

# Pulse Compression with Minimum Uncertainty: An Efficient Microwave Medical Imaging Technique

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**Abstract**— A pulse compression technique that gives an optimum contrast and visibility of targets in radar-based medical imaging is presented. A smoothing window for microwave beamforming technique which more properly alleviates the effect of abrupt truncation in finite length signals with the aid of the uncertainty principle is utilized. It is found that using a closer output signal shape to the Gaussian pulse results in a lower uncertainty and ambiguity in the reconstructed images. Hence, when the back-scattered signal passes through a window whose uncertainty is the least, the visibility of the target in the imaged domain will be the highest with high signal-to-noise ratio and fine resolution in microwave medical imaging. The accumulation of the above properties together increases the chance of detecting any abnormality in the human body at early stages and thus resulting in a higher chance of survival. The idea is tested on a real-sized head model surrounded by an array of dipoles operating across the band 1.3-1.4 GHz. The results are compared with the most commonly used beamforming techniques to show the achieved improvements in practice.

## 1 INTRODUCTION

In medical imaging, it is well known that there is a direct relation between the chance of survival and the early detection of cancerous and abnormal tissues. This fact has led the research on microwave imaging toward methods with fine resolution, minimum transmitted power level, maximum signal to noise ratio (SNR) and low localization error. To the knowledge of the authors, there has not been a proposed technique for microwave imaging which simultaneously possesses the above four key factors.

Reviewing the literature shows that the technique in [1] considers the directivity of imaging antennas in the reconstructed images to give fine resolution, though the transmitted power, localization error and SNR are not optimized in the method. The method in [2] implements a technique to lower down the required power in medical imaging applications, but the method does not improve the resolution, localization error and SNR. Moreover, the approach in [3] proposes an assessment algorithm by which only the high SNR responses are acquired to reconstruct the image. That method, however, does not improve the resolution, localization error and power level. While some approaches applied pulse compression technique to simultaneously satisfy three out of the four factors mentioned above, the localization error has not been efficiently minimized yet [4], [5].

In pulse compression, the sidelobe level (SLL) (either in the transmitted or received pulse) needs to be low in order to yield an image in which the contrast and visibility of targets are distinguished

well enough and no target is masked out. To this end, smoothing windows are used to reduce the SLL versus the peak power. The window reduces the discontinuities appearing at the edges of the spectrum of the signal and correspondingly reduces the loss of energy at the edges. Clearly, this smoothing process reduces the signal bandwidth and thus the resolution by neglecting some frequency components. This is a tradeoff and the Kaiser window [6] is nearly optimal for this purpose due to its adjustable parameter between resolution and SLL.

In this paper, the shortcomings on Kaiser window i.e. its computational complexity and non-theoretical optimum point between resolution and SLL lead us towards another window which is adjustable and optimum, but less computationally complex. To this end, the normalized Gaussian wave packet satisfying the minimum uncertainty principle is a proper candidate to perform as a smoothing window. This function benefits from the minimum ambiguity and therefore it optimally adjusts the resolution and SLL to have minimum uncertainty and maximum visibility in the reconstructed image.

Rest of the paper is organized as follows. In section 2, the Kaiser and Gaussian windows are objectively compared. Then, Section 3 applies the two windows to a same head imaging problem and the results are compared to show the advantages of Gaussian window. Finally, conclusions are made in Section 4.

## 2 Theory

In this section, the theory of pulse compression and Kaiser window is briefly reviewed. Following this, the uncertainty principle and Gaussian window are discussed and the theoretical differences between them are shown.

### 2.1 Pulse compression

In pulse compression, the following linear FM pulse  $s(t)$  is used in the transmitting mode

$$s(t) = \text{rect}\left(\frac{t}{2T}\right)e^{j\pi Kt^2} \quad (1)$$

for which the frequency ( $f = \frac{1}{2\pi} \frac{d}{dt}(j\pi Kt^2)$ ) is a linear function of time as

$$f = Kt \quad (2)$$

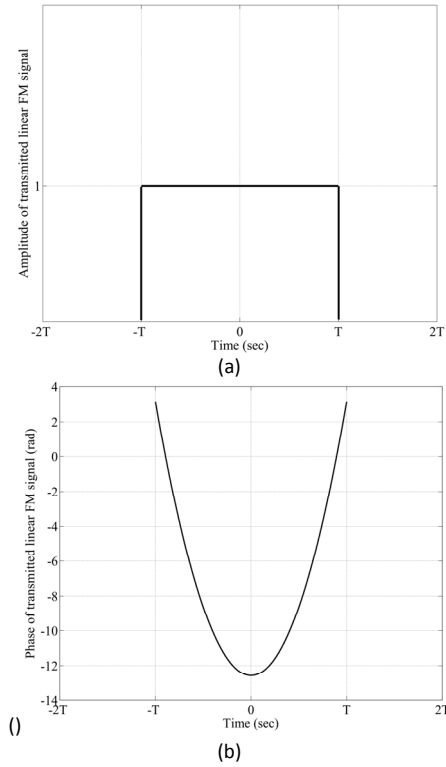


Figure 1: The amplitude (a) and phase (b) of transmitted linear FM signal with  $K=5$ .

where  $t$ ,  $2T$  and  $K$  are the variable of time, pulse duration and linear FM rate (Hz/s), respectively. Figs. 1 show the amplitude and phase for (1) with  $K=5$ . Since all the required data from a tissue e.g. position, shape and reflectivity are derived by the scattered signals measured by the imaging antennas, these scattered signals must be strong enough in terms of power level (considering safety levels) and SNR, and possess fine resolution. To obtain a fine resolution, a short pulse should be sent and received. On the other hand, the SNR of the signal must be kept high to accurately reconstruct the image. Therefore, these two qualities are often in conflict with respect to each other. While the SNR can be increased by leveling up the transmitted signal power, the safety limitations lead us toward using an alternative option, i.e. a longer pulse for the same purpose. This longer pulse possesses poor resolution, unless the pulse compression technique is utilized in the receiver to compress the pulse to a desired resolution. This is done if the received signals, which are scattered from the human tissues, are passed through a convolution (matched) filter with the following impulse response

$$h(t) = s^*(-t) \quad (3)$$

where  $*$  is the complex conjugate operator.

The output signal used to reconstruct the image is compressed in the order of hundreds or even thousands with respect to the transmitted signal. As

every real target has a different phase, the compressed output signal is complex in practice, and both of the amplitude and phase of the compressed signal are utilized in the image reconstruction. The filter is only matched to the signal, not to the noise, and thus is robust versus noise media.

## 2.2 Kaiser window

Since the SLL of the above filter is relatively high (-13 dB), it is necessary to improve the SLL in order to have high visibility for every target. This is usually done by applying the Kaiser window, defined as follows [6]

$$w(t) = \frac{I_0(\beta\sqrt{1-(t/T)^2})}{I_0(\beta)} \rightarrow |t| \leq T \quad (4)$$

where  $\beta$  is the adjustment factor between resolution and SLL and  $I_0$  is the zeroth-order Bessel function of the first kind. The nearly optimum value of the adjustment factor is commonly 2.5 to give -21 dB SLL and 1.18 times larger resolution than the non-windowing case. As seen, the resolution has not been degraded noticeably, while the SLL has been improved significantly.

Clearly, using (4) at each receiver requires considerable computational time since the Bessel function has a complicated form. Also, in some applications, we require better SLL at the cost of slightly degrading the resolution.

## 2.3 Gaussian window

According to the uncertainty principle, higher SLL means more resolution and vice versa. This phenomenon has been experienced before by applying the Kaiser window. However, it is not still clear what kind of window and what value of the corresponding adjustment factor are theoretically the most optimized ones. The Heisenberg's uncertainty principle in physics implies that the highest precision with which the position and momentum of a target can be measured is satisfied by the following Gaussian wave packet [7]

$$G(t) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{1}{4}} \exp\left(-\frac{t^2}{4\sigma^2}\right) \exp(jk_0 t) \quad (5)$$

where  $\sigma$  is the uncertainty and  $\hbar k_0$  is the mean of momentum with  $\hbar$  as the reduced Planck's constant. In other words, a closer measured signal to (5) indicates less uncertainty in the target localization.

By weighting the received signal using (5) instead of (4), the tradeoff between the resolution and SLL reaches its optimum point. Since every window is a real-valued function, the absolute value of (5) is utilized as follows

$$|G(t)| = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{1}{4}} \exp\left(-\frac{t^2}{4\sigma^2}\right) \quad (6)$$

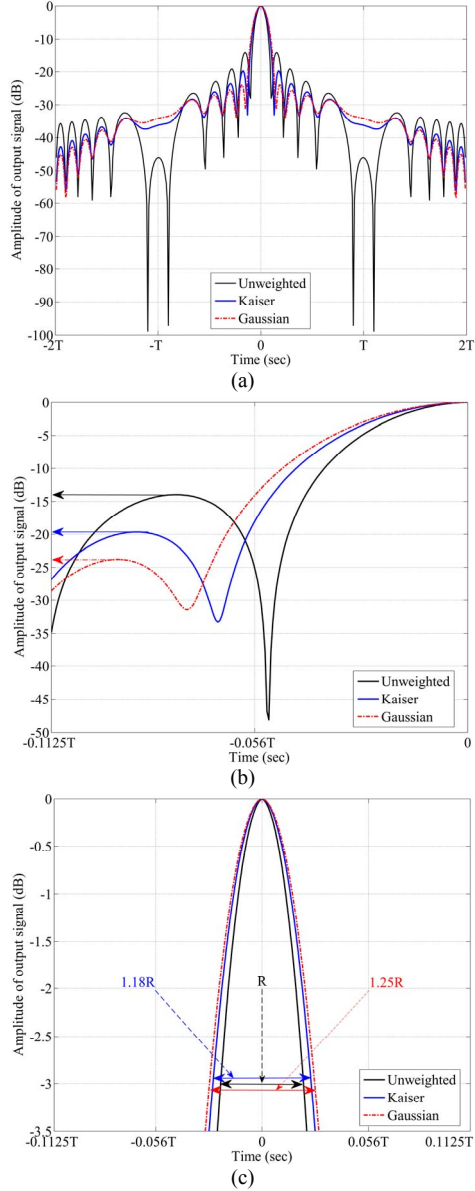


Figure 2: (a) The compressed output signals with and without Kaiser and Gaussian windows which has been zoomed in (b) for a better comparison on SLL and in (c) on resolution (R).

The window must be normalized to avoid wrong weightings. Thus,  $\sigma = \frac{1}{\sqrt{2\pi}}$  is the correct value giving the Gaussian window as follows

$$w(t) = \exp\left(-\frac{\pi}{2}t^2\right) \quad (7)$$

Figs. 2 show a quantitative comparison between (4) and (7) in terms of SLL and resolution. No adjustment on  $\beta$  can exactly match the Kaiser window on the Gaussian one. The adjustment on the resolution increases the SLL, while the adjustment on SLL degrades the resolution. Hence, the optimum

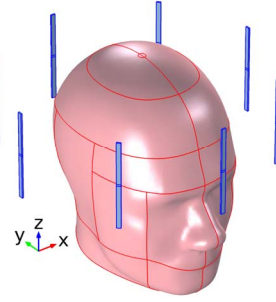


Figure 3: The simulation environment.

value for SLL and resolution is derived by the minimum uncertainty principle through Gaussian window which is not as complicated as Kaiser one.

### 3 Gaussian with Respect to Kaiser Window in Practice

To test the proposed technique in comparison with others, the real-sized head model with average effective permittivity 40 is utilized. A circular array of dipole antennas with 15 cm radius resonating at 1.35 GHz are used to successively illuminate the head and record the corresponding S-parameters. The transmitted pulse has 100 MHz bandwidth, from 1.3 to 1.4 GHz. The resolution is 1 mm and the target is a sphere with 1 cm radius and relative permittivity 42 (early case bleeding), located at the backside of the head. This close value to 40, is selected to show the ability and limitations of Kaiser and Gaussian windows.

The recorded signals are compressed by convolving in (3) and then passed through both the Kaiser (with  $\beta = 2.5$ ) and Gaussian windows. The delay-and-sum algorithm [8] is used to reconstruct the images from these outputs. Figs. 4 show the reconstructed images using the two windows. As seen, the higher SLL in Kaiser window decreases the visibility and contrast of the target, while the Gaussian window holds the optimum point between resolution and SLL to give the maximum visibility and contrast. This fact is more obvious in Figs. 4b and 4c where the edges of the target as highly scattering regions have more contrast using Gaussian window than the Kaiser one. The computational time for the whole reconstruction process is 31 minutes using Gaussian window and 36 minutes using Kaiser one. Consequently, in image reconstruction problems where the high precision in visibility and low computational time are required, e.g. [9]-[14], Gaussian window may be a wiser choice than the Kaiser one.

### 4 Conclusions

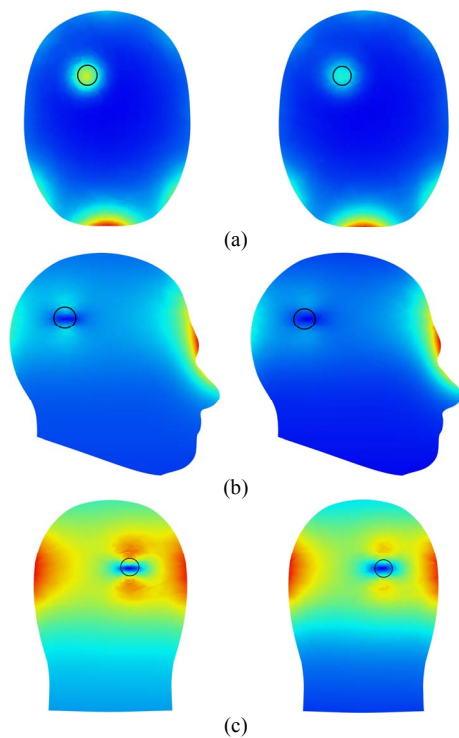


Figure 4: Normalized reconstructed head images using Gaussian (left) and Kaiser (right) windows in top (a), side (b) and front (c) views. The black circle shows the exact location of the target.

A pulse compression technique for medical imaging with optimum compromise between resolution and side-lobe level using Gaussian window has been presented. The proposed technique satisfies the minimum uncertainty principle and thus gives maximum visibility for every target with a lower computational time. This method is successfully tested in a head imaging problem. The comparison with an existing method proves superiority of the presented approach.

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