f). Fig. 1g shows the von Mises equivalent plastic strain plot corresponding to the microstructure shown in Fig. 1d. Although some regions in the microstructure have reversed to austenite (Fig. 1d), plastic strains created during martensite formation are retained in the reversed austenite [28]. This plot also shows that a martensitic lath-like structure is inherited by the reversed austenite.

Fig. 2 shows variations in volume fraction of martensite with dimensionless time  $t^*$ . During the heating stage of each thermal cycle, the martensite volume fraction decreases due to reversion of martensite to austenite. The volume fraction of martensite that reverts to austenite decreases with increasing number of thermal cycles, as also observed in microstructures (Fig. 1), and suggests an increased stability of martensite. It also suggests that the



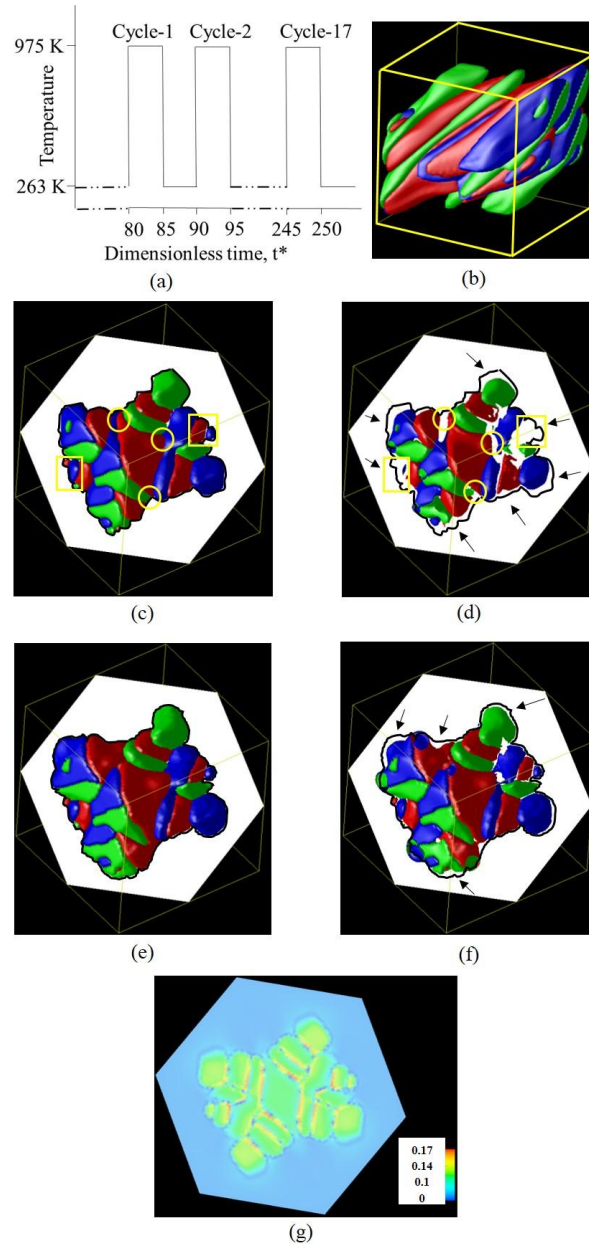


Figure 1: (a) Schematic of the simulated thermal cycling process. Microstructures at (b)  $t^* = 80$  (side view) (c)  $t^* = 80$  (top view) (d)  $t^* = 85$  (e)  $t^* = 190$  and (f)  $t^* = 195$  and (g) von Mises equivalent plastic strain plot of the microstructure in (d). Martensite variants are shown in red, blue and green. Austenite on the  $(111)^\gamma$  plane is shown in white. Arrows point towards the areas where reversion occurs.

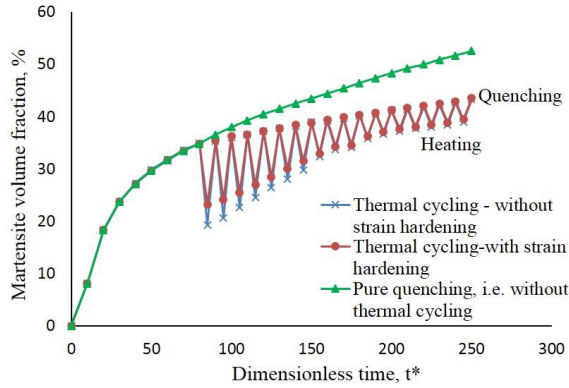


Figure 2: Variation in martensite volume fraction.

surrounding retained austenite resists the reversion of martensite, suggesting an increased strength and stability of austenite.

Fig. 3 shows variations of the mean von Mises equivalent stress, strain and plastic strain in austenite and martensite, with the number of thermal cycles. The mean von Mises equivalent stress (Fig. 3a) in martensite decreases with increasing number of thermal cycles, due to the relaxation provided by reversion during heating. The internal stresses in martensite can add to the driving force for formation of austenite and can lead to reversion of martensite [28]. However, in the present case the equivalent stress in martensite decreases, which implies less driving force for formation of austenite. Hence martensite becomes more stable with increasing number of thermal cycles (Fig. 2). The von Mises equivalent stress in austenite does not show significant variations, although it slightly increases compared to the as-quenched case (cycle-0).

The mean von Mises equivalent strain (Fig. 3b) in martensite decreases with increasing number of thermal cycles, due to stress relaxation as ex-

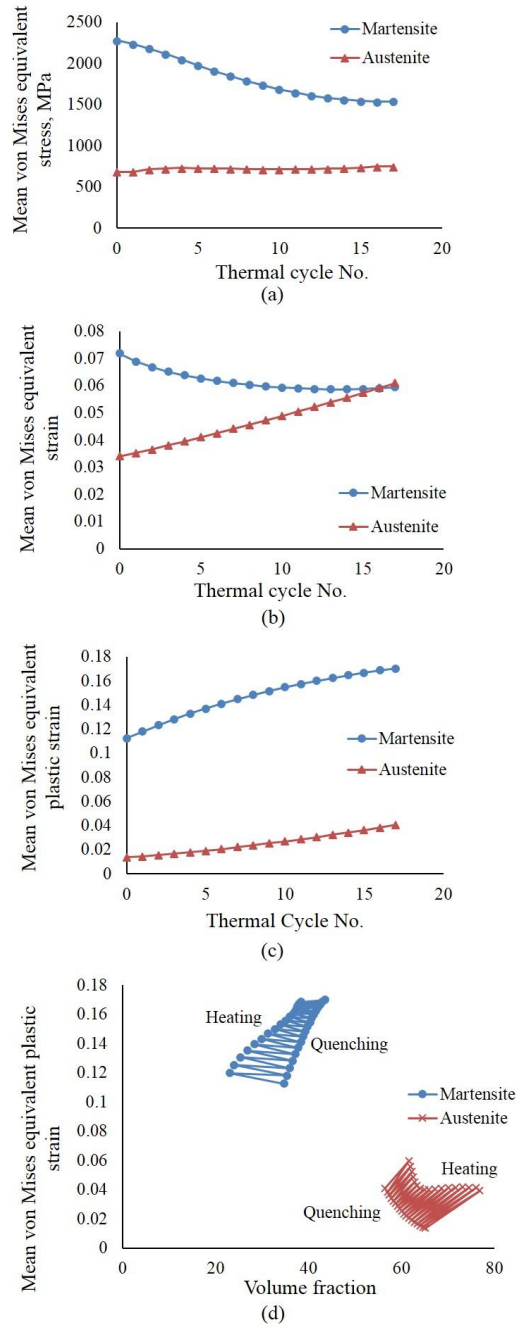


Figure 3: Variation of mean von Mises equivalent (a) stress (b) strain (c) plastic strain with thermal cycles (d) variation of mean von Mises equivalent plastic strain with volume fraction.

plained above. However, von Mises equivalent strain in austenite increases significantly. The increased strain in austenite increases the strain energy, which opposes the chemical energy (thermodynamic driving force for martensite formation) and thus martensite formation becomes more difficult. This requires an increased driving force, which implies a decrease in  $M_s$  temperature. This is in good agreement with the experimental study of Hidalgo and Santofimia, who observed that  $M_s$  temperature of Fe-C-Si steel decreases with increasing number of thermal cycles due to strengthening of austenite [10]. Lee has also observed that the  $M_s$  temperature of an Fe-Mn alloy decreases with increasing number of thermal cycles [35].

In the experimental work presented in Ref. [10], the strengthening of austenite is mainly attributed to grain refinement. **In the present simulations the prior austenite grain size is kept constant during thermal cycling. Although some regions in the prior austenite grain transform to martensite and back to reversed austenite (Fig. 1d) and give rise to finer austenite grains (grain refinement), the variations in austenite strength due to grain refinement is not considered in this work. Hence grain refinement can-not be the reason for strengthening of austenite.** Alternatively, plasticity and strain hardening contribute to the strengthening of austenite. The mean von Mises equivalent plastic strains (dislocation density) in both austenite and martensite increase with increasing number of thermal cycles (Fig. 3c). Hidalgo and Santofimia observed that dislocation density increases with decreasing grain size due to thermal cycling of an Fe-C-Si steel [10]. Several other experimental studies also showed that dislocation density increases with thermal cycling of steels

[13, 14, 35]. Lee [35] has reported that the increase in dislocation density increases the shear stress, which requires larger driving force for martensite formation and thereby decreases the  $M_s$  temperature.

The increase in plastic strains, i.e. dislocation density, leads to strain hardening and thereby contributes to the strengthening of austenite. The effect of strain hardening on strength of austenite can be seen in Fig. 2. In the absence of strain hardening, the ‘weaker’ austenite facilitates reversion of martensite more easily compared to the ‘stronger’ strain hardened austenite. In the absence of strain hardening, several thermal cycles are needed for accumulation of plastic strains and for the material to be strong enough to resist reversion of martensite. In the presence of strain hardening, the strain hardening of the material provides an extra resistance to reversion of martensite already from the initial stages of thermal cycling. Thus during the initial stages of thermal cycling, there is a visible difference between the martensite volume fractions obtained in the two cases (Fig. 2). However, during the later stages of thermal cycling, the material in both cases is strong enough to resist reversion and hence the difference in volume fractions obtained in the two cases decreases.

During reversion in a given thermal cycle, the equivalent plastic strain in austenite increases more compared to that in martensite (Fig. 3d). In the 301-type steel studied in the present work, reversion occurs through a shear mechanism and therefore dislocations are inherited from martensite to reversed austenite [2, 5, 28]. Moreover, since the highly dislocated martensite is less effective in accommodating plastic strains during reversion, austenite deforms more compared to martensite, which is in agreement with Ref. [10].

## 4. Conclusions

The phase-field simulations show that martensite becomes more stable with increasing number of thermal cycles, due to decreasing stresses in martensite and due to strengthening of austenite. Austenite strengthening is due to increase in plastic strain, i.e. dislocation density, which leads to strain hardening of the material. The strain in austenite increases with increasing number of thermal cycles and increases the energy barrier for martensitic transformation, implying a decreased  $M_s$  temperature. The results are in good agreement with experimental observations, such as increase in dislocation density and strengthening of austenite with increasing number of thermal cycles [10, 35]. Lee [35] has reported that the increase in dislocation density increases the shear stress and thereby decreases the  $M_s$  temperature.

In reality, austenite grains (typically around  $10 - 50 \mu$ ) contain lath martensite that forms in blocks and packets, which also act as barriers for dislocation motion. It is not possible to observe this hierarchic pattern formation in the simulated microstructures, due to the small grain size of  $1 \mu$ . Nevertheless, an encouraging correlation between the trends in the present simulations and experimental works is obtained, which highlights the effect of thermal cycling on strengthening of stainless steels. In the future, it would be interesting to model the effect of thermal cycling on martensitic transformation in polycrystalline steels.

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