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Investigating Coherent Structures in the Standard Turbulence Models using Proper Orthogonal Decomposition

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Abstract. The wind turbine design standards recommend two different methods to generate turbulent wind for design load analysis, the Kaimal spectra combined with an exponential coherence function and the Mann turbulence model. The two turbulence models can give very different estimates of fatigue life, especially for offshore floating wind turbines. In this study the spatial distributions of the two turbulence models are investigated using Proper Orthogonal Decomposition, which is used to characterize large coherent structures. The main focus has been on the structures that contain the most energy, which are the lowest POD modes. The Mann turbulence model generates coherent structures that stretches in the horizontal direction for the longitudinal component, while the structures found in the Kaimal model are more random in their shape. These differences in the coherent structures at lower frequencies for the two turbulence models can be the reason for differences in fatigue life estimates for wind turbines.

1. Introduction

The size of the wind turbine rotors are continuing to grow, especially for the offshore market, where the size of the components are not as limited as onshore, since vessels can transport heavier and larger parts compared to cars and trains. In addition there are large areas offshore, with good wind conditions, that can be used for wind farms and this makes offshore wind very attractive. In the recent years, the interest for floating offshore wind turbines have increased, and a pilot floating wind farm, the Hywind Scotland project, is planned to be in operation in 2017. However, as the size of the wind turbine rotor increases, the spatial distribution of the wind velocities becomes more important.

Simulations using the SIMA tool developed at MARINTEK show that the mooring response, especially the bridal response, of the floating wind turbines in the Hywind Scotland project are sensitive to the turbulence model used [1]. The two turbulence models used were the models recommended by the wind turbine design standard, IEC 61400-1 [2]. The first model, the Kaimal spectrum and exponential coherence model, where the spectral densities and coherence are explicitly given, is often referred to as the Kaimal model and can be generated by TurbSim. The other model, the Mann turbulence model, which is based on a three-dimensional velocity tensor, can be generated by the DTU Mann64 generator [3]. The spectral density used in the Mann model is fitted to the Kaimal spectrum, and is therefore the same as used in the Kaimal model, and the difference between the two models is the spatial distribution of the velocities in the wind box.

The dynamics of a floating wind turbine are different from a bottom fixed wind turbine, and especially the coherent structures at the lower frequencies wind are important for the mooring line response. In the study [1] it was found that aero-elastic simulations using the Kaimal model yields almost twice as high fatigue life as when using the Mann model. The largest differences in response was found at the lower frequencies for the mooring line and connecting bridle. Since the wind spectrum in both turbulence models are the same, the spatial distribution of the wind velocities are likely the reason for the differences seen in the response. In order to investigate the large coherent structures in the turbulent wind field, Proper Orthogonal Decomposition, POD, is applied. Previous studies using POD to describe inflow turbulence and wind turbine loads has shown that a small number of inflow POD modes can account for the low frequency energy in the wind turbine loads [4]. The aim of this study is to investigate the coherence in these two turbulence models, and focusing especially at the low frequencies.

2. Methodology

2.1. Turbulence models

The wind turbine design standard, IEC 61400-1, recommends two different wind turbulence models to be used; the Kaimal spectrum and exponential coherence model and the Mann turbulence model. These will be referred to as the Mann model and the Kaimal model in this study. Both turbulence models are fitted to the same spectral density and the turbulence intensity is similar, the difference is in the spatial distribution of the velocities.

In the Kaimal model, an exponential coherence function is used in conjunction with the wind spectrum in the longitudinal direction, to describe the spatial distribution. The exponential function used is:

$$Coh(r,f) = e^{\left[-12\left(\left(\frac{fr}{U_{hub}}\right)^2+\left(0.12\frac{r}{L_c}\right)^2\right)^{1/2}\right]} \tag{1}$$

where f is the frequency in Hertz, r is the separation distance between the two points in meter, L_c is the coherence scale parameter in meter and U_{hub} is the wind speed in meter per second at hub height.

The Kaimal model does not physically model the evolving eddies, this is however done by the Mann model. It assumes that the isotropic von Karman energy spectrum is rapidly distorted by a uniform, mean velocity shear. The description of this model is given by a three dimensional velocity tensor that is given in the IEC 61400-1 [2], and the derivation of the model is given by Mann in [5]. The wind fields will be generated using TurbSim [6] for the Kaimal model and the Mann 64bit turbulence generator for the Mann model [3].

2.2. Turbulence intensity and coherence

In this section the turbulence intensity at the middle of the grid is presented. This point is chosen since the hub height in an aero-elastic simulation program (e.g. HAWC2 and FAST) is normally at the center of the turbulent wind field. For the Mann model, the number of grid points must be a number that can be written as an exponential number with 2 as the exponent. Therefore the number of grid points needs to be an even number of grid points used in the simulation and there will be no grid point in the middle of the wind field. Six simulations of 10-min duration was generated for each turbulence model at each wind speed. Figure 1 shows the mean turbulence intensity of the four grid points closest to the centre point. The dots indicate each 10-min simulation, and the solid line shows the mean value of these simulations at each wind speed.

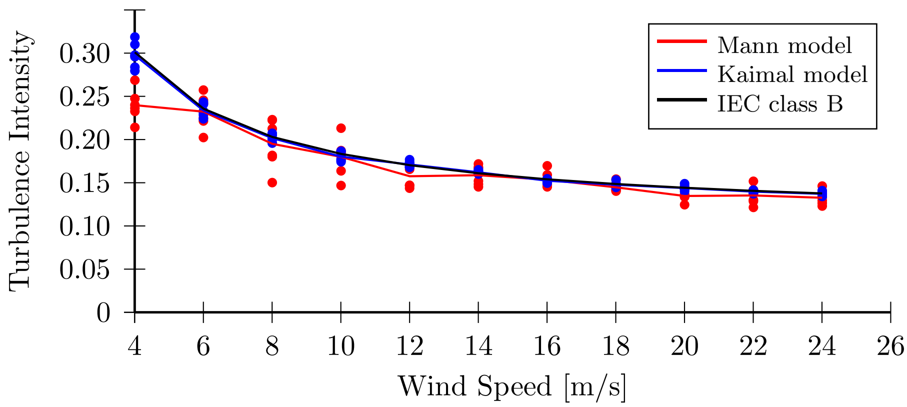


Figure 1. Turbulence intensity in the generated wind files. The dots indicate the value for each of the 10-min simulations, while the solid line is the mean value of the six simulations at each wind speed.

The wind simulations will be based on stochastic simulations and the turbulence levels will be different relative to the recommended IEC turbulence levels, however the mean level between the six simulations is close to the targeted IEC turbulence value. The turbulence intensity level used is the class B in the IEC 61400-1 [2], and the resulting turbulence levels matches the IEC class B quite well. The largest difference is seen at the lowest wind speed, where Mann turbulence model generate a lower turbulence level. This is most likely due to the time-step of 0.1 s being too large to generate the turbulent structures at the higher frequencies at this low wind speed. The difference between the two models are larger when investigating the coherence levels.

The real part of the coherence is the co-coherence and the co-coherence of the longitudinal wind velocities at two arbitrary chosen separation distances, 7 m (top graphs) and 56 m (bottom graphs), are shown in Figure 2. The left graphs show the co-coherence results for the Mann model, while the right are for the Kaimal model. The figures are limited to 3 wind speeds and the average of the co-coherence of six simulations across the rotor. Both the co-coherence in the vertical (solid lines) and in the horizontal direction (dashed lines) are shown. Here, the horizontal separation is in the lateral direction.

When the distance is only 7 m, there is no visible difference in the co-coherence for vertical nor in horizontal direction for either turbulence models. For the larger separation distance of 56 m, the Mann turbulence model will have a difference in the vertical and horizontal separation, and the co-coherence is higher in horizontal than in vertical direction. This is due to that the Mann model considers that the turbulent structures are generated by the vertical shear of the mean wind, and the coherence level will therefore be higher in the horizontal direction. The Kaimal turbulence model does not differ much between the vertical and horizontal separation.

The co-coherence is higher for the Kaimal model compared to the Mann model for the large separation, except at small wind speeds. At the very short separation, it is opposite, but the difference is relatively small.

2.3. Proper Orthogonal Decomposition

The implementation of POD modes is based on the description given by Jørgensen et al [7] and a general overview of POD can be found in [8]. In this study we consider all three velocity components ($\mathbf{u} = (u, v, w)$). The velocities, $\mathbf{u}_k$, are organised as n slices and the number of slices is equal to the number of time-steps considered. The mean value, $\mathbf{u}_0$, is often removed [7]

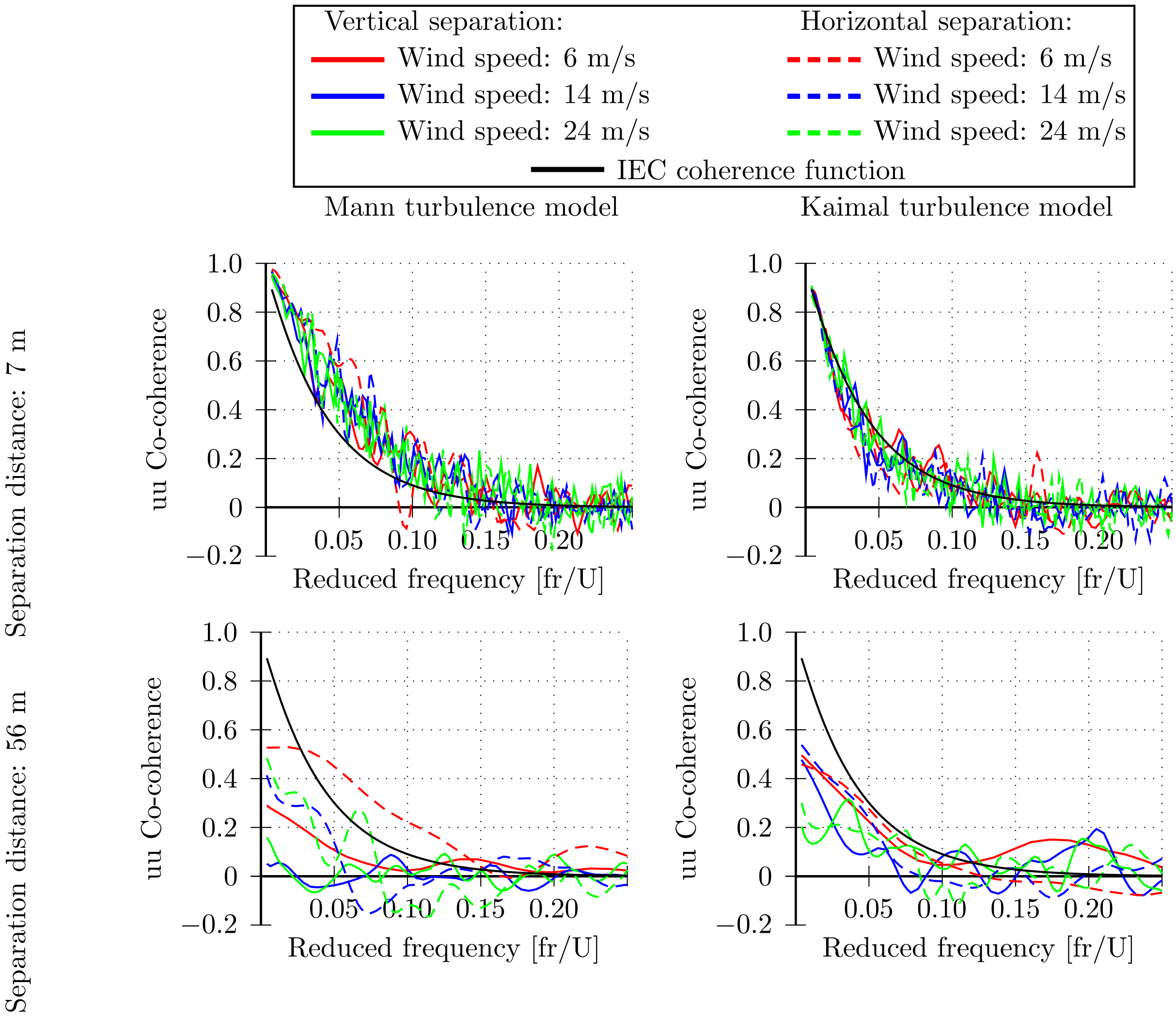


Figure 2. The uu co-coherence for 7 m and 56 m separation at hub height. The solid lines are the vertical separation and the dashed line is the horizontal separation.

[9], so that the velocity vectors are:

$$\mathbf{u}'_k = \mathbf{u}_k - \frac{1}{n}\sum_{i=1}^{n}\mathbf{u}_i = \mathbf{u}'_k - \mathbf{u}_0 \tag{2}$$

The velocity matrix is:

$$\mathbf{U} = [\mathbf{u}'_1 \ldots \mathbf{u}'_n] \tag{3}$$

and the $n \times n$ auto-covariance matrix is defined as $\mathbf{R} = \mathbf{U}^T\mathbf{U}$. An eigenvalue problem of the form:

$$\mathbf{RG} = \mathbf{G\Lambda} \tag{4}$$

where $\mathbf{\Lambda}$ is the eigenvalue matrix with the eigenvalues in decreasing order,

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \lambda_{n-1} \end{pmatrix} \tag{5}$$

and the matrix of orthonormal eigenvectors:

$$\mathbf{G} = [g_1 \cdots g_{n-1}] \tag{6}$$

In order to find the POD modes $\phi_1, \phi_2, \ldots \phi_n$, a matrix product $\mathbf{B} = \mathbf{UG}$ is defined. The matrix of POD modes is found as:

$$\mathbf{\Phi} = \mathbf{B}\mathbf{\Lambda}^{-1/2} \tag{7}$$

It is possible to reconstruct the flowfield, using the POD modes. First the amplitudes are defined as $\mathbf{A} = \mathbf{\Phi}^T\mathbf{U}$. The velocity vectors can now be found as:

$$\mathbf{u_j} = \mathbf{u}_0 + \sum_{k=1}^{n-1} \phi_k a_{kj} \tag{8}$$

where a_{kj} are the elements of the matrix $\mathbf{A}$, k is the POD mode and j is the slice number. Since the POD modes are sorted, the reconstruction of the wind field can be truncated by limiting the number of POD modes included, i.e. only including the first K POD modes:

$$\mathbf{u_j} = \mathbf{u}_0 + \sum_{k=1}^{K} \phi_k a_{kj} \tag{9}$$

3. Results

3.1. POD modes

The eigenvalues of the POD modes are a measurement of the level of turbulent kinetic energy in the various modes, and in Figure 3 the eigenvalues for three different wind speeds are shown. The graphs to the left show the sorted eigenvalues of the modes, and the graphs to the right are the cumulated values. The eigenvalues are normalized such that 1 is the level of energy if all POD modes are included. The number of POD modes is 8192 for all simulations using both the Kaimal model and the Mann model.

As shown in Figure 3, the first POD mode contains around 20 % of the turbulent kinetic energy. This is similar to the first POD mode estimated in [9], where the turbine wakes were studied using Large Eddy Simulations (LES). The mild slope, which indicates that a large part of the turbulent kinetic energy is related to the high frequency part of the wind field, was also seen in the wake study.

For the low wind speed, 6 m/s, the differences in energy levels are small between the two turbulence models, but for the higher wind speeds the difference is increasing. The lower POD modes contain more of the energy using Mann turbulence model compared to the Kaimal turbulence model.

The focus in this study is on the large coherent structures related to the low frequency part of the wind spectra, which are found in the lowest POD modes, as this is where one see the largest differences in response for the floating wind turbine [1]. The four first POD modes for the Mann turbulence model are shown in Figure 4 for a 10-min simulation at 14 m/s wind speed. Since the turbulent structures in the Mann model is influenced by the mean vertical shear, one can see the coherent structures in the POD modes are elongated in the horizontal direction. This was also indicated by the co-coherence values shown in Figure 2 for the separation distance of 56 m, where the coherence was largest in the horizontal direction. The velocities in v- and w-direction also contain large coherent structures for the first POD modes. There seems to be a correlation between the velocities in the velocities in the u- and w-direction, where a high intensity in the u-direction corresponds to a low intensity in the corresponding location for the velocity in the w-direction.

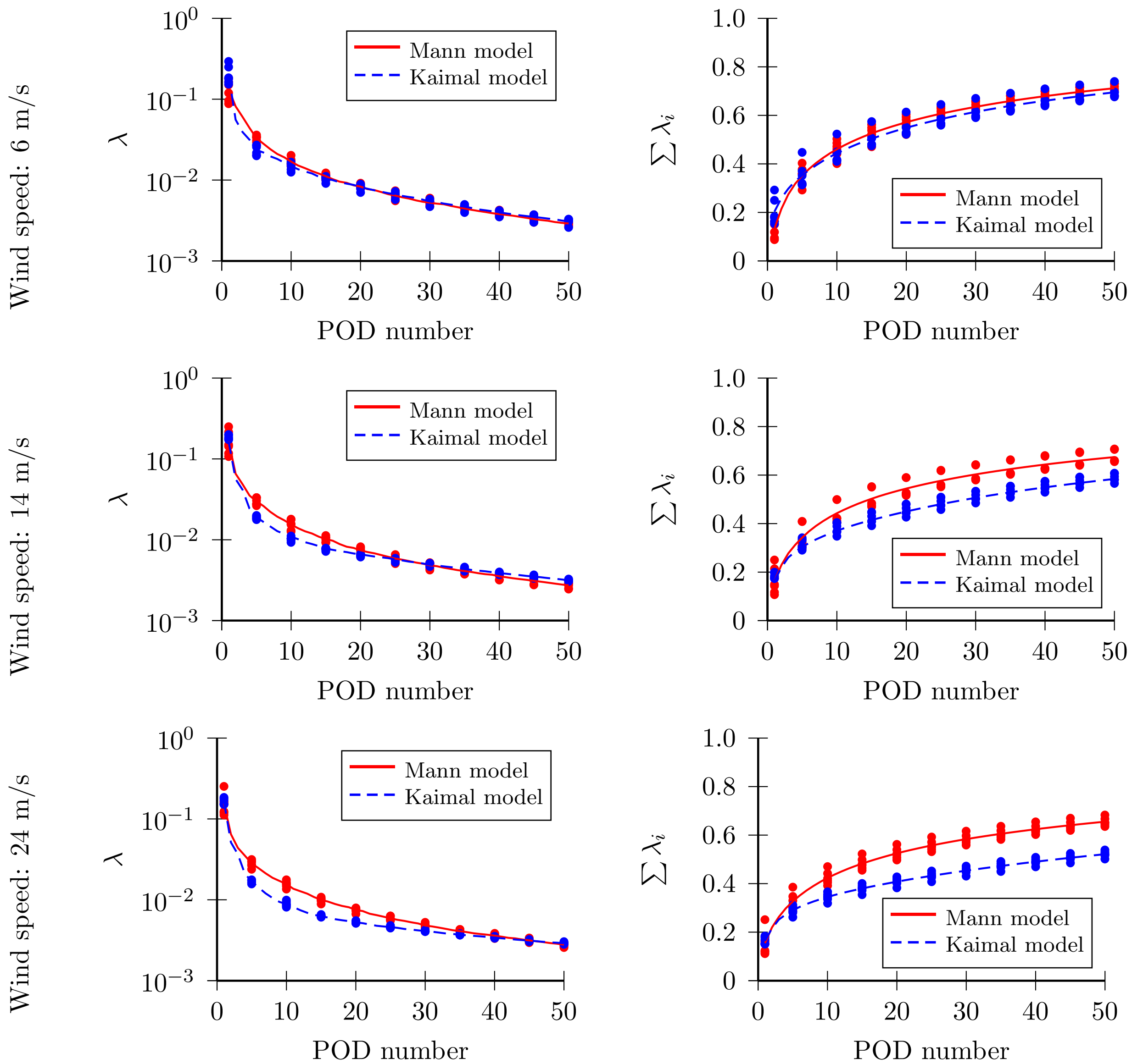
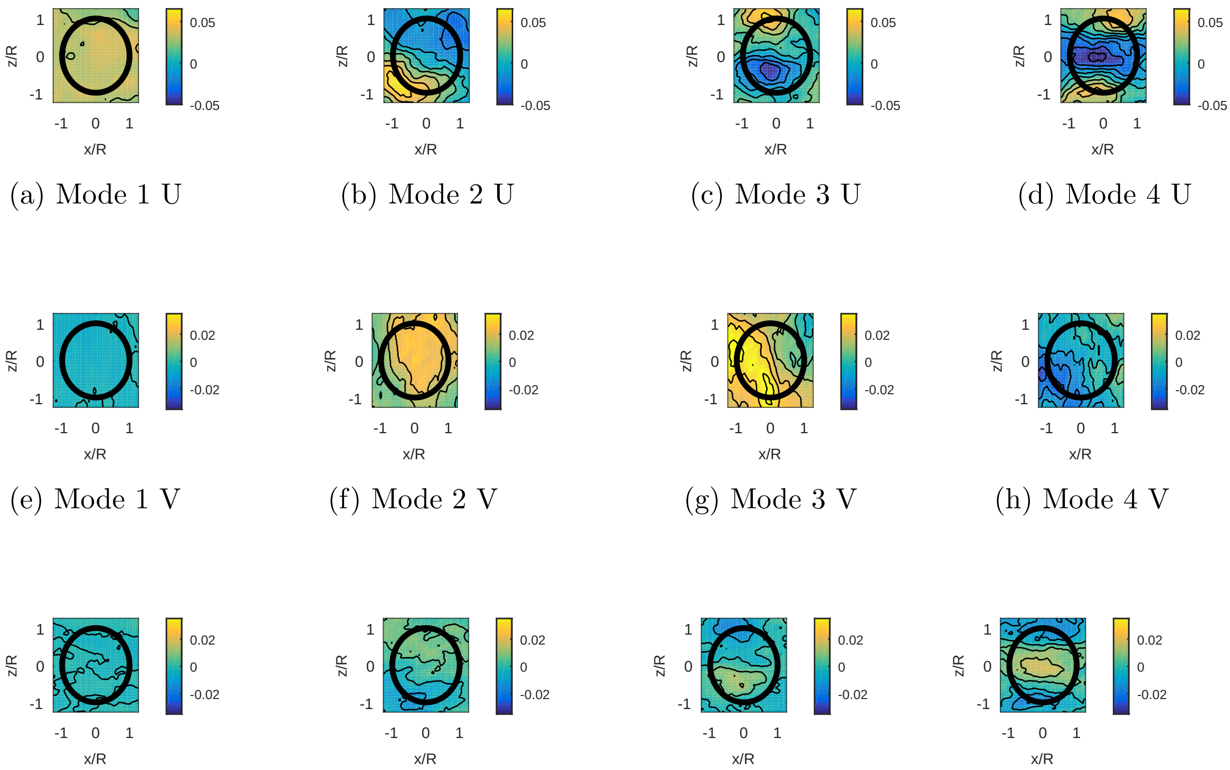


Figure 3. POD eigenvalues for three wind speeds; the sorted POD eigenvalues to the left and cumulated eigenvalues to the right.

The POD modes for the Kaimal turbulence is shown in Figure 5 and similarly to the lower POD modes found using the Mann model, the longitudinal velocities contain large coherent structures. However, the coherent structures in the horizontal and vertical direction (v and w) are quite small, even for the low POD modes. Since the coherence function used for the Kaimal model (Equation 1) is only applied in the longitudinal direction, the lack of larger structures in the low POD mode is reasonable.

Both turbulence models exhibit large coherent structures in the longitudinal direction, but the shapes of these are different. The Mann model had horizontally stretched structures, while the coherent structures in the Kaimal model have shapes that have the maxima and minima on opposite sides of the wind field. In Figure 5, POD mode 2 has a maximum at the top and min at the bottom, while POD mode 3 has a maximum to the left and minimum to the right. The lowest and highest values are therefore not placed within the rotor circle of 178 m that is drawn on the graphs. However, the POD modes for the Mann model have the minima and/or maxima



(a) Mode 1 U (b) Mode 2 U (c) Mode 3 U (d) Mode 4 U

(e) Mode 1 V (f) Mode 2 V (g) Mode 3 V (h) Mode 4 V

(i) Mode 1 W (j) Mode 2 W (k) Mode 3 W (l) Mode 4 W

Figure 4. Spatial POD modes 1-4 for U, V, W for Mann turbulence model. The circle marks the rotor area that has a diameter of 178 m.

within the rotor circle. It should be noted that it is the combined effect of these modes that are important and not the single modes, and these effects can be countered by higher modes that have not been studied.

For the Kaimal model, the coherence function is dependent on the wind speed, and as the wind speed increases, the coherence value is also increasing. The size of coherent structures seen in the modes will therefore be dependent on the wind speed. A visual investigation of the POD modes show that at lower wind speeds, both the Kaimal model and the Mann turbulence model have small eddies in their lower POD modes. The POD modes of the other wind speeds are not shown here for brevity of the article.

Generating wind fields only consisting of POD mode 1, POD modes 1-5, POD modes 1-10 and POD modes 1-50 and comparing to a full wind field, one can find the frequencies in the POD mode ranges. The nondimensional wind spectra of three different wind speeds are shown in Figure 6. The Mann turbulence model is plotted using a solid line, while the Kaimal model is plotted using a dashed line. From these graphs one can see that the lowest POD modes contains the lowest frequencies, and it is these frequencies that we are interested in.

In the top graph, in Figure 6, which shows the full wind field with all POD modes, one can see that the wind spectra energy for the lowest wind speed the higher frequencies in the Mann model is too low. This is because a time stepping of 0.1 s is used, and this is too low to capture these high frequencies in the Mann model. However, in this study the focus is on the lower

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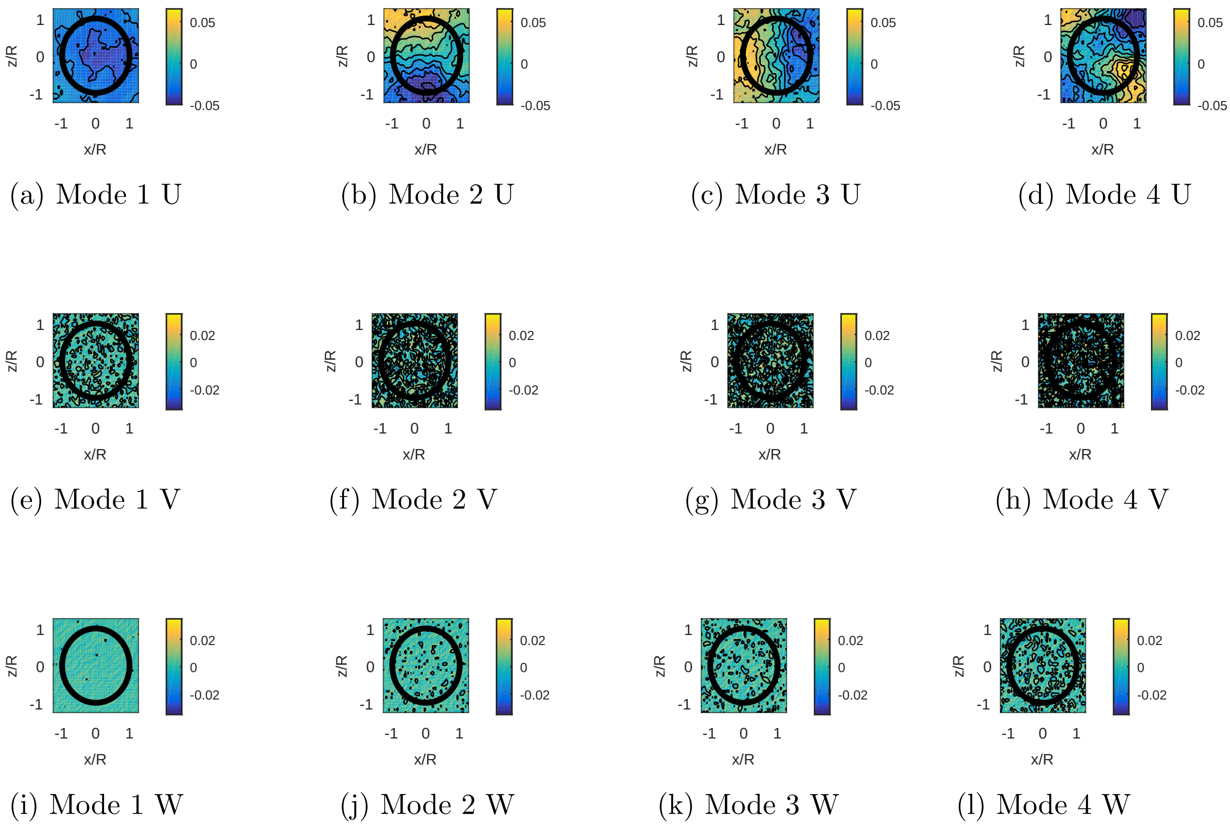


(a) Mode 1 U (b) Mode 2 U (c) Mode 3 U (d) Mode 4 U

(e) Mode 1 V (f) Mode 2 V (g) Mode 3 V (h) Mode 4 V

(i) Mode 1 W (j) Mode 2 W (k) Mode 3 W (l) Mode 4 W

Figure 5. Spatial POD modes 1-4 for U, V, W for turbulent wind field with Kaimal spectra and IEC coherence function. The circle marks the rotor area that has a diameter of 178 m.

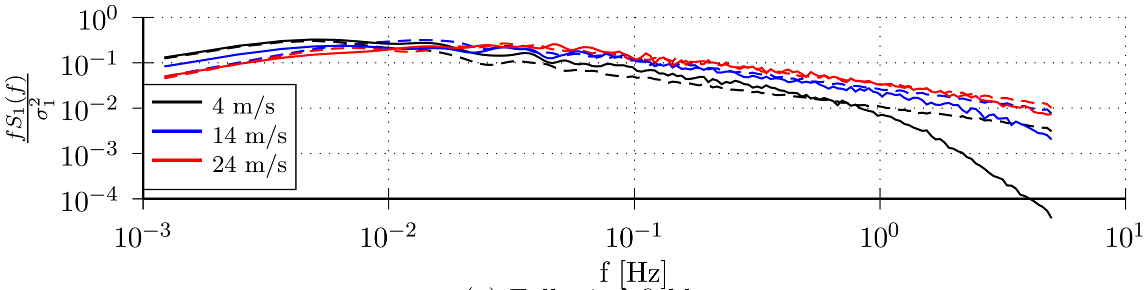
frequencies, since the higher frequencies have little influence on the response [4].

Looking at the wind spectra for the first POD mode in Figure 6b, the difference between the two models for the lowest wind speed is small, while the difference is more visible for the higher wind speeds, 14 m/s and 24 m/s. The energy in the wind spectra is higher for the Kaimal model relative to the Mann model for the two higher wind speeds. However, increasing the number of POD modes in the wind field to 5, as seen in Figure 6(c), the differences between the two models in energy content are very small. The trend is the same for 10 POD modes as well.

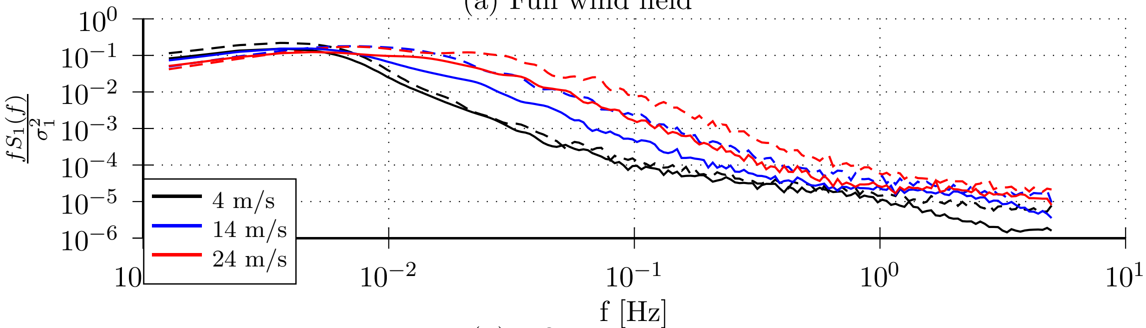
For an even higher number of POD modes, the trend is turning, and the energy in the wind spectra is higher for the Mann model compared to the Kaimal model. This is also true for the vertical and horizontal wind velocities. The same trend, that the energy is high for the Mann turbulence model relative to the Kaimal model, is also seen in the energy levels presented in Figure 3.

4. Conclusion

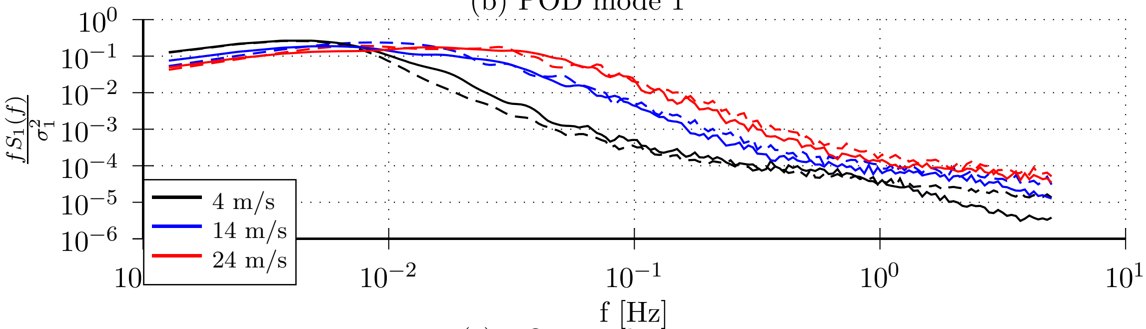
The motivation for looking into the coherence of these two turbulence models was the difference in fatigue life seen for a floating spar wind turbine [1]. The largest differences were found in the mooring line and the yaw responses, where the Mann turbulence model estimated a lower fatigue life compared to the Kaimal turbulence model. Only the lowest POD modes is needed to describe the wind turbine loads at low frequencies [10]. In this study it has been shown that



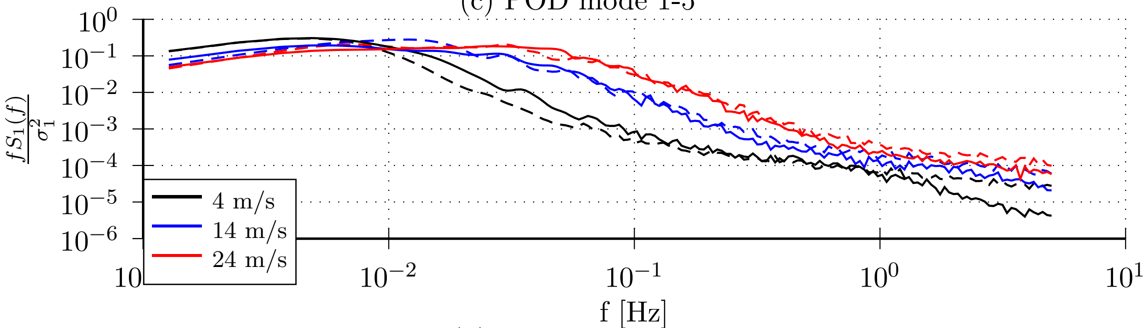
(a) Full wind field



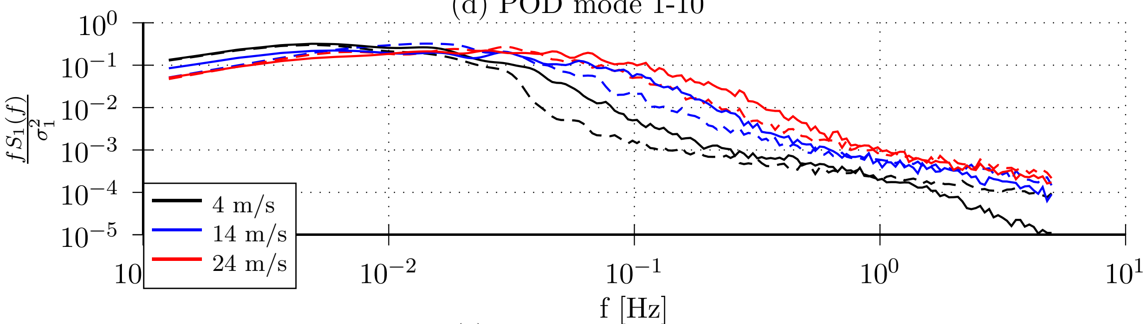
(b) POD mode 1



(c) POD mode 1-5



(d) POD mode 1-10



(e) POD mode 1-50

Figure 6. Nondimensional windspectra for various number of POD modes and three different wind speeds. The solid lines are Mann turbulence model and the dashed lines are the Kaimal turbulence model.

the coherent structures, found in the first POD modes, of the Mann model are more stretched in the horizontal direction, relative to the Kaimal model which have coherent structures of a more dipole character and the maxima and minima at opposite sides of the wind field. The differences in the coherent structures found in the POD modes can be the reason for the increase in mooring line and yaw response for the Mann model. The large difference in fatigue life indicates that it is important to use the correct wind turbulence model, especially for the floating wind turbines. Further work, investigating the effect of the lowest POD modes on the response of a floating wind turbine, will give more information on how the coherent structures influences the response.

This study does not conclude on which turbulence model is the correct to use, however it shows that there is a difference in the large coherent structures generated by the two standard turbulence models. In order to conclude on which is the correct model to use, comparisons with measurements should be performed. A study of the coherence at FINO1 indicates that the Mann turbulence model is closer to the real wind field compared to the Kaimal wind field [11]. However, this study is limited to only one month of measurement, and more comprehensive measurements should be performed. It is expected that comparisons with high fidelity turbulence simulations will indicate the same, since Mann turbulence model is a linearization of the Navier-Stokes equation, while the coherence function used in the Kaimal model is based on an empirical two-point cross spectra, so Mann is expected to provide more realistic results.

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