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Graphene-based THz modulator analyzed by equivalent circuit model

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A THz modulator based on graphene is proposed and analyzed by use of equivalent transmission line of a homogeneous medium and the local anisotropic model of the graphene conductivity. The result calculated by the equivalent circuit is consistent with that obtained by Fresnel transfer matrices. For the modulator proposed here, when the frequency of carrier wave is 0.6THz, the theoretical analysis indicates that the modulation bandwidth is 55.5 kHz and the modulation depth is 81.3% for voltage change from 0V to 50V.

Key –words: THz modulator; graphene; equivalent circuit

1. Introduction: Graphene is composed of a flat single layer of carbon atoms which are tightly packed into a two-dimensional (2D) honeycomb lattice [1]. Since Geim and Novoselov fabricated the graphene by mechanical extraction of graphite pieces in 2004 [2], a series of processes for fabricating graphene have been proposed, especially chemical vapor deposition (CVD) [3].

In recent years, graphene is found to have widespread application prospects in microwave and terahertz (THz) devices, and a series of novel microwave and THz devices based on graphene are proposed [4-10]. Terahertz (THz) modulator is one of the key components of wireless communication. So, designing and analyzing new THz modulators in order to meet future application and development has become to be a hot research direction. In 2012, a novel modulator was designed by Lee et al., which is composed of sapphire substrates and graphene with the working frequency ranging from 1Hz to 100MHz [4]. By using a similar structure, a THz modulator was designed by Sensale-Rodriguez with the bandwidth of 20 kHz and the modulation depth of 64% [5-6], showing a better performance when comparing to the modulator based on metamaterial [7]. Sensale-Rodriguez et al. also proposed terahertz electro-absorption modulators based on graphene plasmonic structure [8], and the results show that reflectance-based device configurations could achieve very high modulation-depth, even ~100%, over a wide frequency range up to tens of THz.

The conductivity of graphene can be tuned by electrostatic and/or magnetostatic bias, and the expression for the conductivity of graphene can be obtained from Kubo formula [11]. In this work, we use the equivalent circuit for analysis of the interaction between electromagnetic field and a thin graphene sheet, based on a local anisotropic model of the graphene conductivity at THz range [11]. It has been shown in Ref. [11] that the proposed equivalent circuit represents a very simple tool for the relevant modulators analysis and design.

In this context, by using the equivalent circuit of graphene and the equivalent transmission line model of S_i/S_iO_2, the equivalent circuit of S_i/S_iO_2thin-film based graphene is established. The graphene-based modulator is composed of graphene sandwiched between quartz substrate and S_i/S_iO_2thin-film. The equivalent circuit of the modulator is established by using the equivalent transmission line of the homogeneous medium and the local anisotropic model of the graphene conductivity. For the conductivity model of graphene is anisotropic in the process of equivalent circuit establishing, the equivalent circuit is widespread applicability.

2. Model and Analysis: The conductivity of graphene sheet can be derived by well-known Kubo formula in the absence of magnetostatic bias [11] :

$$\sigma_{\mathrm{D}}\left(\mu_c\left(E_0\right)\right)=\sigma_{\mathrm{DR}}-j\sigma_{\mathrm{DJ}}=\frac{je^2\left(\omega-j2\Gamma\right)}{\pi\hbar^2}\cdot \left[\frac{1}{\left(\omega-j2\Gamma\right)^2}\int_0^{\infty}\varepsilon\left(\frac{\partial n_F\left(\varepsilon\right)}{\partial\varepsilon}-\frac{\partial n_F\left(-\varepsilon\right)}{\partial\varepsilon}\right)d\varepsilon - \int_0^{\infty}\frac{n_F\left(-\varepsilon\right)-n_F\left(\varepsilon\right)}{\left(\omega-j2\Gamma\right)^2-4\left(\varepsilon/\hbar\right)^2}d\varepsilon\right] \quad (1)$$

The relationship between μ_c and E_0 is shown as:

$$\frac{\varepsilon_0\pi\hbar^2v_F^2}{q_e}E_0=\int_{\Delta}^{+\infty}\varepsilon\left[n_F\left(\varepsilon\right)-n_F\left(\varepsilon+2\mu_c\right)\right]d\varepsilon \quad (2)$$

where, Γ is the phenomenological scattering rate controlled by chemical doping and electric field $E_{0,}$ n_F is the Fermi-Dirac constant distribution given by $n_F(\varepsilon)=\dfrac{1}{1+e^{(\varepsilon-\mu_c)/(k_BT)}}$, ε is energy, V_F is the Fermi velocity in graphene, $\hbar$ is reduced Planck's constant and μ_c is chemical potential. The conductivity of graphene as a function of electric bias is illustrated in Fig. 1, showing that the conductivity σ varies significantly with the gate voltage..

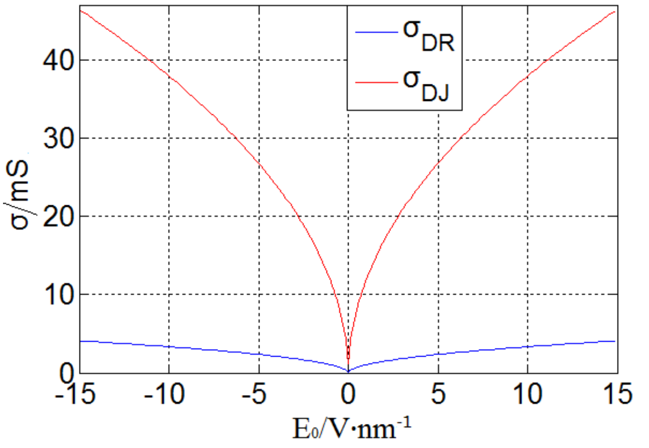

Figure 1 The conductivity of graphene as a function of bias voltage when T=300K，Γ=0.11meV，f=600GHz (σ_{DR} is the real part of the conductivity of graphene, σ_{DJ} is the imaginary part of conductivity of graphene).

The conductivity of graphene can be controlled by electric bias E_0. The conductivity is nearly zero without any electric bias, and the conductivity increases when raising the amplitude of E_0. Therefore a modulator can be designed according to the characteristics of different conductivity with bias voltage.

Based on the dependence of electrostatic bias to graphene conductivity, a THz modulator is proposed as shown in the Fig 2. A graphene film is chemically transfered on a 1.6×1.5 cm^2 quartz substrate with the thickness of l_3. Then a 1.5×1.5 cm^2 Si/SiO$_2$thin-film is put on the single–layer graphene, and the thickness of S_i and S_iO_2 thin-film are l_1 and l_2 respectively. The metal electrode contacts with the graphene film where it is uncovered with the S_i/S_iO_2thin-film. Finally, the S_i thin-film and metal electrode is connected with a signal source. The graphene conductivity is adjusted by the voltage of the signal source, thus the transmission of the carrier source is adjusted.

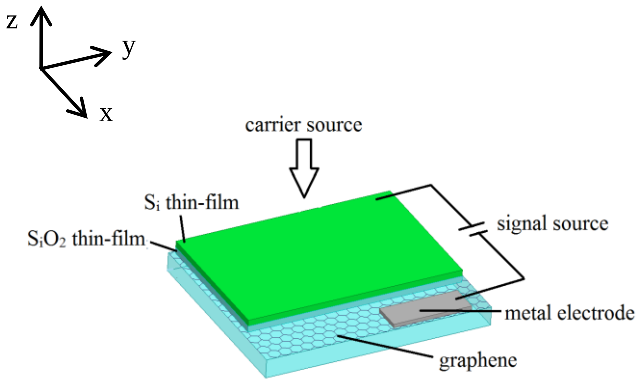

Figure 2 Schematics of the proposed graphene-based THz modulator

In the microwave and THz spectra region, when a plane wave impinges, assuming that the interface between air and S_i thin-film is located at the z=0 plane, by using the equivalent circuit model [11], the equivalent circuit of S_i/S_iO_2 thin-film based graphene can be established, as shown in the Fig 3.

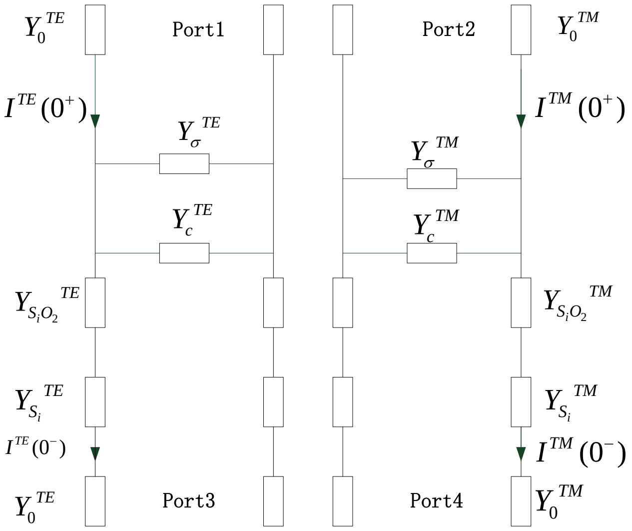

Figure 3 The equivalent circuit for S_i/S_iO_2 thin-film based graphene, where $Y_\sigma^{TE/TM}$ is the characteristic admittance of graphene, $Y_0^{TE/TM}$ is the characteristic admittance of air, $Y_{S_io_2}^{TE/TM}$ and $Y_{S_i}^{TE/TM}$ are the characteristic admittance of S_i/S_iO_2, and $Y_C^{TE/TM}$ is the equivalent admittance of the carrier source, respectively

One can see in the Fig 3 that the equivalent circuit is a four-port network, where ports 1 and 3 represent the input and output of TE waves and ports 2 and 4 represent the input and output of TM waves respectively. In low magnetic field and frequency below 1THz, $Y_c^{TE}=Y_c^{TM}=0$.When a plane normally impinges on the graphene, then the above equivalent circuit can be simplified as shown in the Figure 4.

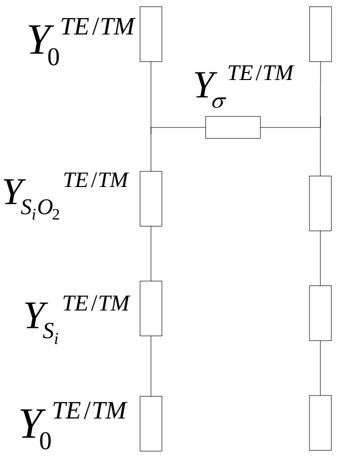

Figure 4 Simplified equivalent circuit of S_i/$S_iO_2$thin-film based graphene

Then the equivalent circuit of the modulator proposed here could be established as shown in the **Error! Reference source not found.**Fig 5(a). Based on the impedance transformation of transmission line, the admittances in the circuit should be equal to a controlling parameter $Y_4^{TE/TM}$, as shown in the Fig 5(b)(c)(d)(e) and (f), where

$$\begin{aligned}
Y_1^{TE/TM} &= Y_{S_iO_2}^{TE/TM}\frac{Y_0^{TE/TM}+jY_{S_iO_2}^{TE/TM}\tan k_{S_iO_2}^{TE/TM}l_3}{Y_{S_iO_2}^{TE/TM}+jY_0^{TE/TM}\tan k_{S_iO_2}^{TE/TM}l_3}\\
Y_2^{TE/TM} &= Y_1^{TE/TM}+Y_\sigma^{TE/TM}\\
Y_3^{TE/TM} &= Y_{S_iO_2}^{TE/TM}\frac{Y_2^{TE/TM}+jY_{S_iO_2}^{TE/TM}\tan k_{S_iO_2}^{TE/TM}l_2}{Y_{S_iO_2}^{TE/TM}+jY_2^{TE/TM}\tan k_{S_iO_2}^{TE/TM}l_2}\\
Y_4^{TE/TM} &= Y_{S_iO_2}^{TE/TM}\frac{Y_3^{TE/TM}+jY_{S_i}^{TE/TM}\tan k_{S_i}^{TE/TM}l_1}{Y_{S_i}^{TE/TM}+jY_3^{TE/TM}\tan k_{S_i}^{TE/TM}l_1}
\end{aligned}\tag{3}$$

The reflection coefficient can be calculated as:

$$\Gamma\left(z=0^+\right)=\frac{Y_0^{TE/TM}-Y_4^{TE/TM}}{Y_0^{TE/TM}+Y_4^{TE/TM}}\tag{4}$$

By using transmission-line matrix, we can also obtain the voltage and current in the interface of quartz substrate and air. Thus, electromagnetic problems are converted to the circuit problems here which greatly simplifies the work of calculation and analysis.

3. Results: The static characteristics of the modulator are calculated when the carrier wave is 0.6THz and l_1, l_2, l_3 is 100nm, 300nm, 128μm, respectively. The obtained results are compared with those which are calculated by Fresnel transfer matrices, see the circles in the Fig 6.

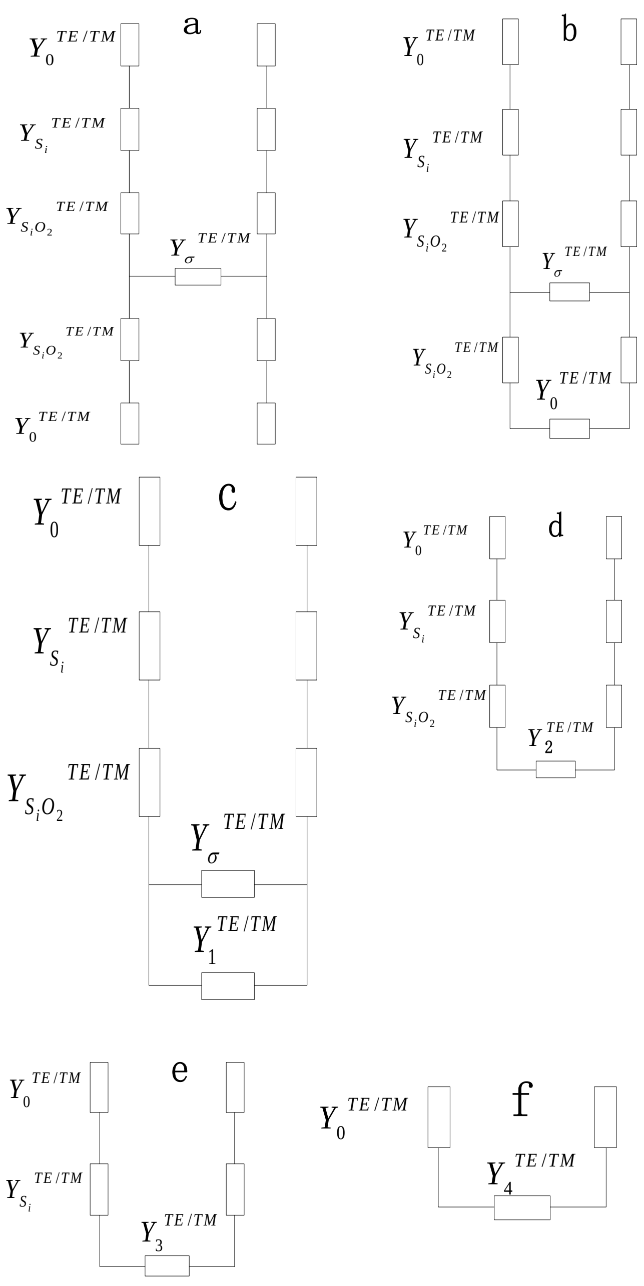

Figure 5 Equivalent circuit of the modulator based on graphene

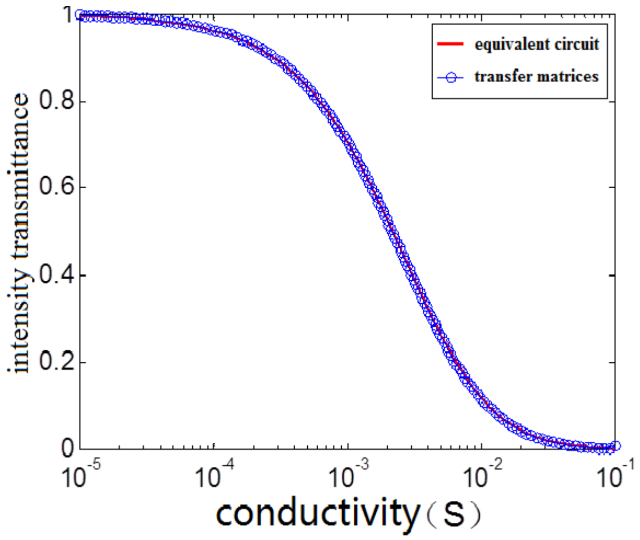

Figure 6 Transmittance as a function of graphene conductivity obtained by the equivalent circuit (the red

solid line) and Fresnel transfer matrices (the blue circles).

It can be seen that the results calculated by the equivalent circuit are in accordance with those from the Fresnel transfer matrices. Compared with the Fresnel transfer matrices, the equivalent circuit is briefer and clearer, and it has wider applicability:

(1) Since the conductivity of graphene is anisotropic with high magnetic field bias or at high frequency. The performance of the device can be calculated by the equivalent circuit under this cirt

(2) The equivalent admittance of a dielectric material is different under oblique incidence condition. Relying on the equivalent circuit, different modes of the plane wave can be analyzed individually.

The most significant change of transmittance occurs for the conductivity ranging from 0.1 mS to 10mS as shown in Fig 6. Since the thickness of Si thin-film is only 300nm, so the conductivity of silicon varies little with the change of voltage, and it has little effect on the final result.

With the aid of the analysis shown above, the relationship between modulation voltage V_g and transmission power ratio P_t can be calculated, which is shown in Fig.7. It can be seen in Fig.7 that when V_g changes from 0 to 50V, the modulation depth (defined as m= $| P_t(50V) - P_t(0V) / P_t(50V)) |$) is 81.3%. The modulation depth strongly depends on the variation of graphene conductivity. Since the thickness of S_iO_2 thin-film which is between the graphene and S_i thin-film is only 300nm, though the variation of V_g is not very large, the variation of graphene conductivity has changed in a large range.

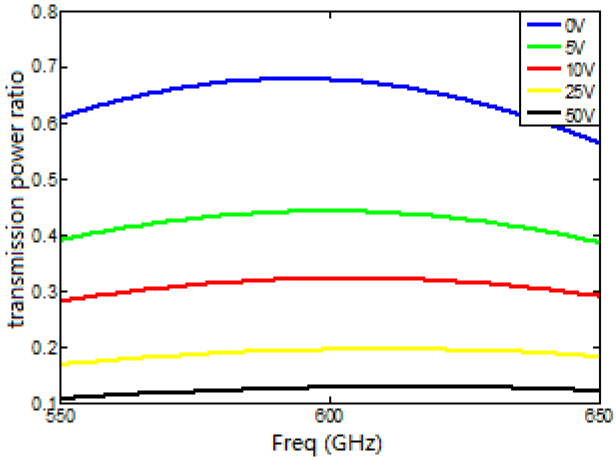

Figure 7 Transmission power ratio under different bias voltage V_g

The equivalent circuit method is used to estimate the RC time constant here. The resistance R is calculated by half of the inverse of the graphene conductance averaged over the voltage bias Vg. The capacitance of this modulator is calculated by parallel plate capacitor model. Here the average resistance of a 1.5×1.5cm2 graphene is calculated as 1000Ω and the capacitance of the SiO2 thin-film is evaluated as 2.59×10-8F [12]. Then the RC time of the modulator is calculated as 25.9μs. The normalized modulation magnitude A is shown in the Fig 8. The result shows that the 3dB bandwidth of the proposed modulator is around 55.5 kHz based on the value of the RC time constant. As long as the frequency of input signal is below the modulation bandwidth, the modulator could operate very well.

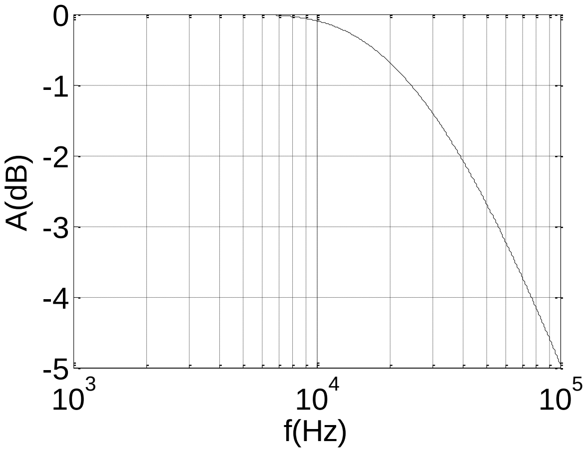

Figure 8 Normalized modulation amplitude

4. Conclusion: A THz modulator based on graphene consisting of a quartz substrate, a single-layer graphene and a Si/SiO2thin-film is proposed and analyzed by the equivalent circuit method. The results show that the modulation bandwidth is 55.5 kHz and the modulation depth is 81.3% for voltage changing from 0V to 50V. The results obtained by the equivalent circuit model are in agreement with those from the Fresnel transfer matrices. Compared with the Fresnel transfer matrices, the equivalent circuit method used here is briefer and cleare, providing an efficient way to analyze the graphene-based devices.

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