Technical University of Denmark

DTU

Three-dimensional submodel for modelling of joints in precast concrete structures

Herfelt, Morten Andersen; Poulsen, Peter Noe; Hoang, Linh Cao; Jensen, Jesper F.

DTU Library
Technical Information Center of Denmark

Three-dimensional submodel for modelling of joints in precast concrete structures

Morten A. Herfelt[1,2], Peter N. Poulsen[2], Linh C. Hoang[2] and Jesper F. Jensen[1]

[1]ALECTIA A/S,
Teknikerbyen 34, 2830 Virum, Denmark

[2]Department of Civil Engineering,
Technical University of Denmark,
Brovej, Building 118, 2800 Kgs. Lyngby, Denmark

Abstract

The shear capacity of in-situ cast joints is crucial to the overall stability of precast concrete structures. The current design is based on empirical formulas, which account for neither the reinforcement layout of the joint nor the three-dimensional stress states present within the joint. In this paper, a semi-analytical submodel is presented. The model is formulated as an optimisation problem within the framework of numerical limit analysis. The presented submodel is capable of describing the key mechanisms within the joint and the model is verified by comparison to existing numerical models. The shear capacity of the joint subjected to triaxial stress states is analysed and the results are discussed.

1   Introduction

The lateral stability of precast concrete structures is often ensured by shear walls. These walls are composed of precast wall panels connected by in-situ cast joints. Horizontal forces, e.g. from wind load, are transferred as in-plane forces and the shear capacity of the panels and joints are crucial to the capacity of the entire structure. This paper will focus on joints between shear walls and slabs or corner joints, which will be subjected to triaxial stress states; something, which is not considered by the current standards (see e.g. European Committee for Standardization, 2005).

The joints typically comprise a concrete core reinforced with U-bar loops or similar. The interface between the precast wall or slab and the in-situ cast joint is often keyed. A continuous reinforcement bar (locking bar) is typically placed in the centre of the joint to enhance the transfer of tension between the pairs of U-bars. Fig. 1 shows two typical joints, namely a joint connecting shear walls and slabs (left) and a corner joint connecting three walls (right).
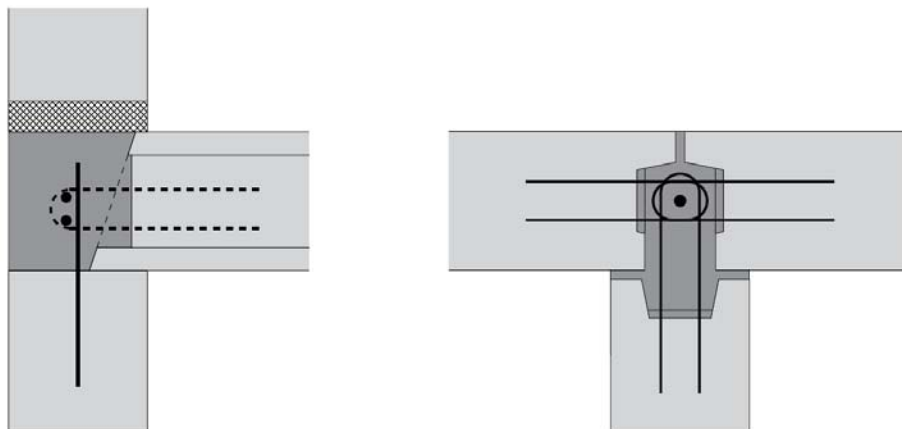
Fig. 1   In-situ cast joints subjected to triaxial stress: Joint connecting shear walls and slabs (left) and corner joint connecting three walls (right).

In practice, the shear capacity of joints is assessed by simple empirical formulas which often lead to a conservative estimate of the capacity. Moreover, the formulas do not consider the effect of triaxial stress conditions. Several authors have investigated the behaviour of in-situ cast joints subjected to in-plane shear (see e.g. Fauchart and Cortini, 1972; Bljuger, 1976; Hansen and Olesen, 1976; Rizkalla et

Proc. of the 11th *fib* International PhD Symposium in Civil Engineering
Aug 29 to 31, 2016, The University of Tokyo, Tokyo, Japan

865

al., 1989); however, to the best knowledge of the authors, no experiments featuring joints subjected to triaxial stress conditions has been published. Upper bound solutions based on analysis of failure mechanisms (Jensen, 1975; Nielsen and Hoang, 2010) as well as lower bound solutions based on strut-and-tie models (Nielsen and Hoang, 2010; fib, 2008) have been used to establish mechanical models for the ultimate limit state design of in-situ cast joints. These models, however, does not cover joints subjected to triaxial stress states.

Numerical limit analysis is an alternative to strut-and-tie models. The method is based on the extremum principles of rigid plastic analysis (Drucker and Prager, 1952) combined with a discretisation known from finite element analysis. Anderheggen and Knöpfel (1972) presented a general formulation and since the 70s several researchers have contributed to the method. A detailed model of keyed joints in plane stress was presented by Herfelt et al. (2016). The model was validated by comparison to experiments and the analysis revealed that the reinforcement layout affects the shear capacity of keyed joints. A simplified semi-analytical submodel was presented by Herfelt et al. (2015) based on the findings of the detailed model. The model was based on the stringer method and used the principle of superposition to combine three models. The submodel was implemented in a numerical limit analysis framework as the yield criterion of a joint element.

This paper will present a semi-analytical model for joints subjected to triaxial stress state. The model will be based on the same principles as the plane stress counterpart (Herfelt et al., 2015), and the scope of the three-dimensional submodel is to be able to model the behaviour of joints subjected to triaxial stress condition. One of the core mechanisms of such joint is the transfer of shear from e.g. a slab to the wall panels; a process, which necessarily leads to triaxial stress states in the centre of the joint. Moreover, the three-dimensional submodel should account for the reinforcement layout which has been shown to affect the shear capacity in plane stress. The 3D submodel should of course be able to replicate the results obtained by the plane stress counterpart, and the model will be validated by comparison existing numerical models. Finally, a parameter study of a corner joint is carried out and the results are discussed.

2 Problem formulation

Lower bound limit analysis can be used to obtain statically admissible stress fields, i.e. a stress field that satisfies equilibrium and does not violate the yield criterion in any point. The scope of the analysis is to maximise the load factor λ, and the general problem can written as follows:

$$\begin{aligned} \text{maximise} \quad & \lambda \\ \text{subject to} \quad & H\beta = R\lambda + R_0 \\ & f(\beta_i) \leq 0, \qquad i = 1, 2, \ldots, m \end{aligned} \qquad (1)$$

Eq. 1 represents a convex optimisation problem since the yield function f is convex. The linear equations ensure equilibrium, while the inequalities ensure that no points violate the yield criterion. H is the global equilibrium and β is the stress vector representing the stress states in m points. The load acting on the structure is comprised of a constant part R_0 and a scalable part $R\lambda$. The problem (1) can be solved efficiently using interior point methods. In this work, (1) will be a semidefinite program (SDP) which is solved using the commercial solver MOSEK. The yield function is in itself a small optimisation problem. For complex structural components, e.g. in-situ cast joints, it may be necessary to use a so-called submodel as the yield criterion.

3 General principles of the submodel

The purpose of the submodel is to model in-situ cast joints subjected to triaxial stress states. The submodel presented in this paper will be capable of treating joints between orthogonal panels and slabs, which covers the vast majority of joints used in precast structures. The submodel is based a variation of the stringer model and consists of several components. Two stringer models are used to transfer in-plane forces between panels or slabs lying in the same plane. A total of 16 corbels are used to transfer shear forces between panels and slabs lying in different planes. Fig. 2 shows a sketch of a repeating unit section of an in-situ cast joint including local coordinate system and geometric parameters.
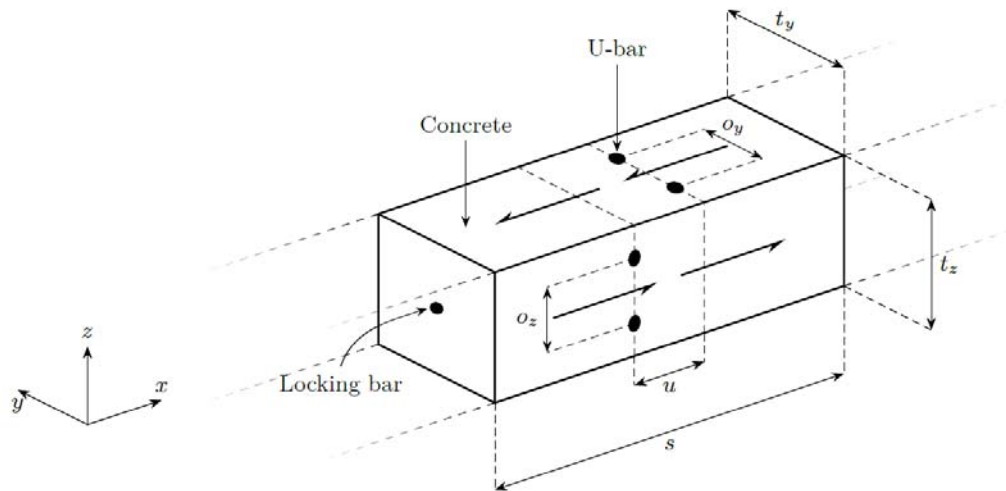
Fig. 2   Sketch of a unit section of an in-situ cast joint subjected to shear stresses.

The parameter u is a measurement of the offset between the two U-bar loops seen in Fig. 2. Ideally, the U-bars should be placed closely together; however, this is not always the case in practice. The overlap, o_z and o_y, as well as distance between the U-bar pairs, s, affects the capacity of the joint heavily. The widths, t_z and t_y, only affects the capacity to a minor degree.

The different components of the submodel each contributes to the combined stress states in the joint. The principle of superposition is used to determine the stress states, which then is checked against the appropriate yield criteria (similarly to the model presented by Herfelt et al., 2015).

3.1   Two-dimensional model

The two-dimensional component is a simplified version of the submodel for plane joints presented by Herfelt et al. (2015), which used three smaller models. For this reason, the equations of the model will not be presented in this paper. This model uses a 3×3 grid of panels combined with both ordinary stringers and so-called compression stringers. The panels are not reinforced and confinement pressure on the boundaries of the model is added instead.
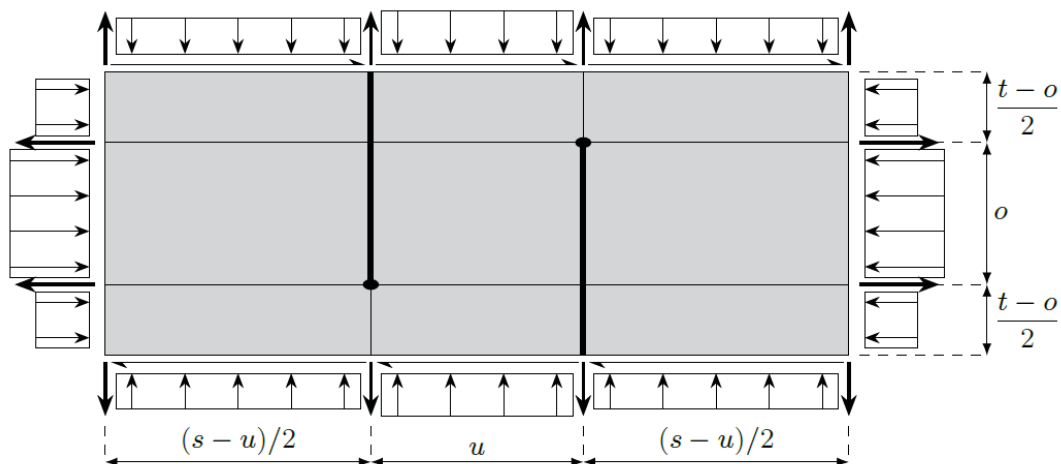

Fig. 3   Model for transfer of in-plane forces between panels or slabs lying in the same plane; the positive direction of forces and stresses is indicated by the arrows.

A set of stress variables describing a plane stress state is associated with each of 9 panels. In the boundaries between the panels, traction equilibrium is required, i.e. normal stresses in the panels are transferred to adjacent panels, while shear stresses can be transferred to either the adjacent panels or stringers. The stringers have a linear variation of the normal force between each node of the model, thus, two stress variables is sufficient to describe the stress state of the stringers.

3.2 Corbels

The corbel components of the submodel are responsible for transforming shear forces to axial forces (acting in the longitudinal direction of the joint) via corbel action. The model is rather simple and consists of one triangular panel, one rectangular panel and three stringers.
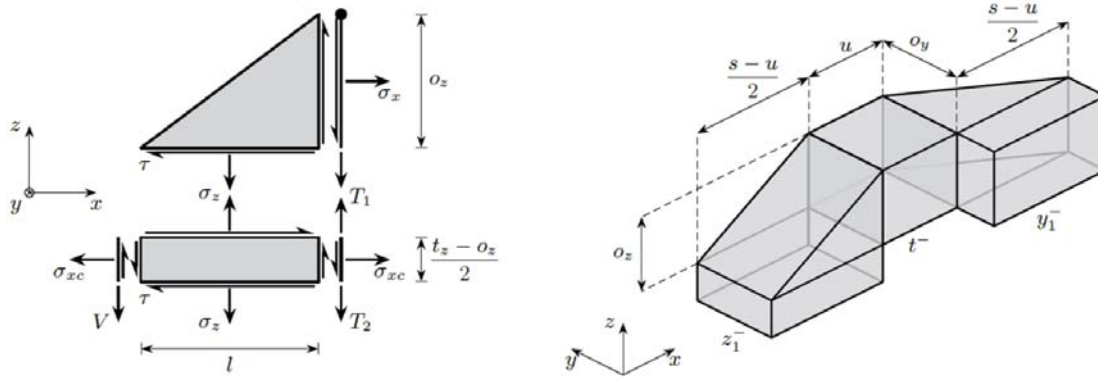

Fig. 4   Left: Corbel component in the zx-plane, the positive direction of forces and stresses is indicated by the arrows. Right: Sketch of the corbels and transfer block in 3D.

Fig. 4 shows an example of a corbel component. The component is located in the zx-plane and its purpose is to transfer a shear stress τ to an axial stress σ_x via the corbel action. The stringer forces, T_1 and T_2, can be positive since the stringer represents the loop reinforcement. V, on the other hand, must be non-positive, since the stringer is a so-called compression stringer. Equilibrium between the two panels and the three stringers seen in Fig. 4 are ensured by the following equations:

$$\tau \frac{t_z - o_z}{2} o_y - V = 0$$
$$-\tau\, o_z o_y - T_1 = 0 \tag{2}$$
$$-\tau \frac{t - o_z}{2} o_y + T_1 - T_2 = 0$$

where the geometric parameters are seen in Fig. 2 and 4. Equilibrium of the panels and the overall model is ensured by the following three equations:

$$-\tau\, l o_y + \sigma_x\, o_z o_y = 0$$
$$-\sigma_z\, l o_y + \tau\, o_z o_y = 0 \tag{3}$$
$$-V - T_2 - \sigma_z\, l o_y = 0$$

Each corbel component requires 6 equilibrium equations and 7 variables. For the corbel components in the xy-plane, the geometric parameters seen in (2) and (3) should of course be changed to the appropriate parameters.

The corbels are combined using a transfer blocks that only carries axial stress in the x-direction. Fig. 5 shows half of the corbels; 8 corbels combined with two transfer blocks:
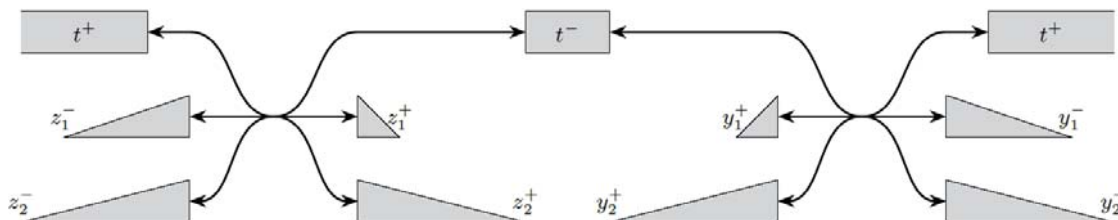
Fig. 5   Concept of transfer of axial forces: The triangles represent corbel components while the squares represent the transfer blocks. Corbels denoted z lie in the zx-plane, while corbels denoted y lie in the xy-plane.

The arrows seen in Fig. 5 indicate how the components of the model are connected, e.g. stresses can be transferred from z_2^- via t^- to y_1^- or y_2^-. The transfer block denoted t^+ *loops* around as seen in Fig.

5. Each set of arrows seen in the figure represent an equilibrium equation, thus, only two equations are needed to ensure equilibrium of the axial forces.

3.3   Yield criterion

To ensure a statically admissible stress field for the submodel, it is necessary to impose an appropriate yield criterion where it is relevant. The many stress states of the many components of the model are combined using the principle of superposition. The core of the joint, i.e. the middle row of the 2D models and the triangular panels of the corbels, will be subjected to triaxial stress condition, and the modified Mohr-Coulomb yield criterion is used for this part of the model:

$$\begin{aligned} \sigma_1 &\leq f_t \\ k\,\sigma_1 - \sigma_3 &\leq f_c \end{aligned} \tag{4}$$

Eq. 4 shows the general form of the modified Mohr-Coulomb yield criterion. σ_1 and σ_3 are the largest and smallest principal stresses, respectively, f_t is the uniaxial tensile strength of the concrete, f_c is the uniaxial compressive strength, and k is a friction parameter chosen as 4 corresponding to an angle of internal friction of approximately 37°.

The bottom and top row of the 2D models (see Fig. 3) as well as the square panel of the corbel components (see Fig. 4, left) experience a two-dimensional stress state and plane stress is assumed. The combined stress states are checked against the modified Mohr-Coulomb yield criterion for plane stress, which is given as:

$$\begin{aligned} \sigma_1 &\leq f_t \\ k\,\sigma_1 - \sigma_2 &\leq f_c \\ -\sigma_2 &\leq f_c \end{aligned} \tag{5}$$

where σ_1 and σ_2 are the largest and smallest principal stress, respectively. The principal stresses are the eigenvalues of the stress tensor, and (4) can be represented as a semidefinite cone (Krabbenhøft, Lyamin, and Sloan, 2008; Bisbos and Pardalos, 2007), which imposes constraints on the eigenvalues of a symmetric matrix. (5) can be implemented as a quadratic cone (see e.g. Bisbos and Pardalos, 2007, Makrodimopoulos and Martin, 2006).

The stringer force in the U-bar loops must be below the tensile strength of the bar, which can be represented as a linear inequality:

$$T \leq A_s f_Y \tag{6}$$

where A_s is the reinforcement area of one U-bar loop, and f_Y is the tensile strength. For the compression stringers, the force is required to be non-positive:

$$V \leq 0 \tag{7}$$

4   Analysis and discussion

In the following section, a single submodel will be analysed. The results for the two-dimensional case will be compared to a numerical model (Herfelt et al., 2015), which has been verified by comparison to experimental results. For the three-dimensional case, the submodel will be subject to a parameter study. The key result of the submodel is the shear capacity, either τ/f_c or τ, where τ is the average shear stress acting on the joint. The shear capacity is affected by the amount of reinforcement in the joint, which is described the mechanical reinforcement degree Φ defined as:

$$\Phi = \frac{A_s f_Y}{s\,t\,f_c} \tag{8}$$

The following parameters will be used for the comparison:

$$f_c = 30 \text{ MPa}, f_t = 0, t_z = t_y = 50 \text{ mm}, o_z = o_y = 40 \text{ mm}, s = 200 \text{ mm}$$

The parameters are similar to the geometry of modern keyed joints and the geometry used for the experiments by Hansen and Olesen (1976). Fig 6 shows the normalised shear capacity for u varying between 0 and s for two different mechanical reinforcement degrees.

Fig. 6 (left) shows that the proposed submodel predicts more or less the same capacity as the original two-dimensional submodel (Herfelt et al., 2015). For $u \cong s/2$, the proposed model predicts a slightly

larger capacity, which is due to the fact that the original submodel assumes a constant shear stress distribution whereas the proposed submodel does not.
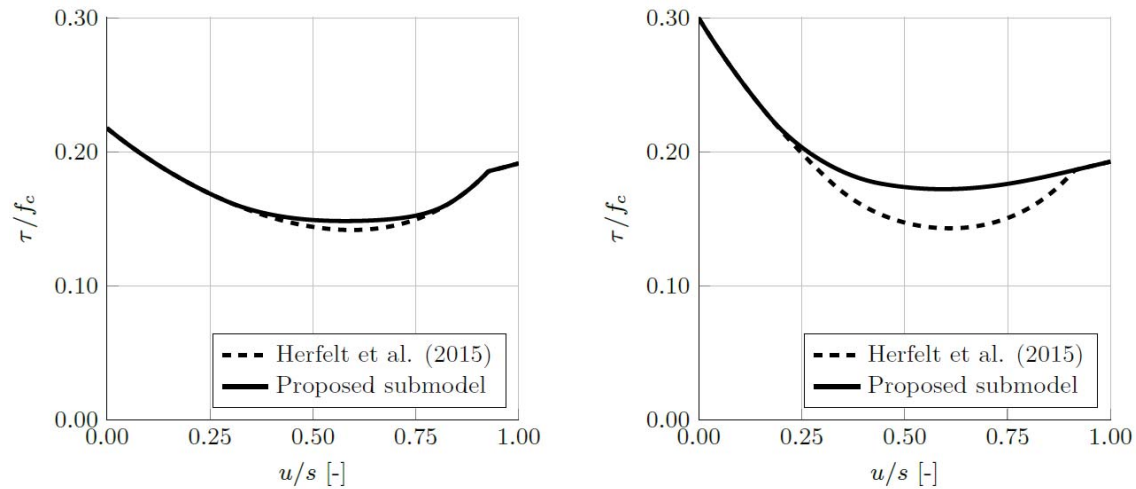

Fig. 6   Comparison of the two-dimensional submodel by Herfelt et al. (2015) and the proposed submodel for $\Phi = 0.050$ (left) and $\Phi = 0.100$ (right).

4.1 Analysis of corner joint

The term "corner joint" covers both joints connecting slabs and shear walls and joints connecting two shear walls in different planes (see Fig. 1). Common for both of these connections is that shear stresses are transferred from one plane to another. This requires mobilisation of the reinforcement and the internal corbels. In this section, a parameter study of a corner joint will be conducted. Unfortunately, no experimental results for this type of connection are available (to the best knowledge of the authors). The following parameters are used for the analysis:

$$f_t = 0, t_z = t_y = 50 \text{ mm}, o_z = o_y = 40 \text{ mm}$$

For the analysis, three different types of loop reinforcement commonly used in the industry are analysed. The distance between the U-bars takes three different values, namely 400 mm, 200 mm, and 100 mm.
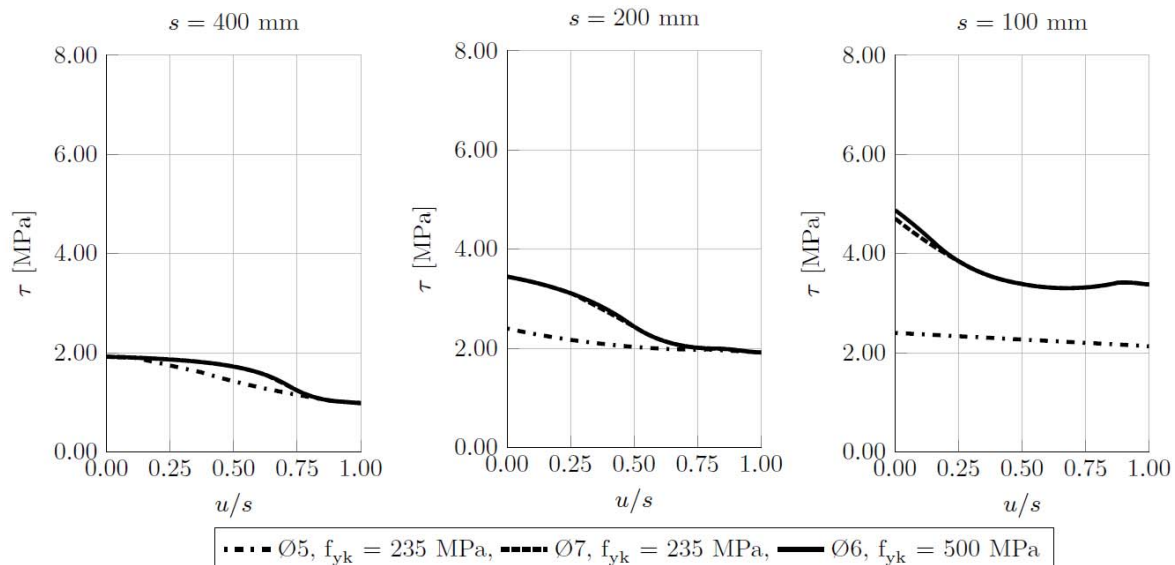

Fig. 7   Shear capacity of the corner joint for three different U-bars, varying U-bar distance and $f_c = 20$ MPa.
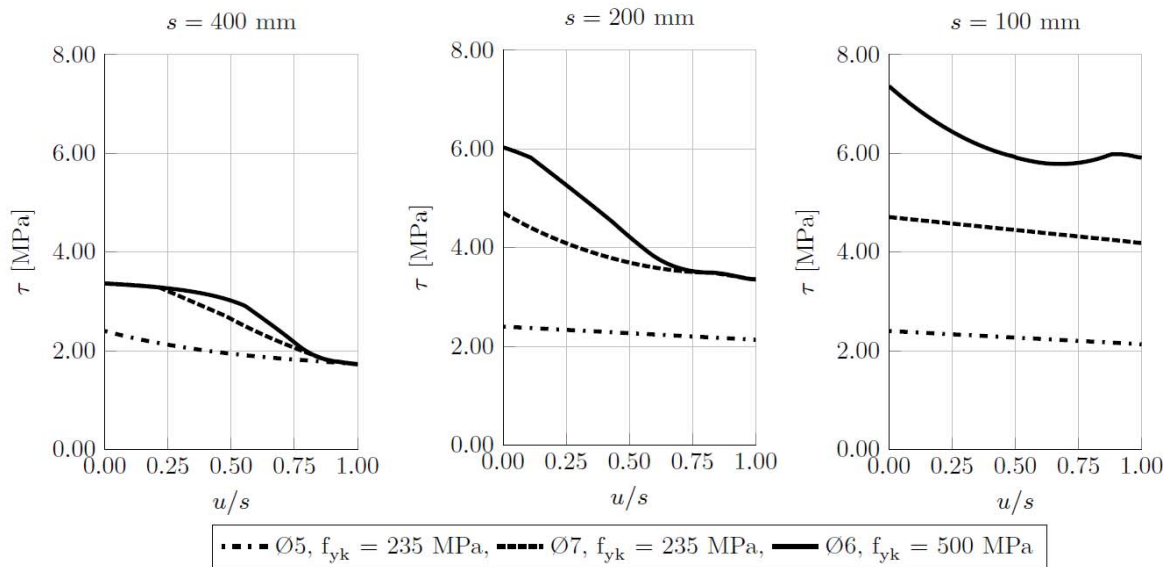
Fig. 8   Shear capacity of the corner joint for three different U-bars, varying U-bar distance and $f_c = 35$ MPa.

Fig. 7 and 8 show that the shear capacity of the corner joint is very dependent on s and u. The strength of the loop reinforcement and the concrete strength are likewise crucial to the capacity. It is observed that in some cases, additional reinforcement does not increase the capacity, which indicates that the failure is happening in the concrete. Fig. 8 shows that a minimum shear strength of approximately 6 MPa can guaranteed by using Ø6 U-bars with $f_{yk} = 500$ MPa per 100 mm.

The smallest U-bar (Ø5) yields a rather limited shear capacity, and the capacity is only affected by the compressive strength of the concrete to a minor degree. This is due to the predefined size of the unit section (see Fig. 2) which limits the size of the corbels.

The current practice for design of joints in precast structures subjected to triaxial stress is somewhat lacking. The joints are often designed as simple keyed joints subjected to plane stress, and the triaxial stress states is simply neglected (European Committee for Standardization, 2005). The model presented in this paper makes it possible account for the triaxial stress state within the joint and provide an estimate of the shear capacity. While the behaviour observed generally is deemed satisfactory, experimental data is needed to verify the model. Together with experiments, the model could be the basis for new design rules for in-situ cast joints.

5   Conclusion

In-situ cast joints are crucial to the lateral stability of precast structures, yet the design of such connections is based on empirical formulas which neglect the triaxial stress states within the joint. In this paper, a semi-analytical model based on the stringer method and the principles of numerical limit analysis and superposition has been proposed.

The behaviour in plane stress is handled by a two-dimensional model, which are combined with several corbels to form the complete model, which can handle triaxal stress. The stress states of the model are checked against appropriate yield criteria; the modified Mohr-Coulomb yield criterion is used for the concrete, while a simple linear criterion is used for the reinforcement and compression stringers.

The model is verified for plane stress by comparison to an already established model. The proposed model performs reasonably and yield slightly larger shear capacity for some reinforcement configurations. A parameter study of the corner joint is conducted, and the shear capacity is analysed for varying parameters. The key parameters are the reinforcement layout (u and s), the compressive strength of the concrete, and the strength of the reinforcement. Experimental data is needed to verify the behaviour of the model subjected to triaxial stress states.

References

Anderheggen, Edoado and Knöpfel, Hans. 1972. "Finite Element Limit Analysis using Linear Programming." International Journal of Solids and Structures 8:381-89.

Bisbos, Christos D. and Pardalos, Panos M. 2007. "Second-Order Cone and Semidefinite Representations of Material Failure Criteria." Journal of Optimization Theory and Applications. 134(2):275-301.

Bljuger, Fiodor. 1976. "Determination of Deformability Characteristics of Vertical Shear Joints in Precast Buildings." Building and Environment. 11:277-82.

Drucker, Daniel C. and Prager, William. 1952. "Soil Mechanics and Plastic Analysis or Limit Design." Q. Appl. Math. 10:157-65.

European Committee for Standardization. 2005. "EN 1992-1-1 Eurocode 2: Design of Concrete Structures – Part 1-1: General Rules and Rules for Buildings". CEN.

Fauchart, J. and Cortini, Primo. 1972. "Étude Expérimentale de Joints Horizontaux Entre Panneaux Préfabriqués pour Murs de Batiments." Annales de L'Institut Technique du batiment et dês Travaux Publics. 300:86-103.

Fib bulletin 43. 2008. "Structural Connections for Precast Concrete Buildings." Fédération Internationale du Béton (fib).

Hansen, Klaus and Olesen, S. Øivind. 1976. "SBI-Report 97: Keyed Shear Joints." Statens Byggeforskningsinstitut (Danish Building Research Institute).

Herfelt, Morten A., Poulsen, Peter N., Hoang, Linh C., and Jensen, Jesper F. 2015. "Lower Bound Element and Submodel for Modelling of Joints between Precast Concrete Panels." Paper presented at the XIII International Conference on Computational Plasticity. Fundamentals and Applications – COMPLAS XIII, Barcelona, September 1-3.

Herfelt, Morten A., Poulsen, Peter N., Hoang, Linh C., and Jensen, Jesper F. 2016. "Numerical Rigid Plastic Analysis of Keyed Shear Joints in Concrete Structures." Structural Concrete. Accepted for publication.

Jensen, Bjarne C. 1975. "On the Ultimate Load of Vertical, Keyed Shear Joints in Large Panel Buildings." Paper presented at the Symposium on Bearing Walls, Warsaw.

Krabbenhøft, Kristian, Lyamin, Andrei V., and Sloan, Scott W. 2008. "Three-dimensional Mohr-Coulomb Limit Analysis using Semidefinite Programming." Communications in Numerical Methods in Engineering. 24(11):1107-19.

Nielsen, Mogens P. and Hoang, Linh C. 2010. *Limit Analysis and Concrete Plasticity, Third Edition*. Taylor & Francis.

Rizkalla, Sarni H., Serette, Reynaud L., Heuvel, J. Scott, and Attogbe, Emmanuel K. 1989. *Multiple Shear Key Connections for Precast Shear Wall Panels*. Prestress Concrete Institute.