Math Ed Res J (2017) 29:237–254
DOI 10.1007/s13394-017-0201-y

ORIGINAL ARTICLE

Using expectancy-value theory to explore aspects of motivation and engagement in inquiry-based learning in primary mathematics

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Received: 31 August 2016 / Revised: 26 February 2017 / Accepted: 3 March 2017 /
Published online: 17 March 2017

Abstract Inquiry-based learning (IBL) is a pedagogical approach in which students address complex, ill-structured problems set in authentic contexts. While IBL is gaining ground in Australia as an instructional practice, there has been little research that considers implications for student motivation and engagement. Expectancy-value theory (Eccles and Wigfield 2002) provides a framework through which children's beliefs about their mathematical competency and their expectation of success are able to be examined and interpreted, alongside students' perceptions of task value. In this paper, Eccles and Wigfield's expectancy-value model has been adopted as a lens to examine a complete unit of mathematical inquiry as undertaken with a class of 9–10-year-old students. Data were sourced from a unit (~10 lessons) based on geometry and geometrical reasoning. The units were videotaped in full, transcribed, and along with field notes and student work samples, subjected to theoretical coding using the dimensions of Eccles and Wigfield's model. The findings provide insight into aspects of IBL that may impact student motivation and engagement. The study is limited to a single unit; however, the results provide a depth of insight into IBL in practice while identifying features of IBL that may be instrumental in bringing about increased motivation and engagement of students in mathematics. Identifying potentially motivating aspects of IBL enable these to be integrated and more closely studied in IBL practises.

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Keywords Expectancy-value motivation theory · Inquiry-based learning · Motivation · Engagement · Primary mathematics

Introduction

In 1989, the National Council of Teachers of Mathematics (NCTM 1989) produced *The Principles and Standards for School Mathematics* to influence teaching reform in the USA. This was in response to a perceived need to improve the standard of mathematics achievement and to combat national assessment data that demonstrated declining engagement levels as students progressed through high school (Carpenter et al. 1981). One way in which these reform goals have been addressed is through the implementation of inquiry-based learning (IBL). IBL is a pedagogical approach in which students address complex problems, interpret and negotiate problem meaning, envisage relevant mathematical knowledge, identify solution pathways, plan and conduct investigations and put forth a defensible solution supported by mathematical evidence. Prior research (Fielding-Wells and Makar 2008) suggests that IBL in primary mathematics may both increase engagement and potentially reverse existing declines in engagement: in particular, enhancing deep learning, increasing interest and decreasing frustration and anxiety.

Student motivation and engagement are strongly related. Motivation affects learning and behaviour by focussing attention towards a particular goal, in turn, leading to an increased energy and effort, an increased initiation of activities and a greater persistence in carrying out those activities. Schunk and Mullen (2012) describe motivation as the process of energising, directing and sustaining activity whereas engagement is the outward and observable outcome of this energy. Engagement may be observed through students' interactions with classroom learning, but underlying motivational influences may be harder to determine (Skilling et al. 2015). Because motivation underpins engagement (Martin 2012), it is important to study motivation to identify these influences.

Limited research into motivational influences on engagement exists. Data drawn at the undergraduate level suggest that students learning mathematics through IBL achieved grades as good as, or higher than, non-IBL comparison students, and that students learning mathematics through IBL were more likely to undertake further mathematics study, suggesting a positive impact on motivation (Kogan and Laursen 2014). However, there was little to suggest what aspects of IBL might have increased motivation, and it is not possible to know whether this would translate to younger students. IBL was found to have a positive impact on the motivation and achievement of ~13-year-old boys with social, emotional and behavioural difficulties (Camenzuli and Buhagiar 2014). With IBL gaining ground in Australia (AAS n.d.), and much research existing that demonstrates the importance of motivation and engagement in leading to achievement outcomes (e.g. Wigfield et al. 2015), it is necessary that the potential for IBL to impact on student motivation be examined. One extensively used framework available for this purpose is Eccles and Wigfield's expectancy-value theory of achievement motivation (EVT) (2002).

The underlying premise of EVT is that achievement–related choices are linked to two sets of beliefs: the individual's expectation of success and the value the individual holds for the options perceived (Eccles et al. 1998). In young children motivation is considered to be developmental, becoming more complex and differentiated over time (Eccles and Wigfield 2002). Research shows that children's beliefs, values and goals relate closely to their performance and choice as they get older. Engagement and motivation are linked in potentially reciprocal ways (Bandura 1997; Eccles et al. 1998); and so experience plays an important role in shaping children's beliefs, values and goals in either positive or negative ways (Dweck 1998). The developmental aspect of motivation further supports the need for motivation research across age levels.

The research described in this paper results from the application of EVT as an analytic framework to examine teacher-student and student-student interactions that took place in a primary classroom during an inquiry-based geometry unit. The aim was to provide insight into, and describe, potential motivational influences that arose during IBL.

Inquiry-based learning

Inquiry-based learning can be considered to be the addressing of problems which are both authentic and ill-defined or complex in nature (Anderson 2002). In real life, the types of problems addressed mathematically are often of this nature and therefore inquiry problems can be considered those that more closely reflect real life; with problems often having many open constraints (Simon 1973). Often such problems are addressed within a social context, for example, they may be addressed in the workplace or may require decision making as a family for example. Accordingly, these situations are most closely adopted in the classroom by having students engage in mathematical problem solving as a community. Thus, the definition of inquiry adopted for this paper is the addressing of *authentic*, *ill-defined problems*.

Authentic problems

With many mathematical problem-solving approaches, students are given problems which are artificially contextualised. In these instances, the teacher often already has a known, preferred response and method of approach. As such, students may not engage authentically, as there is no real necessity to explain method or solution, given that the teacher and fellow students will know what they have done and why (Sandoval and Millwood 2007). Authentic problems are typically more complex, with additional constraints and a purpose that drives the goal of the problem. Within IBL approaches, authentic problems therefore generate an opportunity for students to discuss and modify parameters, identify and justify decisions made, and incorporate these adjustments into their approach. Accordingly, students will likely respond with varied approaches and solutions, and this gives potential for deeper discussion of the more complex responses, along with deeper student engagement with the problems themselves (Fielding-Wells and Makar 2008). The authenticity of the problem addressed by students in the study reported here comes from a student "wondering" about a geometric property that developed into an inquiry question.

Ill-defined problems

Nearly all problems in school mathematics are well-defined and yet most problems encountered beyond the classroom are ill-defined (or ill-structured). A well-defined problem can be described as one in which there is a "systematic way to decide when a proposed solution is acceptable" (Minsky 1961, p. 9). It is likely that the initial state and the end state are also clearly defined, and that even if the methods vary, it is clear when the end state is reached (Reitman 1965). Simon (1973) characterised ill-defined problems as those which have no definitive criteria that would enable the judging of a solution, where the problem space is not defined in a meaningful way, and where any boundaries or constraints can be impacted upon by new alternatives. "To solve an ill-defined problem … whatever it takes to close its open constraints must be sought out or generated by the problem solver himself" (Reitman 1965, p. 164). This puts an additional responsibility onto the student to both define the problem and evaluate the progress of his/her solution and to make adjustments as the solution unfolds.

In traditional mathematics classrooms, expert knowledge is derived from the teacher or the textbook. The vast majority of textbook problems would be considered to be extremely well-defined. From worked examples, to practice questions, to answers provided in the book, there is an expectation of a particular process being followed with a specific answer sought; thus, students are typically "correct" or "incorrect". Inquiry-based learning, however, embeds an expectation that students will be involved in an ongoing re-negotiation of the problem statement and/or solution process as they work (Makar 2012). By working with ill-defined or ill-structured problems in mathematics, students benefit in two ways: from skills they develop (e.g. negotiation of problem definition and meaning, evaluation of progress towards a solution, development or selection of problem methods and tolerance for ambiguity about the outcomes of potential solution methods) and by building beliefs about the value of mathematics (McGregor 2016). Challenges with addressing ill-structured problems arise, however, in that solution pathways are less predictable, require different skills for the teacher and student and depend on a classroom culture which supports intellectual risk (Makar and Fielding-Wells 2011): all aspects which could be thought to impact on motivation and thus engagement.

Motivation and engagement

Fredricks, Blumenfeld and Paris (2004) categorise three dimensions of engagement: affective, behavioural and cognitive. Affective engagement encompasses beliefs, attitudes and emotions as experienced by students. Behavioural engagement can be identified across three categories: positive conduct, school commitment and through measures of effort, persistence, concentration, attention, questioning and communicating (Fredricks et al. 2004). Finally, Connell and Wellborn (1991; cited in Kong et al. 2003) identify cognitive engagement as a measure of psychological investment in learning. This includes a desire to go beyond basic requirements and the desire for challenge. It incorporates flexibility in problem solving, industry and resilience. There is a distinction in cognitive engagement between students' use of surface strategies as distinct from deeper strategies (Kong et al. 2003).

Engagement in learning is strongly influenced by motivation (Pintrich 2003) in that motivation "energises and directs action" (Wigfield et al. 2006, p. 933). In the context of classroom learning, achievement motivation has particular relevance, since it refers specifically to motivation germane to performance on tasks for which the outcomes have significance. Motivation researchers suggest that engagement is observable in the level of energy of an individual's behaviour, and it is the *sources* of this energy that are of most interest to understanding how motivation in learning operates. For example, three motivational factors, or sources of energy, related to cognitive strategy use are self-efficacy, achievement goals and perceived instrumentality (Greene et al. 2004). Motivation enhances cognitive processing and can lead to improved performance (Ormrod 2006). While previous research emphasised the role of an individual's drives, needs and reinforcements in motivation (Eccles et al. 1998; Pintrich and Schunk 2002), contemporary theories, such as EVT, suggest that individual beliefs, values and goals are the key sources of motivation (Eccles 2006; Eccles et al. 1998; Wigfield et al. 2006). From an EVT perspective, motivation is considered to be under the control of the individual and entails cognitive, conscious and affective processes. However, what distinguishes EVT from other conceptions of motivation is the acknowledgement of the broad array of psychological, social and cultural influences at play, as well as the importance of the role of real-world achievement tasks and experiences that shape people's expectancy-related and task value beliefs. For this reason, EVT was selected as a framework for this research.

Theoretical framework: expectancy-value motivation theory

The EVT framework provides three broad motivation-related questions that capture aspects of motivation related to expectancy, values and goals. While these questions assist our understanding of the sources of motivation and the mobilisation of energy to specific tasks and contexts, they can also provide lenses of analysis to illuminate the potential impact of instruction *on* children's motivation.

That is to say, using EVT as an explanatory framework enables the researcher to understand that when teachers set a learning task, the students might ask themselves: "Can I do this task?"; "Do I want to do this task and why?"; and, "What do I have to do to succeed on this task?" (Eccles 2006). The levels of energy and attention that any individual student gives that task would provide an indication of how motivated that child was. If the researcher was to delve more deeply into that child's beliefs about ability in relation to that task, degree of valuing for that task and expectations about what is required to complete that task, then further insights would be provided about the sources of motivation. Since EVT primarily illuminates individual expectancies of success, and intrinsic valuing of the task and setting, from an EVT perspective, we can understand how responses to these questions shape the degree of energy, attention and self-regulated action that is directed towards a task. The focus of this paper is to identify those aspects of instructional practice, within an IBL setting, that have the empirical potential to shape and influence children's learning-related beliefs, values and goals in beneficial ways. To achieve this, we will draw from EVT to devise an analytical framework based on its three central questions.

Can I do this task? Individual beliefs about competency, self-efficacy and expectancies for success

Asking the question, "Can I do this task?", elicits an individual's beliefs about *competence, self-efficacy* and their *expectancies for success*. The research tells us that when a child answers this question affirmatively, they generally try harder, show greater levels of persistence, achieve higher levels of performance and are then more motivated to select other challenging tasks (Wigfield et al. 2006). In this paper, we consider which aspects of IBL, as an instructional practice, can act to build competence and self-efficacy for students and reinforce expectancies for success within mathematics learning.

Do I want to do this task and why? Individual values related to perceived task value, intrinsic values and goals

Asking the question, "Do I want to do this task and why?", connects to individual students' values and interests, their perceived sense of control and autonomy over their environment, and the often influential role that individual values play in deploying energy and attention. A sense of control or autonomy can fuel one's connection to a task and therefore their choice to engage in a task (Grolnick et al. 2002). Values can be strongly predictive of the learning-related choices that students make (Eccles et al. 1998; Feather 1992). Individual values can be complex and highly situational/context specific. *Intrinsic enjoyment* and *interest value* refer to the enjoyment an individual may attain from simply participating in the task, whereas *utility value* connects to an individual's current or future goals (Wigfield et al. 2006). As such, students can find a task motivating based on personal interest or by identifying the value of the task in achieving their short- or long-term aspirations (*utility value*). However, the value of a task is also mediated by the perceived "cost" of doing the task (e.g. completing a homework task may mean missing out on a more highly valued social activity). So perceived task value (its apparent cost) also influences motivation.

What do I have to do to succeed in this task? Individual perceptions of academic and cognitive processes and self-regulation

This question addresses what is required by the task and relates to perceptions of self-efficacy as well as self-regulation of behaviour and cognitive processes (Eccles and Wigfield 2002). Research shows that this is developmentally challenging for younger children (Eccles and Wigfield 2002). Newman's (2002) research suggests that when a learning goal (rather than performance) is emphasised, students are more likely to persist, take initiative and seek help from peers or a teacher. Importantly, in the context of this paper, research suggests that motivation and cognition influence each other, particularly in the context of classrooms where learning and conceptual change occur (Pintrich et al. 1993). For our analysis of instructional practice and classroom learning within an IBL setting, it will be useful to consider the potential relatedness between certain instructional practises and those EVT components that are crucial to achievement success in specific settings, such as self-regulation and adaptive help-seeking practises. Since IBL is characterised by tasks that are ambiguous and designed to promote cognitive dissonance or conceptual challenge (change) as a learning experience, an EVT framed analysis of IBL settings has the potential to provide unique understandings about the relationship between instruction, learner's self-regulation and willingness to seek help when they encounter difficulties (Newman 2002).

Methodology

The analysis described in this paper comes from data in the first author's doctoral study undertaken with the purpose of investigating the development of IBL in primary mathematics classrooms. Design-based research was adopted as a methodology for the larger project, as it lends itself to the implementation, ongoing adjustment and examination of successive iterations of intervention in the classroom (Cobb et al. 2003). This paper undertakes a fresh analysis of the data to investigate the potential of EVT for gaining insight into *how* IBL in a mathematics classroom potentially impacts student motivation in mathematics.

Participants

The research school was a co-educational, metropolitan government primary school in Australia with an average Index of Community Socio-Economic Advantage (ICSEA). The teacher-researcher (first author) was experienced at teaching mathematics through inquiry and shared the class with another experienced IBL teacher. The relevant class was a year 4–5 (aged 8–10) continuing class, meaning that the class and the class teachers remained the same for the 2 years of middle primary. As the school had multiple classes at each year level, students had come into year 4 from any one of the year 4 three classes, some of whom had been working with inquiry and some who had not. The entire class (n = 28 students) was engaged in IBL in year 4 and year 5. The unit of work described here took place in year 5 and was the fifth inquiry unit undertaken by this class collectively. Other aspects of the inquiry project have been reported elsewhere (Fielding-Wells 2014, 2015; Fielding-Wells 2014; Fielding-Wells and Makar 2012, 2013).

Inquiry unit

The research described here draws on a single unit of work from the larger project and addresses the inquiry question, "Can a pyramid have a scalene face?". This question was posed by a student towards the completion of a non-inquiry unit on geometry and was enthusiastically supported by the class. This material addressed in the non-IBL unit included properties of angles (obtuse, acute and right), triangles (isosceles, scalene and equilateral) and 3D shapes (triangular and square based pyramids, cubes, and triangular and rectangular prisms).

Once a question is posed in inquiry, there is a need to envisage the evidence required to answer the question. A strong focus on developing mathematical evidence is important in IBL, and the students were accustomed to this and familiar with an evidence model (Fig. 1) which had been used previously to emphasise the role of evidence in making conclusions. The class worked in small collaborative groups to plan the evidence needed and then shared their ideas as a whole class, identifying multiple representations they planned to use as evidence, including models, nets and labelled diagrams.

Students proceeded to work through their plans with ongoing facilitated discussion to deepen mathematical understanding and vocabulary usage. The students were afforded autonomy; however, regular whole class and group checks were made, and complex issues were brought to class discussion. Mathematical issues that arose and needed teacher input were specifically taught as needed (e.g. the measurement of angles). As students shared their progress, they were encouraged to challenge developing evidence to improve quality and accuracy of representations. Finally, the students presented their claim and evidence, and communicated and justified their solutions to provide their "conclusion".

Data collection and analysis

Data collected consisted of classroom video (fully transcribed) of the unit of work, student work samples and research logs. Video analysis followed a process derived from Powell et al (2003). First, the videos were watched in their entirety to appreciate the unit progression and context. Transcripts were then coded using thematic coding derived from the EVT framework (Table 1). Emphasis in this phase was on identifying all instances of aspects that could be considered pertinent to the sub-topic regardless of the directionality of the influence (i.e. negative or positive influence on motivation). Finally, sections of the transcripts were refined to one or two excerpts, as appropriate, that were deemed illustrative of the sub-topic. No instances that could be considered negative in directionality were noted.

Results

The results section is organised under Eccles' (2006) EVT of motivation achievement framing questions of: "Can I do this?"; "Do I want to do this?"; and "What do I have to do to succeed on this task?". Each of these questions is discussed with examples to illustrate insights into the nature of the teacher-student and student-student interactions in the inquiry classroom and the relationships and messages being promulgated.

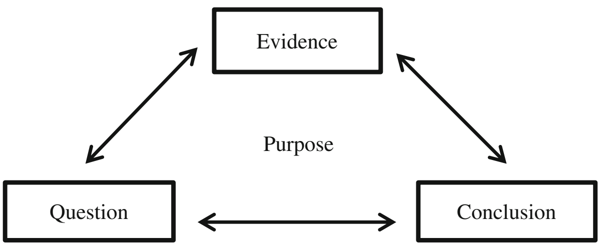

Fig. 1 Evidence model (Fielding-Wells 2010)

Table 1 Thematic coding derived from EVT Framework

EVT questions	Sub-topic
Can I do this task?	• Competency • Self-efficacy • Expectancy of success/failure
Do I want to do this task?	• Intrinsic enjoyment • Interest value • Utility value • Sense of control/autonomy • Cost
What do I have to do to succeed at this task?	• Self-regulation • Adaptive help-seeking

Can I do this?

In this inquiry, the students' belief that they could achieve stemmed from their own perceptions of success and failure as well as the expectations and support of the teacher. These elements are analysed in terms of EVT's concepts of *academic competence, self-efficacy,* and *expectations of academic success or failure*.

Beliefs about competence and self-efficacy

Positive belief about competence and self-efficacy is the key to students feeling they can achieve a task. During inquiry, students are encouraged to share their developments, understandings and learnings with others, predominantly in small groups and then through group representation with the whole class. This enables semi-formed ideas to be built upon and minimises individual focus. Placing the competency focus on the group/whole class may serve to reduce individual concerns. The comment below was unprompted but illustrative of this process:

1. Lucy	Then we would go back and think about it again, and the mistakes that we made. We will probably all have different ways but if someone is in trouble we will all work together as a group on one person's thing until we've got it correct. [Classroom video, 16 May]

Lucy's comment above illustrates the three key elements of the social aspect of inquiry. First, the assumption that both mistakes and rethinking are natural experiences in the solution process in an inquiry. The anticipation of mistakes being made normalises them and redefines errors as consistent with success. Second, the expectation that the thinking in the group would be divergent initially ("We will probably all have different ways"). This suggests that students' individual ideas are valued. Finally, the expectation that students would work together to address the problem. This social focus of the learning also likely reduced individual concerns about competence, as the responsibility for success would rest with the group.

Teachers impact students' sense of task competence and self-efficacy through their expectations. In this classroom, the focus of the teacher was to encourage and support student thinking and to scaffold them towards a conclusion so that all students would achieve.

2. Teacher	So when you work on the inquiry, what I want you to be thinking about is, and here is your question, *Can you have a pyramid with a face that is scalene?* At the end, I am going to ask you for your conclusion. And in your conclusion, you will make a claim, you'll provide me with evidence of that claim and then you're going to explain to me how that evidence is strong enough for you to be able to make that claim. So all we are doing is focussing on this conclusion but I will help you through it and we will look at a few good ones and see how we can strengthen them ok? [Classroom video, 16 May]

In this inquiry unit, the teacher worked to establish high expectations within a climate of support. The excerpt above is an example of how she conveyed her expectations that students would achieve the objective but also acknowledged to the students that they would be supported. In conveying expectations for success, the teacher also articulated her belief that students would achieve. These actions by the teacher likely created a foundation for a student's personal belief that "I can do this".

Expectations of academic success/failure

EVT emphasises the importance of student expectations of success/failure in a response to the question, "Can I do this?". A critical aspect of inquiry is the redefining of students' perceptions of, and attitudes towards, "failure". Throughout inquiry, failure is normalised, as students are encouraged to discuss processes and outcomes and use gained knowledge to lead towards better successive approximations and the development of incremental knowledge. In the excerpt below, the students provide the teacher with an update of their progress. The students do not simply advise they have successfully made a scalene-faced pyramid (which Shana refers to in her last sentence) but share the process, including their unsuccessful attempts, implying a belief that the information about what did not work was as important as identifying what did.

3. Dominica	… this one is our first attempt at a pentagonal based. It did not turn out very well and we decided to stop because when we folded it together the base would fold so straight away we knew it wouldn't work and we also attempted a square based and when we do fold it together again the base will bend.
4. Shana	Also for the triangular based one, we had a few attempts but they didn't work. Mine was the little one and we cut it up just before and this one was actually a failure that I made. It was a little bit off but then I cut and I put those two together (sides) and dotted in the lines and then I measured it and cut it out and it stuck on. [Classroom video, 7 Jun]

In responding to the question "Can I do this?", EVT provided an opportunity for insight into *how* the inquiry may have supported students' sense of competence, self-efficacy and beliefs about success. The features of the inquiry included teacher expectations of success, awareness that the teacher and their peers would support them through the inquiry and re-constructions of beliefs about failure. Believing that one can complete a task, however, is insufficient motivation to undertake the task. Wanting to do a task is also critical.

Do I want to do this?

To address the question "Do I want to do this?", students may consider their own interests, seek intrinsic enjoyment or consider the usefulness of the activity. This category can be analysed in terms of EVT's concepts of *interest and intrinsic value, sense of control and autonomy,* and *utility value*.

Intrinsic enjoyment and/or interest value

Enjoyment and interest are strong motivators for children. This inquiry stemmed from a student's curiosity during a traditional mathematics lesson on shape. From the student asking if a pyramid could have a scalene face, the class requested to adopt the question as their next inquiry. They therefore expressed an interest in not only knowing the answer but also an interest in actively finding out for themselves through the inquiry process.

There were visible indicators throughout the unit that the students were engaged and enjoying the activity. For example, students could have addressed the question with a single pyramid; however, they extended the breadth of their inquiry by desiring to create pyramids with multiple base shapes to further their knowledge:

5. Shana	We want to prove to everybody that it isn't just a square based pyramid. We want to do other things because we think the other groups might only do square based and then we won't know anything extra. [Classroom video, 16 May]

Students further demonstrated their interest through the voluntary contribution of their free time. For example, many students requested the opportunity to remain in the classrooms during breaks to build more pyramids, as well as constructing more in their out-of-school time.

This suggests that there may be potential for IBL to tap into students' interests through the contextual nature of IBL, as well as providing opportunities to present knowledge as problematic, which may tap into young students' intrinsic desire to learn.

Sense of control/autonomy

EVT suggests that higher motivation stems from students having greater control over their own learning. The inquiry unit offered students extended opportunities to manage meaningful aspects of their learning. While the extent and nature of the aspects managed by students in an inquiry may vary, the students are always afforded significant control and autonomy. In this instance, students determined the question and the pathway to solution, but perhaps less usual, they set their own criteria for success. In the interchange below, Lucy started creating a scalene triangle but realised that two sides were so close in measure that she doubted whether it could be still considered scalene:

6. Lucy	(Referring to her drawing) That's an isosceles triangle, Dominica.
7. Dominica:	Then why are you making it?
8. Lucy	I didn't mean to. It just came out as one. (Measuring the sides of the triangle to demonstrate to the group) That is 11.6, that is 11.6. Yep, it is isosceles …
9. Dominica:	(Measures triangle sides of her own diagram to check) That's 6.8 cm that's 6.7. (Lucy checks and group debates whether that the sides are too close to consider it isosceles). [Classroom video, 17 May]

The result of this discussion was a class decision by the students to include the sum of angles in the evidence for each triangle. In instances where the lengths of two sides were exceptionally close, if the sum of angles was off, for example 178 degrees, then the students decided the triangle was not accurate enough to serve as valid evidence.

This excerpt illustrate that students perceived a sense of control over their learning: they did not expect or ask the teacher to set these criteria for them but rather saw it under their control, recognising the need for the class to come to agreement on the criteria.

Utility value

Utility is often considered to be related to the external or extrinsic reward received for engaging with a task. While primary students are aware they are formally graded in mathematics, students did not appear to foreground the grading aspect. While there was no evidence to support this, the absence of evidence is perhaps more telling, with no reference to grading or teacher expectation standards made by the students.

A broader view of utility can be taken as the extent to which learning facilitates students' goals, rather than just assisting students to obtain external rewards. Ainley et al. describe utility in school mathematics as the "construction of meaning for the ways in which mathematical ideas are useful" (2006, p. 30) and identify the rarity with which students work with problems that provide the opportunity to appreciate the utility of mathematics. The extent to which the students saw the task as facilitating their goals bore a stronger relationship to a desire to address the question and thus to meet the goal of determining an answer to a mathematical problem they were curious about.

Students' motivation in response to the question "Do I want to do this and why?" was fuelled in this inquiry by three elements outlined in EVT. First, students' interest and enjoyment of the task motivated them to go beyond task requirements. A sense of control and autonomy to manage their own inquiry, including the setting of their own criteria for success, appeared to be another factor that motivated them to engage in a greater depth of mathematical understanding. Finally, the students' perception of utility of mathematics in addressing questions they were curious about fostered a sense of motivation in their desire to determine an answer. These aspects of EVT therefore may provide a greater understanding of why students engage in inquiry tasks.

What do I have to do to succeed on this task?

An important link between motivation and achievement is children's regulation of their own behaviour (Eccles and Wigfield 2002). EVT provides perspective into motivation in the third question ("What do I have to do to succeed in this task"). Beyond believing that they are able to succeed (question 1) and having the desire to do so (question 2), achieving at a task requires relevant skills and strategies as well as a willingness to seek help when needed. This perspective is critical in inquiry where tasks are often challenging, ambiguous and unfamiliar.

Self-regulation

A key aspect of students' knowing on how to succeed was their foundational understanding that the solution of an inquiry question was expected to include evidence. In this inquiry, students relied on an evidence framework (Fig. 1) that they had been using with progressively more confidence and sophistication over the past four inquiries. In the excerpt below, the group of children self-monitored their group's progress as they directed their search for possible evidence.

10. Salome	So what would count as evidence?
11. Geneva	A model could. If you could get a model with at least one scalene side then it would be evidence because obviously it would be possible.
12. Lee	Maybe a diagram.
13. Sadie	A model because it actually does show us.
14. Salome	(Talking aloud as she writes) "A model of a pyramid with one face that is scalene." And I like Lee's idea about a diagram. A diagram of a pyramid.
15. Teacher	(To the class) Ok, a couple more minutes and I'm going to ask each group to share what they have put down (for ideas of evidence).
16. Salome	And a diagram of a pyramid with one face that is scalene.
17. Geneva	A net.
	(Students debate whether a diagram and a net are the same thing.)
18. Lee	A net of a scalene pyramid.
19. Salome	A net might not be great. We might need to *test* it.
20. Sadie	Yeah, test it because it might be wrong.
21. Lee	A *testable* net.
22. Salome	No, an *already tested* net.
23. Geneva	A *correct* net. [Classroom video, 16 May]

The exchange above illustrates several skills and strategies that students had developed through inquiry which allowed them to self-monitor their progress. First, working collaboratively to seek and apply evidence within an inquiry was a critical strategy in their planning. Through their discussion, the children in this group were developing increasingly sophisticated ideas of evidence that they deemed as valuable: evidence that would help them determine a conclusion and which would persuade the class. The students were working without assistance from the teachers here, suggesting that they were able to self-regulate both the process of their exchange (keeping it productive) and the quality of the outcome. Finally, the students not only sought evidence (a net and a model), but included a process to test their net—implying they were also considering evidence quality and ensuring a process to meet an anticipated standard.

Adaptive help-seeking

When children encounter a challenging unfamiliar situation, exerting effort to continue is often dependent on their willingness to seek help (Newman 2002). If a student's goal is to learn rather than perform, they are more likely to openly express difficulties and acknowledge errors and seek assistance. In the excerpt below, these students are struggling and, as a group, seek help from the teacher.

24. Lee	(Teacher arrives in response to hands raised)We tried with this one (holding up a pyramid) but it is not very good.
25. Teacher	Why has this one not worked out very well?
26. Salome	Because to make it meet at the top they are not actually triangles once we make them the same size. It is sort of not a triangular pyramid at all.
27. Teacher	OK, so let's look at this pyramid here (another attempt by this group). What is wrong with this one?
28. Salome:	Well we were trying to make all the triangles the same.
29. Teacher	What do you mean by all the same?
30. Geneva:	We were trying to make it so they all fit together at the top.
31. Teacher	So are you happy with the way they all fit together?
32. Sadie:	No. It is no good.
33. Teacher	Ok. But could you use this though? Is there something that this could help you to do?
34. Geneva:	We could use it to make another idea.
35. Lee:	You could make an edge like shorter or longer so they do fit.
36. Teacher	OK. So what is the next thing that this group is going to work on? [Classroom video, 17 May]

The conversation highlights a rather typical progression from the unit: the students seek help, and the teacher responds by having students identify the specific problem they are having (line 25, 27, 29), and then uses questioning (line 33) to assist the students to think of their own way forward (line 34–35). In instances where students still experienced difficulty, the teacher would call the class together to collectively contribute ideas. As a last resort, the teacher would provide the minimal guidance required to enable a student or group to "discover" a way forward.

In terms of answering Eccles and Wigfield's third motivation question, two salient aspects were noted. First, students did not worry about whether they knew how to solve a problem from the beginning because they saw all attempts as progressing towards a goal and could self-regulate that progress. Therefore, making an attempt was in itself a productive strategy. Second, when students got stuck or encountered problems, they had multiple ways to seek help that was non-judgemental. Their collaborative group was the most immediate resource for help, and the source students used initially. They also knew that the teacher would regularly stop the class in order to share progress and obstacles. These opportunities to share were frequent and broadened students' access to assistance.

Discussion and conclusion

In classrooms, student motivation comes into play when students are faced with everyday learning tasks and activities. Consciously or sub-consciously, they make decisions about how effectively they will direct their energy and attention, that is, their level of engagement. At the individual level, these decisions are largely influenced, at least initially, by how confident a student may feel about succeeding in the task, the opportunities the task offers for individual input and control, their interest in the task and how prepared the student may feel to address the task. As this analysis has illustrated, IBL fostered an explicit recalibration of students' expectations. The emphasis of IBL on exploration, open-endedness and iterative trial and error that is actively encouraged and supported by the teacher (e.g. lines 24–36) likely offset anxiety or concerns about not succeeding. Explicit acceptance of failure as a valuable part of the learning process (e.g. lines 3–4) reflected other research that highlighted the significance of conceptions of failure in motivation and achievement (Haimovitz and Dweck 2016). In Bandura's (1997) model of expectancies, which in part informs EVT, two kinds of beliefs influence efficacy: beliefs about outcomes (what certain behaviours will lead to) and beliefs about process (whether one can perform the behaviours necessary to produce the outcome). The pedagogy of IBL is such that efficacy (related to both outcomes and process) is bolstered explicitly by the teacher, their peers and through experience as each lesson unfolds (e.g. lines 1, 2). In this way, student beliefs about how well they will do in IBL are socially mediated (as Wigfield et al. 2006, have indicated), shaped with effective intention by the IBL practises. When efficacy is high, motivation improves (Bandura 1997), and cognitive engagement is enhanced (Greene et al. 2004).

However, beyond building confidence in being able to do a task, the brief examples discussed in this paper point to the potential of IBL to generate high levels of motivation to commit to a task. Next to the scaffolding of efficacy, IBL practises offered students a great degree of autonomy and control (e.g. lines 6–9) and opportunities to extend their learning (e.g. line 5). As the extracts in this example illustrate, student choice can drive the selection of a topic, the framing of a problem, the strategies for testing a solution and the development of arguments and reasoning around solutions (even failed ones). The purposes that students have for engaging in a task influence their level of engagement: by encouraging students to select topics that pique their curiosity, student interest is enhanced. From the commencement, the students took responsibility for determining the evidence they thought necessary, sufficient and convincing to answer the question (e.g. lines 10–23). Envisaging the evidence needed to address the question required the students to engage with the mathematics at a deeper level than if they had been told how to proceed. In this respect, students had gained strategies to allow them to envisage the content they would address at a level that exceeded the requirements of the Australian Curriculum: Mathematics (ACARA 2016) through non-standard representations of pyramids, measurement of angles and design of nets. Experiences such as these would likely build students' sense of self-worth which in turn reinforces both efficacy and autonomy (Pintrich 2003), together strengthening student motivation for learning.

Applying EVT (Eccles 2006; Wigfield et al. 2006) to an analysis of teaching and learning highlighted how inquiry-based learning could enhance student motivation in mathematics. Students' beliefs related to competence and expectations of success were reflected in the overarching question "Can I do this?" (Q1). Perceptions about the value of the task in relation to interest, enjoyment and a sense of autonomy were captured in the question "Do I want to do this?" (Q2). Finally, the question, "What do I need to do to succeed?" (Q3), was used to relate examples of self-regulation and adaptive help-seeking to the inquiry classroom. These three questions provided a more nuanced understanding of *how* aspects of IBL can be influential sources of student motivation and therefore have potential to impact engagement.

The research question being addressed was whether EVT could be used as a lens for developing insight into aspects of motivation that would manifest as engagement in IBL, and it would appear that this is the case. Through using the framework to code a unit of inquiry, specific examples became abundantly apparent, especially in terms of perceived success (specifically the reconceptualization of failure as incremental to success) and implicit beliefs about competence; that is, the identifiers related to being *able* to carry out the task. In terms of *wanting* to do the task, there were explicit and implicit examples of increased autonomy and control, intrinsic enjoyment and interest although few, if any, of extrinsic valuing. The latter may also be attributable to the children's age and stage of schooling, where they are not yet valuing "grades" as being of extreme importance for their future success. However, utility value in terms of valuing the development of mathematical skills for their utility will require further exploration, as it is likely to require alternate research methods, such as interview, to elicit deeper understanding about students' views of utility.

While these findings relate to a single class and therefore the extent to generalise is limited, it does appear that there is usefulness to continuing to apply the EVT framework to examine further examples of mathematical inquiry. If aspects of inquiry that have robust potential to motivate students can be identified, there is a potential for both enhancing these in mathematics teaching and learning with inquiry, and also incorporating aspects into more traditional lesson formats.

Acknowledgements The authors wish to acknowledge the participating students. This work was funded by the ARC grants DP120100690 and DP140101511 and an Australian Postgraduate Award.

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