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VALUE OF INFORMATION BY UPDATING OF MODEL UNCERTAINTIES UTILISING PROOF LOADING IN THE CONTEXT OF SERIES AND DANIELS SYSTEMS

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Abstract

In this paper, an approach is presented for the determination of the Value of Information (VoI) in relation to models which can represent structural systems such as e.g. towers, cables, jackets. Stochastic capacities and loads are assumed for the models studied herein. The VoI is obtained with a prior and a pre-posterior decision analysis. The prior decision analysis takes basis in the design phase of the structural system. Pre-posterior decision analysis builds upon modelling results of not yet conducted experiments. In order to perform the prior and pre-posterior Bayesian decision analysis, the expected life-cycle benefit of the considered systems are computed. The difference in the expected benefits relating to the prior and pre-posterior decision analysis leads to the VoI. The system models are probabilistically computed using the Monte Carlo / Importance sampling simulations to estimate their probability of failure. Next to the intrinsic uncertainties in loads and capacities further uncertainties accounting for the model uncertainties are included in the simulations. As an SHM strategy, proof loading is considered and modelled as a process accompanying the construction. The costs of proof loading and probable component failures are considered explicitly. The analyses results point to high Value of Information for component proof loading in systems with a low reliability.

1 INTRODUCTION

It is currently often unclear whether experiments, e.g. proof load testing that provide data on the structural performance are beneficial. A method in order to assess this benefit is the Bayesian decision theory. The decision analysis is based on system models (section 2). The discussed series system could represent a monopile and the Daniels system a jacket substructure. The system models and measurements incorporate model and measurement uncertainties, respectively. In section 3 the model updating is explained and with a short outline of the Bayesian decision theory is given. Section 4 presents the modelling results taking basis in the economics of offshore wind turbines. The paper is concluded and summarised in section 5.

2   SYSTEM MODELS

Various typical structural systems can be represented by generic models. A series system can represent e.g. a tower or single fibre cable consisting of several components. A Daniels system can present e.g. cables, tendons with several fibres or jackets and other truss structures. The failure of the discussed systems is described by the limit state function (Equation 1).

$$R \cdot M_R - S \cdot M_S \leq 0 \tag{1}$$

with the resistance force, R, and the load, S. The model uncertainties for R and S are represented by M_R and M_S respectively. All four random variables are sampled as described in Table 1. The limit state function in equation (1) is used in order to determine the failure probabilities of the systems described in section 2.2 and 2.3.
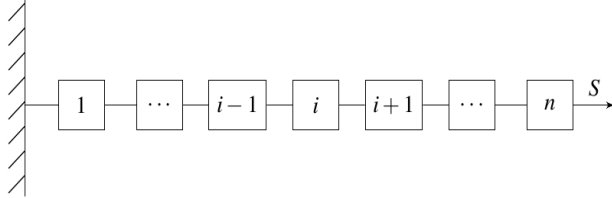
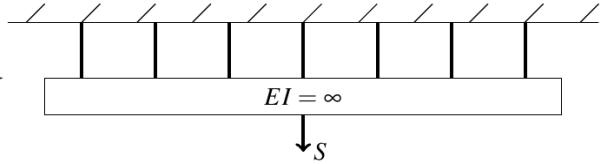### 2.1 SERIES SYSTEMS

Figure 1: Series system.

Figure 2: Daniels system with infinitively stiff bar for the instantaneous load redistribution.

A series system fails if any of its n components fail. Equation 2 follows the definition in [9]. The weakest component determines the capacity of the whole series system.

$$P_F = P\left(\bigcup_{i=1}^{n} F_i\right) \qquad \text{with } F_i = \{R_i \cdot M_{R_i} \leq S \cdot M_S\} \tag{2}$$

2.2 DANIELS SYSTEMS

The so-called Daniels system was first introduced by H. E. Daniels in 1945 [3], Figure 2. It is a parallel system with special properties that make it meaningful in terms of mechanic systems. As described in [6, 9] all components experience the same load, $S_i = S_{tot}/n$, which is the nth fraction of the total load, S_{tot}. If the load exceeds the system's load bearing capacity the weakest component breaks first and the load is equally redistributed among the remaining intact components. The entire system fails when the remaining intact components cannot jointly carry the additional load fraction that was sustained by the last component that broke.

In the presented Daniels systems it is distinguished whether all their components behave perfectly brittle or perfectly ductile. A structural component is called perfectly brittle if it loses its bearing capability completely at failure. A perfectly ductile component maintains its load level after failure.

The probability of failure of a perfectly ductile Daniels system is given by equation (3).

$$P_F = P\left(\sum_{i=1}^{n} R_i \cdot M_{R_i} - S \cdot M_S \leq 0\right) \tag{3}$$

For a perfectly brittle Daniels system the failure probability is determined by equation (4).

$$P_F = P\left(\bigcap_{i=1}^{n}\{(n-i+1)R_i \cdot M_{R_i} - S \cdot M_S \leq 0\}\right) \tag{4}$$

Where the realisations of $R_i \cdot M_{R_i}$ are ordered according to $\hat{R}_1 \cdot \hat{M}_{R_1} \leq \cdots \leq \hat{R}_i \cdot \hat{M}_{R_i} \leq \hat{R}_{i+1} \cdot \hat{M}_{R_{i+1}} \leq \cdots \leq \hat{R}_n \cdot \hat{M}_{R_n}$.

2.3 PROBABILISTIC MODEL SIMULATION
The applied model parameters are described in Table 1. In case of the prior model the variables R_i and S are substituted by $R'_{des,i}$ and S'_{des}, respectively. In the pre-posterior model R_i is substituted by the variable R''_{mix} that follows the distribution explained in section 3.2.

Table 1: Parameters of the structural probabilistic model.

Model parameter	Distribution type	Mean (μ)	Standard deviation (σ)
R_i Resistance	To be substituted by either prior ($R'_{des,i}$) or pre-posterior (proof loading) uncertainties. See section 4.3.		
$R'_{des,i}$ Design resistance	Lognormal	13.4196	1
S'_{des} Design load	Weibull	10	$\sqrt{2}$
R''_{mix} Updated resistance	Mixture distribution	Depend on distribution definition	
M_R uncertainty of R	Lognormal	1.15	$0.15 \cdot \text{mean}(M_R)$
M_S uncertainty of S	Lognormal	1.0	$0.05 \cdot \text{mean}(M_S)$

2.4 SINGLE COMPONENT RELIABILITY
This study is based on a target component reliability of $\beta_t = 2$. This is achieved by pre-defining the mean and the variance of the load, S, as well as the variance of the component strength, $\text{Var}(R_i)$. The mean of R_i is calibrated iteratively so it matches β_t. $\beta = 2$ is approximately equal to a probability of failure $= 0.02275$ [2].

2.5 CORRELATION
Correlation between the random variables has a strong influence on the system reliability. The random variables of the resistance, R_i, are correlated in a range from 0 to 1 in the system models. This is achieved by applying a normal copula which uses the Spearman rank as input correlation. According to [8] the ultimate strength of tubular joints in jacket structures are correlated in a range from 0.2 to 0.9. The wide spread maybe due to varying joint dimensions and welding craftsmanship. As the components are represented by the same models it seems reasonable to assume a correlation between their model uncertainties, M_R, in the discussed systems fully ($\rho_{M_R} = 1$) as well as un-correlated ($\rho_{M_R} = 0$) model uncertainties are simulated in order to compute upper and lower bounds.

3 VALUE OF INFORMATION IN PROOF LOAD TESTING

3.1 DECISION THEORY
The theory of the Value of Information is part of the Bayesian decision theory developed by Raiffa and Schlaifer [10]. In order to obtain the VoI in the context of civil engineered structures with pre-posterior information, the expected value of sample information (EVSI) is to be determined. The EVSI considers experimental results before they have been obtained. In the context of proof load testing, the prior information may be retrieved from the structural design and the pre-posterior information may be obtained through modelling the proof load testing results probabilistically.

A brief explanation of the concept of VoI follows here according to [4, 12, 14] where the value of Information V is quantified as the difference between life-cycle benefit, B_1, as determined by pre-posterior decision analysis and life-cycle benefit, B_0, according to a scenario without proof load utilisation based on prior models (Equation 5).

$$V = B_1 - B_0 \tag{5}$$

The life-cycle benefit, B_1, depends on the outcome of the pre-posterior decision analysis which depends on the choice of the proof load test, structural performance and the impact of undertaken corrective measures. The proof load test influences the result by its capabilities, i.e. load type and its distribution, and cost. In order to choose the highest utility $B_1 = E_X[B(X,s)]$, expressed as the expected value of benefits must be maximised accounting for the decision alternative s (Equation 6).

$$V = \max_{s} E_X[B(X,s)] - B_0 \tag{6}$$

The above described VoI derivation is given in greater detail by [4] or [14]. Some examples of VoI analyses are given in [1]. The value of proof load information may thus relate to increasing benefits or decreasing costs, or in a wider context, to increasing human safety.

3.2 PRIOR AND PRE-POSTERIOR KNOWLEDGE
The prior model incorporates the design model uncertainties, M_S and M_R, of the load and resistance respectively. The prior model uncertainties are defined as suggested by [16] in Table 1. In order to obtain pre-posterior knowledge proof load testing is applied to each component separately which updates the structural resistance, R. Relevant for obtaining the updated knowledge is the proof loading distribution with its parameters listed in Table 2.

Table 2: Parameters of the proof load distribution.

Uncertainty parameter	Distribution type	Mean (μ)	Coefficient of variation
R'_{proof} proof load	Normal	$0.85 \cdot \text{mean}(R'_{\text{des}})$	0.01

UPDATING BY PROOF LOADING
Proof loading for reliability updating has been considered already in 1984 [7], by applying a load to a structure its capacity can be tested and the resistance distribution be updated. The authors in [13] suggest to update the resistance distribution by combining the proof load and

resistance distribution to a posterior mixture distribution, R''_{mix}, which will substitute R'_{des} in the reliability analysis. R''_{mix} consists of two truncated distributions, first it follows the proof load distribution, R'_{proof}, up to a threshold, l, and beyond l it follows the design resistance, R'_{des}. l is chosen such that it is at the intersection point of the PDFs of R'_{proof} and R'_{des} on their increasing flanks. The updating is solely based on survival information as failed components will not be used for construction.

$$R''_{\text{mix}} = \begin{cases} R'_{\text{proof}}, & \text{for } x \leq l \\ R'_{\text{des}}, & \text{for } l < x \end{cases} \tag{7}$$
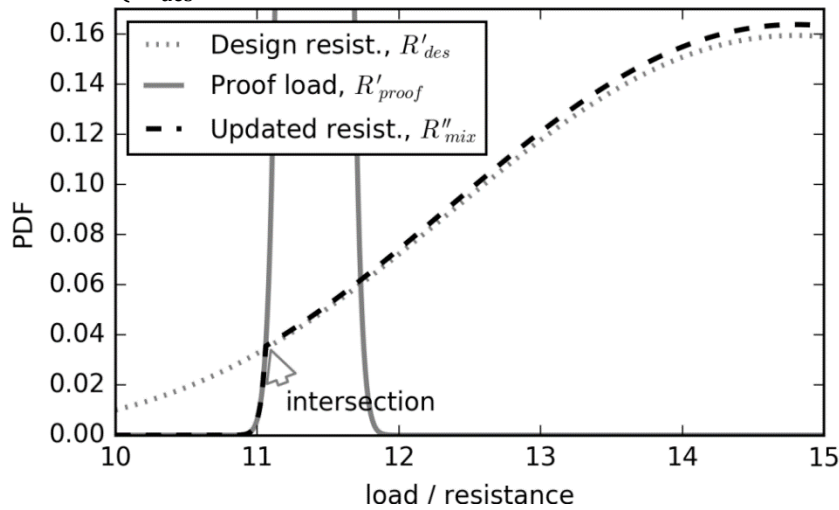

Figure 3: Visualisation of the resistance updating process using probability density functions.

With the chosen distribution parameters the proof load had to be rather high in order to obtain numerically significantly different results. 85% of the mean(R'_{des}) was chosen. With increasing uncertainty in the prior distribution of the resistance model the required proof load becomes smaller in order to achieve significant differences. Such a high proof load is probably best realised by testing single components. The here-in presented models assume that each component is updated separately. The probability of failure during the proof load test estimates to approximately 0.0421. PDFs of the involved distributions are shown in Figure 3.

COMPUTUATION OF THE VALUE OF INFORMATION
In order to estimate the benefit of the presented models, assumption are made based on [15] for offshore wind turbines. Capital expenditures are assumed to be 3.0 M€ / MW. The proof load tested system contributes 600 k€ / MW to the capital expenditures. The income per MWh is 45 €, and the capacity factor 50%, over a service-life of $t = 1, ..., 20$ years and is discounted with the rate $r_d = (1 + 0.025)^{-t}$. The component cost is assumed to be anti-proportional with the amount of components in the system. Expenses for operation & maintenance are not considered. Direct consequences arise from component failure during proof load testing, indirect consequences defined by the capital expenditures and the system failure probability. The costs for proof loading are assumed to be 1% of the system costs. On the bases of [8] the distribution of the component resistance correlation, ρ_{R_i}, is chosen as shown in Figure 4.
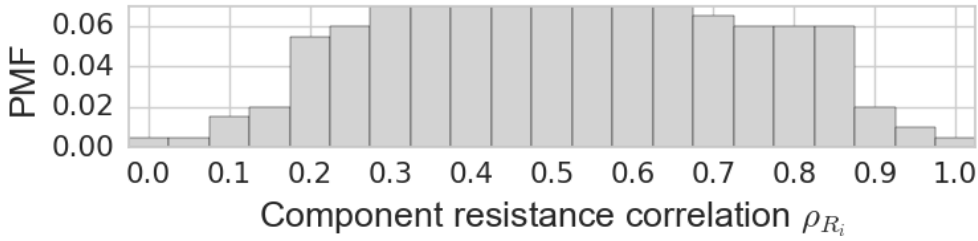
Figure 4: Assumed distribution of component resistance correlation.

The Bayesian pre-posterior decision analysis follows the decision tree in Figure 4. Based on the not yet known test results a component that failed will not be used for construction; a surviving component will be used. Thus failure is not explicitly modelled which results in the simple decision tree in Figure 4. The first chance nodes after the decision represent the probabilities of a certain component resistance correlation. The following chance nodes branch into system failure or survival.
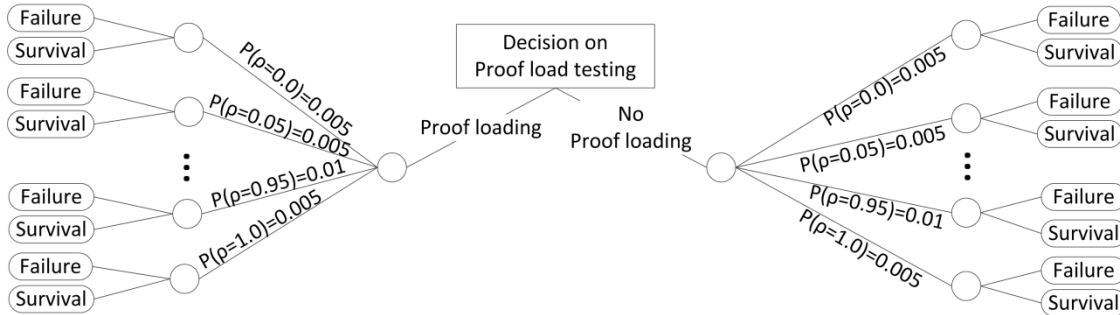

Figure 5: Decision tree. The resistance correlation ρ varies according to Figure 4.

4   MODEL RESULTS
Figures 6 to 8 show the relative change of the reliability index β normalised by the target index $\beta_t = 2$. Figure 9 to 11 display the difference of the proof load test reliability and the design model reliability normalised by the difference in reliability one component gained. The images use bi-linear interpolation. In order to aid the graph interpretation, recognise that a dark colour in the background represents a low value with a light colour contour line for better contrast. A high value is thus shown by a light background colour and a dark contour line.

DESIGN MODEL
For the series (Figure 6) and brittle Daniels (Figure 7) systems it can be observed how the system reliability is reduced with increasing number of components; a higher correlation counter acts on this effect.
In Figure 8 one can observe an increase in reliability with increasing amount of components in ductile Daniels systems. In case of fully correlated model uncertainties the reliability converges faster to its maximum reliability which is only slightly higher than $\beta_t = 2$.
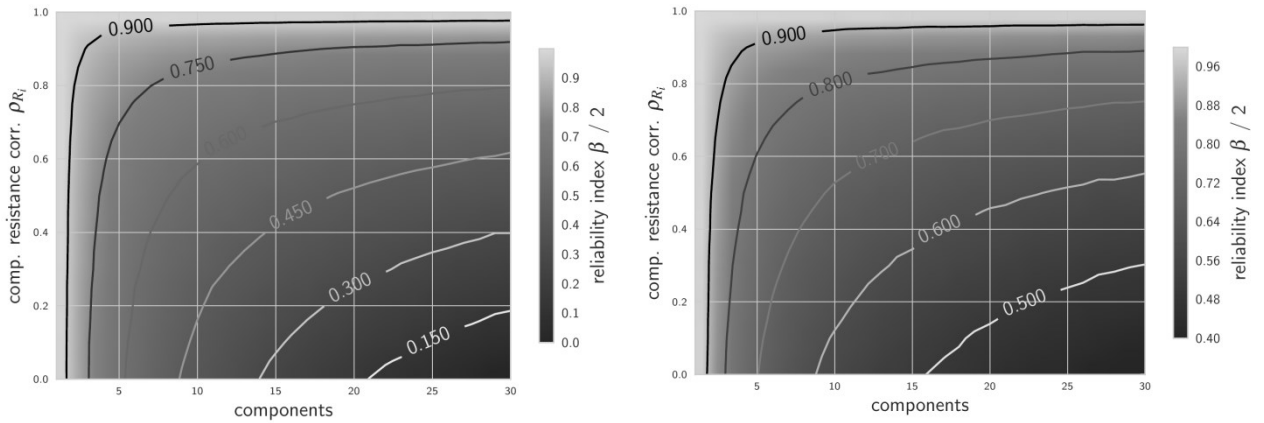
Figure 6: series systems with $\rho_{M_R} = 0$ (left) and with $\rho_{M_R} = 1$ (right)
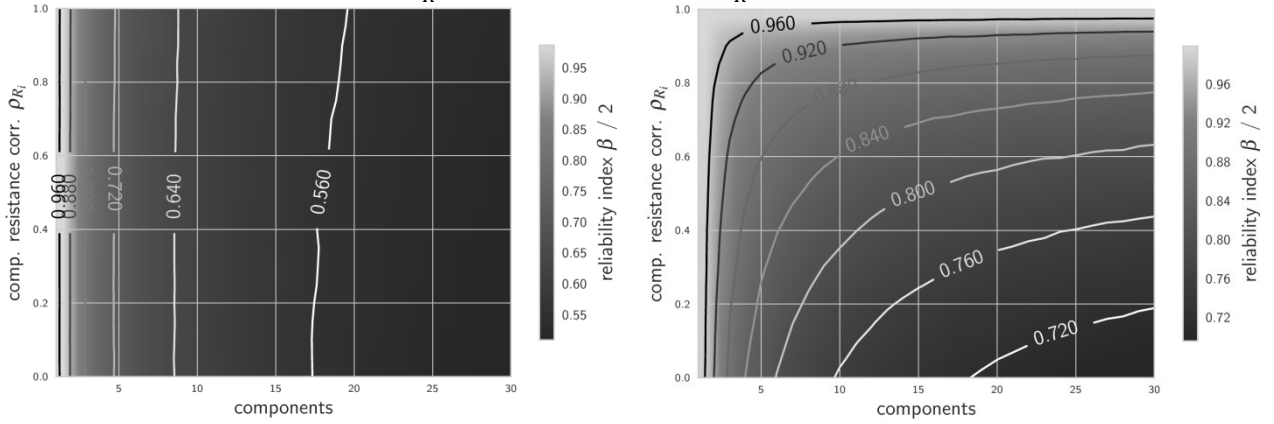

Figure 7: brittle Daniels systems with $\rho_{M_R} = 0$ (left) and with $\rho_{M_R} = 1$ (right)
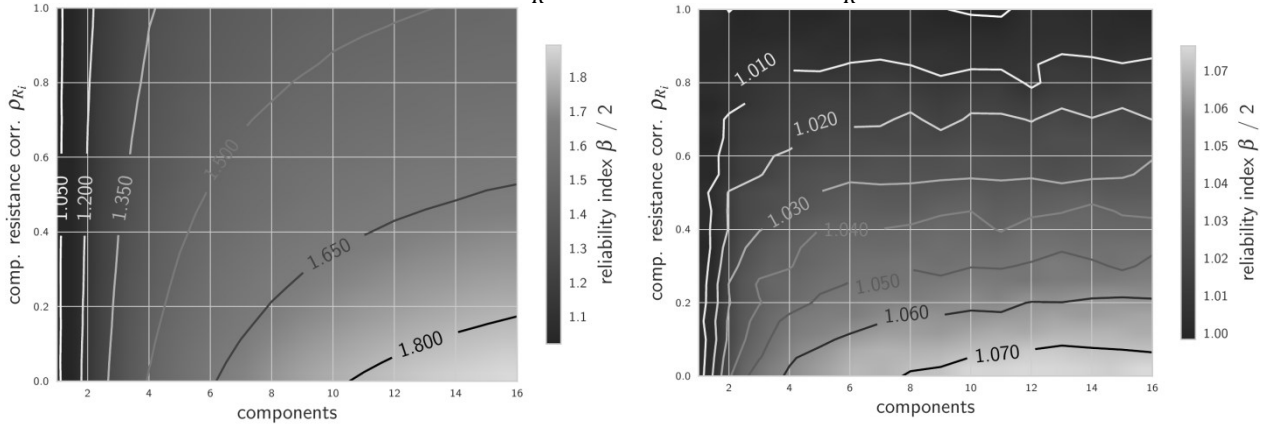

Figure 8: ductile Daniels systems with $\rho_{M_R} = 0$ (left) and with $\rho_{M_R} = 1$ (right)

UPDATED STRUCTURAL MODELS

In the figures 9 to 11 a value larger than 0 represents a gain in reliability through proof load testing, a value larger than 1 indicates gain in the system reliability that is larger than the gain of a single updated component.

In the series system models (Figure 9) the proof load testing increases the reliability estimate with increasing numbers of components and a low correlation of component resistances. Thus the maxima are concentrated in the lower right corner of the graphs. Furthermore the

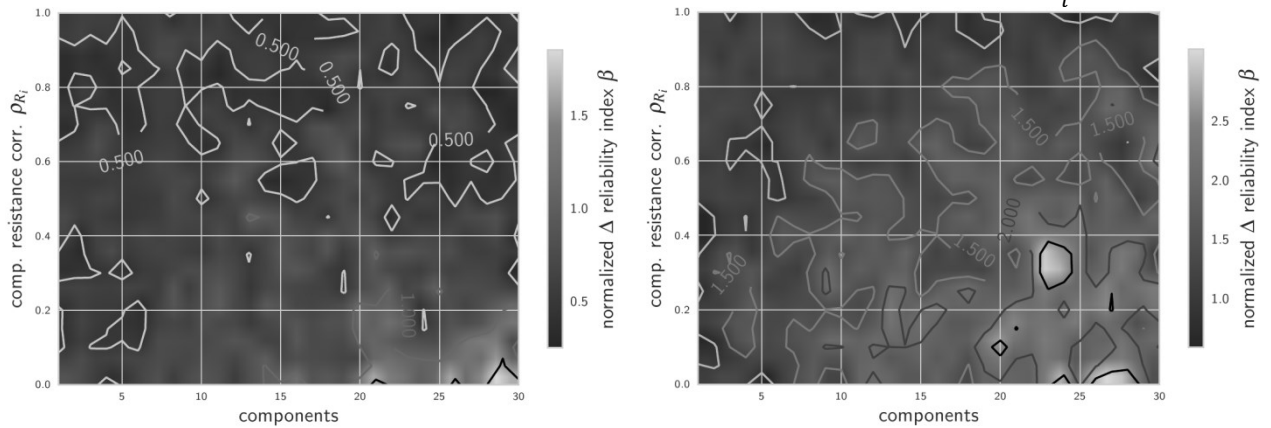reliability increase is stronger when the model uncertainties are correlated with $\rho_{M_{R_i}} = 1$.

Figure 9: updated series systems with $\rho_{M_R} = 0$ (left) and with $\rho_{M_R} = 1$ (right)

Brittle Daniels systems show behaviour similar to that of series systems in case of $\rho_{M_{R_i}} = 0$. The peak reliability gain remains with a large amount of components but moves towards a higher correlation of component resistance.
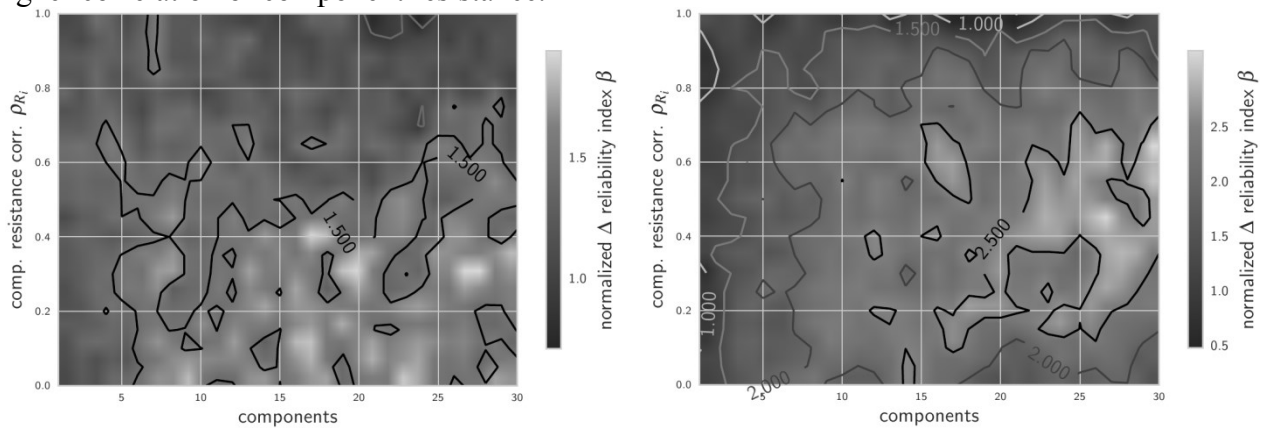

Figure 10: updated brittle Daniels systems with $\rho_{M_R} = 0$ (left) and with $\rho_{M_R} = 1$ (right)

Ductile Daniels systems gain more reliability through proof load testing with high component resistance correlation. This is especially pronounced with uncorrelated model uncertainties.
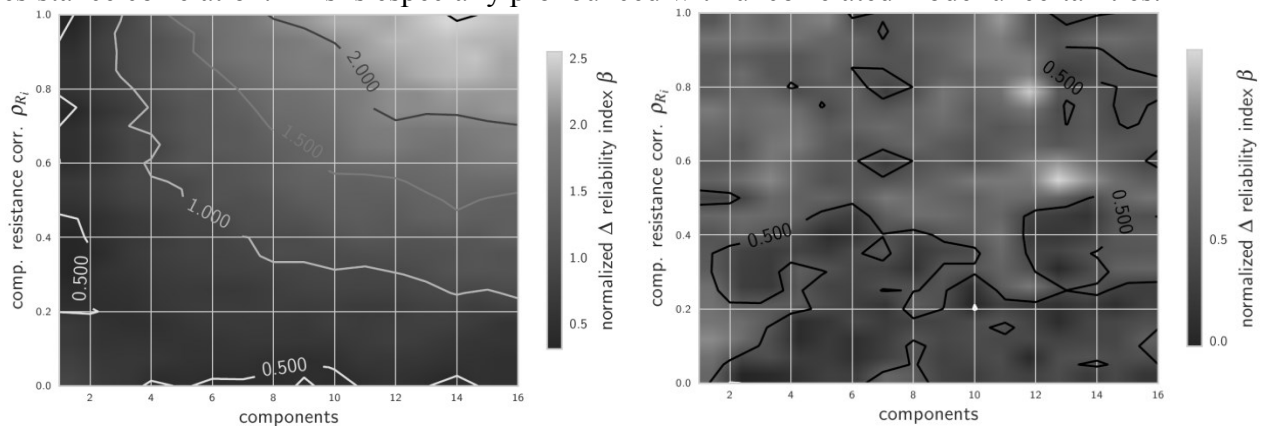

Figure 11: updated ductile Daniels systems with $\rho_{M_R} = 0$ (left) and with $\rho_{M_R} = 1$ (right)

DECISION ANALYSIS

As shown in Figure 13 the VoI of proof load testing for series and brittle Daniels systems become positive for larger numbers for components. The VoI = 0 is exceed between 11 and 16 components and beyond. Proof load testing of ductile Daniels system components does not provide a positive VoI in this study. The increase of the VoI with the number of components is due to lower risk associated with the failure of a component as the price per component is anti-proportional to the amount of components.
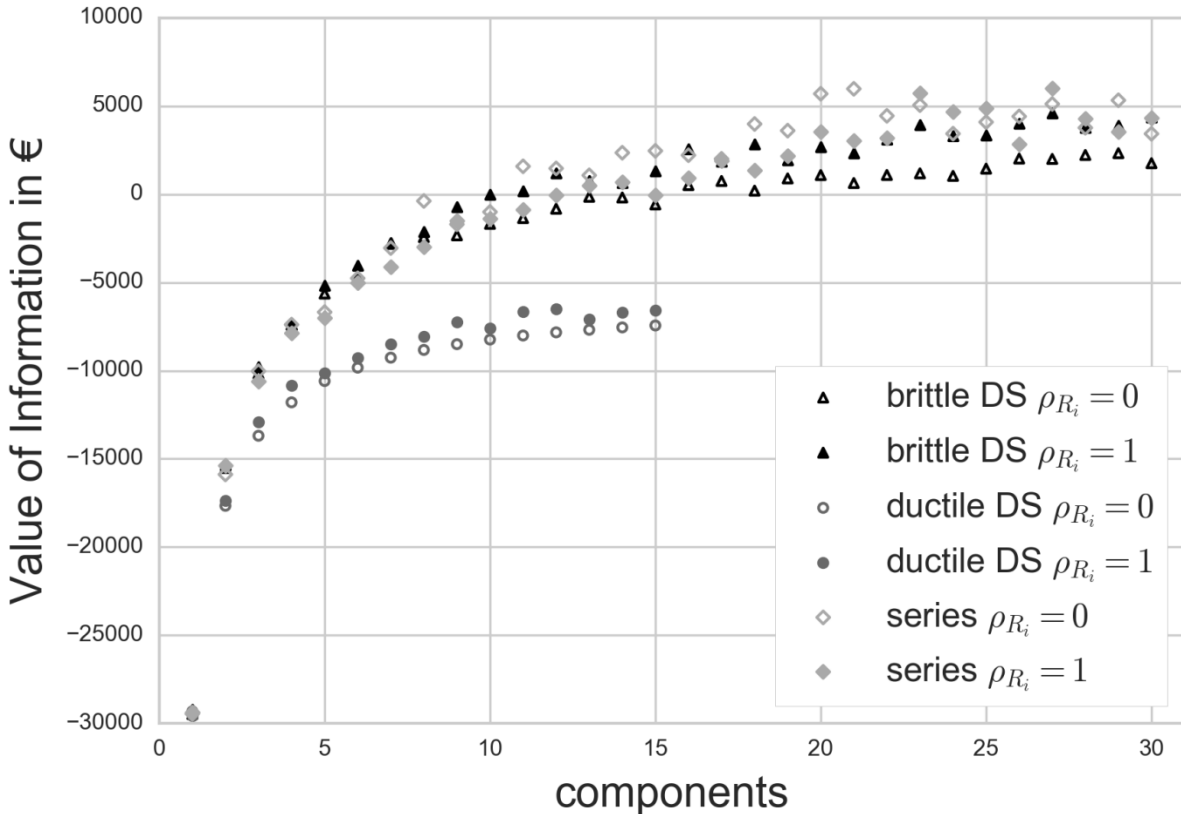

Figure 13: Value of Information obtained by proof load testing.

5 SUMMARY AND CONCLUSION

The study has shown how proof load testing of individual components can be utilised in order to update the system resistance distribution and hence the system's reliability for series and Daniels system models. But for owners and operators of structural systems a more accurate estimation of the system reliability is not sufficient in itself. Before conducting an experiment such as proof load testing it is preferable to assess the value the experiment provides to the owner or operator. In this study the Value of Information was estimated using the Bayesian decision theory in order to assess whether or not proof load testing of single components can add value.

Considering the same type of system the Value of Information is mainly influenced by the risk associated with the proof load tests. The larger the amount of components, the larger is the Value of Information as the individual component price drops in more complex systems. Across systems the Value of Information is changing with the fundamental reliability of the system.

9

From the most reliable system type in this study, the ductile Daniels system, to the least reliable system, the series system, the Value of Information increases. This generic approach requires an adaptation to relevant failure mechanisms for actual applications.

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