

## METHODS &amp; TECHNIQUES

# Intuitive representation of photopolarimetric data using the polarization ellipse

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**ABSTRACT**

Photopolarimetry is the spatial characterization of light polarization. Unlike intensity or wavelength, we are largely insensitive to polarization and therefore find it hard to explore the multidimensional data that photopolarimetry produces (two spatial dimensions plus four polarization dimensions). Many different ways for presenting and exploring this modality of light have been suggested. Most of these ignore circular polarization, include multiple image panes that make correlating structure with polarization difficult, and obscure the main trends with overly detailed information and often misleading colour maps. Here, we suggest a novel way for presenting the main results from photopolarimetric analyses. By superimposing a grid of polarization ellipses onto the RGB image, the full polarization state of each cell is intuitively conveyed to the reader. This method presents linear and circular polarization as well as ellipticity in a graphical manner, does not require multiple panes, facilitates the correlation between structure and polarization, and requires the addition of only three novel colours. We demonstrate its usefulness in a biological context where we believe it would be most relevant.

**KEY WORDS:** Photopolarimetry, Circular polarization, Polarization ellipse, Biology, Biomedicine

**INTRODUCTION**

Photopolarimetric data – spatially resolved information about the polarization of light – are useful for humans. Polarization allows us to gain knowledge about our environment that is inaccessible using light intensity and wavelength alone. Photopolarimetric data are used in ellipsometry (Azzam and Bashara, 1987), remote sensing (Tyo et al., 2006; Halajian and Hallock, 1972; Walraven, 1977; Egan et al., 1991), structure analysis (Zappa et al., 2008), microscopy (Mickols et al., 1985), machine vision (Wolff and Boulton, 1991; Wolff et al., 1997), target detection in scattering media (Tyo et al., 1996; Rowe et al., 1995), astronomy (Hough, 2006) and biology as well as biomedical sciences (York et al., 2014). Many invertebrate (Nilsson et al., 1987; Nilsson and Warrant, 1999; Marshall et al., 1991; Horváth, 2014) and some vertebrate species (Phillips and Waldvogel, 1988; Hawryshyn, 1992) have also invested in polarization vision and use it for navigation (Rossel, 1989; Wehner, 2001; Hawryshyn, 1992; Flamarique and Hawryshyn, 1997; Schwind, 1991; Krapp, 2014; Heinze, 2014; el Jundi et al., 2014), predation (Lythgoe and Hemmings, 1967; Shashar et al., 1995, 1998, 2000, 2002) and communication (Horváth, 2014;

Cronin et al., 2003; Shashar and Cronin, 1996; Shashar and Hanlon, 1997; Shashar et al., 2002; Mäthger and Denton, 2001; Sweeney et al., 2003). Human sensitivity does not, however, extend to functionally discriminating and spatially resolving the polarization of light. Therefore, we have to resort to displaying the polarization of light in some other modality that we do perceive.

Displaying imagery of partial polarization is problematic because (other than the two spatial dimensions,  $x \times y$ ) at least four variables are needed to fully characterize it. Although many different interpretations for polarization exist, one of the most commonly used comprises (see List of symbols and abbreviations): intensity ( $0 < I < \infty$ ), degree of linear polarization ( $0 \leq \text{DoLP} \leq 1$ ), angle of linear polarization ( $-90 \text{ deg} \leq \text{AoLP} < 90 \text{ deg}$ ) and degree of circular polarization ( $-1 \leq \text{DoCP} \leq 1$ ).

Presenting photopolarimetric data has had different approaches (Yemelyanov et al., 2003a,b; Tyo et al., 1998; Wolff et al., 1997; Bernard and Wehner, 1977; Solomon, 1981; Engheta and Pugh, 2002). Although some of these suggestions include unique ways to present the extra dimensions needed (e.g. movement; Yemelyanov et al., 2003b), most photopolarimetric data are presented as a set of three to five image panes, each describing one of the polarization variables using a colour space or gamut available on computers (Fig. 2). This method suffers from a number of disadvantages: (1) circular polarization (or ellipticity) is often ignored; (2) when circular polarization is not ignored, it is hard to discern any information about ellipticity and total polarization; (3) multiple panes are required to display the polarization state in full, making it difficult to correlate structures and locations between the different panes; (4) richly coloured panes obscure rather than convey the main trends in the data; (5) edge artefacts are common and conspicuous, distracting the reader from the main point; and (6) small angular differences in AoLP are difficult to discern.

These shortcomings are especially evident when presenting biological photopolarimetric data. Biologists are often interested in patches and areas with similar polarization (rather than small-scale variations), they require information about circular and elliptical polarization, and they want to correlate the polarization with the body segments or markings on the organism. Here we suggest a new polarimetry representation that both improves on the shortcomings of the traditional method and answers most of the needs biologists and biomedical imaging may have.

**MATERIALS AND METHODS****Polarization ellipse**

In this study, polarization is represented using the polarization ellipse (Collett, 1992). Although the polarization ellipse is mostly used as a graphical representation of purely polarized light (either monochromatic incoherent radiation or a single electromagnetic wave), it can be adapted to describe partly polarized light by scaling the ellipse in proportion to the total polarization of the light (see Fig. 1).

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**List of symbols and abbreviations**

$a$	minor axis of the polarization ellipse
AoLP	angle of linear polarization ( $\theta$ )
$b$	major axis of the polarization ellipse
DoCP	degree of circular polarization
DoLP	degree of linear polarization
$h$	handedness of the polarization ellipse
$I$	intensity
$s_{0...3}$	Stokes parameters
$s_0^P$	polarized component of light
$s_0^{UP}$	unpolarized component of light
$R$	ratio between the minor and major axes
$P$	ratio between the total and polarized light

The shape of the polarization ellipse (its minor and major axes and its orientation angle) are dictated by the Stokes parameters. These four parameters are often combined into a vector, the Stokes vector, which fully describes the polarization state of electromagnetic radiation in a mathematically convenient way. Stokes parameters are also easy to experimentally measure, each resulting from the addition or subtraction of a few light measurements. For instance, the first Stokes parameter simply describes the total intensity of the light (the reader is directed to any optics literature for further details about Stokes mathematics).

Because of the additive property of Stokes vectors, a partially polarized beam of light can be decomposed into its unpolarized and (completely) polarized components:

$$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} s_0^{UP} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} s_0^P \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}, \quad s_0^P = \sqrt{s_1^2 + s_2^2 + s_3^2} \quad (1)$$

where  $s_0^{UP}$  is the unpolarized portion of the light and  $s_0^P$  is the polarized part. We will calculate the shape of the polarization ellipse using these two Stokes vectors. The shape of the ellipse depends on the ratio between the minor and major axes of the ellipse,  $R$ , the orientation angle (i.e. angle of polarization) of the ellipse,  $\theta$ , and its handedness,  $h$ . Following are the mathematical relationships between these variables and the Stokes parameters of a partially polarized beam of light (Collett, 1992):

$$R = \frac{s_0^P - \sqrt{s_1^2 + s_2^2}}{s_0^P + \sqrt{s_1^2 + s_2^2}}, \quad (2)$$

$$\theta = \frac{1}{2} \arctan \frac{s_2}{s_1}, \quad (3)$$

$$h = \text{sign}(s_3). \quad (4)$$

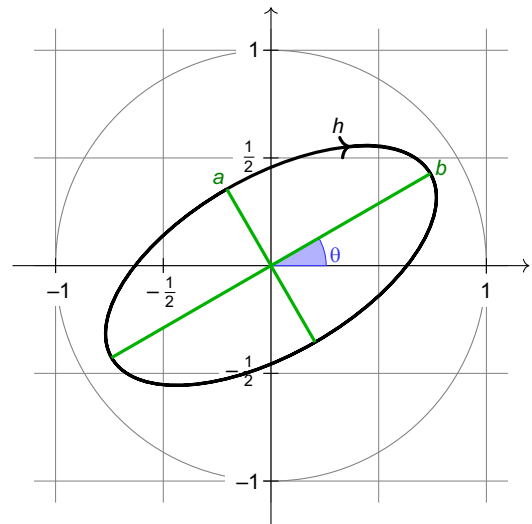
Because the degree of polarization will be depicted by the size of the ellipse, we need to scale the ellipse's axes. Therefore, the minor ( $a$ ) and major ( $b$ ) axes are scaled by the ratio  $P$  of the polarized light intensity,  $s_0^P$ , to the total light intensity,  $s_0$ :

$$P = \frac{s_0^P}{s_0}, \quad (5)$$

$$a = P \cdot R, \quad (6)$$

$$b = P. \quad (7)$$

Note that because both axes are scaled by the polarization ratio  $P$ , the actual intensity of the light (i.e.  $s_0$ ) has no bearing on the shape of the ellipse.



**Fig. 1. The polarization ellipse of partly polarized light.** Here, the Stokes parameters  $[s_1, s_2, s_3]$  are equal to  $[0.15, 0.26, 0.8]$  (arbitrarily chosen values). Using Eqns 3–7, we can calculate that the total polarization is 85%, the angle of polarization ( $\theta$ ) is 30 deg, and the two axes are 0.41 ( $a$ ) and 0.85 ( $b$ ). The handedness ( $h$ ) of the light is indicated by the clockwise turning arrow (the light is right-hand circularly polarized). Notice that because the sum of squares of these stoke parameters must be smaller (or equal) to the square of the first Stokes parameter, the ellipse can never be larger than the grey circle (with a radius equal to 1).

**Photopolarimetry**

The suggested presentation of polarimetric data in this study follows these general steps. (1) Spatially resolved polarimetric data are collected photographically. (2) These data are divided into a grid of cells. (3) At each cell: (a) the mean Stokes vector is used to calculate the parameters of a polarization ellipse, and (b) this ellipse is superimposed on an image of the subject matter at the cell centre.

Although a methodological description of how photopolarimetric data should be collected (i.e. photopolarimetry) is outside the scope of this study, we describe briefly how we collected our photopolarimetric data (for more information, see Horváth, 2014; Wolff, 1997). A Nikon D300 (with a Nikon 105 mm micro lens) was fitted with two rotatable filter rings. One ring had a linear polarizing film while the other had a quarter-wave retarder film (both from American Polarizers, Reading, PA, USA). By rotating these two rings (relative to each other and the camera's objective), the six required measurements for calculating the polarization state were obtained (horizontal, vertical, diagonal, anti-diagonal, right-hand circular and left-hand circular). Rotating the filters and taking the six pictures took approximately 30–60 s. This relatively manual procedure can be made quicker with the use of motorized filters, splitting the image into separate sensors, or etching orthogonal filters onto a thin film that is then aligned to the sensor's pixels (Gruev et al., 2010). To maintain the linear relationship between light intensity and pixel intensity, the images were saved in raw format. Converting the raw NEF files was done with dcraw (an open-source program that reads raw image formats and converts these into the standard PPM and TIFF image formats; <https://www.cybercom.net/~dcoffin/dcraw/>). All image processing was performed in Julia (Bezanson et al., 2014) (an open-source, high-level, high-performance dynamic programming language for technical computing). Superimposing the ellipses on the RGB image was performed with LaTeX (and the TikZ package). All of

the code used in this procedure is freely available on github (<https://github.com/yakir12/polarimetryLab>).

The grid's cell size was chosen so that the resulting cells are not too small to be discernible while at the same time not too big as to include too many imaged structures. Total polarization is encoded by the size of the ellipse (as previously discussed), and handedness is illustrated by the ellipse's colour (two different colours). When the eccentricity of the ellipse is below a certain threshold and its shape is practically a line, handedness is less relevant and the ellipse is coloured with a third colour.

To better assess the functionality of our method, we constructed a custom polarization standard. This standard contained six squares (see Fig. 2): black and white polyester felt at the top, vertically and horizontally oriented polaroids in the middle, and left-hand and right-hand circularly polarized filters at the bottom (all filters are from American Polarizers). Finally, all the animal photography in this report was carried out under the approval and oversight of the University of Queensland Native and Exotic Wildlife and Marine Animals (NEWMA) Animal Ethics Committee.

## RESULTS AND DISCUSSION

The proposed method tackled all of the shortcomings described in the Introduction (disadvantages 1–6). Because information about circular polarization is intrinsic to the polarization ellipse, it is never ignored in our method.

The polarization ellipse intuitively presents information about the ellipticity of the light, allowing the viewer to obtain an overview of this modality of light. We can see, for example, that the two bottom circularly polarized filters in the standard in Fig. 2 slightly differ in ellipticity (i.e. the left-hand circular polarized filter is less elliptical than the right-hand circular polarized filter). However, this difference is not as evident in the traditional presentation of photopolarimetric data in Fig. 2 and would require a comparison of

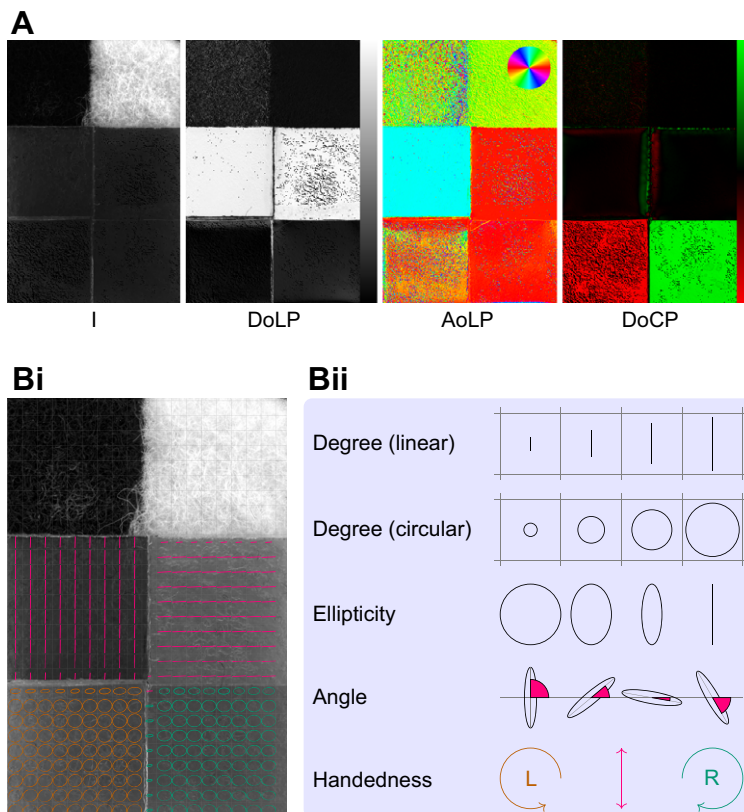
intensities between the DoLP and DoCP panes (note that the DoLP of the right-hand circular polarized filter is indeed higher than that of the left-hand circular polarized filter).

With our method, there is no need for multiple panes to display the polarization state, which facilitates the interpretation of the data. It is easy to correlate polarization with body segment or marking on the subject organism (e.g. the red tail, legs and head of the mantis shrimp shown in Fig. 3).

Only three novel colours are needed to convey information about the polarization in full. Fewer novel colours are needed in cases where the scene contains only linear, left-hand circular or right-hand circular polarization.

Edge artefacts are removed as a result of the averaging that occurs within each grid cell, facilitating natural images where objects constantly move (the fern in Fig. S2) or the stability of the photopolarimetry is compromised. Apart from reducing edge artefacts, averaging results in smaller and less conspicuous ellipses in grid cells that contain dissimilar Stokes vectors (i.e. the differences in the Stokes parameters between the pixels are large). This effect is caused by the fact that the mean degree of polarization of a set of heterogeneous Stokes vectors approaches zero. Therefore, averaging actually highlights patches that have similar polarization properties, facilitating the detection of biologically relevant patches. This is noticeable, for example, in the lack of any ellipses in the dark and noisy background of the beetle in Fig. S3.

Angular data are presented contextually with directional shapes, making any interpretation intuitive and easy. For example, note how easily one can discern the vertical and horizontal AoLP in the two middle polaroid filters in Fig. 2, whereas the equivalent interpretation in the conventional presentation of photopolarimetric data in Fig. 2 requires checking the legend. Additionally, the directionality of the polarization ellipse brings to light novel angular patterns: note the concentric directional pattern of the elliptically



**Fig. 2. A comparison between the conventional and current presentation of photopolarimetric data of a custom-made polarization standard.** (A) Conventional multi-pane presentation of photopolarimetric data. The left pane shows the relative (linear) intensity ( $I$ ) of the light. To the right is the degree of linear polarization (DoLP) and a colour scale where black is linearly unpolarized light and white is 100% linearly polarized light. Next is the angle of linear polarization (AoLP) (notice the circular colour scale at the top right corner). Last to the right is the degree of circular polarization (DoCP) with a colour scale where red is 100% left-hand circularly polarized light, black is circularly unpolarized light and green is 100% right-hand circularly polarized light. (B) The present study's method applied to the same standard. (Bi) The RGB image of the standard was superimposed by a grey grid and polarization ellipses. Polarization that is primarily linear is colour coded as magenta (the threshold was arbitrarily set to an eccentricity of 0.2), left-hand circular polarization as orange, and right hand circular polarization as turquoise. (Bii) The meaning of the shape, orientation and colour of the ellipses.



**Fig. 3. Photopolarimetric data of the stomatopod *Gonodactylaceus falcatus* in a defensive position.** Because most of the polarization is on the red portions of the head, legs and tail of the animal, the left-hand circularly polarized light is coded with red's complimentary colour, cyan; rare right-hand circularly polarized light is coded with yellow and linearly polarized light is magenta. The red colour in the image was overly saturated to better emphasize the correlation between the red pigmentation and the polarization (compare this figure with the traditional presentation of photopolarimetric data in Fig. S1).

polarized light reflecting from the beetle's sides in Fig. S3. Finally, directionality is only discernible when it is most relevant, i.e. when linear polarization is stronger than circular polarization. Notice how the radiating pattern of ellipses is visible at the periphery of the beetle's body in Fig. S3 while any such pattern is not as discernible in the centre of the beetle's body where the polarization is mostly circular.

By removing the colour scales traditionally needed for DoLP, AoLP and DoCP, we reduce interpretation error associated with trying to scale variables over false-colour gamuts (Rogowitz and Treinish, 1998). One can see a clear example of all of these advantages when comparing our suggested presentation of photopolarimetric data in Fig. 3 with the conventional presentation in Fig. S1. It is difficult to get a sense of the location (on the body of the animal), degree and orientation (AoLP) of the ellipticity in the traditional presentation of photopolarimetric data. Edge artefacts are numerous and make the interpretation of the DoLP pane cumbersome. The current method for displaying photopolarimetric data (Fig. 3) overcomes most of these difficulties.

Although this method is useful, it does suffer from a number of limitations. Polarization details that are smaller than the grid cell are not visible because of the averaging within the cell. Smaller cells may help in this respect, but reducing the cell size beyond a certain point may prevent the reader from seeing the polarization ellipses. A

simple solution may be to include a zoomed-in section of the area of interest. As mentioned before, there is an intrinsic relationship between circular polarization and angular variation: when DoCP is much higher than DoLP, the polarization ellipse becomes a circle. Because of this relationship, it may become difficult to gather information about the AoLP in such cases. It is, however, important to consider that for relatively high DoCP, AoLP may not be so relevant. A similar relationship exists with degree of total polarization: the lower the degree of total polarization, the smaller the polarization ellipse is going to be and the harder it will be to see the shape of the ellipse. The same logic can be applied here as well: polarization characteristics are less important at very low polarizations. Finally, it is important to note that this method does not attempt (and consequently fails) to simultaneously give the reader access to the raw data and present the main trends in the data. Instead, this method concentrates on the latter task – efficiently presenting the reader with the main photopolarimetric trends.

Apart from adjusting the grid cell size, this method can be modified in other ways to better suit each case study. The colours encoding handedness and linearity can be adjusted to increase/decrease contrast depending on the existing colours in the background image (note the usage of different colours in Figs 2 and 3). One could couple the opacity of the polarization ellipse to the degree of total polarization to make it even clearer which part of the image is more polarized. Eccentricity can also be made to inversely control the saturation of the polarization ellipse's colours. This makes sense because handedness information is less relevant at high eccentricities. Instead of the current linear relationship between the ellipse parameters and the polarization state (e.g. ellipse size and degree of total polarization), one could employ a nonlinear relationship. Although the exact mechanism of polarization vision is not fully understood, a logarithmic relationship between the degree of polarization and the size of the polarization ellipse may better emulate a biological visual system (How and Marshall, 2014). One can divide the parameters of the polarization ellipse into a small number of discrete categories to facilitate and accelerate the comparison between available polarization states in the image. Finally, because the polarization data are not conveyed by the colours or intensities of the background image, the colours and intensities of the image of the subject matter may be adjusted to maximally suit the presentation of the data. These adjustments may include enhanced colour saturation (the red colouring of the mantis shrimp in Fig. 3 was saturated to enhance the apparent correlation between red pigmentation and circular polarization), white balance, different brightness responses or ranges, colorblind suited, anaglyph 3D effects, etc.

We suggested an efficient way of displaying the main trends in photopolarimetric data by superimposing a grid of polarization ellipses on the image. This method improves on the more traditional way of displaying photopolarimetric data. Most of these improvements will attract scientists in biology and biomedicine, but may also be relevant in other disciplines.

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#### Competing interests

The authors declare no competing or financial interests.

#### Author contributions

Conceptualization of ideas, Y.L.G. and N.J.M.; methodology, Y.L.G.; software, Y.L.G.; formal analysis, Y.L.G.; resources, Y.L.G. and N.J.M.; writing – original draft, Y.L.G.; writing – reviewing and editing, Y.L.G. and N.J.M.; visualization, Y.L.G.; supervision, N.J.M.; funding acquisition, N.J.M.

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## Supplementary information

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