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# Generation Expansion Planning with Large Amounts of Wind Power via Decision-Dependent Stochastic Programming

Yiduo Zhan, Qipeng P. Zheng, *Member, IEEE*, Jianhui Wang, *Senior Member, IEEE*, Pierre Pinson, *Senior Member, IEEE*

**Abstract**—Power generation expansion planning needs to deal with future uncertainties carefully, given that the invested generation assets will be in operation for a long time. Many stochastic programming models have been proposed to tackle this challenge. However, most previous works assume predetermined future uncertainties (i.e., fixed random outcomes with given probabilities). In several recent studies of generation assets’ planning (e.g., thermal versus renewable), new findings show that the investment decisions could affect the future uncertainties as well. To this end, this paper proposes a multistage, decision-dependent stochastic optimization model for long-term, large-scale generation expansion planning where large amounts of wind power are involved. In the decision-dependent model, the future uncertainties are not only affecting but also affected by the current decisions. In particular, the probability distribution function is determined by not only input parameters but also decision variables. To deal with the nonlinear constraints in our model, a quasi-exact solution approach is then introduced to reformulate the multistage stochastic investment model to a mixed-integer linear programming (MILP) model. The wind penetration, investment decisions, and the optimality of the decision-dependent model are evaluated in a series of multistage case studies. The results show that the proposed decision-dependent model provides effective optimization solutions for long-term generation expansion planning.

**Index Terms**—Decision-Dependent, Stochastic, Wind, Mixed Integer Programming, Power Generation, Expansion Planning, Long-term, Endogenous Uncertainties.

## NOMENCLATURE

### A. Sets and Indices

- $a(n)$  The *ancestor* of node  $n$ .
- $i$  Index for types of generator: 1 for thermal, 2 for wind.
- $j$  Index for stage,  $j = 1, \dots, J$ .
- $l$  Index for the binary variables introduced for linearization,  $l = 1, \dots, L$ .
- $\mathcal{N}$  The complete set all nodes of the scenario tree.
- $\mathcal{N}^-$  The set of nodes excluding the one in the first stage.
- $\mathcal{N}'$  The set of nodes excluding those in stage  $J$ .
- $n$  Index for each node  $n \in \mathcal{N}$ .
- $S_n$  The *successor* set of node  $n$  in the next stage.

### B. Parameters

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- $B_i$  Levelized investment cost of thermal or wind  $i \in \{1, 2\}$ , in \$Billion/TWh.
- $b_i$  Unit investment (overnight capital) cost of thermal or wind,  $i \in \{1, 2\}$ , in \$Billion/TW.
- $\beta_i$  Capacity factor of thermal or wind  $i \in \{1, 2\}$ .
- $c_i$  Levelized electricity operation and maintenance cost  $i \in \{1, 2\}$ , in \$Billion/TWh.
- $D^n$  The elastic demand in scenario tree node  $\forall n \in \mathcal{N}$ , in TWh.
- $\delta^n$  Price variation level at node  $n$ ,  $\forall n \in \mathcal{N}$ .
- $E$  Elasticity level of demand.
- $e$  Incremental level between demands of two consecutive stages.
- $H$  Number of hours in a planning stage, in Hour.
- $L$  The number of binary variables used to represent the probability at each node of the scenario tree.
- $M_l$  A large number to bound all continuous decisions.
- $p^n$  Market price offered in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in \$Billion/TWh.
- $Rev^n$  Total revenue from selling electricity in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in \$Billion.

### C. Variables

- $\alpha_i^n$  Future investment on capacity of thermal/wind in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in TW.
- $CO^n$  Total operation cost in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in \$Billion.
- $\epsilon^n$  The error term when using binary variables to represent the continuous probability value at node  $n$ .
- $g_i^n$  Thermal or wind power production in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in TWh.
- $IC^n$  Total investment cost in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in \$Billion.
- $Prob^n$  Probability function of scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ .
- $R^n$  The weighted average total profitability of power generation asset composition in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ .
- $SW^n$  The profit in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in \$Billion.
- $\theta^n$  The decision variable introduced to replace the bilinear term containing the error term associated with the probability at node  $n$ .
- $x_i^n$  Installed capacity for thermal/wind producer in scenario tree node  $n$ ,  $\forall n \in \mathcal{N}$ , in TW.
- $z_l^n$  The  $l^{\text{th}}$  binary variable used to represent the probability at node  $n$ .
- $\zeta_l^n$  The continuous decision variable introduced to replace the bilinear term composed of the  $l^{\text{th}}$  binary variable and a continuous variable representing the current profit.

## I. INTRODUCTION

IN recent years, power generation via fossil fuels has become a main source of air pollution. The development of renewable energy becomes a potential solution to maintain a sustainable future environment. Since 1997, the world’s total wind power generation capacity has been increasing at an average rate of more than 25% per year. It is considered a green alternative to fossil fuels due to its incomparable features of being plentiful, renewable, and widely distributed, and producing no greenhouse gases during operations [1]. However, because of the variable and uncertain behavior of wind, generation planning with high penetration of wind power

becomes more difficult. In order to make the most effective and efficient investment decisions, dealing with the increasing uncertainty in generation planning that involves large-scale wind power is an urgent and challenging task.

To address these uncertainties, stochastic programming is one of the most popular approaches applied in power system generation planning problems [2], where uncertainties are described by random variables with some predetermined probability distributions. Stochastic programming approaches have been explored by a lot of researches not only in the short term generation operations [3] but also in the long-term expansion planning [4, 5]. However, most previous works were designed to deal with *exogenous uncertainty* [6], where the probability distributions of uncertain factors are predetermined and fixed before the optimization process. In other words, the stochastic programming models with *exogenous* uncertainties, such as electricity prices, are usually formulated based on the assumption that future random electricity prices are independent of the investment decisions at the current stage. However, in real-world generation planning, the decision variables in the current stage also play an important role influencing the future uncertainties. The study in [7] shows that the decisions on power plant expansion are affected by several variable criteria including capital costs, current costs, budget deduction, and electricity prices. It is discovered in [8] that different risk-aversion levels result in different investment strategies in wind facilities. In the study in [9], the maximum social welfare is achieved when the electricity price is varying according to a user's energy demands. It is shown in [10] that different installed capacities of wind power will influence the entire power system, especially when a large amount of wind power is involved. The probability distribution of future electricity prices is affected by the level of wind power. All of these research findings indicate that decision variables play an important role in determining the uncertain process at later stages. Hence, in order to consider the *endogenous uncertainties*, we adopt a decision-dependent approach that takes into account decision variables in determining the distributions of the uncertain process. In the operations research field, several studies have utilized this decision-dependent approach to deal with *endogenous* uncertainties. Among the many approaches, a hybrid mixed-integer disjunctive programming approach has been presented in [6] to address a class of stochastic programs with decision-dependent uncertainties. In [11], a decision-dependent approach was applied to a mixed-integer stochastic programming model where the timing of information discovery can be influenced by decisions.

In this paper, we propose a long-term planning model through a multistage, decision-dependent, stochastic nonlinear programming approach. We take advantage of the decision-dependent process where the probability distributions of electricity prices depend on the key decision variables: the installed capacities of different types of generation assets. Our proposed model contains bilinear terms that make the optimization process computationally very challenging. This model, known as the bilinear program (BLP), belongs to the class of hard nonconvex nonlinear programs where functions are twice continuously differentiable [12]. Previous studies proposed

many theoretical and algorithmic approaches for acquiring the optimality of BLPs, such as deterministic branch-and-bound [13], branch-and-contract global optimization algorithm [14], reformulation-linearization technique [15], Lagrangian relaxation [16], automatic symbolic reformulation procedure [17], linear cutting plane algorithms [18], effective heuristic algorithms [19] and etc. However, there are also limitations among some of the existing approaches that prevent their direct applications to our model. For example, some approaches only work with special BLPs (e.g., disjoint BLP [13], BLP with nonlinear constraints [14, 18]), some approaches only converge under certain conditions (e.g., the zero duality gap conditions for Lagrangian relaxation [16]), some approaches may not always converge to a global optimum [19]. Nevertheless, the quasi-exact solution algorithm in [20] uses a straightforward linearization mechanism that does not have the aforementioned limitations. Borrowing the method that a modern computer represents any fractional number by using binary variables, the quasi-exact approach ensures that the MILP (Mixed Integer Linear Programming) problem is equivalent to the original problem when a large number of binary variables are used to ensure the accurate representation of the fractional numbers. Hence, the new resulting MILP (Mixed Integer Linear Programming) problem can be solved conveniently and efficiently by using any off-the-shelf MILP commercial solver, but still attains a high level of accuracy as shown in our numerical results. Using this model and the solution approach, we study the impact of the investments of large-scale wind generation on long-term generation expansion planning.

The structure of this paper is the following. Section II states model assumptions. Then, the mathematical formulation is described in details in Section III. The quasi-exact solution approach using discretization and linearization is discussed in Section IV. The results from our numerical case studies to validate the model and the solution method are presented in Section V. Finally, Section VI presents conclusions.

## II. MODEL ASSUMPTIONS

In this section, we discuss the settings and assumptions of our model. It is assumed that the power system consists of two types of generators: thermal and wind. The model considers a planning horizon of 4 stages, each of which spans 5 years. This model can be also applied to compute under other lengths of planning horizon by changing the values of the parameters without loss of generality. Since the temporal variability of load and wind power is mainly due to meteorological fluctuations of seasons and hours of the day [10], in our long-term model these factors have a relative short-term effect, and thus can use the average values.

Expansion planning models deal with the long-term investment problem, where long-term load growth and price trends are the main drivers for investment decisions. Given the size of the system and the multi-stage nature of the investment decisions, even simplified investment problems may become extremely complex and large-scale optimization problems if all operational constraints, as well as the stochastic, dynamic characteristics of the renewable generation and load are considered. Such operational details may result in a large bi-level

(or tri-level) optimization problem, e.g., in [21]. In addition, it is often seen that results may not be that different from the case where some of operational constraints are simplified or ignored. For example, the commitment for thermal units can be absent. Besides, from the market perspective, similar considerations would hold for the modeling of all types of strategic behaviors of market competitors and the potential resulting equilibria as in [21]. Hence, in many of the previous studies, network effects which may be of less importance, have not been explicitly included (e.g., in the European context as discussed in [21] and [22]). Other cases without modeling network effects include short-term models [23, 24] and long-term models [10]. In addition, regardless of the network, the overall average electricity price has a negative relationship with the wind penetration [10]. Therefore, in this paper, since we focus on a new decision-dependent modeling approach, where the development of future uncertain prices depends on current investment decisions, we assume an aggregated level of operations and uncertain prices without explicitly modeling the network effects.

#### A. Planning Horizon and Scenario Tree Settings of the Uncertain Electricity Price

Our study is aiming at long-term modeling where generation expansions are usually conducted via multiple steps/stages. We use a rooted scenario tree with multiple stages and branches to represent the planning horizon with uncertainty. There are two key features in the scenario tree: a time horizon divided into discrete stages, and each node (except the leaf) has several child nodes with different outcomes that represent the different realizations of uncertainty. For simplicity, these stages,  $j \in \{1, 2, \dots, J\}$ , occur at evenly spaced increments of time. We denote the complete set of *nodes* of the scenario tree by  $\mathcal{N}$ , each of which represents a potential state of the market price of electricity. We also use the set  $\mathcal{N}^- = \mathcal{N} \setminus \{1\}$  to represent all nodes starting from stage  $j = 2$ . For every node  $n \in \mathcal{N}^-$  in stage  $j$ , we denote its unique *ancestor* node as  $a(n)$  at stage  $j - 1$ . In contrast,  $\mathcal{S}_n$  denotes the set of *successors* of node  $n$  in stage  $j + 1$ . Hence, we can use  $\mathcal{S}_{a(n)}$  to denote the set of nodes that share the same ancestor node  $a(n)$  in stage  $j$ . The root node is denoted as  $n = 1$  which is in stage  $j = 1$ .

We assume that the electricity prices (at different time periods) are uncertain, and model them by using discrete random variables. At each ancestor node  $a(n)$  (at time  $j$ ), each node in the child node set  $\mathcal{S}_{a(n)}$  is corresponding to an outcome/realization of the discrete random electricity price (at time  $j + 1$ ). We use  $p^{a(n)}$  and  $p^n$  to represent the prices in ancestor node  $a(n)$  and node  $n$ , respectively. Then,  $\delta^n$ , a prefixed parameter, is used to generate the outcome/realization of price at node  $n$ , through the equation,

$$p^n = p^{a(n)} \cdot (1 + \delta^n), \quad \forall n \in \mathcal{S}_{a(n)}. \quad (1)$$

For different nodes,  $\delta^n$  is chosen differently. For example, in a binary tree, the two child nodes of  $a(n)$  can have opposite values, e.g.,  $\pm 5\%$ , to represent an increase and a decrease.

#### B. Modeling the Decision-Dependent Probability

While a power market is embracing more deregulation and competition, electricity prices and demands are directly influenced by the mix of the power generation capacity as in [25, 26, 27]. Wind generation's marginal cost (excluding its maintenance cost) is usually considered as zero. Hence penetration of wind power will undoubtedly decrease the electricity price. However, electricity prices are also considered as uncertain in many long-term expansion planning researches. It is important to link the price uncertainty with the expansion planning decisions. As discussed in Section I, one of the key features of our decision-dependent stochastic model is the decision-dependent probability distribution, which is modeled by a function of decision variables. In this paper, we discretize the electricity price in a known and fixed sample range. In addition, we assume that the probabilities associated with given levels of electricity prices are not input parameters but are dependent on the investment decisions, as evidenced in the previous literature [7, 9, 10, 28, 29]. For example, researchers found that the average electricity price would decrease as the share of wind power in the generation portfolio increases. Moreover, a low-electricity-price scenario is more likely to happen if wind power's share is increasing. The opposite occurs for a high-electricity-price scenario. This is largely due to the fact that wind-power generation, compared to thermal generation, has a lower combined generation plus maintenance cost ( $c_i$ ) for every megawatt hour of electricity it generates, even when we factor in the levelized investment cost ( $B_i$ ) [30]. To model these findings, we propose a decision-dependent model to link probabilities of uncertain electricity price outcomes with investment decisions.

In our proposed model, we assume that the probability associated with any electricity price outcome (at node  $n$ ) is a multivariate function of the possible future electricity price itself, generation portfolio (including both wind and thermal power capacity), combined generation and maintenance cost and levelized investment cost (per megawatt hour). In the scenario tree, every node (e.g.,  $n$  representing a price outcome) is associated with a transition probability from its parent  $a(n)$ . As investment is a vital factor driving the electricity prices, for the decision-dependent uncertain electricity price, we assume that there is a positive relationship between the likelihood of a price outcome and its return or profitability on the investment. Motivated by [31], this probability is modeled as follows,

$$Prob^n = \frac{\sum_{i \in \{1,2\}} \frac{x_i^n (p^n - c_i - B_i)}{B_i (x_1^n + x_2^n)}}{\sum_{t \in \mathcal{S}_{a(n)}} \sum_{i \in \{1,2\}} \frac{x_i^t (p^n - c_i - B_i)}{B_i (x_1^t + x_2^t)}}, \quad \forall n \in \mathcal{N}^-. \quad (2)$$

where  $\mathcal{S}_{a(n)}$  is the set of nodes having the same parent node,  $a(n)$ . Based on the real-world data (see Table I from [30]), it is clear that  $Prob^n \geq 0$ . In addition,  $\sum_{t \in \mathcal{S}_{a(n)}} Prob^t = 1$ . As  $B_i$  is the levelized investment cost, we can use  $\frac{(p^n - c_i - B_i)}{B_i}$  as a measure for the rate of the return on the investment of generation type  $i$  when price is  $p^n$ , and then  $\sum_{i \in \{1,2\}} \frac{x_i^n}{x_1^n + x_2^n} \cdot \frac{p^n - c_i - B_i}{B_i}$  can be considered as a measure for the average rate of return on the total generation capacity. Equation (2) then defines the transition probability of a specific price outcome

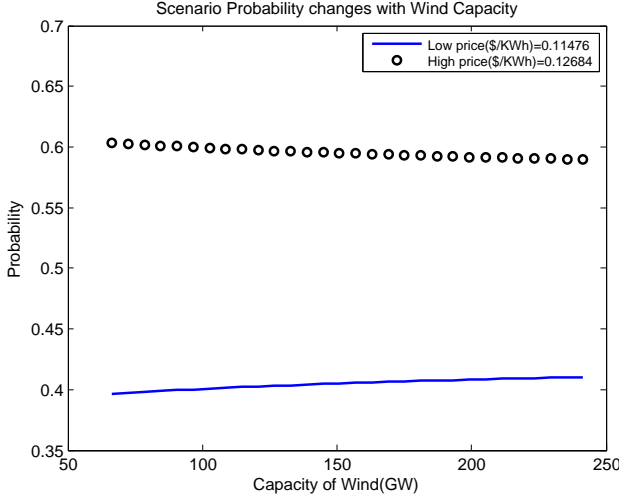


Fig. 1. Price Probabilities vs. the Wind Capacity of the Power System

$p^n$  (i.e., node  $n$  in the scenario tree) from its parent node  $a(n)$ . It is positively related to the return or profitability on the composition of the total generation asset. Note that Equation (2) presents the transition probability as a function of the market price  $p^n$  and the generation capacity  $x_i^n$ ,  $\forall i$ . Unlike the traditional stochastic programming models with exogenous uncertainties where probability is fixed as a model's input parameters, the decision-dependent probability changes according to the investment decisions, making the model a decision-dependent stochastic programming model. In a nutshell, investment decisions affect the random price through influencing the probabilities of the outcomes while having a prefixed/known sample space of the price. Figure 1 shows an example of the two-outcome probability curves that change in response to the installed wind capacity (i.e., the generation capacity mix as the thermal capacity is fixed) according to our model (2). The vertical axis represents the probability value, and the horizontal axis is the installed capacity of wind generation. The top and the bottom curves are representing the probability values under high-price and low-price outcomes respectively while perturbing the wind generation capacity from 50 GW to 250 GW. The thermal capacity is fixed at 306 GW. The two price outcomes used are 0.12684 \$/Kwh and 0.11476 \$/Kwh. All other data regarding levelized investment costs ( $B_i$ ) and combined generation plus maintenance costs ( $c_i$ ) for both types of generations are obtained from EIA data [30] as shown in Table I. From the plot, we can observe that the probability is affected by both the price level and the wind capacity (i.e., its share of the total capacity as the thermal capacity is fixed). The two discrete outcome's probabilities are reflected by two separate curves on the plot. When the wind capacity is small, the high-price outcome has a higher probability. When the wind capacity is increasing, the high-price outcome's probability starts to decrease and the low-price outcome's probability starts to increase. In addition, the expected value of electricity price decreases while wind capacity penetration increases.

All of the above observations can be shown as corollaries

of the following theorem.

**Theorem 1.** *In the two-outcome case with a high price outcome ( $P^H$ ) and a low price outcome ( $P^L$ ), the ratio between the corresponding probabilities,  $Prob^H/Prob^L$ , is a decreasing function of wind generation capacity,  $x_2$ .*

*Proof:* By plugging in the the formulas of probabilities of high and low prices defined in (2), we can have

$$\frac{Prob^H}{Prob^L} = \frac{P^H - c_2 - B_2}{P^L - c_2 - B_2} + \frac{x_1(c_1 + B_1 - c_2 - B_2)(P^H - P^L)}{x_1(P^L - c_1 - B_1)(P^L - c_2 - B_2) + x_2(P^L - c_2 - B_2)^2}$$

Indices 1 and 2 are denoting thermal and wind generation respectively. Based on EIA data [30] (see Table I),  $c_1 + B_1 - c_2 - B_2 > 0$ , i.e., that the thermal generation has a higher total sum of the levelized operations cost and the levelized investment cost. Also we know that  $P^H - P^L > 0$ , and then when  $x_2$  increases, the ratio  $\frac{Prob^H}{Prob^L}$  decreases. ■

**Corollary 1.**  *$Prob^H$  is a decreasing function of  $x_2$ .*

*Proof:* We know that  $Prob^H + Prob^L = 1$ . Hence  $\frac{\partial Prob^H}{\partial x_2} = -\frac{\partial Prob^L}{\partial x_2}$ . By Theorem 1, we know that  $\frac{\partial \frac{Prob^H}{Prob^L}}{\partial x_2} < 0$ . In addition,

$$\begin{aligned} \frac{\partial \frac{Prob^H}{Prob^L}}{\partial x_2} &= \frac{\frac{\partial Prob^H}{\partial x_2} Prob^L - \frac{\partial Prob^L}{\partial x_2} Prob^H}{(Prob^L)^2} \\ &= \frac{\frac{\partial Prob^H}{\partial x_2}}{(Prob^L)^2} \end{aligned}$$

Hence,  $\frac{\partial Prob^H}{\partial x_2} < 0$ . This also means that  $Prob^L$  is an increasing function of  $x_2$ . ■

**Corollary 2.** *The average electricity price is a decreasing function of wind generation capacity, i.e.,  $x_2$ .*

*Proof:* Let  $AVP$  denote the expected price, and then  $AVP = Prob^H P^H + Prob^L P^L$ . The partial derivative with respect to  $x_2$  is

$$\begin{aligned} \frac{\partial AVP}{\partial x_2} &= P^H \frac{\partial Prob^H}{\partial x_2} + P^L \frac{\partial Prob^L}{\partial x_2} \\ &= \frac{\partial Prob^H}{\partial x_2} (P^H - P^L) < 0 \end{aligned}$$

Hence, when  $x_2$  increases, the expected price decreases. ■

This price dependence on capacity is consistent with the results from other studies on real-world power systems. The long-term wind power investment study from [10] indicates that the average expected value of price decreases as wind farms are added. Similarly, the study from [29] also shows a decrease of average price due to the increasing wind power. Therefore, as mentioned in many studies, the price dependency on wind capacity is important for investors in evaluating the economic effects of power generation investments.

### C. Generating the Market Demands as Inputs to the Model

Since the early seminal study of US electricity demand [32],

electricity price and demand are founded closely linked. The relationship is depicted by the elasticity equations between electricity demand and price. Various research efforts have been taken to understand this relationship at both national and regional levels [33, 34, 35]. In this paper, we generate the demands as input parameters by assuming that the load/demand is affected by the electricity price variation. When price is increasing, the customer's desire to consume is lower, and therefore the load demand should be decreasing. Conversely, a large amount of demands could be stimulated by cheap prices. In this paper, we assume an elasticity model based on the well-known Tellis's econometric model [36]. The elasticity of demand to price is defined as  $E = (\Delta D/\Delta p) \cdot (p/D)$ , where  $p$  is the market price,  $E$  is the elasticity level, and  $D$  is the elastic demand. Based on this elasticity relation, we generate the demands in each node by using the following numerical expression of demand variation with respect to the corresponding price outcome ( $p^n$ ), i.e.,

$$D^n = \frac{D^{a(n)}}{(p^{a(n)})^E} \cdot (p^n)^E \cdot (1 + e). \quad (3)$$

In addition, this equation constructs the connection between  $D^n$  and  $D^{a(n)}$ , which are the demand in node  $n$  and in its ancestor node  $a(n)$ , respectively. The parameter  $E$  represents the elasticity which is usually a negative value between  $-0.13$  and  $-0.15$  based on the study in [34], which covered the data of electricity price and demand relationship in US for more than two decades. Because  $E$  is chosen greater than  $-1$  but negative, meaning the demand is not change much while price is varying, it is generally considered as inelastic (as opposed to the perfectly inelastic case, i.e.,  $E = 0$ ). The incremental level representing other factors (e.g., population growth, new electricity appliances) between demands in node  $n$  and its direct ancestor  $a(n)$  is represented by  $e$ .

### III. MATHEMATICAL FORMULATION

The objective of power generation expansion planning is to maximize the total expected profit (based on the whole scenario tree), which is calculated as the difference between total revenue ( $Rev$ ) and the total cost. The total revenue at each node  $n$  can be calculated by  $Rev^n = p^n D^n$ . The total cost consists of two parts: the total investment cost and the total operational cost (fuel costs plus maintenance costs). In our model, the investment costs ( $IC^n$ ) are calculated on all nodes except the nodes associated with the last stage  $J$ . This is because the investment decisions are made to accommodate the future power system operations, and we assume an invested infrastructure at the current time period will be available starting from the next time period. For convenience, we use  $\mathcal{N}'$  to denote the set of nodes having investment costs. The operating costs are calculated at each node of the scenario tree and include both generation costs (mainly thermal generators) and maintenance costs (both generation types). Hence, the objective function is a weighted sum of these revenues and costs, where the weights are simply the probabilities of the associated nodes in the scenario tree. Then we propose a

multistage stochastic investment [MSI] model as follows,

$$\max SW^1 \quad (4a)$$

$$\text{s.t.} \quad (2)$$

$$SW^{a(n)} = -IC^{a(n)} + \sum_{t \in \mathcal{S}_{a(n)}} Prob^t \cdot$$

$$(Rev^t - CO^t + SW^t), \forall n \in \mathcal{N}^- \quad (4b)$$

$$IC^n = \sum_{i \in \{1,2\}} b_i \alpha_i^n, \forall n \in \mathcal{N}' \quad (4c)$$

$$CO^n = \sum_{i \in \{1,2\}} c_i g_i^n, \forall n \in \mathcal{N}^- \quad (4d)$$

$$x_i^n = x_i^{a(n)} + \alpha_i^{a(n)}, i \in \{1,2\}, \forall n \in \mathcal{N}^- \quad (4e)$$

$$g_i^n \leq H \beta_i x_i^n, i \in \{1,2\}, \forall n \in \mathcal{N}^- \quad (4f)$$

$$g_1^n + g_2^n = D^n, \forall n \in \mathcal{N}^- \quad (4g)$$

$$g_i^n, \alpha_i^n, w_i^n, Prob^n, x_i^n \geq 0, \forall i, n \in \mathcal{N} \quad (4h)$$

The decision variables  $\alpha_i^n$ ,  $x_i^n$ , and  $w_i^n$  respectively represent the future invested capacity, currently total, installed, cumulative capacity, and electricity generation of type  $i$  at node  $n$ . The cost parameter  $b_i$ ,  $B_i$ ,  $c_i$ , represent the unit investment cost, levelized investment cost, and levelized operation and maintenance cost of generation type  $i$ , respectively. Note that  $x_i^0$  is the initial installed capacities, which are given as parameters for both types of generators. The objective function (4a) has only one term:  $SW^1$ , which represents the total expected profit of the whole planning horizon, being calculated in a recursive way. Constraint (4b) defines the profit of the ancestor node  $a(n)$  in stage  $j-1$  that includes two terms: the investment cost  $IC^{a(n)}$  and the expected total cost of node  $n$ 's successors. The expected cost part consists of three terms: the operation cost  $CO^t$ , the revenue  $Rev^t$ , and the profit  $SW^t$  at node  $t$ , which is the immediate successor of node  $a(n)$ . Constraint (4c) defines the investment cost  $IC^n$ , which is determined by unit investment cost  $b_i$  and the new generation capacity  $\alpha_i^n$ . The operational cost  $CO^n$ , given by constraint (4d), is determined by production level  $g_i^n$ . We assume that capacity expansion investment decisions made at time  $j$  will be ready to use at time  $j+1$ . Then the relation between current installed capacity and the future investment amount is given by constraint (4e). The power generation amount is also limited by the capacity factor in (4f). The capacity factors  $\beta_i$  represent the average ratio of currently installed capacity that can be utilized for generation. The power generation amount is enforced by (4g) to meet the load demand. According to Section II-B, the decision-dependent price-capacity settings are included in (2) to capture the decision-dependent probability distributions.

### IV. SOLUTION APPROACH

Since the constraints (4b) and (2) contain bilinear terms and fractional terms of decision variables, [MSI] is therefore a nonlinear optimization model. We first rewrite constraint (2) to be  $Prob^n \cdot \sum_{t \in \mathcal{S}_{a(n)}} R^t = R^n$  to eliminate the fractional terms defining the probabilities, where  $R^n = \sum_{i \in \{1,2\}} \frac{x_i^n (p^n - c_i - B_i)}{B_i (x_1^n + x_2^n)}$ . In this way, the constraints (4b) and (2) both contain bilinear terms,  $\sum_{n \in \mathcal{S}_{a(n)}} Prob^n \cdot$

$(Rev^n - CO^n + SW^n)$  and  $Prob^n \cdot \sum_{t \in \mathcal{S}_a(n)} R^t$ . A bilinear term is the product of two decision variables and therefore makes the problem nonconvex and hence difficult to solve. As discussed in Section I, linear-reformulation is widely used to solve nonconvex nonlinear optimization problems [15, 37, 38, 39]. We employ a quasi-exact method [20] to deal with the bilinear terms of our model. This method has a close link to Meyer and Floudas's [15] reformulation-linearization technique that reformulates the bilinear program (BLP) into mixed-integer linear programs (MILP). In both methods, the BLP is augmented with a set of binary variables. However, note that in our BLP model, the bilinear terms are very special which contain a continuous variable between 0 and 1, i.e.,  $Prob^n$ . The quasi-exact method is specifically designed for this particular format. Unlike the reformulation-linearization technique that needs additional linear relaxation preprocessing, the quasi-exact approach uses a more straightforward approach that directly transforms the BLP to a series of bilinear products containing a binary variable and a continuous variable. Eventually, these products can be further linearized and therefore transformed to a series of mixed-integer linear programs. As the result, the constraints with bilinear terms can be formulated as a series of mixed-integer linear constraints.

This is because the quasi-exact approach is specifically designed to deal with bilinear terms that contains a fractional number between 0 and 1. We represent the variable  $Prob^n$  via a series of binary variables. Eventually, the [MSI] model could be transformed to be a mixed-integer linear programming (MILP) problem, which can be solved conveniently by a state-of-the-art MILP solver.

#### A. Solving the Bilinear Model through the Discretization-Linearization Procedure

Given the definition of probability ( $Prob^n$ ), it can only take a value between 0 and 1. In a modern computer system, any fractional number or variable  $x$  that is between 0 and 1 is represented by a series of binary variables  $z_l \in \{0, 1\}$  [20], i.e.,  $x = \sum_{l=0}^L 2^{-l} z_l + \epsilon$  where  $L$  is the number of binary variables needed, and is related to the degree of accuracy.  $\epsilon$  is the nonnegative error term. Its value is confined by  $L$  as  $\epsilon < 2^{-L}$ . Clearly, the more binary variables being used, the more accurate this approximation becomes.

Using the same approach, the variable  $Prob^n$  can be discretized as follows,

$$Prob^n = \sum_{l=0}^L 2^{-l} z_l^n + \epsilon^n, \quad \forall n \in \mathcal{N}^- \quad (5)$$

Substituting  $Prob^n$  in model [MSI] with the expression in (5), we have a new expression for constraints (4b) and (2):

$$SW^{a(n)} = -IC^{a(n)} + \sum_{t \in \mathcal{S}_a(n)} \left( \sum_{l=0}^L 2^{-l} z_l^t + \epsilon^t \right) \cdot (Rev^t - CO^t + SW^t), \quad \forall n \in \mathcal{N}^-, \quad (6a)$$

$$\sum_{t \in \mathcal{S}_a(n)} R^t \cdot \left( \sum_{l=0}^L 2^{-l} z_l^n + \epsilon^n \right) = R^n, \quad \forall n \in \mathcal{N}^- \quad (6b)$$

However, both  $z_l^n$  and  $\epsilon^n$  are variables, and there still exist bilinear terms in (6a) and (6b). These bilinear terms have the

same format: a binary variable multiplied by a continuous variable. This type of bilinear terms can be easily linearized by introducing additional constraints and a big number,  $M_l$  [20]. For constraint (6a), we introduce a new variable  $\zeta_l^n$  to replace the bilinear term:

$$\zeta_l^n = z_l^n \cdot (Rev^n - CO^n + SW^n), \quad \forall n \in \mathcal{N}^-, l \quad (7)$$

Furthermore, we can replace the above equation with following equivalent constraints:

$$0 \leq \zeta_l^n \leq Rev^n - CO^n + SW^n, \quad \forall n \in \mathcal{N}^-, l \quad (8a)$$

$$(Rev^n - CO^n + SW^n) - M_l(1 - z_l^n) \leq \zeta_l^n \leq M_l z_l^n, \quad \forall n \in \mathcal{N}^-, l \quad (8b)$$

where  $M_l$  is a large number to bound the variables. For another term on the right side of constraint (6a),  $\epsilon^n \cdot (Rev^n - CO^n + SW^n)$ , there still exist bilinear terms with two continuous variables. However, this value is extremely small when enough binary variables (i.e., a large value for  $L$ ) are used to represent the probability. As discussed in the previous part of this section, the range of the error term while representing the probability is:  $0 \leq \epsilon^n < 2^{-L}$ . Hence, we can introduce a new variable  $\theta^n$  to represent the remaining bilinear term without losing accuracy by including the following constraint,

$$0 \leq \theta^n \leq 2^{-L} \cdot (Rev^n - CO^n + SW^n), \quad \forall n \in \mathcal{N}^-. \quad (9)$$

Combining equation (8) and equation (9), we can replace the nonlinear constraint (6a) with the following linear constraints,

$$SW^{a(n)} = -IC^{a(n)} + \sum_{t \in \mathcal{S}_a(n)} \left( \sum_{l=0}^L 2^{-l} \zeta_l^t + \theta^t \right), \quad \forall n \in \mathcal{N}^-, \quad (10a)$$

$$0 \leq \theta^n \leq 2^{-L} \cdot (Rev^n - CO^n + SW^n), \quad \forall n \in \mathcal{N}^-, l \quad (10b)$$

$$0 \leq \zeta_l^n \leq Rev^n - CO^n + SW^n, \quad \forall n \in \mathcal{N}^-, l \quad (10c)$$

$$(Rev^n - CO^n + SW^n) - M_l(1 - z_l^n) \leq \zeta_l^n \leq M_l z_l^n, \quad \forall n \in \mathcal{N}^-, \quad (10d)$$

Similarly, constraint (6b) can be converted as,

$$R^n = \sum_{l=0}^L 2^{-l} \eta_l^n + \sigma^n, \quad \forall n \in \mathcal{N}^-, \quad (11a)$$

$$0 \leq \sigma^n \leq 2^{-L} \cdot \sum_{t \in \mathcal{S}_a(n)} R^t, \quad \forall n \in \mathcal{N}^-, l \quad (11b)$$

$$0 \leq \eta_l^n \leq \sum_{t \in \mathcal{S}_a(n)} R^t, \quad \forall n \in \mathcal{N}^-, l \quad (11c)$$

$$\sum_{t \in \mathcal{S}_a(n)} R^t - M_l(1 - z_l^n) \leq \eta_l^n \leq M_l z_l^n, \quad \forall n \in \mathcal{N}^-, \quad (11d)$$

Because this quasi-exact solution process uses the error range of  $[0, 2^{-L})$  to replace the error term  $\epsilon^n$ , it is an approximation approach. Hence, the accuracy of our model depends on the number of binary variables used ( $L$ ). So does the computational difficulty, but negatively. Therefore, it is important to find a proper value of  $L$  to obtain high accuracy in a short computational time. This will be discussed in Section V-B.

#### B. The Multi-stage Stochastic Mixed-Integer Linear Model

After the bilinear terms are discretized and therefore linearized, the bilinear constraints (4b) and (2) from [MSI] are replaced by the mixed-integer linear constraints (10) and (11). A multistage stochastic mixed-integer linear model [MSMIL] is therefore formulated as shown below:

$$\begin{aligned} \min \quad & SW^1 & (12a) \\ \text{s.t.} \quad & (4c) - (4g), (4h), (10), (11) & (12b) \\ & z_l^n \in \{0, 1\}, \forall n \in \mathcal{N}^-, l & (12c) \end{aligned}$$

## V. NUMERICAL EXPERIMENTS

In this section, we present numerical experiments and analyze the results on generation expansion planning. Our model and algorithm are tested in a four-stage ( $J = 4$ ) scenario tree. At first, Section V-A introduces the experimental settings as well as input data for our model. The fidelity of the quasi-exact approach is verified in Section V-B via a series of computational experiments to show the relation between the number of binary variables and the relative approximation error. Then, Section V-C tests the applicability of our quasi-exact approach via a series of comparisons with an existing commercial solver. Finally, the results of numerical experiments are discussed and analyzed in Section V-D, V-E, and V-F. The computational model is programmed in C++ by calling the commercial MILP solver ILOG CPLEX 12.5. All experiments are implemented on a personal computer, which has quad Intel Core i7 processors with CPU at 3.40 GHz and a RAM space of 8GB.

### A. Data Preparation

The input data for our model are acquired from US EIA [30], as shown in Table I. We adopt the data of the conventional coal generator as the thermal generator, and the data of onshore wind farm as the wind generator. Compared to thermal generators, the wind generators have lower operation and investment costs. However, on the other hand, the capacity factor, which represents the average utilization of the total capacity, is lower for wind generator than that of thermal generators because of the nature of the unstable wind speed. Thus, with consideration of the capacity factor, the actual effective cost of investment and maintenance of wind is higher than that of thermal. The detailed data are shown in Table I. As mentioned in Section II-A, the market price is an uncertain parameter. The retail price at stage 1 (year 2015) is set at \$0.15/KWh. The variation level  $\delta$  of price outcomes is adjusted according to experiment settings. As mentioned in Section II-C, the load demand changes elastically with respect to the market price. The initial stage demand  $D_0$  is set to be 12303.8 TWh. The number of hours  $H$  is set to be 43750 hours as we assume each stage spans 5 years. The elasticity index  $E$ , which reflects the correlation between demand and price, is accordingly set to be  $-0.15$ . The demand increasing level  $e$  is adjustable with different experiment settings.

### B. Accuracy of Model Approximation by MILP

Through a series of computations, we study the accuracy of our quasi-exact linearized approximation approach with different values of binary variable ( $L$ ). The input data of our calculation comes from Section V-A, and we set the price variation level to be zero, that is, there is no difference between

TABLE I  
INPUT PARAMETERS [30]

Parameter	Thermal	Wind	Unit
$x^0$	0.306	0.0604	TW
$c$	0.0345	0.013	Billion\$ /TWh.
$B$	0.06	0.0641	Billion\$ /TWh.
$b$	3292	2213	Billion\$ /TW
$\beta$	0.85	0.30	N/A

different price outcomes (i.e., nodes in the scenario tree). It is obvious that the probability  $Prob^n$  of each node should be equal to each other, which is 0.5 for the two-node outcome. In this case, the [MSI] model could be linearized by setting the variable  $Prob^n$  to be a fixed value 0.5. Since  $Prob^s$  is no longer a variable, this simplified [MSI] model becomes a linear and deterministic model. Therefore, we can eliminate approximation, and solve the simplified [MSI] model with a linear solver, providing a benchmark. Without the quasi-exact linearization process which brings in approximation, the optimization result of this deterministic model provides a benchmark for estimating the relative error of the quasi-exact linearized approximation approach.

Table II lists the error level, the relative error, the optimal profit, and computational time given to a series of numbers of binary variables ( $L$ ). The error level is defined as  $2^{-L}$ . The relative error is defined as the percentage difference between the profit  $SW$  of the deterministic model and the one from the quasi-exact approximation approach  $SW_L^A$  (using  $SW$  as the base), that is,  $|SW_L^A - SW|/SW \times 100\%$ .

We notice in Table II that as  $L$  increases, the computational time rises significantly, whereas the relative error decreases dramatically. When  $L = 20$ , the relative error is at the same level as the result of BARON. As a result,  $L = 20$  is chosen as the initial approximation setting in the later case studies.

TABLE II  
NUMBER OF BINARY VARIABLES AND ERROR

L	$2^{-L}$	Profit(10 <sup>9</sup> \$)	Relative Error	Time(s)
Deterministic Result:		4320.9214*	0	0.047
BARON's Result:		4320.9252	$3.87 \times 10^{-5}$	136.91
5	$3.13 \times 10^{-2}$	4599.6700	$2.84 \times 10^{-0}$	0.27
10	$9.77 \times 10^{-4}$	4329.4600	$8.71 \times 10^{-2}$	0.61
15	$3.05 \times 10^{-5}$	4321.1900	$2.74 \times 10^{-3}$	5.43
20	$9.54 \times 10^{-7}$	4320.9297	$8.50 \times 10^{-5}$	15.21
25	$2.98 \times 10^{-8}$	4320.9223	$8.77 \times 10^{-6}$	25.17
30	$9.31 \times 10^{-10}$	4320.9216	$1.02 \times 10^{-7}$	92.32

\*: Computed from simplified [MSI] model with  $Prob^s = 0.5$

### C. Computation Comparison with Nonlinear Solver

To compare our quasi-exact approach with existing nonlinear solver, we embedded the bilinear [MSI] model in the global solver BARON. BARON is a state-of-the-art commercial software for solving nonconvex optimization problems to global optimality. We use BARON as a benchmark to test the applicability of our proposed approach. When solving an optimization problem, BARON reports an optimal solution (lower bound) and an upper bound. It declares global optimality



when the corresponding optimality gap is less than a certain threshold.

In the following tests, we conducted the same numerical experiments in Section V-D by using both BARON and the proposed quasi-exact approach. The optimality gap of BARON is set at  $10^{-6}$ . We report the relative difference between the optimal values from the quasi-exact approach and BARON. The computational time is also reported along with the results. Both solvers are implemented on the same personal computer.

TABLE III  
COMPUTATION COMPARISON UNDER DIFFERENT PRICE VARIATION LEVELS

Price Uncertainty Level	Profit ( $10^9$ \$)		Relative Difference (%)	Time (sec)	
	Quasi-exact	BARON		Quasi-exact	BARON
0%	4320.930	4320.92	0.00%	15.21	247.75
2%	4327.731	4327.68	0.00%	52.59	475.24
4%	4354.652	4354.65	0.00%	25.12	159.07
6%	4401.900	4401.89	0.00%	45.98	159.00
8%	4467.879	4467.88	0.00%	85.53	207.21
10%	4553.001	4552.99	0.00%	161.29	238.01
12%	4658.699	4658.69	0.00%	156.00	603.87
14%	4785.292	4785.27	0.00%	175.31	608.28
16%	4929.447	4929.52	0.00%	172.88	996.42
18%	5093.699	5093.69	0.00%	126.74	493.31
20%	5278.769	5278.91	0.00%	284.35	588.53

TABLE IV  
COMPUTATION COMPARISON UNDER DIFFERENT INCREMENTAL LEVELS FOR DEMANDS

Incremental Demand Level	Profit ( $10^9$ \$)		Relative Difference (%)	Time (sec)	
	Quasi-exact	BARON		Quasi-exact	BARON
0.0%	4484.35	4484.34	0.00%	144.30	125.56
0.2%	4498.00	4498.00	0.00%	102.17	304.03
0.4%	4511.57	4511.69	0.00%	95.72	313.03
0.6%	4525.44	4525.42	0.00%	101.92	454.07
0.8%	4539.19	4539.19	0.00%	111.45	279.68
1.0%	4553.00	4552.99	0.00%	48.33	238.01
1.2%	4566.85	4566.83	0.00%	374.16	503.07
1.4%	4580.71	4580.71	0.00%	94.44	360.57
1.6%	4592.87	4592.85	0.00%	221.90	477.34
1.8%	4604.58	4604.58	0.00%	121.11	606.18
2.0%	4616.34	4616.33	0.00%	334.81	521.48

In addition, we perform two series of tests. Firstly, we fix the demand incremental level, and change the price variation level and compare the computational differences between BARON and quasi-exact approach. The results is presented in Table III. Table IV presented the computational differences when we fix the price variation level but perturb the demand incremental level. From the results in Table III and Table IV, we notice that the relative difference in the optimal value between the proposed quasi-exact approach and BARON is always less than 0.01%. Hence, the optimality gaps are about at the same level for both solvers. This indicates that the quasi-exact method can provide equally accurate results as BARON, but with much less computational time on average.

In terms of computational time, the proposed quasi-exact approach finishes the computation in a shorter time in most of the cases. The quasi-exact method is able to acquire optimal results within 15 to 374 seconds under different price variation levels. On the other hand, the solution time of BARON varies

greatly from 125 to 996 seconds for different cases. Especially, when the price variation level or increment demand level is getting larger, the computational time of BARON increases dramatically. This indicates that the quasi-exact method has a much more stable performance than BARON. In addition, it is notable that the computational time is not monotonically increasing while we are increasing the demand incremental level and the variation level of the price uncertainty. The quasi-exact model is a mixed integer linear program. With different data inputs (but the problem size is the same), the cutting planes from the solver might have different strengths and the branch-and-bound procedure might take different paths. Hence it is not predictable if high incremental level or price variation level means more computational time.

#### D. Analysis under Different Prices and Demands

The uncertain market price is one of the factors that affects the investment decision. In this case study, we try to understand the economic effects of market price under price variation levels from  $\pm 0\%$  to  $\pm 20\%$ . Table V shows the results of this case study, including average market price, demand, profit, and wind penetration at each uncertainty price level. The average market price is calculated as the weighted average of market prices in all nodes. Wind capacity penetration is introduced to quantify the share of wind generators in the total power system's capacity as follows,

$$\text{Wind Capacity Penetration} = \frac{\text{Installed Wind Capacity}}{\text{Total Capacity}} \times 100\%$$

TABLE V  
OPTIMIZATION RESULT UNDER DIFFERENT PRICE VARIATION LEVELS

Price Uncertainty Level	Average Price ( $10^9$ \$/TWh)	Demand (TWh)	Profit ( $10^9$ \$)	Wind Penetration (%)
0%	0.150	49422.6	4320.93	16.150%
2%	0.152	49306.2	4327.73	16.194%
4%	0.155	49181.3	4354.65	16.159%
6%	0.158	49048.3	4401.90	16.158%
8%	0.161	48907.5	4467.88	16.134%
10%	0.165	48759.1	4553.00	16.094%
12%	0.169	48603.4	4658.70	16.053%
14%	0.173	48440.7	4785.29	16.014%
16%	0.177	48274.3	4927.45	15.982%
18%	0.182	48102.3	5093.70	15.941%
20%	0.187	47924.5	5278.77	15.940%

In Table V, we see that as the variation level of the uncertain price increases, the average market price increases and demand decreases. It is because the demand is affected by the elastic relationship with the market price; thus, the demand shrinks as the price increases. We also notice that the wind penetration level decreases as the market price increases. In the investment problem of power systems, the more wind power we have, the lower the electricity price will be because of the price elasticity curve and the zero marginal cost of wind power. In this paper, prices (outcomes) are set as input parameters. Hence, when the average price increases, the wind power penetration is expected to decrease. This is in line with the previous literature

[10, 25, 29]. Note that we show the data on average price, demand, and wind penetration. They are the average of all nodes in the scenario tree. However, the total installed capacity is not necessarily monotonically decreasing. This is because a larger decrease of price will lead to a larger increase of demand (based on the elasticity equation), and the investment in the parent node needs to cover the larger demand (from the low price-outcome node) in stochastic programming, causing the total capacity to increase. The profit is increasing as the average price is getting higher. The increment of the average price from  $\$0.150/KWh$  to  $\$0.187/KWh$  makes the profit increase by 22%. This increase in profit is attributed to two causes, i.e., the increasing revenue and the decreasing cost. On one hand, even with a small amount of demand decrease (3.03%), the large increase of market price (24%) appears to increase total revenue. On the other hand, lower demand results in a reduced cost in production and investment.

The demand is also an important factor that influences the generation expansion decisions, as shown in Table VI. To analyze the effect of the increasing demand on the power system, we conduct numerical experiments under different incremental levels of the demand while the price variation level is fixed at  $\pm 10\%$ . The results are shown in Table VI. It illustrates that wind penetration and the profit are correlated outputs: they both change according to different demands. As the demand increases, the wind penetration decreases and profit increases. When the demand increases, the needs for infrastructure expansion increase, which leads to more investments. The investment decisions tend to invest in less wind which has higher investment cost.

TABLE VI  
OPTIMIZATION RESULT UNDER DIFFERENT INCREMENTAL DEMAND LEVELS

Demand Increment Level	Demand (TWh)	Profit ( $10^9$ \$)	Wind Penetration
0.0%	48039.2	4484.35	16.25%
0.2%	48182.4	4498.00	16.22%
0.4%	48326.0	4511.57	16.19%
0.6%	48470.0	4525.44	16.16%
0.8%	48614.3	4539.19	16.12%
1.0%	48759.1	4553.00	16.09%
1.2%	48904.2	4566.85	16.06%
1.4%	49049.7	4580.71	16.03%
1.6%	49195.6	4592.87	15.98%
1.8%	49341.9	4604.58	15.91%
2.0%	49488.5	4616.34	15.85%

### E. Investment Decision Analysis

To illustrate the result of the [MSMIL] model graphically, we plot the optimization decisions in scenario trees, as shown in Figure 2. In Case 1, 2 and 3, the price's variation level is fixed at  $\pm 10\%$ . The incremental demand level is 1% in Case 1 and 2% in Case 2 and 3. Case 1 and 2 use the decision-dependent probability model from Section II-B, the probability in Case 3 is set to be 0.5. The numbers above/below each branch represent the probability ( $Prob^n$ ) of the outcomes. The two numbers within the parentheses represent the investment decisions ( $\alpha_i^n$ ) for thermal and wind generation, respectively.

The numbers in each node represent the node number of the scenario tree. For the two child nodes of the same parent node, the market price in upper node is higher than the price in lower node.

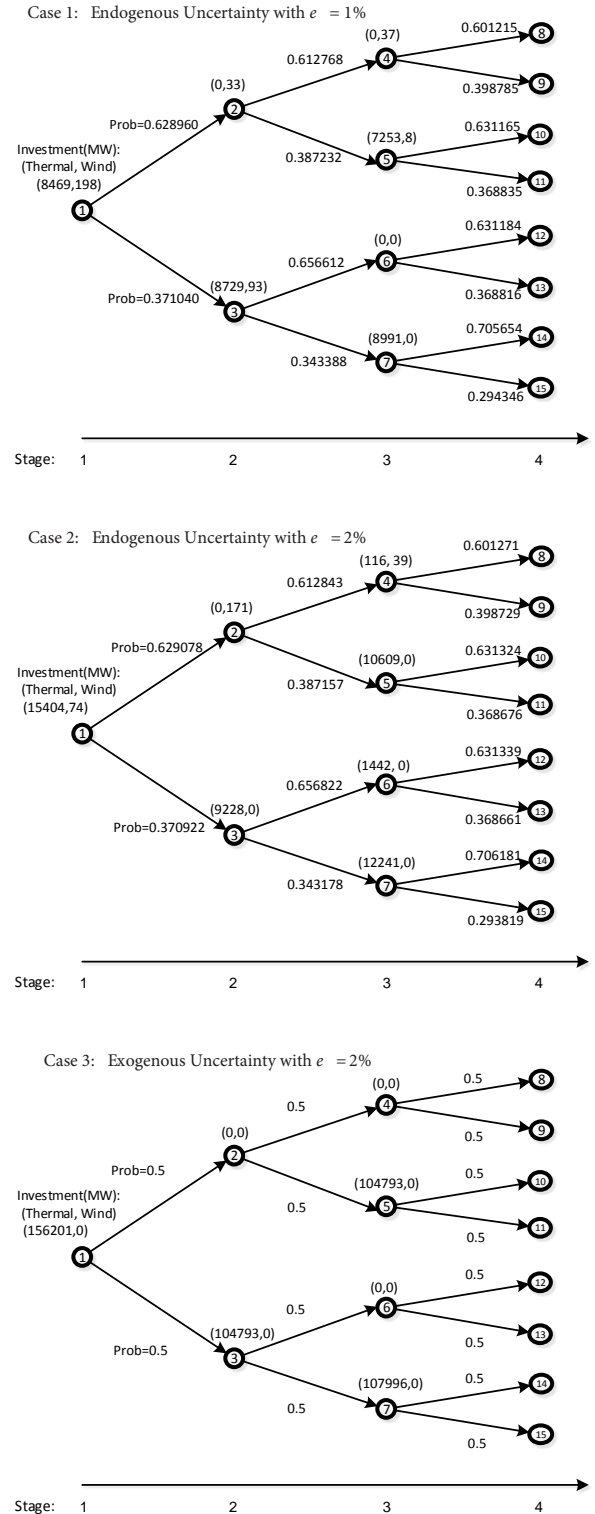


Fig. 2. Investment Decisions and Probability at Each Outcome

Considering Case 1 and 2, from Section II-B, we can see

that the probability distribution is affected simultaneously by both the market price and investment decisions. In order to highlight the effect of investment decisions, the market price level is set the same between the first two cases in Figure 2. For each outcome, when the price level is given, the investment, on the other hand, plays an important role in determining the probability distributions. The following example shows how the investment decisions influence the decision-dependent probability distributions. While comparing the stage 2 (node 2,3) in Case 1 and 2, we note that, even though the price levels are the same in both cases, the probability distributions are different between Cases 1 and 2 ( $\{0.628960, 0.371040\}$  vs.  $\{0.629078, 0.370922\}$ ). This is because the investment in node 1 increases the wind capacity in stage 2 for both cases, but the amount of wind power investment in Case 1 is larger than Case 2 (198MW vs. 74MW). This makes the wind capacity in stage 2 of Case 1 larger than that in Case 2. Thus, this causes the probability distribution difference between the two cases. As a result, Figure 2 shows that investment decisions in stage 1 shift the decision-dependent probability distributions in stage 2.

From Section V-A, we already know that the thermal generator has a low-cost investment advantage over wind. From the result in Case 3, we notice that without decision-dependent process, the traditional optimization decisions will focus all on the thermal generator for future investment. However, attributing to the decision dependent process, the results in both Case 1 and 2 show the investment decisions involve both thermal and wind generators.

#### F. Decision-Dependent Analysis

To examine the effectiveness of the decision-dependent approach, we introduce a term, the value of decision-dependent stochastic programming solution (VDDSS). It is extended from the concept of the value of stochastic programming solution (VSS). Unlike the VSS that compares a stochastic approach to a deterministic approach, the VDDSS evaluates the decision-dependent approach over the traditional stochastic approach (with exogenous uncertainty). To calculate the VDDSS, we first compute the optimal solution from a traditional stochastic model that uses the same input parameters and a prefixed probability distribution (e.g., uniformly distributed). Then, this solution is plugged into the decision-dependent formulation and the objective function value is then acquired. Finally, the VDDSS is calculated as the difference between the optimal objective value from decision-dependent approach and the objective value by using the traditional stochastic model solution in the decision-dependent model.

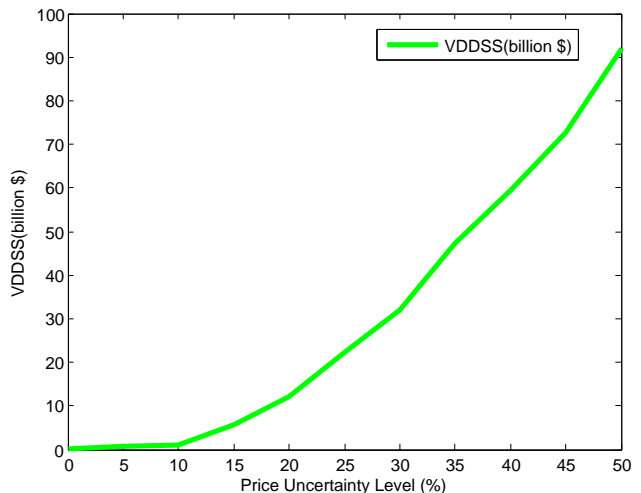


Fig. 3. The Value of Decision-dependent Stochastic Programming Solution

Figure 3 shows that as price uncertainty changes from 0% to 50%, the VDDSS increases dramatically. When the price variation level is equal to zero, we observe that VDDSS is also equal to zero. This is because they both reduce to the same deterministic model. We observe that the VDDSS is greater than zero which indicates that the optimal solution from decision-dependent [MSMIL] formulation provides a larger profit than the one from the traditional stochastic programming approach. Moreover, as the price variation level increases (i.e., the difference of input parameters is greater between outcomes), the VDDSS is greater. Hence the decision-dependent approach outperforms the traditional stochastic programming approach especially when the price variation level is high. From these results, we can conclude that it is important to take into account the decision-dependent approach in evaluating the economics of long-term generation expansion planning where tremendous uncertainty exists and interplays with the investment decisions.

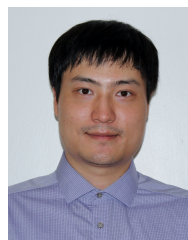
#### VI. CONCLUSION

In this paper, we introduce a decision-dependent stochastic programming model for long-term power generation expansion planning, where probabilities of price outcomes are variables dependent on investment decisions. We develop an optimization strategy to maximize the total profit. To solve this nonlinear stochastic program, a quasi-exact solution approach is then adopted to reformulate the multistage, stochastic, nonlinear model to a MILP model, which is solved by CPLEX. The analysis of different prices and demands shows that the wind penetration and the profit are strongly related to both price and demand. From the analysis of investment decisions, we discover that generation expansion investment plays an important role in determining the probability distribution. We also analyze and compare the solutions from the decision-dependent model against those from the stochastic model with exogenous uncertainties. The numerical results show that it is very important to take into account the proposed decision-dependent approach in evaluating the economics of long-term generation expansion planning. We conclude that the

proposed decision-dependent stochastic programming model, which adopts the decision-dependent probabilities, can provide effective optimization information on investment for long-term generation expansion planning. As operational constraints and network effects in some cases can be the driving factors affecting the investment decisions for generation expansion, we will take a different approach to develop new models with endogenous uncertainties while including the operational details of system constraints in our future research endeavors. Applications of high-performance computing (HPC) may also be needed.

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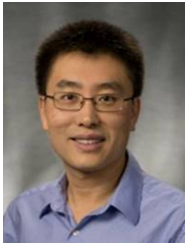


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