THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

A General Equilibrium Analysis of the Eaton and Kortum (2002) Trade Model

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A thesis submitted for the degree of Doctor of Philosophy at
The University of Queensland in 2016
School of Economics

Abstract

This thesis examines the national welfare consequences of trade liberalization, in the form of changes to import tariff rates, within the Eaton and Kortum (2002) trade model, a 2-sector Ricardian trade model with a novel stochastic specification of national productivity.

We first provide a thorough characterization of the general equilibria found in both the mobile labor and immobile labor variants of the Eaton and Kortum (2002) trade model, followed by proofs of the existence and uniqueness of these equilibria.

Given the multiple sectors of trade specified in Eaton and Kortum (2002), it is possible for countries to experience trade surpluses in one particular sector of trade, so long as they are balanced by deficits in the other sectors. Welfare consequences of sector specific trade liberalization measures depends on the sectoral trade balance. This thesis provides a characterization of the pattern of national trade balances in the Eaton and Kortum (2002) trade model, and demonstrates that when some regularity conditions are met, countries can be totally ordered by the number of other countries with which they enjoy trade surpluses.

Having established the features of the general equilibrium of the Eaton and Kortum trade model, the thesis next examines how the establishment of preferential trade agreements influences national welfare within the context of the model. We do so by consider the elemental case of a country unilaterally changing the tariff rate imposed on imports from some other country, and mathematically describe the amplification process that propogates, via trade in intermediate goods, the initial price shock resulting from the unilateral tariff change to the rest of the world.

It was found that in the mobile labor variant of the Eaton and Kortum (2002) model, any country increasing import tariff rates would increase prices in every country, causing all countries to substitute away from imports towards domestic suppliers. Furthermore, global free trade agreements would not be stable within this context, as every country has an incentive to unilaterally deviate from such arrangements by imposing import tariffs. It was also found that rules punishing such unilateral deviations by allowing punitive import tariffs to be imposed against the offending country would be ineffective in removing the incentives for deviation.

Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

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Publications during candidature

No publications.

Publications included in this thesis

No publications included.

Contributions by others to the thesis

An extract of Part V of the thesis has been submitted to the Journal of Economic Theory, with Dr Shino Takayama and Dr Juyoung Cheong as co-authors, and is currently under review.

As such, Drs. Takayama and Cheong had significant input into the introduction of this thesis (an extract of which also served as the introduction to the jointly submitted paper) as well as Part V of the thesis - in particular providing the motivation, design and interpretation of the numerical simulations contained in Section 8.

Statement of parts of the thesis submitted to qualify for the award of another degree

None.

Acknowledgements

I would like to thank the University of Queensland for the financial support provided by the University of Queensland International Scholarship.

I would next like to express my deepest gratitude to the following scholars:

Dr. Juyoung Cheong: For showing a deep interest in my area of research, and steering me towards a wealth of literature relevant to my thesis.

Prof. Andrew McLennan: For his incisive comments, which has saved me from many potential blind alleys of inquiry.

Dr. Shino Takayama: For providing the initial idea for the thesis, and the countless hours of discussion that have shaped this thesis towards its final form. I would also like to thank her for her kindness, advice, and the many opportunities she has provided, in expanding my exposure to academic life.

On a personal note, I especially thank my wife **Phoebe Tan** for the years of loving support, as well as for her bemused tolerance of a husband prone to muttering greek letters to himself.

I must also express my deep respect and appreciation for my parents, **Thomas** and **Christina Yeo**. They have never lost faith in me and always provided me with everything I ever needed. I regret the years we have spent apart and hope to find some small way of repaying my debt to them.

Finally, over the course of preparing this thesis, we welcomed my daughter **Penelope** into the world. She has brightened my evenings after each exhausting day and provided me with fresh motivation to complete my thesis. I only hope she would have the good fortune to meet as many good people in her life, as I have had over the course of my studies in the University of Queensland.

Keywords

trade, tariffs, preferential trade agreements, welfare

Australian and New Zealand Standard Research Classifications (ANZSRC)

ANZSRC code: 140104 Microeconomic Theory, 50%

ANZSRC code: 140210 International Economics and International Finance, 50%

Fields of Research (FoR) Classification

FoR code: 1401, Economics Theory, 50%

FoR code: 1402, Applied Economics, 50%

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Part I

Introduction

What are the welfare consequences of preferential trade liberalization, not only for the countries involved, but also for those countries excluded? Further, if trade liberalization affects the welfare of both groups, then through what channels? With the surge in preferential trade agreements (PTAs) in the past two decades, these questions are frequently posed by trade economists and policymakers, and generate considerable public interest.

This thesis aims to examine this issue, through the prism of the Eaton and Kortum (2002) trade model. We do so by first characterizing the general equilibrium of the Eaton and Korum (2002) trade model, and subsequently by performing a comparative statics analysis of tariff barriers.

With respect to the characterization of the general equilibrium in Eaton and Kortum (2002), we first prove that the general equilibrium in both mobile and immobile labor scenarios of the Eaton and Kortum (2002) always exists, and that the mobile labor general equilibrium is always unique. The sufficient conditions for the uniqueness of the general equilibrium in the immobile labor equilibrium is also provided.[1]

One implication of having 2 sectors in the Eaton and Kortum (2002) model is that countries can experience sectoral trade surpluses or deficits, and that welfare changes due to terms-of-trade effects depends crucially on sectoral trade balance. We provide a result that shows that countries can be totally ordered by the number of other countries with which they run trade surpluses, provided that some regularity conditions on bilateral trade costs are met. This provides a characterization of the structure of trade surpluses and deficits in the Eaton and Kortum (2002) trade model, which might be useful in analyzing the welfare effects of trade liberalization measures.

The general equilibrium characterization portion of the thesis is largely technical in nature. In the comparative statics portion of the thesis, we consider the implications of preferential trade agreements in trade models utilizing the Eaton and Kortum (2002) trade framework.

In his pioneering study, Viner (1950) suggests that if trade liberalization is preferential (for example, through the formation of free trade blocs), it could be either welfare improving or welfare deteriorating for member countries; however, preferential trade agreements would unambiguously harm nonmember countries because of trade diversion. Since Viner, the ambiguity of the direction of welfare changes for member countries has lead to an extensive literature on theoretical and empirical studies on member country welfare (e.g Richardson (1993); Krishna (1998); Trefler (2004)). However, the literature has paid relatively less attention to welfare implications for nonmember countries, instead largely focusing on the magnitude of the trade diversion effects in empirical studies (e.g. Clausing (2001); Ghosh and Yamarik (2004); Magee (2008)).

[1] While Eaton and Kortum (2002) was noted for introducing a novel trade model, the paper was primarily focused on providing numerical estimates of welfare implications of preferential trade agreements, as opposed to characterizing the features of the new trade model. Hence, proofs of existence and uniqueness of the Eaton and Kortum (2002) trade model were not provided in the original paper.

There are some exceptions which examined welfare implications of preferential trade agreements on nonmember countries. Using a three-country three-good model, Mundell (1964) shows that when two countries implement a preferential trade agreement by lowering tariffs on imports from each other, the excluded country reduces its price to offset trade diversion, thus worsening its terms-of-trade. Chang and Winters (2002) also use a simple strategic pricing model with differentiated products to show that a PTA may induce a significant reduction in the export prices of nonmember countries to member countries, again worsening the terms-of-trade of non-member countries. Unlike these studies, which investigated the endogenous terms-of-trade changes induced by the PTA as proxies for changes in national welfare,[2] in this thesis we directly study the welfare effects of a PTA, conducting a comparative static analysis of equilibrium welfare with respect to the tariff changes.

Further, Kemp and Wan (1976) studied the implications of customs unions, and showed that when there are tariff concessions by or transfers from member countries, each country whether a member or not would not be made worse off by the formation of the union. Endoh, Hamada and Shimomura (2013) also show that in a two-good three-country world, tariff concessions or transfers are necessary for a Pareto-improving PTA unless one of the member countries of the PTA is an entrepot.

In contrast to these studies, this thesis would show that in the Eaton and Kortum general equilibrium model, nonmember countries can also passively gain from a PTA, even in the absence of tariff concessions or transfers from member countries. Nonmember countries potentially gain through trade of intermediate goods. When one country decreases import tariffs, the prices of imported intermediate goods in that country decreases, bringing down manufacturing costs of all final and intermediate goods in the same country. Much of the lower cost intermediate goods is exported, resulting in lower costs of production everywhere else.[3] Our theoretical analysis demonstrates how trade in intermediate goods amplifies and propagates the effects of local tariff changes. We show then that when even just one country reduces tariffs on imports from another country, every country would find domestically sourced products becomes more relatively more costly than imports, causing every country to substitute towards imports, indicating that PTAs leads surely to trade creation in every country, within the context of the Eaton and Kortum (2002) trade model.

This thesis contributes to two strands of the international trade literature. The first strand follows Eaton and Kortum (2012), and could be described as a well-developed literature that uses the Eaton-Kortum trade framework to analyze the welfare and trade effects of trade liberalization.[4] Recently, Arko-

[2] Another strand of trade literature examines the tariff changes in member countries as being endogenously determined, providing further implications for the terms-of-trade of nonmember countries. For instance, Bond, Syropoulos and Winters (1996) argue that the optimum tariffs for PTA members against the excluded countries increase when trade barriers between the PTA members fall, while Ornelas (2005) shows that a PTA reduces the incentive to lobby for higher external tariffs because of rent destruction in import-competing sectors. Conversely, Gawande, Krishna and Olarreaga (2015) generalized Grossman and Helpman (1994), empirically evaluating the relative importance of three different factors that motivate redistributive government policy, namely, tariff revenues, consumer welfare, and producer profits. They show that developing countries with weak tax systems often weigh tariff revenue more heavily, whereas more developed countries weigh producer interests most heavily, but still assign significance to tariff revenues.

[3] The importance of intermediate goods has been increasing in international trade. For example, Feenstra and Hanson (1996) showed that the share of imported intermediate inputs in the US increased from 5.3% to 11.6% between 1972 and 1990. Hummels, Ishii and Yi (2001) found that vertical specialization accounts for some 21% of trade in some OECD countries and emerging markets, and that it increased by about 30% between 1970 and 1990.

[4] For a comprehensive survey, see Eaton and Kortum (2012). Some examples of extensions of the basic Eaton and Kortum model would be Caliendo and Parro (2015), Parro (2013) and Ramondo and Rodriguez-Clare (2013).

lakis, Costinot and Rodríguez-Clare (2012) show that for a range of Ricardian trade models, changes in welfare resulting from changes in trade patterns can be computed using only the two aggregate statistics, own-trade share and trade elasticity; and demonstrate that these results can be extended to models applying the Eaton and Kortum framework. Alvarez and Lucas (2007) examined a version of the Eaton and Kortum model in which intermediate goods are tradable but final goods are not; and in which every country experiences balanced trade by assumption. Within this framework, they show the sufficient conditions for the existence of a unique equilibrium. Caliendo and Parro (2015) extend the Eaton and Kortum model by introducing inter-sectoral linkages through trade in intermediate goods, and evaluate the North American Free Trade Agreement (NAFTA).

In this thesis, we also use the Eaton and Kortum trade framework to examine welfare consequences of a PTA. This thesis differs from the papers discussed above in several aspects. First, Arkolakis, Costinot and Rodríguez-Clare (2012)'s welfare analysis considered the impacts of external shocks in a world without import tariffs, while our analysis specifically allows for tariffs and studies the motivations of countries with respect to the implementation of import tariffs. We show that while the main result in ibid. still holds in our analysis of a no-tariff revenue world, the introduction of tariff revenues may alter the welfare consequences of a PTA such that own-trade shares and trade elasticity would no longer be sufficient statistics for computing welfare changes triggered by external shocks. Furthermore, our analysis aims to disentangle the underlying mechanisms of welfare changes in trade liberalization within the Eaton and Kortum-framework, while the focus in ibid. was to find common features of welfare predictions across various trade models.

Another important difference is sectoral trade imbalances. Alvarez and Lucas (2007) studied the Eaton and Kortum trade framework with one tradable sector under the assumption of balanced trade. In a single period model, national income must be equal to expenditure. When there is only one tradeable sector in the model, trade must necessarily balance within this sector for each country. However as pointed out by Caliendo and Parro (2015), in general within the Eaton and Kortum-framework with multiple sectors, a country's imports in any particular sector do not necessarily equal exports in the same sector. The two-sector Eaton and Kortum-model provides a simple analytically tractable framework to consider how sectoral trade-imbalances influence the welfare consequences of a PTA, and we showed that trade surpluses and deficits have significant impact on changes to national welfare.

The second strand of the literature that we contribute to is the one of PTAs and network formation games. In the literature of trade theory, PTAs are sometimes modelled as networks, with countries considered as nodes, and reciprocal free trade arrangements forming links between countries. Network formation games are then used to study the stability of PTAs. Recently Furusawa and Konishi (2007) showed that global free trade is potentially a stable network following the definition of Jackson and Wolinsky (1996): a network in which (i) no country has an incentive to cut a link with another; and (ii) for any unlinked pair of countries, at least one of them has no incentive to form a link with the other. Goyal and Joshi (2006) also prove a similar result, such that if countries are symmetric, a complete network in which every pair of countries has a PTA, i.e., global free trade, is consistent with the incentives of individual countries.

The literature considering PTAs as network formation games typically constructs a simple partial equilibrium setting to analyze the stability of PTAs. Departing from this literature, we employ the Eaton and Kortum's general equilibrium framework, and show that a global free trade network is not stable. Starting with the scenario of global free trade, we show that increasing tariffs on imports is welfare improving for any country. In our analysis, changes to national welfare induced by a country increasing import tariff rates originate from three sources: changes in tariff revenues, changes in the patterns of trade, and the terms-of-trade effect. Taking each effect independently, an increase in national tariff revenues increases the country's purchasing power and hence national welfare. A diversion of purchases away from more efficient foreign producers towards domestic producers is welfare decreasing, and an increase in the price of the country's exports relative to its imports is welfare increasing. By imposing tariff barriers on imports, which include intermediate goods, the country forces up production costs and hence, given competitive product markets, the prices of its exports, resulting in an improvement in the terms of trade. We show that in the Eaton and Kortom model, this improvement in the terms-of-trade, combined with the increase in tariff revenues, is sufficient to overcome the negative consequences of reduced trade. This results in a net national welfare gain.

The remainder of the paper is structured as follows. Part II through IV concerns the characterization of the general equilibrium of the Eaton and Kortum trade model. Part II describes the Eaton and Kortum (2002) model, explains the foundations of the model of stochastic production efficiencies underpinning it, and defines the general equilibria in mobile and immobile variants of the model. Part III provides the proofs of existence and uniqueness of the equilibria. In Part IV, the potential for sectoral trade imbalance is discussed and a characterization of the international structure of national trade balance is provided. Part V provides comparative statics analysis of trade liberalization policies in the mobile labor scenario of the Eaton and Kortum model. Part VI concludes.[5]

[5] For interested readers, the Appendix contained in Part VII contains analysis of welfare implications of trade liberalization in the *immobile* labor scenario of the Eaton and Kortum trade model. Given the mathematical complexity (brought on by endogenously determined wages) and a paucity of general results for the immobile labor scenario that are substantially different from that of the mobile labor scenario, this analysis was omitted from the main body of the thesis.

Part II

The Eaton and Kortum (2002) Model of Trade

In Ricardian trade models, different countries are assumed to have different levels of efficiency in the production of different goods. Trade between countries is generated by differences in productivity across countries and goods. If all countries are equally productive in every good, incentives for international trade would not exist.

Ricardian trade models typically rely on the assumptions of perfectly competitive markets, constant returns to scale (CRS) production technologies and a single primary (non-produced) factor of production, labor. The Eaton-Kortum trade model as a model in the Ricardian framework exhibits all these features.

At its core, the Eaton and Kortum trade model is a variant of the Dornbusch, Fischer and Samuelson (1977) trade model. The key innovative feature of the Eaton and Kortum trade model is a set of country specific stochastic functions determining how productivity differs across goods in each country. This along with a constant elasticity of substitution (CES) aggregation function combining the continuum of tradeable goods into a composite final good allows the extension of the 2 country Dornbusch, Fischer and Samuelson (DFS) trade model to accommodate an arbitrary number of countries.[6]

The overall general equilibrium model used in the Eaton and Kortum (2002) needs to be carefully distinguished from the key idea in the paper, which is a partial equilibrium model of trade with the above-mentioned stochastic production functions and CES aggregation function, specifying how prices and patterns of trade are determined in the presence of international trade costs, and heterogeneity of productivity across goods and countries. We shall use the term 'Eaton and Kortum trade framework' to denote price and trade flow determining process.

We begin examining the Eaton and Kortum trade framework by first describing the theoretical foundations of Eaton and Kortum's specification of stochastic production technology in Section 1. In Section 2, the Eaton and Kortum trade framework would be described and characterized, after which in Section 3 the Eaton and Kortum trade framework would be augmented with additional assumptions on

[6]The basic Ricardian model starts with 2 tradeable goods and 2 trading countries. From a historical perspective, extending the basic model to accommodate an arbitrary set of tradeable goods and an arbitrary number of trading countries hadn't been a trivial problem.

Some of the issues complicating the extension of Ricardian trade models to many goods and countries can be found in Chipman (1965). The theoretical issues stems primarily in the complexity of obtaining general results for patterns of specialization of national production when there are a large number of distinct goods.

The Dornbusch, Fischer and Samuelson (1977) trade model overcame some of these problems by specifying a continuum $\Omega = [0, 1]$ of uncountably infinite tradeable goods, simplying the problem of 'comparing' comparative advantage across goods and countries, by describing relative national productive efficiencies as a continuous function of goods in the domain $[0, 1]$. However, it had been difficult to extend the DFS model of trade to analysis beyond 2 countries.

Wilson (1980) provides a good overview of the difficulties encountered, and sufficient conditions on measures of cross-country relative efficiencies of production allowing the extension of the DFS trade model to an arbitary number of countries.

the determinants of national income and costs of production to obtain the 'mobile' and 'immobile' labor variants of Eaton and Kortum (2002) general equilibrium model.

This part of the thesis introduces no new features to the Eaton and Kortum (2002) model, seeking only to provide a comprehensive introduction to the model.

As previously noted, the Eaton and Kortum (2002) paper was largely focused on application: estimating the effects of historical preferential trade agreements based on empirical data. Given this emphasis, formal development and analysis of the model introduced in Eaton and Kortum (2002) were not presented in the original paper.

The following sections provide a formal development of the model starting from the fundamental model assumptions stated or implied in Eaton and Kortum (2002), deriving the functions describing model variables such as prices, patterns of trade and national welfare. We also provide formal definitions of the general equilibria in the Eaton and Kortum (2002) model, which would be key to the proofs of existence and uniqueness of equilibria found in Part III. As a aide to comparative statics analysis found in Part V, we deviate from the common presentation of the Eaton and Kortum (2002) model by restating the model in linear algebraic form. As noted in the text, for some key model propositions, we provide proofs which we believe to be more concise and intuitive than what is found in the literature.

1 Research, Blueprints and Stochastic Productivity

In the context of the Eaton and Kortum trade framework, a blueprint or an idea, b, is a production technique for some good ω to be employed in some country n. The key feature of a blueprint b is the efficiency $z_n(\omega, b)$ with which one unit of inputs could be converted into output when following the production technique specified in the blueprint.

Under the assumption of constant returns-to-scale production, using some blueprint b for the production of good ω in country n yields output $y_n(\omega)$ as a linear function of the quantity of inputs used $k_n(\omega)$ such that

$$y_n(\omega) = z_n(\omega, b) \cdot k_n(\omega).$$

The efficiency of the blueprint $z_n(\omega, b)$ is simply the quantity of output that 1 unit of input yields.

Let $\mathcal{B}_n(\omega, t)$ denote the set of blueprints for producing good ω possessed by country n at some time t. The blueprint $b^*(t) \in B_n(\omega, t)$ actually deployed by country n in production of good ω is the blueprint with greatest efficiency, i.e. $b^*(t) = \arg\max_{b \in B_n(\omega,t)} \{z_n(\omega, b)\}$.

Eaton and Kortum (2001) introduced the key defining feature of the Eaton and Kortum trade framework: that the efficiency $z_n(\omega) := z_n(\omega, b^*(t))$ with which each country n actually produces each good ω is specified as a random variable independently drawn from a Frechet distribution with a country specific scale parameter[7] T_n, interpreted as the level of accumulated production technology in the

[7]A Frechet distributed random variable X with scale parameter s and shape parameter θ would have a cumulative distribution function, $\mathbb{P}(X \leq x) = \exp(-s^\theta x^{-\theta})$. Hence strictly speaking, $T_n^{\frac{1}{\theta}}$ is the scale parameter for the distribution of efficiency $z_n(\omega)$. However, we refer to T_n as the scale parameter for convenience.

country, and a global shape parameter θ which describes that degree of heterogeneity of production efficiencies between various goods.

Even though Ricardian trade models are single period models, for the purpose of modeling the accumulation of blueprints and the quality of the blueprint actually employed - $z_n(\omega, b^*(\tau))$, it is assumed that time is continuous, indexed as $t \in (-\infty, \tau)$ where τ is the final period in which the trade analysis takes place.

Let $R_n(t) \geq 0$ denote the research intensity in country n in period t, and $\bar{a}$ the productivity of research. The time between the arrival of new blueprints for any good ω in country n is an exponentially distributed random variable with arrival rate $\bar{a}R_n(t)$, and each new blueprint is added to the library $B_n(\omega, t)$ with no possibility of being forgotten. The beneficial effects of research intensity is not good specific, in the sense that increases in research efforts in country n increases the rate with which new blueprints are generated for every good.

By the final period τ, the number of ideas accumulated in $B_n(\omega, \tau)$ is the result of a non-homogeneous Poisson process, with expected *number* of accumulated ideas being $\bar{a}T_n(\tau)$, where $T_n(\tau) := \int_{-\infty}^{\tau} R_n(t)dt$ summarizes the history of research intensity in country n.[8] The total *number* of blueprints accumulated by final time τ, is thus Poisson distributed with parameter $\bar{a}T_n(\tau)$.

The *efficiency* of each new blueprint is independently drawn from a Pareto distribution with shape parameter θ and minimum value $\underline{z}$; the probability a blueprint b has efficiency $z_n(\omega, n)$ exceeding some level z is given as $\mathbb{P}(z_n(\omega, b) > z) = (z/\underline{z})^{-\theta}$ for $z > \underline{z}$ and 1 otherwise. Of all blueprints accumulated, the expected proportion with efficiency greater than z would be $(z/\underline{z})^{-\theta}$. A higher level of θ means that the expected efficiency of new blueprints is low.

Let $\mathcal{C}_n(z) := |\{b \in \mathcal{B}_n(\omega) \mid z_n(\omega, b) > z\}|$ denote the number of blueprints accumulated for good ω in country n with efficiency exceeding $z \in (\underline{z}, \infty)$, and notice that $\mathcal{C}_n(\underline{z})$ is the total number of blueprints, regardless of efficiency. It can be easily shown[9] that the number of blueprints accumulated by time τ with efficiency of at least $z \in (\underline{z}, \infty)$ is Poisson distributed with parameter $\bar{a}T_n(\tau)\,(z/\underline{z})^{-\theta}$.

[8] As an aside, to ensure that $T_n(\tau)$ is bounded, assume that $\lim_{t\to-\infty} R_n(t) = 0$. Intuitively, this means that there is no research at the start of history, and every country started with a tabula rasa.

[9] Since the efficiency of each blueprint is independently and identically distributed, the distribution of $\mathcal{C}_n(z)$ conditioned on the total number of blueprints $C_n(\underline{z})$ is binomial if $\mathcal{C}_n(z) < \mathcal{C}_n(\underline{z})$.

$$\mathbb{P}\Big(\mathcal{C}_n(z) = k \,|\, \mathcal{C}_n(\underline{z}) = i\Big) = \begin{cases} 0 & k > \mathcal{C}_n(\underline{z}) \\ \frac{i!}{k!(i-k)!} p^k (1-p)^{i-k} & k \leq \mathcal{C}_n(\underline{z}), \end{cases}$$

where here $p := \mathbb{P}\big(z_n(\omega, b) > z\big)$ is the probability that a blueprint $b \in \mathcal{B}_n(\omega)$ has greater efficiency than z.

With $\mathcal{C}_n(\underline{z}) \sim \text{Pois}(\lambda)$ where $\lambda = \bar{a}T_n(\tau)$, the unconditional probability mass function of $\mathcal{C}_n(z)$ is therefore

$$\begin{aligned}
\mathbb{P}\Big(\mathcal{C}_n(z) = k\Big) &= \sum_{i=k}^{\infty} \mathbb{P}\Big(C_n(\underline{z}) = i\Big)\mathbb{P}\Big(\mathcal{C}_n(z) = k \,|\, \mathcal{C}_n(\underline{z}) = i\Big) \\
&= \sum_{i=k}^{\infty} \left(\frac{\lambda^i e^{-\lambda}}{i!}\right)\left(\frac{i!}{k!\,(i-k)!}\right) p^k (1-p)^{i-k} \\
&= \frac{(\lambda p)^k e^{-\lambda}}{k!} \sum_{i=k}^{\infty} \frac{(\lambda(1-p))^{i-k} e^{-\lambda(1-p)}}{(i-k)!} \\
&= \frac{(\lambda p)^k e^{-\lambda}}{k!}
\end{aligned}$$

In order to consider blueprints with all possible efficiencies in $(0, \infty)$, Eaton and Kortum (2010) normalized $\bar{a}\underline{z}^{\theta} = 1$ and considered the limiting case as the minimum possible efficiency $\underline{z}$ tends towards 0. With these conditions in place, we have $\mathcal{C}_n(z) \sim \text{Pois}\left(T_n(\tau)z^{-\theta}\right)$, with

$$\mathbb{P}\big(C_n(z) = k\big) = \exp\left(T_n(\tau)z^{-\theta}\right)\frac{\left(T_n(\tau)z^{-\theta}\right)^k}{k!}. \tag{1}$$

For any country n and good τ, we can rank the blueprints accumulated in the library $\mathcal{B}_n(\omega, \tau)$ in terms of efficiency, such that $b^{(k)}$ denotes the blueprint with the k^{th} highest efficiency, i.e. $b^{(1)} = \arg\max_{b\in\mathcal{B}_n(\omega)} \{z_n(\omega, b)\}$ and $z_n(\omega, b^{(1)}) \geq z_n(\omega, b^{(2)}) \geq \ldots$. Denote by $z_n^{(k)} := z_n(\omega, b^{(k)})$ the k^{th} highest level of efficiency which could be found. It might be fruitful for future research to derive the probability distribution of $z_n^{(k)}$. Eaton and Kortum (2010) provides a similar derivation focused on production costs rather than production efficiency spanning several pages. The argument below provides a more concise, and hopefully a more intuitive, presentation focused on production efficiency.

For an arbitrary level of efficiency z, the event $\mathcal{C}_n(z) = 0$ implies that no blueprint has an efficiency level greater than z, hence it must be the case that $z^{(1)} \leq z$. On the other hand, should the event $\mathcal{C}_n(z) = 1$ occur, then there must exist some blueprint $b^{(1)}$ with efficiency level $z^{(1)} > z$, while every other blueprint, including the second most efficient blueprint, must have efficiency level less than z. Hence $\mathcal{C}_n(z) = 1$ implies that $z^{(1)} > z \geq z^{(2)}$. By similar reasoning, for any natural number k and positive real number z, the event $\mathcal{C}_n(z) = k$ is equivalent to the event $z^{(k)} > z > z^{(k+1)}$, which in probability terms can be written as $\mathbb{P}\big(\mathcal{C}_n(z) = k\big) = \mathbb{P}\big(z^{(k)} > z > z^{(k+1)}\big)$.

Lemma 1. *Let blueprints in $\mathcal{B}_n(\omega, \tau)$ be indexed by descending order of efficiency, such that $z_n^{(k)} := z_n(\omega, b^{(k)})$ is the efficiency level of the k^{th} most efficient blueprint in producing good ω available at time τ in country n. Then, the distribution of $z_n^{(k)}$ is given as*

$$\mathbb{P}\big(z_n^{(k)} \leq z\big) = \exp\left(T_n(\tau)z^{-\theta}\right)\sum_{i=0}^{k-1}\frac{\left(T_n(\tau)z^{-\theta}\right)^i}{i!} \tag{2}$$

Proof. We shall argue that $\mathbb{P}(z^{(k)} \leq z) = \sum_{i=0}^{k-1}\mathbb{P}(\mathcal{C}_n(z) = i)$.

For any arbitrary event A, let $\{A\}$ denote the set all of possible outcomes in which the event A occurs. Consider the set of outcomes in which the k^{th} ranked efficiency level is less than z, i.e. $\left\{z^{(k)} \leq z\right\}$. It must always be the case for the next higher ranked efficiency level $z^{(k-1)}$, that either $z^{(k-1)} \leq z$ or $z^{(k-1)} > z$ but not both. Hence we can partition the set of events where $z^{(k)} \leq z$ into the a set of events in which $z^{(k)} \leq z$ and $z^{(k-1)} \leq z$, and another set in which $z^{(k)} \leq z < z^{(k-1)}$, such that

$$\left\{z^{(k)} \leq z\right\} = \left\{z^{(k)} \leq z < z^{(k-1)}\right\}\bigcup\left(\left\{z^{(k)} \leq z\right\}\bigcap\left\{z^{(k-1)} \leq z\right\}\right).$$

By definition, $z^{(k)} \leq z^{(k-1)}$, hence $z^{(k)} \leq z$ whenever $z^{(k-1)} \leq z$, such that $\left\{z^{(k)} \leq z\right\} \subseteq \left\{z^{(k-1)} \leq z\right\}$.

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with the last equality since $\sum_{i=k}^{\infty}\frac{(\lambda(1-p))^{i-k}e^{-\lambda(1-p)}}{(i-k)!} = \mathbb{P}(X \geq 0) = 1$ for some random variable $X \sim \text{Pois}\left(\lambda\left(1-p\right)\right)$.

It follows immediately that the number of blueprints with efficiency greater than $z \in (\underline{z}, \infty)$ is also Poisson distributed with parameter $\bar{a}T_n(\tau)\left(\frac{z}{\underline{z}}\right)^{-\theta}$.

This immediately implies that $\left\{z^{(k)} \leq z\right\} \bigcap \left\{z^{(k-1)} \leq z\right\} = \left\{z^{(k-1)} \leq z\right\}$.

We can therefore write the probability that $z^{(k)} \leq z$ as

$$\begin{aligned}
\mathbb{P}\big(z^{(k)} \leq z\big) &= \mathbb{P}\big(z^{(k)} \leq z < z^{(k-1)}\big) + \mathbb{P}\big(z^{(k-1)} < z\big) \\
&= \mathbb{P}\left(\mathcal{C}_n(z) = k-1\right) + \mathbb{P}\big(z^{(k-1)} < z\big) \\
&= \sum_{i=0}^{k-1} \mathbb{P}\left(\mathcal{C}_n(z) = i-1\right). \ (*)
\end{aligned}$$

We complete the proof by substituting the probability mass function for $\mathcal{C}_n(z)$ provided by equation 1 into equation $(*)$ above. $\square$

The distribution of higher indexed (lower efficiency) blueprints might be useful, depending on the structure of the market for good ω and how blueprints are distributed between different firms. For example, if Bertrand competition characterizes the market for good ω and each firm is exogeneously endowed with only one unique blueprint, the production costs and hence the efficiency level of higher indexed blueprints would be relevant in the determination of the resulting market outcome.

However, in the Eaton and Kortum trade framework, it is implicitly assumed that in country n at time τ, every firm has access to all blueprints available in the library $\mathcal{B}_n(\omega,\tau)$. Perfect competition in product markets ensures that every firm would chose to employ the most efficient blueprint $b_n^{(1)}$ hence the efficiency level of the method actually used to produce good ω at time τ in country n is $z_n(\omega,\tau) = z_n^{(1)}$.

Using Lemma 1, we obtain the result that production efficiency $z_n(\omega,\tau)$ for any individual good ω is a Frechet distributed random variable with a country specific scale parameter $T_n(\tau)$ capturing the history of research in country n, and a global shape parameter θ which captures the variability of efficiency of blueprints.

We now turn to stating the Eaton-Kortum trade framework in full.

2 The Eaton and Kortum Trade Framework

Let $\mathcal{N} := \{1, 2, ..., N\}$ denote the set of N trading countries in the global economy. As with DFS trade models, there is the continuum of uncountably infinite tradeable goods indexed $\omega \in \Omega := [0, 1]$. In addition, the Eaton-Kortum trade model makes the following assumptions common to Ricardian trade models:

1. Labor is the only primary (non-produced) input into production. Labor is immobile between countries but are perfectly mobile between production of different goods within countries. Every country is endowed with a fixed pool of labor force L_n. Each household supplies 1 unit of labor and a country's labor force L_n could also be interpreted as the country's population.

2. Perfectly competitive product and labor markets.

3. Every country has the ability to produce any tradeable good in Ω with constant returns-to-scale production technologies, such that for $k_n(\omega)$ units of a bundle of inputs into the production of good ω, producers in country n would be able to produce

$$y_n(\omega) = z_n(\omega) \cdot k_n(\omega) \tag{3}$$

units of good ω, where $z_n(\omega)$ is a exogeneously determined level of productive efficiency. Let c_n refer to the cost of one unit of production inputs in country n.[10] Perfect competition in product markets ensures that the factory-gate price $p_{nn}(\omega)$ of good ω equal marginal costs of production, with

$$p_{nn}(\omega) = \frac{c_n}{z_n(\omega)}.$$

4. Costs of trading between countries follows the Samuelson Iceberg assumption: delivering 1 unit of some good ω from country i to country n requires the production of $\bar{d}_{ni} \geq 1$ units in country i, with the excess $\bar{d}_{ni} - 1$ dissipated in the act of transportation from i to n. The unit price of good ω produced in country i and delivered to country n unencumbered by import tariffs would hence be

$$\bar{p}_{ni}(\omega) = \bar{d}_{ni} p_{ii}(\omega). \tag{4}$$

If an ad-valorem tariff rate t_{ni} were imposed by country n on imports from country i, the final country n price inclusive of import tariffs would be

$$p_{ni}(\omega) = d_{ni} p_{ii}(\omega) = \frac{d_{ni} c_i}{z_i(\omega)}, \tag{5}$$

where $d_{ni} = (1 + t_{ni})\,\bar{d}_{ni}$ could be interpreted as the total costs of trade from country i to country n or the 'economic' distance from country i and n, with $\bar{d}_{ni}$ interpreted as the 'geographical' distance associated with factors like physical distance, cultural affinity etc, and $(1 + t_{ni})$ being the tariff barrier to trade.

It is assumed that the triangular inequality holds for trade costs, such that for all $n, i, j \in \mathcal{N}$, it is always the case that $d_{ni} d_{ij} \geq d_{nj}$. This assumption ensures that it is never cheaper to route trade through a third country.

In the Eaton and Kortum trade model, tradeable goods in Ω are valued only in the form of a final composite good, aggregated with a constant elasticity of substitution (CES) function

$$G_n = \left[\int_{\omega \in \Omega} q_n(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{1-\sigma}}, \tag{6}$$

where $q_n(\omega)$ is quantity of an individual tradeable good $\omega \in \Omega$ used in the production of G_n units of

[10] The assumption here is that the production of every manufacturing good Ω requires the same input bundle.. As would be later discussed, with the specification of the Eaton and Kortum trade model examined in this thesis, the production function for each good $\omega \in \Omega$ is Cobb-Douglas, with Total Factor Productivity $z_n(\omega)$, taking as inputs labor and intermediate goods with β and $(1 - \beta)$ being the labor and intermediate good output elasticities (equivalently cost shares). For a given wage rate and intermediate good price in each country, the production of every good would require labor and intermediate goods in the same proportions, hence the assumption of a uniform production input bundle and national input cost c_n.

the final composite good.

Buyers in each country wish to minimize the cost of the final composite good. For each good $\omega \in \Omega$, buyers in country n are faced with potential suppliers from every country $i \in \mathcal{N}$ each offering a price $p_{ni}(\omega)$. Cost minimizing buyers would choose the lowest priced supplier, and the realized price of good[11] ω in country n would be

$$p_n(\omega) := \min_{i \in \mathcal{N}} p_{ni}(\omega).$$

Taking as given $\{p_n(\omega)\}_{\omega \in \Omega}$, the set of country n realized prices, solving the optimal demand problem for the composite good given expenditure X_n yields

$$G_n(q^*) = \frac{X_n}{p_n}$$

where $q^* := \{q_n^*(\omega)\}_{\omega \in [0,1]}$ is the optimum basket of tradeable goods; $q_n^*(\omega_0)$ the optimal demand for for each good $\omega_0 \in \Omega$ given by

$$q_n^*(\omega_0) = \frac{p_n(\omega_0)^{-\sigma}}{p_n^{1-\sigma}} X_n; \tag{7}$$

and

$$p_n := \left[\int_0^1 p_n(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \tag{8}$$

is the CES price index which is interpreted as the unit cost of the composite good.

The discussion up to this point has been of a conventional Ricardian trade model with a CES demand function. The main contribution of Eaton and Kortum in the extending the DFS trade model is the parsimonious specification of heterogeneous production efficiencies $z_n(\omega)$ across countries and goods. Two parameters, a country specific measure of technological prowess T_n, and a global measure of variability of manufacturing efficiencies θ, fully specifies the differences in national productivities that gives rise to international trade.

For each country $n \in \mathcal{N}$ and each good $\omega \in \Omega$, the level of efficiency $z_n(\omega)$ with which country n produces good ω, is an independently distributed random variable with a Frechet distribution. The probability distribution of $z_n(\omega)$, is given by the cumulative distribution function

$$\mathbb{P}(z_n(\omega) \leq z) = \exp\left(-T_n z^{-\theta}\right). \tag{9}$$

As earlier discussed in Section 1, the parameter T_n could be interpreted as the level of accumulated technology in country n. A higher level of national technology T_n would result in higher expected level of efficiency with which goods could be produced in country n.

The parameter θ is defined as a global parameter that measures the effectiveness of research effort. High values of θ implies that research is ineffective, in the sense that high efficiency blueprints arrive at a low rate and are relatively uncommon. Hence, θ could be interpreted as a measure of the dispersion of productive efficiency with greater values of θ associated lower likelihood of goods being produced

[11] With the stochastic nature of prices p_{ni} (as discussed further on in the text), the term 'realized price' might be *mistakenly* taken to mean the observed value of a randomly distributed price. In the context of this section, 'realized price' of good ω, $p_n(\omega)$ should instead be simply understood as the price that buyers in country n actually pay for good ω.

with very high efficiency. High values of θ is associated with less weight in the upper tail of the probability distribution of $z_n(\omega)$ for all countries and goods. Some authors interpret T_n as the level of absolute advantage for country n and θ as the level to which the effects of comparative advantage operates in the global market place.

From an analytical perspective, it is more intuitive to consider the random variable $z_n(\omega)$ in terms of an exponentially distributed random variable $z_n(\omega)^{-\theta} \sim Exp(T_n)$.[12] Since production efficiencies (or equivalently total factor productivity) can be expressed as exponentially distributed random variables, and production functions are constant returns to scale, it is unsurprising that tradeable goods prices can also be expressed as exponentially distributed random variables.

Proposition 1. *Taking exporting country i's national production inputs costs c_i as given,*

1. *$p_{ni}(\omega)$ the price of a good ω produced in country i when sold in country n, can be expressed as an exponentially distributed random variable, such that $p_{ni}(\omega)^\theta \sim Exp(\phi_{ni})$ where $\phi_{ni} := T_i c_i^{-\theta} d_{ni}^{-\theta}$.*
2. *$p_n(\omega) := \min_{i\in\mathcal{N}} p_{ni}(\omega)$ the realized price of good ω in country n, can also be expressed as an exponentially distributed random variable, such that $p_n(\omega)^\theta \sim Exp(\Phi_n)$ where $\Phi_n := \sum_{i\in\mathcal{N}} \phi_{ni}$.*

Proof. (Item 1) From equation 5 and $z_n(\omega)^{-\theta} \sim Exp(T_n)$, we have

$$\begin{aligned} \mathbb{P}(p_{ni}^\theta \leq p) &= \mathbb{P}\Big(\Big(\frac{c_i d_{ni}}{z_i(\omega)}\Big)^\theta \leq p\Big) \\ &= \mathbb{P}\Big(z_i(\omega)^{-\theta} \leq p\,(c_i d_{ni})^{-\theta}\Big) \\ &= 1 - \exp\Big(-T_i\,(c_i d_{ni})^{-\theta}\,p\Big). \end{aligned}$$

(Item 2) By the definition of $p_n(\omega) = \min_{i\in\mathcal{N}} p_{ni}(\omega)$, it is the case that $p_n(\omega) > p$ for some $p > 0$ only if $p_{ni}(\omega) > p$ for all $i \in \mathcal{N}$, so we can write

$$\begin{aligned} \mathbb{P}\Big(p_n(\omega)^\theta \leq p\Big) &= 1 - \mathbb{P}\Big(p_n(\omega)^\theta > p\Big) \\ &= 1 - \mathbb{P}\Big(\bigcap_{i\in\mathcal{N}} p_{ni}(\omega)^\theta > p\Big). \quad (*) \end{aligned}$$

Since $\{z_i(\omega)\}_{i\in N}$ are independently distributed random variables, the set of prices $\{p_{ni}(\omega)^\theta\}_{i\in\mathcal{N}}$ is

[12] To see this, simple note that

$$\begin{aligned} \mathbb{P}(z_n(\omega)^{-\theta} \leq z) &= \mathbb{P}(z_n(\omega) > z^{-\frac{1}{\theta}}) \\ &= 1 - \mathbb{P}(z_n(\omega) \leq z^{-\frac{1}{\theta}}) \\ &= 1 - \exp(-T_n z). \end{aligned}$$

also a set of independent random variables, in which case we must have

$$\begin{aligned}\mathbb{P}\Big(\bigcap_{i\in\mathcal{N}} p_{ni}(\omega)^\theta > p\Big) &= \prod_{i\in\mathcal{N}} \mathbb{P}(p_{ni}(\omega)^\theta > p)\\ &= \prod_{i\in\mathcal{N}} \exp\Big(-\phi_{ni}\Big)\\ &= \exp\Big(-\sum_{i\in\mathcal{N}}\phi_{ni}\Big), \quad (**)\end{aligned}$$

with the second equality from Item 1 of the Proposition. Substituting $(**)$ into $(*)$ gives the required result. □

For every pair of countries $n, i \in \mathcal{N}$, let $\Omega_{ni} \subseteq \Omega$ denote the set of goods country n sources from country i, and $\pi_{ni} := \frac{|\Omega_{ni}|}{|\Omega|}$ denote the proportion of tradeable goods in Ω for which country i is the lowest cost provider to country n. Equivalently, π_{ni} denotes the proportion of tradeable goods that country n purchases from country i.

Given stochastic national productive efficiencies and tradeable good prices $p_{ni}(\omega)$, it is not possible to *exactly* identify *which* particular goods would be in Ω_{ni}. However the stochastic nature of prices in the Eaton and Kortum trade model is not a drawback, but in fact provides the model with desirable mathematical qualities.

For one, it is easy to provide a measure of the range of goods that one country sources from another. As there is an infinite number of goods in the continuum $\Omega = [0, 1]$, Borel's Law of Large Numbers implies that the *probability* that country i is the lowest cost provider of a specific good to country n, is also π_{ni}, the *proportion* of goods for which country i is the lowest cost provider to country n.

Proposition 2. *The fraction of tradeable goods in Ω that country n sources from country i is given as*

$$\pi_{ni} = \frac{\phi_{ni}}{\Phi_n}.$$

Proof. It is a well known result that for two exponentially distributed random variables, $a \sim Exp(A)$ and $b \sim Exp(B)$, we have $\mathbb{P}(a = \min\{a, b\}) = \frac{A}{A+B}$.

Fix some arbitrary $\omega \in \Omega$. In the context of this proof, let $a = p_{ni}(\omega)^\theta \sim Exp(\phi_{ni})$ and $b = \min_{j\in\mathcal{N}\setminus i} p_{nj}(\omega)^\theta$ where $\mathcal{N}\setminus i$ refers to set of countries in $\mathcal{N}$ excluding country i. From Item 2 of Proposition 1, we have $b \sim Exp(\Phi_n - \phi_{ni})$. The probability that country i is the lowest priced supplier to country n is then

$$\begin{aligned}\mathbb{P}\big(p_{ni}(\omega) = \min_{j\in\mathcal{N}} p_{nj}(\omega)\big) &= \mathbb{P}\big(p_{ni}(\omega)^\theta = \min_{j\in\mathcal{N}} p_{nj}(\omega)^\theta\big)\\ &= \mathbb{P}\Big(p_{ni}(\omega)^\theta = \Big\{p_{ni}(\omega), \min_{j\in\mathcal{N}\setminus i} p_{nj}(\omega)^\theta\Big\}\Big)\\ &= \mathbb{P}(a = \min\{a, b\})\\ &= \frac{\phi_{ni}}{\Phi_n}.\end{aligned}$$

Pick some arbitrary positive integer M. Let $X_1, ..., X_M$ be a number of Bernoulli trials in which goods $\omega_1, ..., \omega_M$ are randomly picked without replacement from Ω, and tested to see if $\omega_m \in \Omega_{ni}$, for $m = 1, ..., M$. Let $X_m = 1$ if $\omega_m \in \Omega_{ni}$ and 0 otherwise, and $C_M(\Omega_{ni}) = \sum_{m=1}^{M} X_m$ be the count of successes after M attempts, such that $\frac{C_M(\Omega_{ni})}{M}$ is the proportion of $\omega_1, ..., \omega_M$ found to be in Ω_{ni}. The Borel's law of large numbers states that $\lim_{M\to\infty} \frac{C_M(\Omega_{ni})}{M} = \mathbb{P}(\omega_m \in \Omega_{ni})$. Suppose every good $\omega \in \Omega$ were so tested, then $M \to \infty$ as there are an infinite number of goods in Ω, and $\frac{C_M(\Omega_{ni})}{M}$ must thus be the proportion of $\omega \in \Omega$ that are in Ω_{ni} such that $\frac{C_M(\Omega_{ni})}{M} = \frac{|\Omega_{ni}|}{|\Omega|} = \pi_{ni}$. Then since $\mathbb{P}(\omega \in \Omega_{ni}) = \mathbb{P}\big(p_{ni}(\omega) = \min_{j\in\mathcal{N}} p_{nj}(\omega)\big)$, we have the desired result, $\pi_{ni} = \frac{\phi_{ni}}{\Phi_n}$. □

Another mathematically convenient feature of the Eaton and Kortum framework is that the definition of the realized prices as the minimum of independent exponentially distributed random variables also implies that the distribution of good prices $p_n(\omega)$ in a country does not depend on where the goods are sourced from.

Proposition 3. *The distribution of the realized price of good ω in country n, $p_n(\omega)$, does not depend on the source of ω, such that for all $n, i \in \mathcal{N}$ and $\omega \in \Omega$,*

$$\mathbb{P}\big(p_n^\theta(\omega) \leq p \,|\, \omega \in \Omega_{ni}\big) = \mathbb{P}\big(p_n^\theta(\omega) \leq p\big).$$

Proof. Since $\min_{j\neq i} \left\{p_{nj}(\omega)^\theta\right\} \sim Exp(\Phi_n - \phi_{ni})$ and $p_{ni}(\omega)^\theta \sim Exp(\phi_{ni})$ are independent random variables, and $p_n(\omega)^\theta = p_{ni}(\omega)^\theta$ if and only if $p_{ni}(\omega)^\theta \leq \min_{j\neq i}^{N} \left\{p_{nj}(\omega)^\theta\right\}$, we can write the joint probability of the events $p_n^\theta \leq p$ and $p_n(\omega)^\theta = p_{ni}(\omega)^\theta$ (which is equivalent to $\omega \in \Omega_{ni}$) as

$$\begin{aligned}
\mathbb{P}\Big(\big[p_n^\theta \leq p\big] \;\&\; \big[p_{ni}(\omega)^\theta = p_n^\theta\big] \Big) &= \mathbb{P}\Big(\big[p_{ni}^\theta \leq p\big] \;\&\; \Big[p_{ni}(\omega)^\theta \leq \min_{j\neq i} \left\{p_{nj}^\theta\right\}\Big] \Big) \\
&= \int_0^p \mathbb{P}\Big(\min_{j\neq i} \left\{p_{nj}^\theta\right\} > z\Big)\, d\mathbb{P}(p_{ni}^\theta \leq z) \\
&= \int_0^p \exp\Big(-\left(\Phi_n - \phi_{ni}\right) z\Big) \cdot \phi_{ni} \exp(-\phi_{ni} z) dz \\
&= \frac{\phi_{ni}}{\Phi_n} \int_0^p \Phi_n \exp\left(-\Phi_n z\right) dz \\
&= \pi_{ni} \mathbb{P}\big(p_n^\theta \leq p\big).
\end{aligned}$$

Recalling that $\pi_{ni} = \mathbb{P}(p_n^\theta = p_{ni}^\theta)$, we immediately notice that

$$\begin{aligned}
\mathbb{P}(p_n^\theta \leq p \,|\, p_n^\theta = p_{ni}^\theta) &= \frac{\mathbb{P}\Big(\big[p_n^\theta \leq p\big] \;\&\; \big[p_{ni}(\omega)^\theta = p_n^\theta\big] \Big)}{\mathbb{P}(p_n^\theta = p_{ni}^\theta)} \\
&= \mathbb{P}(p_n^\theta \leq p),
\end{aligned}$$

as required. □

Finally, the specification of realized prices as continuously distributed random variables implies that it is always possible to sort goods in Ω such that prices of tradeable goods could be expressed as a continuous function of goods in $\Omega = [0, 1]$. As shall be shown, this gives a tractable specification of both the national CES price index, as well as how a country allocates import expenditures among different suppliers.

We first turn our attention to specifying realized prices $p_n(\omega)$ as a continuous function of goods $\omega \in \Omega$ and solving for the CES price index p_n.

Proposition 4. *It is possible to sort goods $\omega \in \Omega = [0,1]$ such that realized prices in each country $n \in \mathcal{N}$ is written as*

$$p_n(\omega) = \left(-\frac{\ln(1-\omega)}{\Phi_n}\right)^{\frac{1}{\theta}}, \tag{10}$$

a continuous and strictly increasing function of ω.

Then the country n price of the composite good is given as

$$p_n = \gamma \Phi_n^{-\frac{1}{\theta}}, \tag{11}$$

where $\gamma := \left[\Gamma(1 + \frac{1-\sigma}{\theta})\right]^{\frac{1}{1-\sigma}}$ and Γ is the gamma function.

Proof. For any realization of prices $\{p_n(\omega)\}_{n\in\mathcal{N}}$, sort the set of goods $\omega \in \Omega = [0,1]$ in ascending order of $p_n(\omega)$. Fix some arbitrary good $\omega_0 \in [0,1]$. Since goods in Ω have been sorted by prices, we would have $p_n(\omega) \le p_n(\omega_0)$ for all $\omega \in [0, \omega_0]$. The proportion of goods with lower prices than ω_0 is $\frac{|[0,\omega_0]|}{|\Omega|} = \omega_0$ as $\Omega = [0,1]$ implies that $|\Omega| = 1$.

Let $F : \mathbb{R}_+ \to [0,1]$ denote the cumulative distribution function of $p_n(\omega)$, such that $F\big(p_n(\omega_0)\big) := \mathbb{P}(p_n(\omega) \le p_n(\omega_0))$. By Proposition 1, it is the case that $p_n(\omega)^\theta \sim Exp(\Phi_n)$, and bearing in mind that $\theta > 0$, we have

$$\begin{aligned} F\big(p(\omega_0)\big) &= \mathbb{P}\big(p_n(\omega) \le p_n(\omega_0)\big) \\ &= \mathbb{P}\big(p_n(\omega)^\theta \le p_n(\omega_0)^\theta\big) \\ &= 1 - \exp(-\Phi p(\omega_0)^\theta). \end{aligned}$$

By Borel's law of large numbers, the proportion of goods ω with $p_n(\omega) \le p_n(\omega_0)$ equals the probability $F(p_n(\omega_0)$. We can therefore write

$$F\big(p_n(\omega_0)\big) = \omega_0.$$

By the definition of function F as a cumulative probability distribution function, F is a continuous one-to-one function mapping from the set of realized prices to the set of goods $[0,1]$. The inverse function $F^{-1} : [0,1] \to \mathbb{R}_+$ therefore exists. Hence, for any good $\omega_0 \in [0,1]$, the function mapping from the set of goods into prices could be written as

$$p_n(\omega_0) = F^{-1}(\omega_0) = \left(-\frac{\ln(1-\omega_0)}{\Phi_n}\right)^{\frac{1}{\theta}}. \tag{12}$$

This gives the first part of the proposition.

To show the second part, substitute equation 10 into the CES price index in equation 8 giving

$$
\begin{aligned}
p_n &= \left[\int_0^1 p_n(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \Phi_n^{-\frac{1}{\theta}} \left[\int_0^1 \left(-\ln(1-\omega)\right)^{\frac{1-\sigma}{\theta}} d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \Phi_n^{-\frac{1}{\theta}} \left[\int_0^\infty y^{\frac{1-\sigma}{\theta}} e^{-y} dy\right]^{\frac{1}{1-\sigma}} \\
&= \gamma \Phi_n^{-\frac{1}{\theta}},
\end{aligned}
\tag{13}
$$

with the third equality due to a change of variable $y = -\ln(1-\omega)$, and $\gamma := \left[\Gamma(1 + \frac{1-\sigma}{\theta})\right]^{\frac{1}{1-\sigma}}$ where Γ is the gamma function. □

Having established national prices of tradeable goods, we shall now focus on patterns of trade: how a country allocates its expenditure on goods across imports from other countries. Proposition 2 established the range of goods a country n might optimally choose to source from some country i. The next proposition tells us that π_{ni} is not only the proportion of the *types* of goods that country n sources from country i, but is also the fraction of country n's *expenditure* on tradeable goods spent on goods produced in country i.

Proposition 5. *Let $X_{ni} = \int_{\omega \in \Omega_{ni}} p_n(\omega) q_n^*(\omega)\, d\omega$ denote the country n's expenditure on goods produced in country i, and recall that X_n is country n's total expenditure on tradeable goods, such that $\frac{X_{ni}}{X_n}$ is interpreted as the proportion of country n's expenditure on tradeable goods spent on goods produced in country i. It is the case that*

$$\frac{X_{ni}}{X_n} = \pi_{ni}$$

where $\pi_{ni} = \frac{\phi_{ni}}{\Phi_n}$ as defined in Proposition 2.

Proof. Fix some arbitrary country $n \in \mathcal{N}$. The optimal expenditure by country n on goods sourced from any country $i \in \mathcal{N}$ is found by summing up on optimal expenditures $p_n(\omega) q_n^*(\omega)$ for all goods $\omega \in \Omega_{ni}$. We write this as

$$X_{ni} = \int_{\omega \in \Omega_{ni}} p_n(\omega) q_n^*(\omega)\, d\omega. \quad (*)$$

Given national prices $p_n(\omega)$ for all goods $\omega \in \Omega$ and nominal national expenditure X_n, equation 7 gives the optimal quantity demanded $q_n^*(\omega_0)$ for any good $\omega_0 \in \Omega$. Substituting equation 7 into equation $(*)$ and dividing throughout by X_n gives the proportion of country n's expenditures spent on imports from country i as

$$\frac{X_{ni}}{X_n} = \frac{\int_{\omega_0 \in \Omega_{ni}} p_n(\omega_0)^{1-\sigma} d\omega_0}{p_n^{1-\sigma}}. \quad (**)$$

We now wish to solve the integral $\int_{\omega_0 \in \Omega_{ni}} p_n(\omega_0)^{1-\sigma} d\omega_0$. In order to do so, we shall derive $p_n(\omega_0)$ as a continuous function of $\omega_0 \in \Omega_{ni}$.

Sort the set of goods $\omega \in \Omega = [0,1]$ by countries of origin such that $[0, \pi_{ni}] = \Omega_{ni}$ is the set of goods country n sources from country i. (Recall that $\pi_{ni} = \frac{|\Omega_{ni}|}{|\Omega|}$ is the proportion of goods in $\Omega = [0,1]$ that are in Ω_{ni}.)

Fix some arbitrary good $\omega_0 \in \Omega_{ni} = [0, \pi_{ni}]$. Sorting the set $\Omega_{ni} = [0, \pi_{ni}]$ in ascending order by price, such that we have $[0, \omega_0] \subset [0, \pi_{ni}]$ as the set of goods imported by n from i with prices less or equal to that $p_n(\omega_0)$. The proportion of goods in $[0, \pi_{ni}]$ with prices less than $p_n(\omega_0)$ would obviously be $\frac{\omega_0}{\pi_{ni}}$.

By Proposition 3, the distribution of the price $p_n(\omega)$ of any good is independent of its source, hence for any $\omega \in \Omega_{ni} = [0, \pi_{ni}]$, we have $p_n(\omega)^\theta \sim Exp(\Phi_n)$ and it immediately follows that

$$\mathbb{P}(p_n(\omega) \leq p_n(\omega_0)) = 1 - \exp(-\Phi_n p_n(\omega_0)^\theta).$$

Since there are an infinite number of goods in $\Omega_{ni} = [0, \pi_{ni}]$, the Borel's law of large number[13] shows that the probability that some good in Ω_{ni} has a lower price than $p_n(\omega_0)$ is also the proportion of goods in $[0, \pi_{ni}]$ that has prices lower than $\omega_0 \in [0, \pi_{ni}]$. This gives the equality

$$1 - \exp(-\Phi_n p_n(\omega_0)^\theta) = \frac{\omega_0}{\pi_{ni}}.$$

By similar argument as in the proof of Proposition 4, we then can write the prices of any good $\omega_0 \in \Omega_{ni} = [0, \pi_{ni}]$ as a continuous function of ω_0, such that

$$p_n^i(\omega_0) = \left(-\frac{\ln(1 - \frac{\omega_0}{\pi_{ni}})}{\Phi_n}\right)^{\frac{1}{\theta}}. \quad (***)$$

Notice the use of the superscript i to distinguish $p_n^i(\omega)$, the function giving prices of goods conditioned on the goods being sourced from country i, from $p_n(\omega)$ which is the 'general' unconditional price function of tradeable goods in Ω.

Having sorted Ω such that $\Omega_{ni} = [0, \pi_{ni}]$, substituting $(***)$ into $(**)$ gives

$$\begin{aligned} \int_{\omega \in \Omega_{ni}} p_n(\omega_0)^{1-\sigma} d\omega_0 &= \int_0^{\pi_{ni}} \left(-\frac{\ln(1 - \frac{\omega_0}{\pi_{ni}})}{\Phi_n}\right)^{\frac{1-\sigma}{\theta}} d\omega_0 \\ &= \pi_{ni} \int_0^1 \left(-\frac{\ln(1-\omega)}{\Phi_n}\right)^{\frac{1-\sigma}{\theta}} d\omega, \end{aligned} \quad (14)$$

with the second equality due to a change of variable $\omega = \frac{\omega_0}{\pi_{ni}}$. In the proof of Proposition 4, equation

[13]The argument here in employing Borel's law of large numbers, that the probability $\mathbb{P}(p_n(\omega) \leq p_n(\omega_0))$ also gives the proportion of goods ω with prices lower than that of ω_0, that is identical to the argument employed in the second part of Proposition 2.

13 showed that $\int_0^1 \left(-\frac{\ln(1-\omega)}{\Phi_n}\right)^{\frac{1-\sigma}{\theta}} d\omega = p_n^{1-\sigma}$.Hence we obtain the desired result

$$\frac{X_{ni}}{X_n} = \pi_{ni}\frac{\int_0^1 \left(-\frac{\ln(1-\omega)}{\Phi_n}\right)^{\frac{1}{\theta}} d\omega}{p_n^{1-\sigma}} = \pi_{ni}$$

and π_{ni} is not only the proportion of goods in Ω that are in Ω_{ni}, but also the proportion of country n's expenditures on tradeable goods is spent on country i sourced goods.[14] □

We would conclude this section with a final proposition, that if the triangular inequality for trade costs holds, a country exports a good only if the country produces the good for domestic use. Intuitively, a country is sufficiently efficient in producing a good ω to be competitive in foreign markets, only if it is competitive in its own domestic market for the good.

Proposition 6. *For any good $\omega \in \Omega$, and country $n \in \mathcal{N}$,*

$$\Omega_{in} \subseteq \Omega_{nn},$$

for all $i \in \mathcal{N}$. Then it immediately follows that

$$\pi_{in} \leq \pi_{nn},$$

and that the no-trade-arbitrage condition holds such that

$$p_i < p_n d_{in}$$

for all $i, n \in \mathcal{N}$.

Proof. Take some good $\omega^* \in \Omega_{in}$ for some countries $i, n \in \mathcal{N}$. Then it must be the case that $p_{in}(\omega^*) \leq p_{ij}(\omega^*)$ for all $j \in \mathcal{N}$, in turn implying that

$$\tfrac{z_n(\omega^*)}{c_n} / \tfrac{z_j(\omega^*)}{c_j} \geq \frac{d_{in}}{d_{ij}}, \quad (*)$$

which can be economically interpreted as meaning that country n is most competitive in supplying good ω^* to country i only if country n's comparative advantage in the production of good ω^* relative to any other country is sufficiently great as to overcome the relative trade costs of supplying into country i's product market.

The triangular inequality assumption in trade costs ensures that $d_{in}d_{nj} \geq d_{ij}$ or equivalently, $\frac{d_{in}}{d_{ij}} \leq d_{nj}$. Combining this with equation $(*)$ gives

$$\tfrac{z_n(\omega)}{c_n} / \tfrac{z_j(\omega)}{c_j} \geq d_{nj},\ \forall j \in \mathcal{N},$$

[14] The basic idea for the proof is that the independence of distribution of prices of a good from the source of the good implies that as a function, p_n^i is identical to p_n, just compressed from the domain $[0,1]$ into the subset $[0, \pi_{ni}]$. The area under the curve for $\left(p_n^i\right)^{1-\sigma}$ in the domain $[0, \pi_{ni}]$ then is simply the area under the curve for $p_n^{1-\sigma}$ over $[0,1]$ scaled down by π_{ni}.

or equivalently, $p_{nn}(\omega) \leq p_{nj}(\omega)$ for all $j \in \mathcal{N}$ and $\omega^* \in \Omega_{nn}$.

Since we have shown that $\Omega_{in} \subseteq \Omega_{nn}$, it must be the case that $\pi_{in} = |\Omega_{in}| \leq |\Omega_{nn}| = \pi_{nn}$.

To show that the no-trade-arbitrage condition holds, simply note that since $\pi_{jn} \leq \pi_{nn}$, $\pi_{jn} = \frac{\phi_{jn}}{\Phi_j}$, $\phi_{jn} = T_j c_j^{-\theta} d_{jn}^{-\theta}$, and $\Phi_j = \gamma^{-\theta} p_j^{-\theta}$ for all $j \in \mathcal{N}$, we have

$$\frac{\pi_{in}}{\pi_{nn}} = \left(\frac{d_{in} p_n}{p_i}\right)^{-\theta} \leq 1$$

or equivalently $d_{in} p_n \geq p_i$. □

The no-trade-arbitrage statement of Proposition 6, that $p_n d_{in} \geq p_i$ for all $n, i \in \mathcal{N}$, simply means that no trader could earn a positive profit by buying the final composite good in country n and ship it to country i at cost $p_n d_{in}$ for resale at price p_i. The proof of Proposition 6 also highlights the critical importance of the triangular inequality in trade costs in Ricardian trade models.

Recall that for each good $\omega \in \Omega$, cost-minimizing buyers in country n chose from a menu of prices $\{p_{ii}(\omega) d_{ni}\}_{i \in \mathcal{N}}$ and picks the supplier offering the lowest 'direct' price[15] $p_{jj}(\omega) d_{nj} = \min_{i \in \mathcal{N}} \{p_{ii}(\omega) d_{ni}\}$, resulting in the realized price of good ω in country n as being $p_n(\omega) = p_{jj}(\omega) d_{nj}$. The results from analysis above all follow from this characterization of national prices of tradeable goods.

The triangular inequality assumption in trade costs ensures that there is no possibility of obtaining good ω at an even lower price by routing the purchase of the good through a third country. Suppose a buyer in country n instead decides to route a purchase of 1 unit of good ω from country j through some third country k. She would have to pay $d_{kj} p_{jj}(\omega)$ for the good to be delivered to country k, and an additional $(d_{nk} - 1)\, d_{kj} p_{jj}(\omega)$ for transporting the good from country k to country n. The total cost of purchasing good ω from j through third party k would be $d_{nk} d_{kj} p_{jj}(\omega)$. If the triangular inequality holds, than $d_{nk} d_{kj} \geq d_{nj}$ and it would never be cheaper for the country n buyer to route the purchase through a third country.

On other hand, suppose the triangular inequality does not hold, and $d_{nk} d_{kj} < d_{nj}$. If this were the case, then $p_{jj}(\omega) d_{nk} d_{kj} < p_{jj}(\omega) d_{nj}$ and the country n buyer should not purchase the good directly from the manufacturer in country j, and the realized price of good ω in country n need not correspond to $\min_{i \in \mathcal{N}} \{p_{ii}(\omega) d_{ni}\}$. In fact, buyers in every country would need to consider the possibility of routing the trade not only through third countries, but other possibilities involving any number of intermediate countries. The immediate implication of course is that Proposition 1 would not hold, and neither any of the other propositions as they depend in part on the condition that $p_n(\omega) = \min_{i \in \mathcal{N}} \{p_{ii}(\omega) d_{ni}\}$.

Yet in reality it is often cheaper to route goods through trade hubs or entrepots like Singapore, in the process of transporting a good from the supplier to its final customer, so it would be interesting, at least in future work, to see how a relaxation of the triangular inequality might be accommodated in the Eaton and Kortum framework.

The Eaton and Kortum trade model as it has been so far developed in this section is a partial equilibrium

[15] The use of the term 'direct' price might require some explanation. From the perspective of the buyer in country n, $p_{jj}(\omega) d_{nj}$ represents the cost of acquiring good ω directly from the supplier in country j at factory gate price $p_{jj}(\omega)$ and paying $p_{jj}(\omega)(d_{nj} - 1)$ in transportation costs, hence the use of the term 'direct' price $p_{jj}(\omega) d_{nj}$.

model as determinants of production input costs c_n and national expenditures on tradeable goods X_n has yet to be defined. We have only explored the general features of Ricardian models of trade employing the Eaton and Kortum specification of stochastic technology, and shown that Propositions 1, 2, 3, 4, 5 and 6 describe the key features of prices of tradeable goods and patterns of trade that must necessarily hold in any such models. The next section develops the general equilibrium model employed in Eaton and Kortum (2002) by specifying how costs and expenditures are determined.

3 General Equilibrium Model in Eaton and Kortum (2002)

A key goal in trade analysis is to examine how national welfare changes in response to changes in *trade flows*. As the term implies trade flows both captures the direction of trade or trade *patterns* - who produces what for sale to whom, and trade *volumes* - of each good traded, how much in either monetary or quantitative terms is transacted. Explaining how trade flows might be affected by changes in variables such as barriers to trade and changes in technologies employed in production and transportation would provide insight into the consequent effects on national welfare.

In order to conduct a general equilibrium analysis of trade flows and national welfare employing the Eaton and Kortum trade framework, the model needs to be closed by specifying the determinants of production costs, national income and expenditures on tradeable goods.

In this section, we describe how the Eaton and Kortum trade framework was augmented to obtain the general equilibrium model employed in Eaton and Kortum (2002).

In the Eaton and Kortum (2002) general equilibrium model, as with Ricardian trade models in general, labor is immobile between countries. There are 2 sectors of tradeable goods, 'manufacturing' and 'non-manufacturing'. Each country $n \in \mathcal{N}$ is exogeneously endowed with a labor force $\bar{L}_n$. The labor force is distributed between employment in the manufacturing sector L_n^M and the non-manufacturing sector L_n^O. Perfectly competitive labor markets ensure full employment such that $\bar{L}_n = L_N^M + L_n^O$.

International trade in the non-manufacturing good is costless and tariff barriers do not exist. Hence the law of one price applies in the non-manufacturing sector and the price of the non-manufacturing good is the same in every country. It is natural therefore to designate the non-manufacturing good as the numeraire good. The only factor of production for non-manufacturing goods is labor. Let w_n^O denote the national non-manufacturing wage rate, such that $Y_n^O = w_n^O L_n^O$ denotes the national non-manufacturing income.

Production and trade in the manufacturing sector is governed by the Eaton and Kortum trade framework as described in Section 2. That is to say, there are infinitely many manufacturing goods $\omega \in \Omega$, aggregated into a final composite manufacturing good by the CES function specified in equation 6. Assumptions 1 through 4 stated at the start of Section 2 applies to the manufacturing sector, as would Propositions 1 through 6.

Alongside labor, production of manufacturing goods takes the composite manufacturing good as intermediate inputs. The manufacturing production function has a Cobb-Douglas specification, with β being the share of labor income and the remainder $(1 - \beta)$ being the cost share of intermediate goods.

The basket of production inputs going into the manufacturing production (as stated in equation 3) is thus given as $k_n(\omega) = L_n^M(\omega)^\beta G_n^M(\omega)^{1-\beta}$ where $L_n^M(\omega)$[16] is the manufacturing labor force, and $G_n^M(\omega)$ is the units of the composite manufacturing good employed in country n on the production of good ω respectively.

Given cost minimization by producers of manufacturing goods, the optimized cost of each input basket in country n is

$$c_n = (w_n)^\beta (p_n)^{1-\beta} \tag{15}$$

where w_n is the national manufacturing wage rate and p_n is the national price of the composite good as given in equation 11 in Proposition 4.

Given national manufacturing revenue Q_n, national manufacturing labor income is

$$Y_n^M = w_n L_n^M = \beta Q_n \tag{16}$$

and the national expenditure on manufacturing goods as intermediate inouts is $(1-\beta) Q_n$.

Substituting the equation 15 into the findings of Section 2 we obtain the following results for prices and trade patterns in the manufacturing sector:

$$\begin{aligned} \phi_{ni}(\mathbf{p},\mathbf{w}) &= T_i \left(w_i^\beta p_i^{1-\beta} d_{ni} \right)^{-\theta} && (17) \\ \Phi_n(\mathbf{p},\mathbf{w}) &= \sum_{i\in\mathcal{N}} \phi_{ni}(\mathbf{p},\mathbf{w}) && (18) \\ p_n(\mathbf{p},\mathbf{w}) &= \gamma \Phi_n(\mathbf{p},\mathbf{w})^{-\frac{1}{\theta}} && (19) \\ \pi_{ni}(\mathbf{p},\mathbf{w}) &= \frac{\phi_{ni}(\mathbf{p},\mathbf{w})}{\Phi_n(\mathbf{p},\mathbf{w})} && (20) \end{aligned}$$

where $\mathbf{p} = (p_1, ..., p_N)$ and $\mathbf{w} = (w_1, ..., w_N)$ denote the vectors of national manufacturing prices (or more specifically, national prices of the manufacturing composite good) and national manufacturing wages respectively. Notice that a consequence of the role of manufacturing goods as intermediate goods is the inter-dependency of national manufacturing prices.

National welfare is determined by the level of real consumption of both non-manufacturing goods C_n^O and of the manufacturing composite good[17] $\frac{C_n^M}{p_n}$. The national aggregate welfare function for each country is Cobb-Douglas, stated as

$$W_n = \left(\frac{C_n^M}{p_n}\right)^\alpha \left(C_n^O\right)^{1-\alpha}.$$

where α is the share of national consumption expenditure spent on consumption of the final manufacturing good.

Given the static nature of Ricardian trade models in general, it is necessarily the case that nominal

[16] Total manufacturing labor employment is $L_n^M = \int_{\omega\in\Omega} L_n^M(\omega) d\omega$.

[17] C_n^M and C_n^O are the nominal expenditures on the consumption of the composite manufacturing good and the non-manufacturing goods respectively. Hence $\frac{C_n^M}{p_n}$ is real consumption of the manufacturing good. C_n^O is also real consumption of the non-manufacturing good as the non-manufacturing good, as the numeraire good, has the price of 1 in every country.

national income Y_n equals nominal expenditures on consumption $C_n^M + C_n^O$. Then optimal national demand for consumption goods, subject to the national budget constraint, gives optimized national aggregate welfare[18] as

$$W_n = \frac{Y_n}{p_n^\alpha}. \tag{21}$$

The national expenditure on non-manufacturing goods is $C_n^O = (1-\alpha) Y_n$, and the national expenditure on manufacturing good consumption is $C_n^M = \alpha Y_n$.

Total national expenditure on manufacturing goods X_n includes both expenditure on manufacturing goods as consumption goods and as intermediate goods into manufacturing production, hence

$$X_n = (1-\beta) Q_n + \alpha Y_n. \tag{22}$$

From Proposition 5, country n's expenditure on goods sourced from country i is $X_{ni} = \pi_{ni} X_n$. If country n imposes an ad-valorem tariff t_{ni} on imports from country i, this expenditure X_{ni} would consist of 2 parts: Q_{in} the before-tariffs manufacturing revenue earned by country i, and $TR_{ni} = t_{ni} Q_{in}$ the tariff revenue collected by country n (and subsequently rebated in full to households in country n). Since $X_{ni} = TR_{ni} + Q_{in} = (1 + t_{ni}) Q_{in}$, we have

$$\begin{aligned} Q_{in} &= \frac{1}{1+t_{ni}} \pi_{ni} X_n \\ TR_{ni} &= \frac{t_{ni}}{1+t_{ni}} \pi_{ni} X_n. \end{aligned}$$

Total national manufacturing revenue $Q_n = \sum_{i \in \mathcal{N}} Q_{ni}$ is

$$Q_n = \sum_{i \in \mathcal{N}} \frac{\pi_{in}}{1+t_{in}} X_i. \tag{23}$$

Total national tariff revenue $TR_n = \sum_{i \in \mathcal{N}} TR_{ni}$ is

$$TR_n = X_n \sum_{i \in \mathcal{N}} \frac{t_{ni} \pi_{ni}}{1+t_{ni}}. \tag{24}$$

Since the national tariff revenue is rebated in full to households in the same country, total national income is the sum of manufacturing and non-manufacturing incomes and tariff revenues on imports of manufacturing goods, such that

$$Y_n = Y_n^O + Y_n^M + TR_n.$$

Equations 19 and 20 show that manufacturing wage rates w_n affects manufacturing prices and patterns of trade. Eaton and Kortum (2002) provided 2 specifications of intra-sectoral labor mobility,

[18] Optimized level of national aggregate welfare should be $W_n = \frac{1}{A} \frac{Y_n}{p_n^\alpha (p_n^O)^{1-\alpha}}$ where $A = \alpha^\alpha (1-\alpha)^{1-\alpha}$ and p_n^O is national price of non-manufacturing goods. Since we are primarily interested in relative changes in national welfare, we can safely ignore the constant term A by setting it to 1. Since the non-manufacturing good is designated the numeraire good, we have $p_n^O = 1$ for all countries n.

termed the 'mobile' and 'immobile' labor scenarios respectively, with different implications for the determination of national manufacturing wages and the resulting equilibria.

3.1 Mobile Labor Scenario

In the mobile labor scenario, labor is fully mobile between the manufacturing and non-manufacturing sectors, and perfectly competitive national labor markets ensure the equality of wages between the two sectors within each country. National wages w_n are completely determined by exogeneously given productivity in the non-manufacturing sector,[19] which when combined with fixed national labor supply $\bar{L}_n$ results in total national labor income being fixed at

$$Y_n^O + Y_n^M = w_n \bar{L}_n.$$

National income in the mobile labor scenario is then

$$Y_n = w_n \bar{L}_n + TR_n. \tag{25}$$

Given exogeneously given national wages and labor supply, national income only varies with tariff revenues in the mobile labor scenario.

National expenditure on manufacturing goods in the mobile labor scenario is then

$$X_n = (1 - \beta) Q_n + \alpha TR_n + \alpha w_n \bar{L}_n. \tag{26}$$

In the Eaton and Kortum (2002) general equilibrium model, only two markets matter, the manufacturing product market and the national labor market. The national budget constraint ensures that if these two markets clear, the non-manufacturing market must clear as well.

In the mobile labor equilibrium, the national labor market outcome is exogeneously determined by the externally given national wages and labor supply. The only market that is not completely determined by external parameters is the manufacturing product market. Consequently, analysis of the mobile labor case need only consider developments in the manufacturing sector.

The discussion of the Eaton and Kortum trade model necessarily involves vectors of variables such as manufacturing prices and wages, and matrices of variables such as expenditure shares, but also vectors and matrices of functions of such variables. To render such notations intelligible, and to avoid too many standalone symbols, we here offer the following definitions:

- Let $\mathbf{M}(z_{ni})$ denote a square $N \times N$ matrix with z_{ni} being the element on the n^{th} row and i^{th} column, i.e. the first subscript refers to the row index while the latter denotes the column index. It follows immediately from this definition that $\mathbf{M}(z_{in}) = \mathbf{M}(z_{ni})^\top$, where $\mathbf{M}(z_{ni})^\top$ refers to the transpose of the matrix $\mathbf{M}(z_{ni})$.

[19] To ensure that that non-manufacturing productivity determines national labor wages in each country, it is assumed that labor supply in each country is sufficiently large such that non-manufacturing production occurs in each country.

- Let $\mathbf{L}\left(z_n\right) = \left(z_1, ..., z_N\right)^T$ denote a $N \times 1$ column vector with the k^{th} element being z_k, and
- $\mathbf{D}\left(z_n\right)$ denote a $N \times N$ diagonal matrix, with all off-diagonal elements being 0, and the k^{th} diagonal element being the k^{th} element from the vector $\mathbf{L}\left(z_n\right)$.

For example,

$$\mathbf{M}\left(z_{ni}\right) = \begin{bmatrix} z_{11} & z_{12} & ... & z_{1N} \\ z_{21} & z_{22} & ... & z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ z_{N1} & z_{N2} & ... & z_{NN} \end{bmatrix}; \quad \mathbf{D}\left(z_n\right) = \begin{bmatrix} z_1 & 0 & ... & 0 \\ 0 & z_2 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & z_N \end{bmatrix}.$$

- Let $\mathbf{1}_N := (1, 1, ..., 1)^\top$ and $\mathbf{0}_N := (0, 0, ..., 0)^\top$ refer to N dimensional vectors entirely composed of 1's and 0's respectively.
- Finally, let $\iota_k := (0, .., 0, 1, 0, ..., 0)^\top$ denote a N dimensional k^{th} unit vector, i.e. a vector with 1 in the k^{th} position and 0's everywhere else.

Cost minimization by buyers of the composite manufacturing good in each country $n \in \mathcal{N}$ requires that national manufacturing prices p_n satisfy equation 19, and import expenditure shares on manufacturing goods π_{ni} satisfy equation 20. Equivalently, the vectorof national manufacturing prices $\mathbf{p} = (p_1, ..., p_N)$ must satisfy

$$\mathbf{p} = \mathbf{L}\big(\Phi_n(\mathbf{p})^{-\frac{1}{\theta}}\big), \tag{27}$$

where $\Phi_n(\mathbf{p})$ is given by equation 18.

The matrix of expenditure shares must satisfy

$$\Pi(\mathbf{p}) = \mathbf{M}\Bigg(\frac{T_i\left(d_{ni} w_i^\beta p_i^{1-\beta}\right)^{-\theta}}{p_n^{-\theta}}\Bigg), \tag{28}$$

where $\Pi(\mathbf{p}) := \mathbf{M}\big(\pi_{ni}(\mathbf{p})\big)$.

Notice that we have removed the reference to manufacturing wages in the arguments to the prices, expenditure shares and Φ_n, because in the mobile labor scenario, manufacturing wages are exogenous parameters.

In linear algebraic form, the vector of national tariff revenue $\mathbf{L}(TR_n)$ can be written as a function of expenditure shares in $\Pi := \mathbf{M}(\pi_{ni})$, tariff rates in $\mathbf{M}(t_{ni})$ and the vector of national manufacturing expenditures $\mathbf{L}(X_n)$

$$\mathbf{L}\big(TR_n(\mathbf{p})\big) = \mathbf{Dt}(\mathbf{p}) \cdot \mathbf{L}\big(X_n(\mathbf{p})\big), \tag{29}$$

where $\mathbf{Dt}(\mathbf{p}) := \mathbf{D}\big(\sum_{i \in \mathcal{N}} \pi_{ni}(\mathbf{p}) \frac{t_{ni}}{1+t_{ni}}\big)$.

The vector of national manufacturing revenues (or equivalently nominal output) can be written

$$\mathbf{L}\big(Q_n(\mathbf{p})\big) = \Pi\mathbf{t}(\mathbf{p})^\top \cdot \mathbf{L}\big(X_n(\mathbf{p})\big), \tag{30}$$

where $\Pi\mathbf{t}(\mathbf{p})^\top := \mathbf{M}\big(\frac{1}{1+t_{ni}}\pi_{ni}(\mathbf{p})\big)^\top$.

Substituting equations 29 and 30 into the the vector form of equation 26, $\mathbf{L}(X_n) = (1-\beta)\,\mathbf{L}(Q_n) + \alpha\mathbf{L}(TR_n) + \alpha\mathbf{L}(w_n\bar{L}_n)$, shows that the vector of national expenditures on manufacturing goods must satisfy

$$\big[\mathbf{I} - (1-\beta)\,\Pi\mathbf{t}(\mathbf{p})^\top - \alpha\mathbf{D}\mathbf{t}(\mathbf{p})\big]\,\mathbf{L}\big(X_n(\mathbf{p})\big) = \alpha\mathbf{L}(w_n\bar{L}_n). \tag{31}$$

The equilibrium conditions for the mobile labor scenario is summarized in the following definition.

Definition 1. Given model parameters α, β, θ, $\mathbf{L}(T_n)$, $\mathbf{L}(w_n)$, $\mathbf{L}(\bar{L}_n)$**,** $\mathbf{M}(d_{ni})$ and $\mathbf{M}(t_{ni})$, an equilibrium in the *mobile labor* scenario of the Eaton and Kortum (2002) trade model consists of

1. A price vector $\mathbf{p} \in \mathbb{R}^N_{++}$, satisfying equation 27. *(Prices)*

2. A matrix of expenditure shares $\Pi(\mathbf{p})$ satisfying equation 28. *(Pattern of Trade)*

3. A vector of national manufacturing expenditures $\mathbf{L}\big(X_n(\mathbf{p})\big)$ satisfying equation 31. *(Scale of Trade)*

Since expenditure shares $\Pi(\mathbf{p})$ is a function of prices $\mathbf{p}$, there must be a unique matrix of equilibrium expenditure shares if the vector of equilibrium prices $\mathbf{p}$ is unique.

By an argument almost identical to the proof of Lemma 3 below, it can be easily shown that the matrix $\big[\mathbf{I} - (1-\beta)\,\Pi\mathbf{t}(\mathbf{p})^\top - \alpha\mathbf{D}\mathbf{t}(\mathbf{p})\big]$ is invertible in which case equation 31 has a unique solution for any vector of prices $\mathbf{p}$, and consequently the vector of manufacturing expenditures $\mathbf{L}\big(X_n(\mathbf{p})\big)$ is a function of the vector of prices $\mathbf{p}$ as well.

Since manufacturing expenditure shares $\Pi(\mathbf{p})$ and national manufacturing expenditures $\mathbf{L}\big(X_n(\mathbf{p})\big)$ are completely determined by manufacturing prices $\mathbf{p}$, it follows that the mobile labor equilibrium, if it exists, is completely characterized by the equilibrium vector of manufacturing prices.

3.2 Immobile Labor Scenario

In the immobile labor scenario, labor is perfectly immobile between the non-manufacturing and manufacturing sectors within each country.

In this scenario, non-manufacturing output (or equivalently income) is an exogeneously given parameter $\bar{Y}^O_n$, and manufacturing labor supply L^M_n is also exogeneously determined.

Manufacturing wages w_n is endogenously determined by manufacturing revenues Q_n. From equation 16, we have $w_n = \beta\frac{Q_n}{L^M_n}$.

National income then would be

$$Y_n = \beta Q_n + \bar{Y}^O_n + TR_n. \tag{32}$$

Unlike in the mobile labor scenario, national income varies not only with tariff revenues, but also with manufacturing revenues as well.

Manufacturing labor and product market outcomes are determined simultaneously, as manufacturing prices and wages are mutually determined endogenous variables in the immobile labor scenario.

As in the mobile labor scenario, cost minimizing buyers of manufacturing goods require that manufacturing prices satisfy

$$\mathbf{p} = \gamma \mathbf{L}\Big(\Phi_n(\mathbf{p},\mathbf{w})^{-\frac{1}{\theta}}\Big), \tag{33}$$

where $\Phi_n(\mathbf{p},\mathbf{w})$ is defined by equation 18, $\mathbf{p} = (p_1, ..., p_N)$ is the vector of national manufacturing prices and $\mathbf{w} = (w_1, ..., w_N)$ is the vector of national manufacturing wages.

Manufacturing expenditure shares must satisfy

$$\Pi(\mathbf{p},\mathbf{w}) = \mathbf{M}\Big(\frac{T_i\left(d_{ni}w_i^{\beta}p_i^{1-\beta}\right)^{-\theta}}{p_n^{-\theta}}\Big). \tag{34}$$

Tariff and manufacturing revenues are given as

$$\mathbf{L}\big(TR_n(\mathbf{p},\mathbf{w})\big) = \mathbf{Dt}(\mathbf{p},\mathbf{w}) \cdot \mathbf{L}\big(X_n(\mathbf{p},\mathbf{w})\big), \tag{35}$$

$$\mathbf{L}\big(Q_n(\mathbf{p},\mathbf{w})\big) = \Pi\mathbf{t}(\mathbf{p},\mathbf{w})^{\top} \cdot \mathbf{L}\big(X_n(\mathbf{p},\mathbf{w})\big) \tag{36}$$

where $\mathbf{Dt}(\mathbf{p},\mathbf{w}) := \mathbf{D}\big(\sum_{i\in\mathcal{N}}\pi_{ni}(\mathbf{p},\mathbf{w})\frac{t_{ni}}{1+t_{ni}}\big)$ and $\Pi\mathbf{t}(\mathbf{p},\mathbf{w}) := \mathbf{M}\Big(\frac{1}{1+t_{ni}}\pi_{ni}(\mathbf{p},\mathbf{w})\Big)$.

Given national income equation 32 and national manufacturing expenditures in equation 22, the vector of national manufacturing expenditures is

$$\mathbf{L}\Big(X_n(\mathbf{p},\mathbf{w})\Big) = \chi\mathbf{L}\Big(Q_n(\mathbf{p},\mathbf{w})\Big) + \alpha\mathbf{L}\Big(TR_n(\mathbf{p},\mathbf{w})\Big) + \alpha\mathbf{L}\Big(\bar{Y}_n^O\Big),$$

where $\chi := 1 - \beta + \alpha\beta$.

Substituting equations 35 and 36 into the above shows that national manufacturing expenditure must satisfy

$$\left[\mathbf{I} - \chi\mathbf{Z}(\mathbf{p},\mathbf{w})^{\top}\right]\mathbf{L}\Big(X_n(\mathbf{p},\mathbf{w})\Big) = \alpha\mathbf{L}\Big(\bar{Y}_n^O\Big), \tag{37}$$

where $\mathbf{Z}(\mathbf{p},\mathbf{w}) := \Pi\mathbf{t}(\mathbf{p},\mathbf{w}) - \frac{\alpha}{\chi}\mathbf{Dt}(\mathbf{p},\mathbf{w})$ for notational convenience.

If the inverse of the matrix $[\mathbf{I} - \chi\mathbf{Z}(\mathbf{p},\mathbf{w})]$ exists, the vector of national manufacturing revenues is given by

$$\mathbf{L}\Big(Q_n(\mathbf{p},\mathbf{w})\Big) = \alpha\Pi\mathbf{t}(\mathbf{p},\mathbf{w})^{\top}\left[\mathbf{I} - \chi\mathbf{Z}(\mathbf{p},\mathbf{w})^{\top}\right]^{-1}\mathbf{L}\Big(\bar{Y}_n^O\Big). \tag{38}$$

Finally since the vector of manufacturing wages is $\mathbf{L}(w_n) = \beta\mathbf{D}\Big(L_n^M\Big)^{-1}\mathbf{L}\big(Q_n\big)$, the vector of manufacturing wages must satisfy

$$\mathbf{w} = \alpha\beta\mathbf{D}(L_n^M)^{-1}\Pi\mathbf{t}(\mathbf{p},\mathbf{w})^{\top}\left[\mathbf{I} - \chi\mathbf{Z}(\mathbf{p},\mathbf{w})^{\top}\right]^{-1}\mathbf{L}\Big(\bar{Y}_n^O\Big). \tag{39}$$

The equilibrium conditions for the immobile labor scenario are summarized below.

Definition 2. Given model parameters α, β, θ, $\mathbf{L}(T_n)$, $\mathbf{L}(L_n^M)$, $\mathbf{L}(\bar{Y}_n^O)$**,** $\mathbf{M}(d_{ni})$ and $\mathbf{M}(t_{ni})$, an equilibrium in the *immobile labor* scenario of the Eaton and Kortum (2002) trade model consists of

1. A manufacturing price vector $\mathbf{p} \in \mathbb{R}^N_{++}$, satisfying equation 33. *(Manufacturing Prices)*
2. A manufacturing wage vector $\mathbf{w} \in \mathbb{R}^N_{++}$, satisfying equation 39. (*Manufacturing Wages)*
3. A matrix of expenditure shares $\Pi(\mathbf{p}, \mathbf{w})$ satisfying equation 34. *(Pattern of Trade)*
4. A vector of national manufacturing expenditures $\mathbf{L}\Big(X_n(\mathbf{p}, \mathbf{w})\Big)$ satisfying equation 37. *(Scale of Trade)*

We have fully characterized the general equilibria described in Eaton and Kortum (2002). The next part of the thesis establishes the existence and uniqueness of the general equilibria in both scenarios of the Eaton and Kortum (2002) trade model.

Part III

Existence and Uniqueness of Equilibrium

Given the empirical focus of Eaton and Kortum's 2002 paper, the necessary and sufficient conditions for the existence or the uniqueness of general equilibria described in their paper were not discussed.

While Alvarez and Lucas (2007) provided existence and uniqueness proofs of equilibrium in their variant of the Eaton and Kortum model, their model differs in some significant ways from the original proposed in Eaton and Kortum (2002).

For example, while all manufacturing goods are traded in the Eaton and Kortum model, in the Alvarez and Lucas variant, only manufacturing intermediate inputs are traded while consumption goods can only be produced domestically. Furthermore, in the Eaton and Kortum (2002) model, there are 2 sectors of traded goods, manufacturing and non-manufacturing. In the Lucas and Alvarez (2007), there is only one. Finally, while in the mobile labor scenario of the Eaton and Kortum model it is possible for labor to move freely between the 2 different sectors of production in response to changes in foreign and domestic demand, this is not the case in the Alvarez and Lucas model since there is only one single sector of traded output with a fixed labor endowment.

In short, the Alvarez and Lucas (2007) model differs substantially from the original model specified in Eaton and Kortum (2002), such that proofs of existence and uniqueness provided by Lucas and Alvarez cannot be directly applied to the general equilibria in the Eaton and Kortum model.

This part of the thesis aims to address the gap by providing rigorous proofs of existence and uniqueness of the general equilibria described in the original Eaton and Kortum (2002) paper. However, it should be noted here that since the immobile labor scenario of the Eaton and Kortum model and the Alvarez and Lucas model share the feature of fixed labor endownment in the sector that relies on intermediate inputs (i.e. the manufacturing sector in the Eaton and Kortum model), the proof of uniqueness of equilibrium in the immobile labor scenario of the Eaton and Kortum model presented here is an adaptation of Alvarez and Lucas's proof of uniqueness of equilibrium along with the associated sufficient conditions, from their single sector model to the multi-sectoral model of Eaton and Kortum.

However, it should be noted that our proof that the Eaton and Kortum immobile labor equilibrium exists unconditionally is unrelated to the work presented in Alvarez and Lucas.

To our best knowledge as well, there are no close parallels in Alvarez and Lucas to our existence and uniqueness results for the mobile labor equilibrium - the topic of the next section.

4 Existence and Uniqueness of Mobile Labor Equilibrium

In this section, we prove the existence and the uniqueness of the mobile labor equilibrium by showing that - taking as given any vector of national manufacturing wages $\mathbf{w} = (w_1, ..., w_N)$, and model parameters α, β, θ, $\mathbf{L}(T_n)$, $\mathbf{M}(t_{ni})$ and $\mathbf{M}(d_{ni})$ - there exists one and only one possible equilibrium vector of prices, $\mathbf{p} = (p_1, ..., p_N) \in \mathbb{R}^N_{++}$ satisfying

$$\mathbf{p} = \mathbf{L}\Big(\Phi_n(\mathbf{p})^{-\frac{1}{\theta}}\Big) \tag{40}$$

where $\Phi_n(\mathbf{p}) := \sum_{i\in\mathcal{N}} T_i \left(w_i^{\beta} p_i^{1-\beta} d_{ni}\right)^{-\theta}$.

In the discussion directly following Definition 1, it was noted that since manufacturing expenditure shares and national manufacturing expenditures are completely determined by manufacturing prices, the vector of manufacturing prices **p** completely characterizes the mobile labor equilibrium, and that the existence and uniqueness of a price vector **p** that satisfies equation 40 (which is identical to equation 27) automatically implies the existence and uniqueness of the mobile labor equilibrium.

For ease of notation, let $\tilde{\mathbf{p}} := (\ln p_1, ..., \ln p_N)$, $g_n(\tilde{\mathbf{p}}) := -\frac{1}{\theta}\ln\Phi_n(\mathbf{p})$ and $\mathbf{G}(\tilde{\mathbf{p}}) = (g_1(\tilde{\mathbf{p}}), ..., g_N(\tilde{\mathbf{p}}))$. The strategy towards the proof is to first demonstrate that the function **G** is a contraction in $\mathbb{R}^N$, in which case there must exist a unique fixed point $\tilde{\mathbf{p}} \in \mathbb{R}^N$ which satisfy $\tilde{\mathbf{p}} = \mathbf{G}(\tilde{\mathbf{p}})$. Then since the natural logarithm is a one-to-one function, there must exist a unique $\mathbf{p} \in \mathbb{R}^N_{++}$ satisfying equation 40. We first derive the Jacobian matrix of $\mathbf{G}(\tilde{\mathbf{p}})$ in Lemma 2.

Lemma 2. *For any $n, i \in \mathcal{N}$, the partial derivative of Φ_n with respect to p_i is given as*

$$\frac{\partial \ln g_n(\tilde{\mathbf{p}})}{\partial \tilde{p}_i} = (1-\beta)\,\pi_{ni}(\mathbf{p}) \tag{41}$$

where $\pi_{ni}(\mathbf{p}) = \frac{T_i\left(w_i^{\beta} p_i^{1-\beta} d_{ni}\right)^{-\theta}}{\Phi_n(\mathbf{p})}$ is the share of country n's expenditures on manufacturing goods spent on imports from country i.

Organized in matrix form, the Jacobian matrix of $\mathbf{G}(\tilde{\mathbf{p}})$ is given as

$$\mathbf{M}\Big(\frac{\partial g_n(\tilde{\mathbf{p}})}{\partial \tilde{p}_i}\Big) = (1-\beta)\,\Pi(\mathbf{p}),$$

noting that the matrix of expenditure shares $\Pi(\mathbf{p}) = \mathbf{M}\Big(\pi_{ni}(\mathbf{p})\Big)$ is a right-stochastic matrix.

Proof. Taking manufacturing wages and model parameters as given, the matrix of manufacturing expenditures shares is a function of national manufacturing prices, written as $\Pi(\mathbf{p})$. The sum of each row of $\Pi(\mathbf{p})$, as a sum of shares, must be unity; and $\Pi(\mathbf{p})$ is a right stochastic matrix. This could be confirmed by noting that the sum of the n^{th} row of $\Pi(\mathbf{p})$ is $\sum_{j\in\mathcal{N}} \pi_{nj}(\mathbf{p}) = \frac{\sum_{i\in\mathcal{N}} T_j\left(w_j^{\beta} p_j^{1-\beta} d_{nj}\right)^{-\theta}}{\Phi_n} = 1$ by the definition of $\Phi_n(\mathbf{p})$ given in equation 40.

Taking the natural logarithm of each expenditure share term, we have $\ln \pi_{nj}(\mathbf{p}) = \ln T_j \left(w_j^{-\beta} d_{nj}\right)^{-\theta} - \theta\,(1-\beta)\ln p_j - \ln\Phi_n$. Since $\ln T_j \left(w_j^{-\beta} d_{nj}\right)^{-\theta}$ is a function of constant terms, we have for each $i \in \mathcal{N}$,

$$\frac{\partial \ln \pi_{nj}}{\partial \ln p_i} = -\theta\,(1-\beta)\,\frac{\partial \ln p_j}{\partial \ln p_i} - \frac{\partial \ln \Phi_n}{\partial \ln p_i}. \;\; (*)$$

Since $\sum_{j\in\mathcal{N}}\pi_{nj}(\mathbf{p})=1$, and using equation $(*)$, it must be the case that

$$\begin{aligned} 0 &= \frac{\partial}{\partial\ln p_i}\left(\sum_{j\in\mathcal{N}}\pi_{nj}(\mathbf{p})\right) \\ &= \sum_{j\in\mathcal{N}}\pi_{nj}\frac{\partial\ln\pi_{nj}(\mathbf{p})}{\partial\ln p_i} \\ &= -\theta\left(1-\beta\right)\pi_{ni}+\frac{\partial\ln\Phi_n(\mathbf{p})}{\partial\ln p_i} \quad (**) \end{aligned}$$

Rearranging $(**)$, we obtain the required result that

$$\frac{\partial g_n(\tilde{\mathbf{p}})}{\partial\tilde{p}_i}=-\frac{1}{\theta}\frac{\partial\ln\Phi_n(\mathbf{p})}{\partial\ln p_i}=(1-\beta)\,\pi_{ni}(\mathbf{p}).$$

□

The following proof that **G** is a contraction closely follows the proof of Theorem 1 in Alvarez and Lucas (2007). In the spirit that simplicity is next to godliness, it is hoped that the proof presented here is a slight improvement on Alvarez and Lucas in the sense that it does *not* require the use of the Blackwell sufficient conditions for a contraction.[20]

Theorem 1. *For any combination of manufacturing wages and model parameters, the function* $\mathbf{G}:\mathbb{R}^N\rightarrow\mathbb{R}^N$ *is a contraction, since for all* $\tilde{\mathbf{p}}^{'},\tilde{\mathbf{p}}^{''}\in\mathbb{R}^N$*, it is always the case that*

$$\|\mathbf{G}(\tilde{\mathbf{p}}^{'})-\mathbf{G}(\tilde{\mathbf{p}}^{''})\|_\infty\leq(1-\beta)\,\|\tilde{\mathbf{p}}^{'}-\tilde{\mathbf{p}}^{''}\|_\infty,$$

where $\|\cdot\|_\infty$ *is the supremum norm, and* $(1-\beta)<1$ *.*

Then by the Banach fixed point theorem, there must exist a unique $\tilde{\mathbf{p}}^*\in\mathbb{R}^N$ *that is a fixed point of* **G***, in which case* $\mathbf{p}^*=(\exp\tilde{p}_1^*,...,\exp\tilde{p}_N^*)$ *is the unique vector of manufacturing prices satisfying equation 40.*

It follows immediately that:

1. *The mobile labor equilibrium vector of manufacturing prices is uniquely determined by manufacturing wages and model parameters, and the mobile labor equilibrium is unique.*
2. *In the immobile labor scenario, holding model parameters constant,*
   (a) *the vector of national manufacturing prices* $\mathbf{p}(\mathbf{w})$
   (b) *the matrix of expenditure shares* $\Pi(\mathbf{p}(\mathbf{w}),\mathbf{w})$

   are both completely determined by manufacturing wages, and are continuous functions of the vector of manufacturing wages $\mathbf{w}$.

[20] This proof also corrects a slight mathematical error in Alvarez and Lucas (2007) in which it seemed to be implied that for a pair of vectors $\tilde{\mathbf{p}}^{'},\tilde{\mathbf{p}}^{''}\in\mathbb{R}^N$, the difference $a=\tilde{\mathbf{p}}^{'}-\tilde{\mathbf{p}}^{''}$ is a scalar.

Proof. For each $n, i \in \mathcal{N}$, the expenditure share $\pi_{ni}(\mathbf{p})$ is obviously continuous for all $\mathbf{p} \in \mathbb{R}^N_{++}$ and $\mathbf{p} = \mathbf{L}(\exp \tilde{p}_n)$ is obviously continuous in $\tilde{\mathbf{p}} \in \mathbb{R}^N$, in which case $\frac{\partial \ln g_n(\tilde{\mathbf{p}})}{\partial \tilde{p}_i} = (1-\beta)\,\pi_{ni}(\mathbf{p})$ must be a continuous function for all $\tilde{\mathbf{p}} \in \mathbb{R}^N$. We have thus shown that for each $n \in \mathcal{N}$, $g_n(\tilde{\mathbf{p}})$ is continuously differentiable in $\mathbb{R}^N_{++}$.

Since $g_n(\tilde{\mathbf{p}})$ is continuously differentiable, for each $\tilde{\mathbf{p}} \in \mathbb{R}^N$ and $\mathbf{a} = (a_1, ..., a_N) \in \mathbb{R}^N$, there must exist some $v \in [0, 1]$, such that

$$\begin{aligned} g_n(\tilde{\mathbf{p}} + \mathbf{a}) - g_n(\tilde{\mathbf{p}}) &= \sum_{i=1}^{N} \frac{\partial g_n(\tilde{\mathbf{p}} + v\mathbf{a})}{\partial \tilde{p}_i} a_i \\ &= (1-\beta) \sum_{i=1}^{N} \pi_{ni}(\tilde{\mathbf{p}} + v\mathbf{a}) \cdot a_i. \\ &\leq (1-\beta) \max_{i \in \mathcal{N}} a_i. \end{aligned}$$

This is true for $g_n(\tilde{\mathbf{p}})$ for all $n \in \mathcal{N}$, including $\max_{n\in\mathcal{N}} g_n(\tilde{\mathbf{p}})$, in which case

$$\|\mathbf{G}(\tilde{\mathbf{p}} + \mathbf{a}) - \mathbf{G}(\tilde{\mathbf{p}})\|_\infty = \max_{n\in\mathcal{N}} g_n(\tilde{\mathbf{p}}) \leq (1-\beta) \max_{i\in\mathcal{N}} a_i = (1-\beta)\|\mathbf{a}\|_\infty,$$

and we obtain the desired result that $\mathbf{G}$ is a contraction in $\mathbb{R}^N$.

Item 2 (a) and (b) of the theorem might require further elaboration.

Given that the price vector in the immobile labor scenario must also satisfy equation 40,[21] it is clear that the immobile labor price vector must uniquely determined by, and hence an implicit function of manufacturing wages, written as $\mathbf{p}(\mathbf{w})$. This results in Item 2 (a).

Taking model parameters as given, each element $\pi_{ni}(\mathbf{p}, \mathbf{w}) = \frac{T_i\left(d_{ni} w_i^\beta p_i^{1-\beta}\right)}{p_n^{-\theta}}$ of the expenditure share matrix is a function of both the vector of manufacturing prices $\mathbf{p} = (p_1, ..., p_N)$ and manufacturing wages $\mathbf{w} = (w_1, ..., w_N)$. But since the price vector is entirely determined by manufacturing wages, the same must also be true of expenditure shares, and we write $\pi_{ni}(\mathbf{w}) := \pi_{ni}(\mathbf{p}(\mathbf{w}), \mathbf{w})$ to show that expenditure share π_{ni} is an implicit function of the vector of manufacturing wages.

In the proof of Proposition 48, it would be shown that the Jacobian of manufacturing prices with respect to wages, if it exists, is given as

$$\mathbf{M}\Big(\frac{\partial \ln p_n}{\partial \ln w_i}\Big) = \beta\,[\mathbf{I} - (1-\beta)\,\Pi(\mathbf{w})]^{-1}\,\Pi(\mathbf{w}).$$

As it would be demonstrated in the proof of Proposition 48, the inverse matrix $[\mathbf{I} - (1-\beta)\,\Pi(\mathbf{w})]^{-1}$ exists for all $\mathbf{w} \in \mathbb{R}^N_{+}$, and hence manufacturing prices are differentiable everywhere with respect to manufacturing wages. It follows immediately that $\mathbf{p}(\mathbf{w})$ is a continuous function of manufacturing wages, in which case $\Pi(\mathbf{w}) = \mathbf{M}\Big(\frac{T_i(d_{ni} w_i p_i(\mathbf{w}))^{-\theta}}{p_n(\mathbf{w})^{-\theta}}\Big)$ is also a continuous function of wages. □

Having established that a unique general equilibrium of the mobile labor scenario must always exist, we turn our attention to the immobile labor equilibrium in the next section.

[21] Notice that equations 27 and 33 are have identical definitions.

5 Existence of the Immobile Labor Equilibrium

In this section, we shall show that that an equilibrium vector of manufacturing labor wages must always exist in the immobile labor scenario of the Eaton and Kortum trade model.

Theorem 1 showed that manufacturing prices $\mathbf{p}(\mathbf{w})$ and expenditure shares $\Pi(\mathbf{w})$ are completely determined by manufacturing wages in the immobile labor scenario. As Lemma 3 would show, the inverse matrix $\left[\mathbf{I} - \chi\mathbf{Z}(\mathbf{p}(\mathbf{w}), \mathbf{w})^\top\right]^{-1}$ exists for all $\mathbf{w} \in \mathbb{R}^N_+$, in which case the vector of manufacturing expenditures would also be completely determined by manufacturing wages.

If an equilibrium vector of manufacturing exists, then equilibrium manufacturing prices, expenditure shares and expenditures satisfying conditions 1, 3 and 4 of Definition must exist as well. Just as the equilibrium vector of manufacturing prices completely characterizes the mobile labor equilibrium, an equilibrium in the immobile labor scenario would be completely characterized by the equilibrium vector of manufacturing wages.

Hence demonstrating that there exists a equilibrium vector of wages satisfying equation 39 is sufficient in proving the existence of the immobile labor equilibrium.

An equilibrium vector of wages $\mathbf{w}^* := \mathbf{L}(w^*_n)$ in the immobile labor equilibrium must satisfy[22]

$$\mathbf{w}^* = \alpha\beta\mathbf{D}(L^M_n)^{-1}\Pi\mathbf{t}(\mathbf{w}^*)^\top \left[\mathbf{I} - \chi\mathbf{Z}(\mathbf{w}^*)^\top\right]^{-1} \mathbf{L}\left(\bar{Y}^O_n\right). \tag{42}$$

The existence of an equilibrium in the immobile labor scenario of the Eaton and Kortum trade model relies on the existence of a fixed point for equation 42. We shall proceed with the proof of existence of the immobile labor equilibrium in the following manner:

- We shall first show that there exists a continuous function $\mathbf{g} : \mathbb{R}^{N\times N} \to \mathbb{R}^N_+$ mapping from the set of all possible trade share matrices to wage vectors, and a continuous function $\mathbf{f} : \mathbb{R}^N_+ \to \mathbb{R}^{N\times N}$ mapping from the set of all possible wage vectors to trade share matrices, such that the composite $\mathbf{f} \circ \mathbf{g} : \mathbb{R}^N_+ \to \mathbb{R}^N_+$ is the function of wages on the right hand side of equation 42.
- We shall then show that the composite function $\mathbf{g} \circ \mathbf{f}$ with a suitable convex and compact domain is a continuous self-map in which case the application of the Brouwer fixed point theorem would show that there must exist a trade share matrix Π^* which is a fixed point of $\mathbf{g} \circ \mathbf{f}$. It would then follow that $\mathbf{w}^* = g(\Pi^*)$ is a fixed point of $g \circ f$ and hence an equilibrium wage vector in the immobile labor scenario of the Eaton and Kortum trade model.

We start by discussing the appropriate space of trade share matrices. Every row of any arbitrary $N \times N$ right stochastic matrix is an N-dimensional probability vector. Every N dimensional probability vector $\Pi_{n*} := (\pi_{n1}, ..., \pi_{nN})$ can be represented as a point in a $N-1$ dimensional simplex $S_N := \left\{(\pi_{n1}, ..., \pi_{nN}) \in \mathbb{R}^N_+ : \sum_i \pi_{ni} = 1\right\}$, which is obviously a convex, closed and bounded subset of $\mathbb{R}^N$. Hence every right stochastic stochastic matrix must exist in $S^N_N := \prod_{n=1}^N \left\{(\pi_{n1}, ..., \pi_{nN}) \in \mathbb{R}^N_+ : \sum_i \pi_{ni} = 1\right\}$, which as the product of convex and compact sets, is also a convex and compact subset of $\mathbb{R}^{N\times N}$. Since

[22]Since expenditure shares are completely determined by manufacturing wages, we drop the vector of manufacturing prices $\mathbf{p}$ from the arguments to functions in the discussion below.

every trade share matrix Π is also by definition a right stochastic matrix, we have $\Pi(\mathbf{w}) \in S_N^N$ for any possible wage vector $\mathbf{w} \in \mathbb{R}_{++}^N$.

Taking model parameters such as α, β, $\mathbf{L}(L_n^M)$ and $\mathbf{L}(Y_n^O)$ as given, let $\mathbf{g} : S_N^N \to \mathbb{R}_{++}^N$ be the continuous mapping from the set of all possible trade share matrices to wage vectors, such that for any $\Pi = \mathbf{M}(\pi_{ni}) \in S_N^N$ the function $\mathbf{g}$ must satisfy

$$\mathbf{g}(\Pi) := \alpha\beta\mathbf{D}(L_n^M)^{-1}\Pi\mathbf{t}^\top \left[\mathbf{I} - \chi\tilde{\mathbf{Z}}(\Pi)^\top\right]^{-1} \mathbf{L}\Big(\bar{Y}_n^O\Big) \tag{43}$$

where for notational convenience, we define $\tilde{\mathbf{Z}}(\Pi) := \mathbf{M}\Big(\frac{\pi_{ni}}{1+t_{ni}}\Big) + \frac{\alpha}{\chi}\mathbf{D}\Big(\sum_{i\in\mathcal{N}} \frac{t_{ni}\pi_{ni}}{1+t_{ni}}\Big)$. It would be trivial to show that $\mathbf{Z}(\mathbf{w}) = \tilde{\mathbf{Z}}\big(\Pi(\mathbf{w})\big)$.[23]

The n^{th} element of $\mathbf{g}(\Pi)$ could be interpreted as the manufacturing wages generated in country n given trade share matrix Π. The basic idea here is that given any trade share matrix Π, the term $\mathbf{D}(L_n^M)\mathbf{g}(\Pi) = \mathbf{L}(\mathbf{g}_n(\Pi) \cdot L_n^M)$ could be interpreted as the vector of national manufacturing revenues generated by the pattern of trade described in Π and global non-manufacturing income.

The term $\chi\mathbf{Z}(\Pi)^\top\mathbf{D}(L_n^M)\mathbf{g}(\Pi)$ represents the demand for manufacturing output recursively generated within the manufacturing sector, driven by the demand for manufacturing output both as intermediate goods and as consumption goods for manufacturing labor. The vector $\alpha\mathbf{M}\Big(\frac{\pi_{ni}}{1+t_{ni}}\Big)^\top\mathbf{L}(Y_n^O)$ represents the demand for each country's manufacturing output generated by exogenous global non-manufacturing income, which is simply calculated as total manufacturing output net of demand for manufacturing goods generated from within the manufacturing sector itself.

The relationship above only indicates a correspondence between trade share matrices and national manufacturing wages, which is insufficient for our purpose. The following lemmas demonstrates that national manufacturing wages are uniquely determined by the pattern of trade captured by the matrix of trade shares.

Lemma 3. *The matrix* $\mathbf{I} - \chi\tilde{\mathbf{Z}}^\top = \mathbf{I} - \chi\mathbf{M}\Big(\frac{\pi_{ni}}{1+t_{ni}}\Big)^\top - \alpha\mathbf{D}\Big(\sum_{i\neq n}^N \frac{t_{ni}\pi_{ni}}{1+t_{ni}}\Big)$ *is non-singular for all possible trade share matrices* $\mathbf{M}(\pi_{ni})$ *and import tariff rates* $\mathbf{M}(t_{ni})$.

Proof. Since $t_{nn} = 0$ for each country n, sum of the elements of n^{th} row of the matrix $\mathbf{Z}$ can be written as

$$\pi_{nn} + \sum_{i\neq n}^N \pi_{ni}\left((1-\tau_{ni}) + \frac{\alpha}{\chi}\tau_{ni}\right) = \pi_{nn} + \sum_{i\neq n}^N \pi_{ni}\left(1 - \left(1 - \frac{\alpha}{\chi}\right)\tau_{ni}\right)$$

where $\tau_{ni} := \frac{t_{ni}}{1+t_{ni}} < 1$ is the proportion of country n's total expenditure on imports from country i collected as import tariffs.

Since $\beta \leq 1$ and $\tau_{ni} < 1$, we have $1 - \left(1 - \frac{\alpha}{\chi}\right)\tau_{ni} < 1$ for each $n, i \in \mathcal{N}$ and

$$\pi_{nn} + \sum_{i\neq n}^N \pi_{ni}\left(1 - \left(1 - \frac{\alpha}{\chi}\right)\tau_{ni}\right) < \sum_{i=1}^N \pi_{ni} = 1,$$

[23]This ensures that the results of Lemma 3 also applies to $\mathbf{Z}$.

such that every row of the matrix $\tilde{\mathbf{Z}}$ sums to less than one. Since it is also the case that every element in $\tilde{\mathbf{Z}}$ is positive, there must always exist some right stochastic matrix Π^* such that $\Pi^* \geq \tilde{\mathbf{Z}}$, and that

$$\lim_{m\to\infty} \tilde{\mathbf{Z}}^m \leq \lim_{m\to\infty} (\Pi^*)^m .$$

Since Π^* is a stochastic matrix, the Perron-Frobenius theorem ensures that $\lim_{m\to\infty} (\Pi^*)^m$ exists and is also a stochastic matrix with each element bounded between 0 and 1. Then since $\chi \in (0,1)$ such that $\lim_{m\to\infty} \chi^m = 0$, we have

$$\lim_{m\to\infty} \chi^m \tilde{\mathbf{Z}}^m \leq \lim_{m\to\infty} \chi^m \lim_{m\to\infty} (\Pi^*)^m = \mathbf{0}_{N\times N},$$

where $\mathbf{0}_{N\times N}$ refers to an $N \times N$ matrix with every element equaling 0.

Then $\mathbf{I} + \sum_{m=1}^{\infty} \chi^m \tilde{\mathbf{Z}}^m = \left[\mathbf{I} - \chi\tilde{\mathbf{Z}}\right]^{-1}$, since for any $M \in \mathbb{N}$,

$$\left[\mathbf{I} + \sum_{m=1}^{M-1} \chi^m \tilde{\mathbf{Z}}^m\right] [\mathbf{I} - \chi\tilde{\mathbf{Z}}] = \mathbf{I} + \chi^M \tilde{\mathbf{Z}}^M,$$

and $\left[\mathbf{I} + \lim_{M\to\infty} \sum_{m=1}^{M-1} \chi^m \tilde{\mathbf{Z}}^m\right] \left[\mathbf{I} - \chi\tilde{\mathbf{Z}}\right] = \mathbf{I}$. We have thus shown that $\left[\mathbf{I} - \chi\tilde{\mathbf{Z}}\right]^{-1}$ exists, in which case, the inverse of $\left[\mathbf{I} - \chi\tilde{\mathbf{Z}}^\top\right]$ exists as well and equals $\sum_{m=0}^{\infty} \chi^m \left(\tilde{\mathbf{Z}}^\top\right)^m$. $\square$

Lemma 4. $\mathbf{g}(\Pi)$ *is a continuous function from* S_N^N *to* $\mathbb{R}_{++}^N$.

Proof. Since $[\mathbf{I} - \chi\mathbf{Z}]^{-1}$ is non-singular and $\mathbf{D}(L_n^M)^{-1}$ is also obviously non-singular and the vector of non-manufacturing income $\mathbf{L}(Y_n^O)$ is exogeneously given, then the vector of wages derived from trade shares $\mathbf{g}(\Pi)$ must be uniquely determined by the matrix of trade shares Π, and hence is a function (and not just a correspondence) of the matrix of trade shares Π. Since $\mathbf{g}$ is a composition of linear functions, each of which is continuous in Π, $\mathbf{g}(\Pi)$ must also be a continuous function of Π. $\square$

For any given wage vector $\mathbf{w} \in \mathbb{R}_{++}^N$, let $\mathbf{f} : \mathbb{R}_{+}^N \to S_N^N$ denote the function mapping from the wage vector into a matrix of trade shares, such that

$$\mathbf{f}(\mathbf{w}) = \mathbf{M}\Big(\pi_{ni}(\mathbf{w})\Big) = \mathbf{D}\big(p_n(\mathbf{w})^\theta\big)\mathbf{M}(d_{ni}^{-\theta})\mathbf{D}\big(T_n w_n^{-\theta\beta} p_n(\mathbf{w})\big),$$

where $p_n(\mathbf{w})$ is defined by equation 11. It should be obvious that $\mathbf{f}$ is a continuous function of $\mathbf{w}$. Noting that the element of $\mathbf{f}(\mathbf{w})$ on the n^{th} row and i^{th} column is simply $\pi_{ni}(\mathbf{w})$ as defined in Proposition 2, a quick visual comparison between the functions $\mathbf{g}$ and equation 42 would confirm that the right hand side of equation 42 is the composite function $\mathbf{g} \circ \mathbf{f}$. We can now proceed with the proof of the existence of the equilibrium vector of wages $\mathbf{w}^*$ as fixed point of equation 42.

Proposition 7. *There exists at least one fixed point* $\Pi^* \in S_N^N$ *of the function* $\mathbf{f} \circ \mathbf{g}$ *such that*

$$\Pi^* = \mathbf{f} \circ \mathbf{g}(\Pi^*).$$

There then must exist at least a vector of manufacturing wages $\mathbf{w}^* = \mathbf{g}(\Pi^*)$ *satisfying*

$$\mathbf{w}^* = \mathbf{g} \circ \mathbf{f}(\mathbf{w}^*).$$

There must therefore exist at least one equilibrium vector of wages $\mathbf{w}^*$ *satisfying Condition 4 of Definition 2. It follows immediately that there exists at least one equilibrium in the immobile scenario of the Eaton and Kortum (2002) trade model.*

Proof. Since the functions $\mathbf{g} : S_N^N \to \mathbb{R}_+^N$ and $\mathbf{f} : \mathbb{R}_+^N \to S_N^N$ are continuous, and $S_N^N \subset \mathbb{R}^{N \times N}$ is convex and compact, it follows that the composite function $\mathbf{f} \circ \mathbf{g} : S_N^N \to S_N^N$ is a continuous self-map. Hence by the Brouwer's fixed point theorem,[24] there must exist some $\Pi^* \in S_N^N$ such that $\Pi^* = \mathbf{f} \circ \mathbf{g}(\Pi^*)$, and some $\mathbf{w}^* \in \mathbb{R}_+^N$ such that $\mathbf{w}^* = \mathbf{g}(\Pi^*)$. Then $\mathbf{w}^*$ must be a fixed point of $\mathbf{g} \circ \mathbf{f}$ since $\mathbf{w}^* = \mathbf{g}(\Pi^*) = \mathbf{g}\left(\mathbf{f} \circ \mathbf{g}(\Pi^*)\right) = \mathbf{g} \circ \mathbf{f}\left(\mathbf{g}(\Pi^*)\right) = \mathbf{g} \circ \mathbf{f}(\mathbf{w}^*)$. □

Since $\mathbf{g} \circ \mathbf{f}$ is the function on the right-hand-side of equation 42, Proposition 7 demonstrates that a fixed point of equation 42 exists, and consequently, there must always exist an equilibrium of the immobile labor equilibrium.

At first glance, it might appear to be more intuitive to directly show that the function $\mathbf{g} \circ \mathbf{f}$ has a fixed point of manufacturing wages, i.e. to directly show that there exists a fixed point of manufacturing wages satisfying equation 42. The problem with that approach is that the space of 'valid' manufacturing wages is difficult to define and shown to be convex, whereas the space of matrices of expenditure shares as probability matrices, is well defined. This hopefully explains the slightly convoluted approach in this proof of the existence of the immobile labor equilibrium.

In order to analyze the general equilibrium in the immobile labor scenario, it would be important to establish the conditions under which the equilibrium is unique such that comparative statics analysis would be meaningful. The next section examines the sufficient conditions for uniqueness of the immobile labor equilibrium.

6 Uniqueness of the Immobile Labor Equilibrium

In this section, we show that the immobile labor equilibrium, as characterized by the equilibrium vector of wages, must be unique so long as certain conditions on the maximum trade costs and import tariffs rates are met. The method by which we do so is to first characterize the global economy in the immobile labor scenario as a pure exchange economy with national *manufacturing labor* and the *non-manufacturing goods* as the traded goods. We shall next examine how patterns of trade are influenced by changes in the prices of these 'goods', before obtaining proof of uniqueness of the equilibrium by showing that the set of traded goods are gross substitutes for one another.

[24] Strictly speaking, $S_N^N \subset \mathbb{R}^{N \times N}$ is not a compact and convex subset of a Euclidean space, but is instead a compact and convex subset of a vector space. In that sense, it might be more appropriate to employ the Schauder fixed point theorem for the proof, but that simply adds an additional layer of mathematical notation that is not essential to the basic proof. We might simply consider rewriting S_N^N as a subset of $\mathbb{R}^{N^2}$ where k^{th} collection of N elements represents the simplex in which the k^{th} row of the $\mathbf{M}(\pi_{ni})$ exists.

We begin by stating the sufficient conditions for a unique immobile labor equilibrium, in the following assumption.

Assumption 1. *The following 2 conditions hold jointly:*

(a) It is the case that $d_{ni}^{-\theta} d_{ij}^{-\theta} > (1-\beta) d_{nj}^{-\theta}$ *for all* $n, i, j \in \mathcal{N}$.

(b) Each country $n \in \mathcal{N}$ *imposes a uniform tariff rate on imports, regardless of source, such that* $t_{ni} = \bar{t}_n$ *for each* $i \neq n$, *and* $\bar{t}_n$ *is sufficiently small.*

In the immobile labor scenario, since each country is endowed with an exogeneously determined manufacturing labor supply L_n^M and non-manufacturing output Y_n^O, it is appropriate to think about the immobile labor Eaton-Kortum trade model as a pure exchange economy, in which each country trades the services of its manufacturing labor force and stocks of non-manufacturing goods in exchange for those of every other country.

The only prices that matters from this perspective are national manufacturing wages $\{w_n\}_{n \in \mathcal{N}}$, and the price of non-manufacturing goods p^O. The heterogeneity of national production efficiency across the set of manufacturing goods implies that different countries' manufacturing labor forces are not perfect substitutes. Furthermore, costly international trade in goods implies transaction costs for the 'trade' in labor services. Both are factors explaining why manufacturing wages might differ between countries. While the non-manufacturing good was previously designated as the numeraire good, for the purpose of this section we shall allow the common price of non-manufacturing goods across all countries to freely vary in $\mathbb{R}_{++}$.[25]

Taking the vector of national wages $\mathbf{w} = \{w_n\}_{n \in \mathcal{N}}$ and non-manufacturing price p^O as given, the national manufacturing income for each country n is $w_n L_n^M$, and the value of the national non-manufacturing endowment (or equivalently the national non-manufacturing revenue or national non-manufacturing labor income) is $p^O Y_n^O$. Total national nominal income is the sum of national labor incomes and tariff income $TR_n(\mathbf{w}, p^O) + p^O Y_n^O + w_n L_n^M$.

Since β is the proportion of national manufacturing revenues paid to labor, national manufacturing revenues would be $Q_n^R(\mathbf{w}) = \frac{w_n L_n^M}{\beta}$, and with the remainder $(1-\beta)\, Q_n^R(\mathbf{w})$ being spent on manufacturing intermediate inputs.

With $X_n(\mathbf{w}, p^O)$ being total national expenditure on manufacturing goods, $TR_n(\mathbf{w}, p^O)$ being national tariff revenues collected on imports of manufacturing goods, net payments to suppliers excluding

[25] Recall that costs of trading non-manufacturing goods between countries was assumed to be costless, in which case the law of one price must apply. The price of non-manufacturing goods must be the same in each country.

The purpose of allowing p^O to freely vary is to ensure that aggregate demand functions for each country's manufacturing labor, and non-manufacturing goods is homogeneous of degree zero in the set of prices $\{w_1, ..., w_N, p_O\}$.

More intuitively, we allow p^O to vary freely to simplify the proof showing the set of goods under discussion are gross substitutes. For example, if we set p^O to be the numeraire good with value always equalling 1, then in order to demonstrate that the non-manufacturing good is a gross substitute for each of the other countries' manufacturing labor, we need to decrease all manufacturing labor wages by the same proportion simultaneously, and demonstrate that excess demand for non-manufacturing goods decreases while excess demand for labor increases everywhere.

On the other hand, treating p^O as an 'ordinary' variable price allows a textbook proof of gross substitution. In short, allowing p^O to freely vary in this section is simply a mathematical convenience.

tariffs is $X_n(\mathbf{w}, p^O) - TR_n(\mathbf{w}, p^O)$. The national manufacturing trade deficit would simply be the difference between manufacturing net payments and revenues, $X_n(\mathbf{w}, p^O) - TR_n(\mathbf{w}, p^O) - Q_n^R(\mathbf{w}, p^O)$. Since $(1-\alpha)$ is the proportion of national income spent on non-manufacturing goods, the trade surplus in non-manufacturing trade is simply non-manufacturing revenues less domestic consumption of non-manufacturing goods, $p^O Y_n^O - (1-\alpha)\left[TR_n(\mathbf{w}) + p^O Y_n^O + w_n L_n^M\right]$. The national budget constraint implies that any trade surpluses in the manufacturing sector must be counter-balanced by trade deficits in non-manufacturing sector. The equality of national manufacturing trade deficits and non-manufacturing trade surpluses can be solved to yield

$$X_n(\mathbf{w}, p^O) = \chi Q_n^R(\mathbf{w}) + \alpha\left(TR_n(\mathbf{w}, p^O) + p^O Y_n^O\right),$$

where $\chi := 1 - \beta(1-\alpha)$.

By Theorem 1, given a vector of manufacturing wages $\mathbf{w}$, there must exist a unique corresponding matrix of trade shares $\Pi(\mathbf{w}) = \mathbf{M}\Big(\pi_{ni}(\mathbf{w})\Big)$. If Assumption 1 holds, tariff revenues could be written as

$$TR_n(\mathbf{w}) = X_n(\mathbf{w})\bar{\tau}_n\left(1 - \pi_{nn}(\mathbf{w})\right), \tag{44}$$

where $\bar{\tau}_n := \frac{\bar{t}_n}{1+\bar{t}_n}$ is the fraction of expenditures on imports collected as tariffs.

National expenditures on manufacturing goods given the vector of wages $\mathbf{w}$ would be written as

$$X_n(\mathbf{w}, p^O) = \frac{\chi Q_n^R(\mathbf{w}) + \alpha p^O Y_n^O}{1 - \alpha\bar{\tau}_n\left(1 - \pi_{nn}(\mathbf{w})\right)}.$$

Given the vector of national expenditures $\mathbf{L}\Big(X_n(\mathbf{w}, p^O)\Big)$ and matrix of trade shares $\Pi(\mathbf{w})$, the nominal demand for manufacturing goods produced in country n net of tariffs is

$$\begin{aligned} Q_n^D(\mathbf{w}, p^O) &= \sum_{i\in\mathcal{N}} \frac{\pi_{in}(\mathbf{w})}{1+\bar{t}_i} X_i(\mathbf{w}, p^O) \\ &= \sum_{i\in\mathcal{N}} \frac{\pi_{in}(\mathbf{w})}{1+\bar{t}_i}\left(\frac{\chi Q_i^R(\mathbf{w}) + \alpha p^O Y_i^O}{1 - \alpha\bar{\tau}_i\left(1 - \pi_{ii}(\mathbf{w})\right)}\right). \end{aligned} \tag{45}$$

Given a national wage vector $\mathbf{w}$ and corresponding demand for country n's manufacturing output Q_n^D, the implied demand for country n's labor is $L_n^D(\mathbf{w}, p^O) = \beta\frac{Q_n^D}{w_n}$. Substituting this, and the manufacturing revenues identity $Q_n^R(\mathbf{w}) = \frac{w_n L_n^M}{\beta}$ into equation 45, we obtain the aggregate demand for country n's labor as

$$L_n^D(\mathbf{w}, p^O) = \sum_{i\in\mathcal{N}} \frac{\pi_{in}(\mathbf{w})}{1+\bar{t}_i}\left(\frac{\chi\left(\frac{w_i}{w_n}\right)L_i^M + \alpha\beta\left(\frac{p^O}{w_n}\right)Y_i^O}{1 - \alpha\bar{\tau}_i\left(1 - \pi_{ii}(\mathbf{w})\right)}\right). \tag{46}$$

Define $F_n(\mathbf{w}, p^O) := L_n^D(\mathbf{w}, p^O) - L_n^M$ as the aggregate excess demand function for each country's manufacturing labor. The competitive equilibrium in the pure exchange economy requires that the equilibrium wage vector $\mathbf{w}^*$ and non-manufacturing price p^{O*} satisfy $F_n(\mathbf{w}^*, p^{O*}) = 0$ for all countries $n \in \mathcal{N}$. Demand for each country's manufacturing labor equals its endowment, and all national labor markets clear.

We next consider the excess demand function for non-manufacturing goods. Given a manufacturing wage vector $\mathbf{w}$, total global nominal demand for non-manufacturing goods is

$$\sum_{n\in\mathcal{N}} C_n^O(\mathbf{w}, p^O) = (1-\alpha)\sum_n \left(TR_n(\mathbf{w}) + w_n L_n^M + p^O Y_n^O\right).$$

By equations 44 and 45, re-express the above equation as

$$\sum_{n\in\mathcal{N}} C_n^O\left(\mathbf{w}, p^O\right) = (1-\alpha)\sum_{n\in\mathcal{N}} \left[\frac{\bar{\tau}_n\left(1-\pi_{nn}(\mathbf{w})\right)}{1-\alpha\bar{\tau}_n\left(1-\pi_{nn}(\mathbf{w})\right)}\left(\frac{\chi}{\beta}w_n L_n^M + \alpha p^O Y_n^O\right) + w_n L_n^M + p^O Y_n^O\right].$$

The aggregate excess demand function for real quantities of non-manufacturing goods is simply global real demand less real output of non-manufacturing goods,

$$F_O(\mathbf{w}, p^O) = \sum_{n\in\mathcal{N}} \frac{C_n^O(\mathbf{w})}{p^O} - \sum_{n\in\mathcal{N}} Y_n^O, \tag{47}$$

and in the competitive equilibrium, $F_O(\mathbf{w}^*, p^{O*}) = 0$.

We shall now demonstrate that if Assumption 1 holds, then the aggregate excess demand functions in this pure exchange economy exhibit the gross substitute property, in which case there any equilibrium must be unique.

A set of goods are gross substitutes if whenever the price of one good is increased, aggregate demand for that good decreases, and the aggregate demands for all other goods are increased.

In the setting of the Eaton-Kortum immobile labor trade model, the set of 'goods' consists of the national manufacturing labor forces, and the set of prices are national manufacturing wages. To demonstrate that the gross substitute property holds, it simply has to be shown that for any arbitrary vector of national wages $\mathbf{w} = \{w_1, ..., w_N\}$, and any country $k \in \mathcal{N}$, the aggregate excess demand for country k's labor is strictly decreasing with country k's wages w_k, while aggregate excess demand for every other country's labor is strictly increasing, and that in response to an increase in non-manufacturing price p^O, demand for every country's manufacturing labor increases, while aggregate demand for the non-manufacturing good decreases.

We shall rely on the two results presented below.

Proposition 8. *In the immobile labor scenario, given any vector of manufacturing wages* $\mathbf{w}$*, the following statements hold:*

1. *The derivative of trade shares with respect to country k's wages is given as*

$$\frac{\partial \ln \pi_{ni}}{\partial \ln w_k} = \begin{cases} \theta\left(\frac{\partial \ln p_n}{\partial \ln w_k} - (1-\beta)\frac{\partial \ln p_i}{\partial \ln w_k}\right) & \forall i \neq k \\ \theta\left(\frac{\partial \ln p_n}{\partial \ln w_k} - (1-\beta)\frac{\partial \ln p_i}{\partial \ln w_k} - \beta\right) & otherwise, \end{cases} \tag{48}$$

2. *The Jacobian of the manufacturing price vector* $\mathbf{p}(\mathbf{w})$ *with respect to the manufacturing wage*

vector $\mathbf{w}$ *is given as:*

$$\mathbf{M}\Big(\frac{\partial \ln p_n(\mathbf{w})}{\partial \ln w_i}\Big) = \beta \left[\mathbf{I} - (1-\beta)\,\Pi(\mathbf{w})\right]^{-1} \Pi(\mathbf{w}). \tag{49}$$

3. *For all* $n, i \in \mathcal{N}$,

$$\min_i \pi_{ik} < \frac{\partial \ln p_n(\mathbf{w})}{\partial \ln w_k} < \pi_{kk}. \tag{50}$$

4. *For any* $k \in \mathcal{N}$,

   (a) $\frac{\partial \ln \pi_{nk}}{\partial \ln w_k} < 0$ *for every country* $n \in \mathcal{N}$*, and*

   (b) $\frac{\partial \ln \pi_{nn}}{\partial \ln w_k} > 0$ *for every country* $n \neq k$*.*

Furthermore, if Assumption 1(a) holds, it will be the case that

5. $\frac{\partial \ln \pi_{ni}}{\partial \ln w_k} > 0$*,for each country* $i \neq k$*, and every country* $n \in \mathcal{N}$*.*

Proof. (Item 1) For each $n, j \in \mathcal{N}$, the natural logarithm of country n's share of expenditure spent on imports from country j could be written as a function of manufacturing wages as $\ln \pi_{nj}(\mathbf{w}) = \ln T_j d_{nj}^{-\theta} + \theta\left[\ln p_n(\mathbf{w}) - (1-\beta)\ln p_j(\mathbf{w}) - \beta \ln w_j\right]$. Noting that technology level T_j and trade costs d_{nj} are exogeneously given constants, the derivative of $\ln \pi_{nj}$ with respect to $\ln w_i$ for any $i \in \mathcal{N}$ is

$$\frac{\partial \ln \pi_{nj}(\mathbf{w})}{\partial \ln w_i} = \theta\left[\frac{\partial \ln p_n(\mathbf{w})}{\partial \ln w_i} - (1-\beta)\frac{\partial \ln p_j(\mathbf{w})}{\partial \ln w_i} - \beta\frac{\partial \ln w_j}{\partial \ln w_i}\right]. \tag{51}$$

Since $\frac{\partial \ln w_j}{\partial \ln w_i} = 0$ if $j \neq k$, and 1 otherwise, we obtain Item 1 of the proposition.
(Item 2) Since $\sum_{j=1}^{N} \pi_{nj}(\mathbf{w}) = 1$ for any $n \in \mathcal{N}$, taking derivatives with respect to $\ln w_i$ on both sides gives

$$\begin{aligned}
0 &= \sum_{n=1}^{N} \pi_{nj}\frac{\partial \ln \pi_{nj}(\mathbf{w})}{\partial \ln w_i} \\
&= \theta\sum_{n=1}^{N} \pi_{nj}\left[\frac{\partial \ln p_n(\mathbf{w})}{\partial \ln w_i} - (1-\beta)\frac{\partial \ln p_j(\mathbf{w})}{\partial \ln w_i} - \beta\frac{\partial \ln w_j}{\partial \ln w_i}\right] \\
&= \theta\left[\frac{\partial \ln p_n(\mathbf{w})}{\partial \ln w_i} - (1-\beta)\sum_{j=1}^{N} \pi_{nj}\frac{\partial \ln p_j(\mathbf{w})}{\partial \ln w_i} - \beta\pi_{ni}\right]. \quad (*)
\end{aligned}$$

Multiplying both sides of $(*)$ by $\frac{1}{\theta}$ and collecting into matrix form, we obtain

$$\left[\mathbf{I} - (1-\beta)\,\Pi(\mathbf{w})\right]\mathbf{M}\Big(\frac{\partial \ln p_n}{\partial \ln w_i}\Big) = \beta\Pi(\mathbf{w}).$$

By a similar argument as Lemma 3, since $\lim_{m\to\infty} \Pi(\mathbf{w})$ is bounded and $\lim_{m\to\infty}(1-\beta)^m = 0$, it must be the case that $\lim_{m\to\infty}\left[(1-\beta)\,\Pi\right]^m = \mathbf{0}_{N\times N}$ and the inverse matrix $\left[\mathbf{I} - (1-\beta)\,\Pi\right]^{-1} =$

$\sum_{m=0}^{\infty}[(1-\beta)\Pi]^m$ exists and we can write

$$\mathbf{M}\Big(\frac{\partial \ln p_n}{\partial \ln w_i}\Big) = \beta\left[\mathbf{I}-(1-\beta)\,\Pi(\mathbf{w})\right]^{-1}\Pi(\mathbf{w}).$$

This completes the proof for Item 2.

(Item 3) We now demonstrate the bounds of $\frac{\partial \ln p_n}{\partial \ln w_i}$.

Since the k^{th} column of any $N \times N$ matrix can be extracted by post-multiplying it with an N dimensional k^{th} unit vector (notated as ι_k),[26] post-multiplying both sides of equation 49 by ι_k yields $\mathbf{L}(\frac{\partial \ln p_n}{\partial \ln w_k}) = \beta\left[\mathbf{I}-(1-\beta)\,\Pi\right]^{-1}\Pi_{*k}$, where $\Pi_{*k} = \mathbf{L}(\pi_{nk})$ is the k^{th} column of the matrix of trade shares.

The matrix $\left[\mathbf{I}-(1-\beta)\,\Pi\right]^{-1} = \sum_{m=0}^{\infty}(1-\beta)^m\,\Pi^m$ is positive since Π as a stochastic matrix has positive elements, in which case since $\min_{j\in\mathcal{N}}\pi_{ik}\mathbf{1}_N \le \Pi_{*k} \le \max_{i\in\mathcal{N}}\pi_{ik}\mathbf{1}_N$, we have

$$\beta\min_{i\in\mathcal{N}}\pi_{ik}\left[\mathbf{I}-(1-\beta)\,\Pi\right]^{-1}\mathbf{1}_N \le \mathbf{L}\Big(\frac{\partial \ln p_n}{\partial \ln w_k}\Big) \le \beta\max_{i\in\mathcal{N}}\pi_{ik}\left[\mathbf{I}-(1-\beta)\,\Pi\right]^{-1}\mathbf{1}_N.$$

As a right stochastic matrix $\Pi\mathbf{1}_N = \mathbf{1}_N$ hence $\left[\mathbf{I}-(1-\beta)\,\Pi\right]\mathbf{1}_N = \beta\mathbf{1}_N$. Pre-multiplying the relationship with $\left[\mathbf{I}-(1-\beta)\,\Pi\right]^{-1}$ shows that that $\left[\mathbf{I}-(1-\beta)\,\Pi\right]^{-1}\mathbf{1}_N = \frac{1}{\beta}\mathbf{1}_N$, and we obtain $\min_{i\in\mathcal{N}}\pi_{ik}\mathbf{1}_N \le \mathbf{L}\Big(\frac{\partial \ln p_n}{\partial \ln w_k}\Big) \le \max_{i\in\mathcal{N}}\pi_{ik}\mathbf{1}_N$, or equivalently, $\frac{\partial \ln p_n}{\partial \ln w_k} \in [\min_{i\in\mathcal{N}}\pi_{ik}, \max_{i\in\mathcal{N}}\pi_{ik}]$.

Finally, from Proposition 6 we obtain $\pi_{kk} = \max_{i\in\mathcal{N}}\pi_{ik}$ as required in establishing the upper bound of $\frac{\partial \ln p_n}{\partial \ln w_k}$. This completes the proof for Item 3 of the proposition.

(Item 4a) From equation $(*)$, we have for each country $n, k \in \mathcal{N}$,

$$\begin{aligned}\frac{\partial \ln p_n}{\partial \ln w_k} &= \beta\pi_{nk} + (1-\beta)\sum_{j\in\mathcal{N}}\pi_{nj}\frac{\partial \ln p_j}{\partial \ln w_k} \\ &= \pi_{nk}\left(\beta + (1-\beta)\frac{\partial \ln p_k}{\partial \ln w_k}\right) + \sum_{j\neq k}\pi_{nj}\frac{\partial \ln p_j}{\partial \ln w_k}. \quad (*)\end{aligned}$$

Pick $n \in \mathcal{N}$ such that $\frac{\partial \ln p_n}{\partial \ln w_k} = \max_{j\neq k}\frac{\partial \ln p_j}{\partial \ln w_k}$, in which case we have

$$\sum_{j\neq k}\pi_{nj}\frac{\partial \ln p_j}{\partial \ln w_k} < \sum_{j\neq k}\pi_{nj}\frac{\partial \ln p_n}{\partial \ln w_k} = (1-\pi_{nk})\frac{\partial \ln p_n}{\partial \ln w_k}.$$

Substituting the inequality back into the equation $(**)$, and bringing all terms with $\frac{\ln p_n}{\ln w_k}$ to the left hand side and dividing throughout by π_{nk} yields

$$\frac{\partial \ln p_n}{\partial \ln w_k} < \beta + (1-\beta)\frac{\partial \ln p_k}{\partial \ln w_k}.$$

Since $\frac{\partial \ln p_n}{\partial \ln w_k} = \max_{j\neq k}\frac{\partial \ln p_j}{\partial \ln w_k}$ by assumption, it must be the case that $\frac{\partial \ln p_n}{\partial \ln w_k} < \beta + (1-\beta)\frac{\partial \ln p_k}{\partial \ln w_k}$ for all countries $n \neq k$, and by equation 48, $\frac{\partial \ln \pi_{nk}}{\partial \ln w_k} < 0$ for all $n \neq k$.

[26] An N dimensional vector with the k^{th} element being 1, and 0 everywhere else. An example would be the k^{th} column of an N dimensional identity matrix.

From Item 3 of the proposition, we have $\frac{\partial \ln p_k}{\partial \ln w_k} < \pi_{kk} < 1$, hence $\frac{\partial \ln \pi_{kk}}{\partial \ln w_k} = \beta\theta\left(\ln p_k - 1\right) < 0$. We have thus shown that $\frac{\partial \ln \pi_{nk}}{\partial \ln w_k} < 0$ for all $n \in \mathcal{N}$. Item 4(a) of the proposition has thus been demonstrated.

(Item 4b) For $n \neq k$, $\frac{\partial \ln \pi_{nn}}{\partial \ln w_k} = \beta\theta \frac{\partial \ln p_n}{\partial \ln w_k} > 0$ since by Item 3 of the proposition $\frac{\partial \ln p_n}{\partial \ln w_k} > 0$ and the prices are always increasing in wages. This gives Item 4(b) of the proposition.

(Item 5) Pick country l such that $\pi_{lk} := \min_{i\in\mathcal{N}} \pi_{ik}$. By Item 3 of the proposition, $\frac{\partial \ln p_n}{\partial \ln w_k} > \pi_{lk}$ and $\frac{\partial \ln p_i}{\partial \ln w_k} < \pi_{kk}$ for all $n, i \in \mathcal{N}$. Hence

$$\begin{aligned}
\frac{\partial \ln p_n}{\partial \ln w_k} - (1-\beta)\frac{\partial \ln p_i}{\partial \ln w_k} &> \pi_{kk}\left(\frac{\pi_{lk}}{\pi_{kk}} - (1-\beta)\right) \\
&= \pi_{kk}\left(\frac{d_{lk}^{-\theta} p_k^{-\theta}}{p_l^{-\theta}} - (1-\beta)\right) \quad (*)
\end{aligned}$$

With $d_{lk}^{-\theta} p_k^{-\theta} = \sum_{i\in\mathcal{N}} T_i (w_i^{\beta} p_i^{1-\beta})^{-\theta} d_{lk}^{-\theta} d_{ki}^{-\theta}$ and $p_l^{-\theta} = \sum_{i\in\mathcal{N}} T_i (w_i^{\beta} p_i^{1-\beta})^{-\theta} d_{li}^{-\theta}$, if Assumption 1(a) holds such that $d_{lk}^{-\theta} d_{ki}^{-\theta} > (1-\beta)\, d_{li}^{-\theta}$ for all $i \in \mathcal{N}$, then we must have $d_{lk}^{-\theta} p_k^{-\theta} > (1-\beta)\, p_l^{-\theta}$ and

$$\frac{d_{lk}^{-\theta} p_k^{-\theta}}{p_l^{-\theta}} > (1-\beta)\,.$$

Substituting this back into equation $(*)$ shows that $\frac{\partial \ln p_n}{\partial \ln w_k} - (1-\beta)\frac{\partial \ln p_i}{\partial \ln w_k} > 0$ for all n, i, and that

$$\frac{\partial \ln \pi_{ni}}{\partial \ln w_k} = \theta\left(\frac{\partial \ln p_n}{\partial \ln w_k} - (1-\beta)\frac{\partial \ln p_i}{\partial \ln w_k}\right) > 0$$

for all $i \neq k$. This concludes the proof of Item 5 and of the proposition. □

The key items of interest in Proposition 8 are Items 2, 4 and 5. Whenever some country $k's$ wages increases, Item 4(a) tells us that the country's output always becomes less competitive in both domestic and foreign markets. Every country including country k itself would choose to spend a lower proportion of their expenditures on manufacturing goods from k. Item 4(b) says that whenever some other country's manufacturing wages increases, manufacturers in every other country would become more competitive in their domestic markets.

Finally Item 5 shows that Assumption 1(a) ensures that an increase in country k's wages would impose no negative externalities on other countries' competitiveness in trade of manufacturing goods. The economic intuition behind Item 5 is that Assumption 1(a), which could alternatively be specified as $d_{ni}d_{ij} \in [d_{nj}, (1-\beta)^{-\frac{1}{\theta}}\, d_{nj})$, places an upper bound on the possible heterogeneity of trade costs, which in turn limits how greatly expenditure shares on imports from one specific country might differ.

An interesting though tangential implication of Item 2 of Proposition 8 is that changes in national manufacturing price of country n - in reaction to the increase in manufacturing wages in country k - is driven by changes in the costs of imported and domestically produced intermediate goods $\{c_i\}_{i\in\mathcal{N}}$, in proportion to the share of the country's expenditures on imports from each source $\{\pi_{ni}\}_{i\in\mathcal{N}}$, as evidenced by $\frac{\partial \ln p_n}{\partial \ln w_k} = \sum_{i\in\mathcal{N}} \pi_{ni} \frac{\partial \ln c_i}{\partial \ln w_k}$. Another implication, is that the country increasing wages would experience the greatest increase in manufacturing costs, such that $(1-\beta)\frac{\partial \ln c_k}{\partial \ln w_k} > \frac{\partial \ln c_n}{\partial \ln w_k}$ for all $n \neq k$.[27]

Finally, to the focus of this section, the uniqueness of the immobile labor equilibrium.

Theorem 2. *Suppose Assumption 1 holds. Then the countries' manufacturing labor forces and the non-manufacturing good are gross substitutes such that*

1. $\frac{\partial F_k(\mathbf{w})}{\partial w_k} < 0$ *for all* $k \in \mathcal{N}$.
2. $\frac{\partial F_n(\mathbf{w})}{\partial w_k} > 0$ *for all* $n \neq k$.
3. $\frac{\partial F_n(\mathbf{w})}{\partial p^O} > 0$ *for all* $n \in \mathcal{N}$.
4. $\frac{\partial F_O(\mathbf{w})}{\partial p^O} < 0$.

Hence the immobile labor scenario of the Eaton-Kortum trade model has an unique equilibrium vector of manufacturing wages in which non-manufacturing goods serve as the numeraire.

Proof. We shall first establish that that gross substitutability property holds by showing that Items 1 to 4 are true if Assumption 1 holds. The proof that gross substitutability implies uniqueness of

[27]To see that these two claims hold, first let $\mathbf{Z} := [\mathbf{I} - (1-\beta)\Pi]^{-1}$. Then Item 2 of Proposition 8 could be stated as

$$\mathbf{M}\Big(\frac{\partial \ln p_n}{\partial \ln w_i}\Big) = \beta \mathbf{Z}\Pi = \beta \Pi \mathbf{Z}, \quad (*)$$

with the second equality since $\mathbf{Z}\Pi = \sum_{m=1}^{\infty}(1-\beta)^m \Pi^{(m+1)} = \Pi\mathbf{Z}$.

Since $[\mathbf{I} - (1-\beta)\Pi]\mathbf{Z} = \mathbf{I}$, we have $\mathbf{Z} - \mathbf{I} = (1-\beta)\Pi\mathbf{Z}$, in which case we can rewrite the above as

$$\mathbf{M}\Big(\frac{\partial \ln p_n}{\partial \ln w_i}\Big) = \frac{\beta}{1-\beta}[\mathbf{Z} - \mathbf{I}]. \quad (**)$$

Cost of producing manufacturing goods in each country $n \in \mathcal{N}$ is given as $c_n = w_n^{\beta} p_n^{1-\beta}$, hence $\frac{\partial \ln c_n}{\partial \ln w_i} = \beta \frac{\partial \ln w_n}{\partial \ln w_i} + (1-\beta)\frac{\partial \ln p_n}{\partial \ln w_i}$, which in matrix form gives $\mathbf{M}\big(\frac{\partial \ln c_n}{\partial \ln w_i}\big) = \beta\mathbf{I} + (1-\beta)\mathbf{M}\big(\frac{\partial \ln p_n}{\partial \ln w_i}\big)$. Comparison with equation $(**)$ above makes it obvious that

$$\mathbf{M}\big(\frac{\partial \ln c_n}{\partial \ln w_i}\big) = \beta\mathbf{Z}, \quad (***)$$

which in turn yields

1. $\mathbf{M}\Big(\frac{\partial \ln p_n}{\partial \ln w_i}\Big) = \beta\mathbf{Z}\Pi = \Pi\mathbf{M}\Big(\Big(\frac{\partial \ln c_n}{\partial \ln w_i}\Big)$ from $(*)$ and $(***)$. This gives for each $n, k \in \mathcal{N}$, $\frac{\partial \ln p_n}{\partial \ln w_k} = \sum_{j\in\mathcal{N}} \pi_{nj}\frac{\partial \ln c_j}{\partial \ln w_k}$.
2. $(1-\beta)\frac{\partial \ln c_k}{\partial \ln w_k} \geq \frac{\partial \ln c_n}{\partial \ln w_k}$. To see this, post-multiply both side of equation $(***)$ with ι_k, the k^{th} unit vector, to obtain $\mathbf{L}\big(\frac{\partial \ln c_n}{\partial \ln w_k}\big) = \beta\mathbf{L}(z_{nk})$ where z_{nk} refers to the $(n,k)^{th}$ element of $\mathbf{Z}$ and $\mathbf{L}(z_{nk})$ is the k^{th} column of $\mathbf{Z}$. Since $[\mathbf{I} - (1-\beta)\Pi]\mathbf{Z} = \mathbf{I}$ by the definition of $\mathbf{Z}$ as the inverse of $[\mathbf{I} - (1-\beta)\Pi]$, we have $[\mathbf{I} - (1-\beta)\Pi]\mathbf{L}(z_{nk}) = \iota_k$ or equivalently,

$$z_{nk} - (1-\beta)\sum_{j\in\mathcal{N}} \pi_{nj} z_{jk} = \begin{cases} 0 & \forall n \neq k \\ 1 & otherwise. \end{cases}$$

   Suppose that $z_{nk} = \max_{j\in\mathcal{N}} z_{jk}$ for some $n \neq k$. Then $0 = z_{nk} - (1-\beta)\sum_{j\in\mathcal{N}}\pi_{nj}z_{jk} \geq \beta z_{nk}$, with the inequality since $\sum_{j\in\mathcal{N}}\pi_{nj}z_{jk} \leq \max_{j\in\mathcal{N}} z_{jk}$. But $\mathbf{Z} = \sum_{m=0}^{\infty}(1-\beta)^m\Pi^{(m)} > \mathbf{0}_{N\times N}$ since $\Pi \gg \mathbf{0}_{N\times N}$ implies that $\beta z_{nk} > 0$. and we obtain a contradiction. So it must be the case that $z_{kk} = \max_{j\in\mathcal{N}} z_{jk}$, and it follows immediately that for all $n \neq k$, $z_{nk} = (1-\beta)\sum_{j\in\mathcal{N}}\pi_{nj}z_{jk} \leq (1-\beta)z_{kk}$. The rest follows from $\frac{\partial \ln c_n}{\partial \ln w_k} = \beta z_{nk}$.

the equilibrium is subsequently presented for completeness. For the sake of brevity in notation, let $Z_i(\mathbf{w}, \bar{\tau}_i) := 1 - \alpha\bar{\tau}_i\,(1 - \pi_{ii}(\mathbf{w}))$.

Since the excess demand function for each country's labor force is $F_n(\mathbf{w}, p^O) = L_n^D(\mathbf{w}, p^O) - L_n^M$ where L_n^M is an exogeneously given manufacturing labor endowment, the excess demand function varies only with demand for manufacturing labor, such that $\frac{\partial F_n(\mathbf{w},p^O)}{\partial w_k} = \frac{\partial L_n^D(\mathbf{w},p^O)}{\partial w_k}$ for $n \in \mathcal{N}$.

(Item 1) We first examine the effect on 'own' manufacturing demand. From equation 46, we can write

$$L_k^D = \frac{1}{w_k}\sum_{i \neq k}^{N} \frac{\pi_{ik}(\mathbf{w})}{1+\bar{t}_i}\frac{\left(\chi w_i L_i^M + \alpha\beta p^O Y_i^O\right)}{Z_i(\mathbf{w},\bar{\tau}_i)} + \frac{\pi_{kk}(\mathbf{w})}{Z_k(\mathbf{w},\bar{\tau}_k)}\left(\chi L_k^M + \alpha\left(\frac{p^O}{w_k}\right)Y_k^O\right). \quad (*)$$

From Proposition 8, we have $\frac{\partial \pi_{ik}(\mathbf{w})}{\partial w_k} < 0$ for all $i \in \mathcal{N}$ hence the first term on the right hand side of equation $(*)$ is strictly decreasing in w_k. Economically speaking, when country k's manufacturing wages increase while every other country's remains constant, the fact that $\frac{\partial \pi_{ii}}{\partial w_k} > 0$ implies that every other country $i \neq k$ reduces imports as a share of manufacturing expenditures, which in turn reduces national tariff revenues. Since manufacturing and non-manufacturing income, $w_i L_i^M$ and Y_i^O respectively, for these countries remains unchanged, the net effect is that for every country other than k, nominal expenditures on manufacturing goods decrease. This decrease in manufacturing expenditures, combined with the decrease in country k's competitiveness (as evidenced by $\frac{\partial \pi_{ik}}{\partial w_k} < 0$) results in a decrease in country k's nominal *exports*, which taken together with an increase in manufacturing wages w_k means that aggregate foreign demand for country k's labor, written as $\frac{1}{w_k}\sum_{i\neq k}^{N}\frac{\pi_{ik}(\mathbf{w})}{1+\bar{t}_i}\frac{\left(\chi w_i L_i^M+\alpha\beta Y_i^O\right)}{1-\alpha\bar{\tau}_i(1-\pi_{ii}(\mathbf{w}))}$, must decrease.

What about domestic demand for country k's labor? Since $\frac{\partial \pi_{kk}}{\partial w_k} < 0$ and $\frac{\alpha\bar{\tau}_k\pi_{kk}}{Z_k(\mathbf{w},\bar{\tau}_k)} < 1$ implies that

$$\begin{aligned}\frac{\partial}{\partial w_k}\frac{\pi_{kk}(\mathbf{w})}{Z_k(\mathbf{w},\bar{\tau}_k)} &= \frac{\frac{\partial \pi_{kk}}{\partial w_k}}{Z_k(\mathbf{w},\bar{\tau}_k)}\left(1 - \frac{\alpha\bar{\tau}_k\pi_{kk}}{Z_k(\mathbf{w},\bar{\tau}_k)}\right)\\ &< 0,\end{aligned}$$

and the term $\alpha\frac{Y_k^O}{w_k}$ is obviously decreasing in w_k, the second term in equation $(*)$ interpreted as domestic demand for country k's labor must also be decreasing. The economic interpretation is slightly more complicated than is for the case of the other countries. While it is true that $\frac{\partial \pi_{kk}}{\partial w_k} < 0$ implies that country k producers lose domestic market share, reducing demand for local labor, the decrease in the share of domestic production in manufacturing expenditures also implies an increase in imports and consequently an increase in tariff revenues, which along with an increase in manufacturing income $w_k L_k^M$, exerts an upwards force on nominal expenditures. However, the increase in tariff revenue is proportionately smaller than the loss of domestic market share (as seen from $\frac{\alpha\bar{\tau}_k\pi_{kk}}{1-\alpha\bar{\tau}_k(1-\pi_{kk})} < 1$) hence the net effect on domestic demand for local manufacturing labor is negative. Then since an increase in country k's wages decreases both foreign and domestic demand for country k's manufacturing labor, we have

$$\frac{\partial L_k^D}{\partial w_k} < 0,$$

which gives us Item 1 of the theorem.

(Item 2) We now consider the effect of an increase of w_k on demand for other countries' labor. Demand for manufacturing labor of country $n \neq k$, is written as

$$L_k^D = \frac{1}{w_n}\left(\pi_{nn}(\mathbf{w})\frac{\chi w_n L_n^M + \alpha\beta Y_n^O}{Z_n(\mathbf{w},\bar{\tau}_n)} + \frac{\pi_{kn}(\mathbf{w})}{1+\bar{t}_k}\frac{\chi w_k L_k^M + \alpha Y_k^O}{Z_k(\mathbf{w},\bar{\tau}_k)} + \sum_{i\neq k,n}^{N}\frac{\pi_{in}(\mathbf{w})}{1+\bar{t}_i}\frac{\chi w_i L_i^M + \alpha\beta Y_i^O}{Z_i(\mathbf{w},\bar{\tau}_i)}\right), \quad (**)$$

where the first term on the right hand side represents domestic demand for country n's manufacturing labor, the second term demand from country k, and the final term aggregate demand from all other countries.

Since country n labor income, manufacturing and non-manufacturing alike are unchanged, and $\frac{\partial}{\partial w_k}\frac{\pi_{nn}(\mathbf{w})}{Z_n(\mathbf{w},\bar{\tau}_n)} = \frac{\partial\pi_{nn}(\mathbf{w})}{\partial w_k}\frac{1}{Z_n(\mathbf{w},\bar{\tau}_n)}\left(1-\frac{\alpha\bar{\tau}_n}{Z_n(\mathbf{w},\bar{\tau}_n)}\right) > 0$ as $\frac{\partial\pi_{nn}(\mathbf{w})}{\partial w_k} > 0$, it is clear that domestic demand for country n's manufacturing labor is increasing with w_k.

For all other countries $i \neq n$, consider the term $\frac{\pi_{in}(\mathbf{w})}{1-\alpha\bar{\tau}_i(1-\pi_{ii}(\mathbf{w}))}$. It would now be argued that if tariff rates are sufficiently small, i.e. if $\bar{\tau}_i$ is sufficiently small, then $\frac{\partial}{\partial w_k}\frac{\pi_{in}(\mathbf{w})}{1-\alpha\bar{\tau}_i(1-\pi_{ii}(\mathbf{w}))} > 0$. Expanding the partial derivative, we have

$$\frac{\partial}{\partial w_k}\frac{\pi_{in}(\mathbf{w})}{1-\alpha\bar{\tau}_i\left(1-\pi_{ii}(\mathbf{w})\right)} = \frac{1}{Z_i(\mathbf{w},\bar{\tau}_i)}\left(\frac{\partial\pi_{in}(\mathbf{w})}{\partial w_k} - \frac{\alpha\bar{\tau}_i\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w})}{\partial w_k}\right),$$

where $Z_i(\mathbf{w},\bar{\tau}_i) := 1-\alpha\bar{\tau}_i\left(1-\pi_{ii}(\mathbf{w})\right)$.

For each arbitrary $\mathbf{w} \in \mathbb{R}^N_{++}$, since $1-\pi_{ii}(\mathbf{w})$ is bounded below 1, we have $\lim_{\bar{\tau}_i\to 0} Z_i(\mathbf{w},\bar{\tau}_i) = 1$, so it must be the case that $\lim_{\bar{\tau}_i\to 0}\frac{\alpha\bar{\tau}_i\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)} = 0$. Since trade shares are dependent not only on manufacturing wages, but also on import tariffs (through trade costs), we can write trade shares as a function of both wages and country i's import tariff, $\pi_{in}(\mathbf{w},\bar{\tau}_i)$. It is easily shown that $\pi_{in}(\mathbf{w},\bar{\tau}_i)$ is a continuous function of $\bar{\tau}_i$, in which case $\frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)}$ would also be a continuous function of $\bar{\tau}_i$.

Since $\frac{\partial\pi_{in}(\mathbf{w},\bar{\tau}_i)}{\partial w_k} > 0$, $\lim_{\bar{\tau}_i\to 0}\frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)} = 0$, and $\frac{\partial\pi_{in}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$ and $\frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)}$ are both continuous in $\bar{\tau}_i$, we must have $\frac{\partial\pi_{in}(\mathbf{w},\bar{\tau}_i)}{\partial w_k} - \frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k} > 0$ for all $\bar{\tau}_i > 0$ sufficiently small.[28] For each vector of manufacturing wages $\mathbf{w} \in \mathbb{R}^N_{++}$, let $\bar{\tau}_i(\mathbf{w})^* := \sup\left\{\bar{\tau}_i \in \mathbb{R}_{++} : \frac{\partial\pi_{in}(\mathbf{w})}{\partial w_k} - \frac{\alpha\bar{\tau}_i\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w})}{\partial w_k} > 0\right\}$, and let $\bar{\tau}_i^* := \min_{\mathbf{w}\in\mathbb{R}^N_{++}}\{\bar{\tau}_i(\mathbf{w})^*\}$. It must then be the case that for all $\mathbf{w} \in \mathbb{R}^N_{++}$, $\frac{\partial}{\partial w_k}\frac{\pi_{in}(\mathbf{w})}{Z_i(\mathbf{w},\bar{\tau}_i)} > 0$ for all

[28] First note that since $\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$ is bounded, it must be that $\frac{\alpha\bar{\tau}_i\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k} = 0$ for $\bar{\tau}_i = 0$. Furthermore, it is the case that $\frac{\alpha\bar{\tau}_i\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$ is a continuous function of $\bar{\tau}_i$, since $\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$ is ultimately a linear and hence continuous function of expenditure shares, and expenditure shares are continuous functions of tariff rates. By the same reasoning, $\frac{\partial\pi_{in}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$ is also a continuous function of $\bar{\tau}_i$, and so would $A_i(\mathbf{w},\bar{\tau}_i) := \frac{\partial\pi_{in}(\mathbf{w},\bar{\tau}_i)}{\partial w_k} - \frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$. (The notation A_i is applicable only to this footnote.)

Suppose there exists some $\bar{\tau}_i^*$ such that $\frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$ is non-increasing for all $\bar{\tau}_i \in [0,\bar{\tau}_i^*)$. Then since $\frac{\partial\pi_{in}(\mathbf{w},\bar{\tau}_i)}{\partial w_k} > 0$ and $\frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k} \leq 0$ for all $\bar{\tau}_i \in [0,\bar{\tau}_i^*)$, it must be the case $A_i(\mathbf{w},\bar{\tau}_i) > 0$ for all $\bar{\tau}_i \in [0,\bar{\tau}_i^*)$ and we obtain the desired result.

Suppose on the other hand that $\frac{\alpha\bar{\tau}_i\pi_{in}(\mathbf{w},\bar{\tau}_i)}{Z_i(\mathbf{w},\bar{\tau}_i)}\frac{\partial\pi_{ii}(\mathbf{w},\bar{\tau}_i)}{\partial w_k}$ is increasing for all $\bar{\tau}_i \in [0,\bar{\tau}_i^*)$, and that $A_i(\mathbf{w},\bar{\tau}_i^*) < 0$. Then since $A_i(\mathbf{w},0) = 0$, the continuity of A_i in $\bar{\tau}_i$ implies that there must exist some $\bar{\tau}_i^{**} \in (0,\bar{\tau}_i^*)$ such that $A_i(\mathbf{w},\bar{\tau}_i) > 0$ for all $\bar{\tau}_i \in (0,\bar{\tau}_i^{**})$. Once again, we obtain the required result that $A_i(\mathbf{w},\bar{\tau}_i) > 0$ for all $\bar{\tau}_i$ sufficiently small.

$\bar{\tau}_i \in [0, \bar{\tau}_i^*)$.

Since $\frac{\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)}\left(\chi w_n L_n + \alpha\beta Y_n^O\right)$ could be interpreted as country n's nominal expenditure on country i's manufacturing labor, and $\chi w_n L_n + \alpha\beta Y_n^O$ is country n's nominal expenditure on manufacturing labor (regardless of source) driven solely by national labor income, the term $\frac{1}{w_i}\frac{\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)}$ could be interpreted as the multiplier that maps from country n's total national labor income to demand for country i's manufacturing labor. For every country $i \in \mathcal{N}$, we have shown that the multiplier for country n's manufacturing labor $\frac{1}{w_n}\frac{\pi_{in}}{Z_i(\mathbf{w},\bar{\tau}_i)}$ is strictly increasing in w_k for tariff rates $\bar{\tau}_i$ sufficiently small. This combined with the fact that national labor incomes in all countries is non-decreasing (i.e. $\frac{\partial}{\partial w_k}\left(\chi w_i L_i + \alpha\beta Y_i^O\right) \geq 0$ for all countries $i \in \mathcal{N}$) yields $\frac{\partial}{\partial w_k}\frac{\pi_{in}(\mathbf{w})}{w_n}\frac{\chi w_i L_i^M + \alpha\beta Y_i^O}{Z_i(\mathbf{w},\bar{\tau}_i)} > 0$, which in turn immediately gives the desired result that demand for every country other than k's manufacturing labor increases with country k's wage rate, or equivalently $\frac{\partial L_n^D}{\partial w_k} > 0$, for all countries $n \neq k$.

(Item 3) Bearing in mind that own trade shares $\pi_{ii}(\mathbf{w})$ and $Z_i(\mathbf{w},\bar{\tau}_i)$ does not depend on non-manufacturing price p^O, a visual examination of equation 46 immediately gives Item 3 of the theorem. All else being constant, an increase in the non-manufacturing price directly increases nominal national income for all countries while leaving patterns of trade unchanged, resulting in greater nominal demand for every country's manufacturing labor. Since manufacturing wages are constant, this results in greater real demand for every country's manufacturing labor as well.

(Item 4) From equation 47, it is immediately clear that aggregate excess demand for non-manufacturing goods depends only on each country's real non-manufacturing consumption. Real non-manufacturing consumption for each country n is given as

$$\frac{C_n^O}{p^O} = (1-\alpha)\left[\frac{\bar{\tau}_n\left(1-\pi_{nn}(\mathbf{w})\right)}{Z_n(\mathbf{w},\bar{\tau}_n)}\left(\frac{\chi}{\beta}\left(\frac{w_n}{p^O}\right)L_n^M + \alpha Y_n^O\right) + \left(\frac{w_n}{p^O}\right)L_n^M + Y_n^O\right] \quad (***).$$

Once again, an increase in the non-manufacturing price by itself does not change patterns of trade and the term $\frac{\bar{\tau}_n(1-\pi_{nn}(\mathbf{w}))}{Z_n(\mathbf{w},\bar{\tau}_n)}$ remains unaffected by an increase in p^O. The purchasing power of non-manufacturing endowment over non-manufacturing goods is obviously unchanged by an increase in p^O. However, an increase in the non-manufacturing price reduces the purchasing power of manufacturing labor income over non-manufacturing goods as evidenced by an decrease in the term $\left(\frac{w_n}{p^O}\right)$, resulting in a decrease real demand for non-manufacturing goods for every country $n \in \mathcal{N}$. This gives us item 4 of the theorem.

We have demonstrated the relevant goods, each country's manufacturing labor and non-manufacturing goods, are gross substitutes. We shall now demonstrate that there is a unique equilibrium vector of manufacturing wages normalized by setting the non-manufacturing price to unity.

For ease of notation, let $\mathbf{v} := (\mathbf{w}, p^O)$ denote the vector of all manufacturing wages joined with the non-manufacturing price. For expository convenience, refer to $\mathbf{v}$ as the 'vector of prices'. Let $\mathbf{F}(\mathbf{v}) := (F_1(\mathbf{v}), ..., F_N(\mathbf{v}), F_O(\mathbf{v}))$ be the vector of aggregate excess demand functions. Noticing that each aggregate demand function is homogeneous of degree zero in $\left(\mathbf{w}, p^O\right)$, it must also be the case that $\mathbf{F}(\mathbf{v})$ is homogeneous of degree zero in $\mathbf{v}$.

It shall now be shown that if 2 distinct equilibrium price vectors exist, they must be collinear.

Suppose to the contrary there exists 2 distinct, non-collinear equilibrium price vectors, $\mathbf{v}^\bullet = \left(v_1^\bullet, ..., v_{N+1}^\bullet\right)$

and $\mathbf{v}^* = \left(v_1^*, ..., v_{N+1}^*\right)$; such that $\mathbf{v}^\bullet \neq \mathbf{v}^*$, $\mathbf{F}(\mathbf{v}^\bullet) = \mathbf{F}(\mathbf{v}^*) = \mathbf{0}_{N+1}$, and there does not exist any real number ε such that $\mathbf{v}^\bullet = \varepsilon \mathbf{v}^{**}$. Multiply $\mathbf{v}^\bullet$ by the scalar $\frac{v_l^*}{v_l^\bullet} := \min_j \left\{\frac{v_j^*}{v_j^\bullet}\right\}_{j=1,...,N+1}$ to obtain $\mathbf{v}^{**}$, and it would be easy to verify that $\mathbf{v}^* \leq \mathbf{v}^{**}$ with $v_l^* = v_l^{**}$. Since $\mathbf{F}$ is homogeneous of degree zero, $\mathbf{F}(\mathbf{v}^{**}) = \mathbf{F}(\mathbf{v}^\bullet) = \mathbf{0}_{N+1}$, and $\mathbf{v}^{**}$ must also be an equilibrium vector of prices. Since the non-collinearity of $\mathbf{v}^\bullet$ and $\mathbf{v}^*$ implies the non-collinearity of $\mathbf{v}^*$ and $\mathbf{v}^{**}$, there must exist at least one 'good' j for which $v_j^* < v_j^{**}$.

Then since the gross substitute property gives $\frac{\partial F_l(\mathbf{v})}{\partial v_j} > 0$ for all $j \neq l$ and $\mathbf{v} \in \mathbb{R}_{++}^{N+1}$, $v_j^* < v_j^*$ for at least one good j, and $v_l^* = v_l^{**}$, it must be that case that $F_l(\mathbf{v}^*) < F_l(\mathbf{v}^{**})$. This implies that $\mathbf{F}(\mathbf{v}^*) \neq \mathbf{F}(\mathbf{v}^{**})$, and either $\mathbf{v}^*$ or $\mathbf{v}^{**}$ are not equilibrium vectors of prices. A contradiction is obtained as both $\mathbf{v}^*$ and $\mathbf{v}^{**}$ are equilibrium vectors of prices by assumption.

It has thus been shown that two distinct vectors are equilibrium vectors only if they are collinear, in which case given any set of exogenous parameters satisfying Assumption 1 and with sufficiently low import tariff rates, the space of equilibrium price vectors is a single dimensional ray in $\mathbb{R}_{++}^{N+1}$. Consequently, there can only be one unique equilibrium vector normalized by setting non-manufacturing good price $p^O = v_{N+1}$ to 1. $\square$

We conclude this section on the uniqueness of the the immobile labor equilibrium by re-emphasizing that the requirement that Assumption 1 holds is a sufficient but not a necessary condition for uniqueness. Numerous numerical simulations employing randomized parameters have not appeared to yield any evidence of multiple equilibria in any set of parameters, even ones that grossly contradicts Assumption 1, admitting speculation that less restrictive sufficient conditions for uniqueness might exist. Furthermore, while Item 2 of Theorem 2 requires some country specific uniform tariff rate $\bar{t}_n$ to be sufficiently low in order for the equilibrium manufacturing wages (normalized by non-manufacturing good price) to be unique, the continuity of the tariff revenue multiplier $1 - \alpha \sum_i \frac{t_{ni}}{1+t_{ni}} \pi_{ni}$ in each tariff rate t_{ni} suggests the complementary result that the uniqueness of the equilibrium might also be obtained if t_{ni} is sufficiently low for each $n, i \in \mathcal{N}$, even if country n imposes different import tariffs on imports from different countries. This would be assumed to be true for comparative statics analysis in later sections, in order to accommodate preferential trade agreements in which countries lower (or increase) tariff rates on imports from specific trading partners.

Part IV

Sectoral Trade Balance

In the Eaton and Kortum (2002) 2-sector general equilibrium trade model, it is possible for countries to run sectoral trade deficits or surpluses. The national budget constraint only requires that any trade surplus in the manufacturing sector be matched with an identical trade deficit in the non-manufacturing sector such that trade balances across sectors in each country. As would be later demonstrated, sectoral trade imbalances have significant influence on the national welfare consequences of tariff changes, particularly when countries enter into sector specific preferential trade arrangements.

As non-manufacturing production relies only on labor as inputs, national non-manufacturing labor income Y_n^O equals nominal national non-manufacturing revenue. National trade surplus in non-manufacturing trade S_n^O is simply nominal national non-manufacturing revenue Y_n^O less national non-manufacturing consumption $C_n^O = (1-\alpha)\left(Y_n^M + Y_n^O + TR_n\right)$, such that

$$S_n^O = \alpha Y_n^O - (1-\alpha)\left(TR_n + Y_n^M\right).$$

National trade surplus S_n^M in the manufacturing sector is receipts from foreigners less payments to foreigners. Trade surplus could also be calculated as national manufacturing revenues less national manufacturing expenditures net of import tariffs

$$S_n^M = Q_n - (X_n - TR_n),$$

since domestically sourced manufacturing goods net out in the process.

By rearranging equation 22, we have $Q_n - (X_n - TR_n) = (1-\alpha)\left(TR_n + Y_n^M\right) - \alpha Y_n^O$, or equivalently $S_n^M = -S_n^O$. The only requirement of the balanced national budget constraint is that trade aggregated across both sectors is balanced in each country, such that national trade surpluses in manufacturing must be balanced by an equal deficit in non-manufacturing.

To see that trade in each sector need not balance, consider the following example in the mobile labor scenario of the Eaton and Kortum (2002) trade model. Suppose the global economy is composed of two countries in $\mathcal{N} = \{1,2\}$, and that international trade is costless and no tariff barriers exists, with $d_{ni} = 1$ for all $n, i \in \{1,2\}$. Further suppose the 2 countries are identical in every possible way, except for differing levels of manufacturing technology. More specifically, assume that $w_1 = w_2 = 1$, $\bar{L}_1 = \bar{L}_2 = 1$ but $T_1 = 1$ and $T_2 = 3$. For simplicity, set the remaining parameters as $\theta = 2$ and $\alpha = \beta = 1/2$.

Since there are no frictions associated with international trade of manufacturing goods, the Law of One Price applies in the manufacturing sector and manufacturing prices must be the same between the 2 countries, with $p_1 = p_2$. Solving for expenditure shares $\pi_{ni} = \frac{T_i(d_{ni}w_ip_i)^{-\theta}}{\sum_{j\in\mathcal{N}} T_j(d_{nj}w_jp_j)^{-\theta}}$, we obtain

$\pi_{11} = \pi_{21} = \frac{1}{4}$ and $\pi_{12} = \pi_{22} = \frac{3}{4}$,

$$\Pi = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}.$$

Define $\mathbf{Z} = \left[\mathbf{I} - (1-\beta)\,\Pi^\top\right]$. It is easily verified that the inverse of $\mathbf{Z}$ is

$$\mathbf{Z}^{-1} = \begin{pmatrix} \frac{5}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{7}{4} \end{pmatrix}.$$

From equation 31, we have national manufacturing expenditures as $\mathbf{L}(X_n) = \alpha \mathbf{Z}^{-1}\mathbf{L}(w_n \bar{L}_n)$. Calculation gives $X_1 = \frac{3}{4}$ and $X_2 = \frac{5}{4}$. From equation 30, manufacturing revenues are given by $\mathbf{L}(Q_n) = \Pi^\top \mathbf{L}(X_n)$, solving which yields $Q_1 = \frac{1}{2}$ and $Q_2 = \frac{3}{2}$.

Finally, since tariff revenues are 0 for each country, the vector of national manufacturing trade surplus is

$$\begin{pmatrix} S_1^M \\ S_2^M \end{pmatrix} = \begin{pmatrix} Q_1 - X_1 \\ Q_2 - X_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} \\ \frac{1}{4} \end{pmatrix}.$$

Country 2 runs a manufacturing trade surplus while country 1 suffers a correspondence manufacturing trade deficit. It is not necessarily the case that trade in each sector balances.

In this particular scenario, since country 2 has a higher level of productive technology in manufacturing sector relative to country 1, we might intepret this as country 2 having a comparative advantage in the production of manufacturing goods relative to country 1. The flipside of the coin is that country 1 has a comparative advantage in the production of non-manufacturing goods. So it should be unsurprising that country 2 should be a net exporter of manufacturing goods while country 1 is a net exporter in the non-manufacturing sector.

One implication of sectoral trade imbalance is that changes in manufacturing prices would have different welfare implications for countries running manufacturing trade surpluses versus those running deficits. Since the non-manufacturing good serves as the numeraire, an increase in nominal manufacturing prices implies an increase in the price of manufacturing goods relative to the price of non-manufacturing goods. For countries running manufacturing trade surpluses, an increase in price of manufacturing goods represents an increase in the price of exports relative to the price of imports, resulting in welfare gains from improving terms of trade. The converse is true for countries with manufacturing trade deficits.

Since sectoral trade balance has welfare implications in the Eaton and Kortum trade model, it would be useful if manufacturing trade surpluses and deficits could be characterized in some way. One complication is that there are many factors influencing a country's manufacturing trade surplus. Higher levels of manufacturing technology, lower wages (in the mobile labor scenario), greater manufacturing labor supply and smaller non-manufacturing sectors (in the immobile labor scenario) would all be associated greater potential for trade manufacturing surplus. Would a country with low technology but a large labor supply enjoy manufacturing trade surpluses? Heterogeneous costs of transportation further obscure the issue.

The remainder of this section demonstrates an interesting and non-obvious result for manufacturing trade balance: if some costs of international trade is symmetrical is some way, it is always possible to rank countries $n \in \mathcal{N}$ in terms of relative competitiveness, defined as the number of countries against which a country runs a manufacturing trade surplus.

S_{ni}, country n's manufacturing trade surplus with country i, is the excess of revenues received by country n on exports to country i, $Q_{ni} = \frac{X_{in}}{1+t_{in}}$, over the expenditures less tariffs paid by country n on manufacturing imports from country i , $Q_{in} = \frac{X_{ni}}{1+t_{ni}}$, such that

$$S_{ni} = Q_{ni} - Q_{in} = \frac{X_i \pi_{in}}{1+t_{in}} - \frac{X_n \pi_{ni}}{1+t_{ni}}.$$

We shall term $\mathbf{M}(S_{ni}) = \mathbf{M}(Q_{ni}) - \mathbf{M}(Q_{ni})^\top$ the matrix of (manufacturing) trade surpluses and note that $\mathbf{M}(S_{ni})$ is skew-symmetric, i.e. $S_{ni} = -S_{in}$.

Definition 3. We say that country n runs a *trade surplus* with country i or equivalently, country i runs a *trade deficit* with country n if

$$S_{ni} > 0 > S_{in}.$$

We say that a country n's manufacturing trade with country i is balanced if $S_{ni} = 0 = S_{in}$.

The two assumptions below each *independently* provide the sufficient conditions for our final results to hold.

Assumption 2. *It is the case that the geographical distance matrix $\mathbf{M}(\bar{d}_{ni})$ and the matrix of tariff rates $\mathbf{M}(t_{ni})$ is symmetrical, such the matrix of economic distances $\mathbf{M}(d_{ni})$ is symmetrical. I.e.*

$$\begin{aligned} \bar{d}_{ni} &= \bar{d}_{in} \\ t_{ni} &= t_{in} \\ d_{ni} = \bar{d}_{ni}\,(1+t_{ni}) &= \bar{d}_{in}\,(1+t_{in}) = d_{in}. \end{aligned}$$

Assumption 2 is simply the assertion that the costs of trading from n to i is exactly the same as for from i to n, that trade distance is symmetrical.

Assumption 3. *It is the case that for all countries $n, i, j \in \mathcal{N}$, we have*

$$d_{ni} d_{ij} d_{jn} = d_{nj} d_{ji} d_{in}.$$

Assumption 3 asserts another form of symmetry in trade costs. Here, we might use Caliendo and Parro's (2015) model of iceberg trade costs and the Most-favored-nation (MFN) clause of World Trade Organization (WTO) rules to provide a justification for Assumption 3.

Economic trade distance d_{ni} could be decomposed into two factors, tariff barriers to trade $(1 + t_{ni})$ and iceberg trade costs $\bar{d}_{ni}$. Caliendo and Parro (2015) further modeled iceberg trade costs $\bar{d}_{ni}$ as

$$\ln \bar{d}_{ni} = \nu_{ni} + \mu_n + \delta_i + \varepsilon_{ni}.$$

The parameter $\nu_{ni} = \nu_{in}$ reflects symmetric bilateral trade costs like distances, cultural affinity and common borders. μ_n and δ_i respectively refer to importer and exporter fixed effects such as non-tariff barriers to trade, quality of shipping terminals etc. ε_{ni} is an uncorrelated random disturbance term. It is easily verified that

$$\ln\left(\frac{\bar{d}_{ni}\bar{d}_{ij}\bar{d}_{jn}}{\bar{d}_{nj}\bar{d}_{ji}\bar{d}_{in}}\right) = \varepsilon_{ni} + \varepsilon_{ij} + \varepsilon_{jn} - \varepsilon_{nj} - \varepsilon_{ji} - \varepsilon_{nj}.$$

Since the sum of error terms would also have an expected value of 0, consequently the expected value of $\ln\left(\frac{\bar{d}_{ni}\bar{d}_{ij}\bar{d}_{jn}}{\bar{d}_{nj}\bar{d}_{ji}\bar{d}_{in}}\right)$ equals 0. It then follows that the expected value of the ratio of economic distances would be

$$\mathbb{E}\left(\ln\frac{d_{ni}d_{ij}d_{jn}}{d_{nj}d_{ji}d_{in}}\right) = -\theta\ln\frac{(1+t_{ni})(1+t_{ij})(1+t_{jn})}{(1+t_{nj})(1+t_{in})(1+t_{ji})}.$$

Under WTO rules, all WTO member countries (which constitutes the bulk of all countries in the world) extend to one another MFN status. If country n imposes tariffs on imports of t_{ni} on imports from country i with MFN status, it must impose the same import tariffs on goods from MFN country j, such that $t_{nj} = t_{ni}$.

If we suppose all countries mutually confer MFN status to each other (a scenario approximating reality), then it follows that every country imposes the same tariff rate on all imports, such that $\bar{t}_n = t_{ni}$ for all countries $i \neq n$. If this supposition holds, then Assumption 3 follows immediately as then

$$\mathbb{E}\left(\ln\frac{d_{ni}d_{ij}d_{jn}}{d_{nj}d_{ji}d_{in}}\right) = -\theta\ln\frac{(1+\bar{t}_n)(1+\bar{t}_i)(1+\bar{t}_j)}{(1+\bar{t}_n)(1+\bar{t}_i)(1+\bar{t}_j)} = 0,$$

and the ratio $\frac{d_{ni}d_{ij}d_{jn}}{d_{nj}d_{ji}d_{in}}$ is expected to take on a value of 1.

The consequence of the two assumptions is found in the next lemma.

Lemma 5. *Suppose Assumption 2 or Assumption 3 holds. Then for all $n, j, i \in \mathcal{N}$,*

$$\frac{\pi_{nj}\pi_{ji}}{\pi_{ij}\pi_{jn}} = \frac{\pi_{ni}}{\pi_{in}}.$$

Proof. For ease of notation, let $\delta_{ni} = d_{ni}^{-\theta}$ and recall that $\pi_{ni} = \frac{T_i c_i^{-\theta}\delta_{ni}}{p_n^{-\theta}}$. Then

$$\begin{aligned}\frac{\pi_{nj}\pi_{ji}}{\pi_{ij}\pi_{jn}} &= \frac{\left(T_j c_j^{-\theta}\delta_{nj}/p_n^{-\theta}\right)\left(T_i c_i^{-\theta}\delta_{ji}/p_i^{-\theta}\right)}{\left(T_j c_j^{-\theta}\delta_{ij}/p_i^{-\theta}\right)\left(T_n c_n^{-\theta}\delta_{jn}/p_j^{-\theta}\right)}\\ &= \frac{\delta_{nj}\delta_{ji}/\delta_{ni}}{\delta_{ij}\delta_{jn}/\delta_{in}}\cdot\frac{T_i c_i^{-\theta}\delta_{ni}/p_n^{-\theta}}{T_n c_n^{-\theta}\delta_{in}/p_i^{-\theta}}\\ &= \frac{\delta_{nj}\delta_{ji}/\delta_{ni}}{\delta_{ij}\delta_{jn}/\delta_{in}}\frac{\pi_{ni}}{\pi_{in}}.\end{aligned}$$

If Assumptions 2 or 3, then $\frac{\delta_{nj}\delta_{ji}/\delta_{ni}}{\delta_{ij}\delta_{jn}/\delta_{in}} = 1$ as required. □

Lemma 5 could be interpreted as meaning that symmetry in trade costs results in symmetry in expenditure shares. This results in transitivity in the relations of which countries runs trade surpluses with whom, the subject of the following proposition.

Proposition 9. *Suppose Assumption 2 or 3 holds. Then,*

$$S_{jn} \geq 0 \,\&\, S_{ni} \geq 0 \implies S_{ji} \geq 0.$$

Proof. Suppose $S_{jn} \geq 0$ and $S_{ni} \geq 0$.

$S_{jn} \geq 0$ implies that $\frac{\pi_{nj}}{1+t_{nj}} X_n \geq \frac{\pi_{jn}}{1+t_{jn}} X_j$ or equivalently $\frac{X_n}{X_j} \geq \frac{\pi_{jn}}{\pi_{nj}} \frac{1+t_{nj}}{1+t_{jn}}$. Similarly, $S_{ni} \geq 0$ implies that $\frac{X_i}{X_n} \geq \frac{\pi_{ni}}{\pi_{in}} \frac{1+t_{in}}{1+t_{ni}}$.

We then have

$$\frac{X_i}{X_j} = \frac{X_i}{X_n} \frac{X_n}{X_j} \geq \frac{\pi_{ni}}{\pi_{in}} \frac{\pi_{jn}}{\pi_{nj}} \frac{1+t_{in}}{1+t_{ni}} \frac{1+t_{nj}}{1+t_{jn}}.$$

By Assumption 2, we have $\frac{1+t_{in}}{1+t_{ni}} \frac{1+t_{nj}}{1+t_{jn}} = 1$ and $\frac{1+t_{ij}}{1+t_{ji}} = 1$. By Lemma 5, we have $\frac{\pi_{jn}\pi_{ni}}{\pi_{in}\pi_{nj}} = \frac{\pi_{ji}}{\pi_{ij}}$. Hence we have

$$\frac{X_i}{X_j} \geq \frac{\pi_{ji}}{1+t_{ji}} \frac{1+t_{ij}}{\pi_{ij}},$$

or equivalently, $S_{ji} \geq 0$. $\square$

Proposition 9 asserts that if country j experiences a trade surplus with country n, and country n does the same with country i, then country j must also enjoy a trade surplus with country i.

The results of Proposition 9 could be applied to the global economy in characterizing the structure of manufacturing trade surpluses that occurs in equilibrium.

Definition 4. Define by "$\succeq$" the 'relative competitiveness' relation on $\mathcal{N} \times \mathcal{N}$.

For some pair of countries $(n, i) \in \mathcal{N} \times \mathcal{N}$, we say the country n is 'at least as competitive as' country i, denoted by $n \succcurlyeq i$, whenever $S_{ni} \geq 0 \geq S_{in}$, i.e. whenever country n does not run a trade deficit with country i.

We say that country n is 'strictly more competitive than' country i', denoted $n \succ i$, whenever $S_{ni} > 0 > S_{in}$ such that country n runs a trade surplus with country i.

Finally, we say that country n is 'just as competitive' as country i, denoted $n \sim i$ whenever country n and i have balanced trade with one another, such that $S_{ni} = 0 = S_{in}$.

Theorem 3. *If Assumption 2 or 3 holds, the set of countries $\mathcal{N}$ is totally ordered under the 'relative competitiveness' relation, $\succeq$, such that for all $n, i, j \in \mathcal{N}$, we have*

1. *$n \succeq n$ (property of reflexivity)*
2. *$n \succeq i$ or $i \succeq n$ (property of completeness)*
3. *$n \succeq i$ and $i \succeq n$ then $n \sim i$ (property of antisymmetry)*
4. *$j \succeq n$ and $n \succeq i$ then $j \succeq i$ (property of transitivity)*

Then since $(\succeq, \mathcal{N})$ is a totally ordered set, and the set of countries $\mathcal{N}$ has finite elements, we can reindex countries, n_i, for $i = 1, 2, ..., N$, such that

$$n_1 \succeq n_2 \succeq ... \succeq n_N.$$

Hence for each number i=$1, 2, ..., N$, we can find a country n_i that does not have trade deficits with at least $N - i$ countries, and does not have trade surpluses with $i - 1$ countries.

Proof. The 'relative competitiveness' relation, $\succeq$, on $\mathcal{N}$ inherits the totally ordered property from $(\geq, \mathbb{R})$, via the definition of $\succeq$ in terms of trade surpluses.

To demonstrate this, we shall show that each item 1 through 4 of the Theorem holds.

(Item 1: Reflexivity) Each country runs 0 trade surplus with itself, such that $S_{nn} = 0$, hence $n \succeq n$ and the property of reflexivity holds.

(Item 2: Completeness) For each $n, i \in \mathcal{N}$, trade surplus $S_{ni} \in \mathbb{R}$ is defined, with $S_{ni} \geq 0$ or $S_{in} \geq 0$. Hence we have $n \succeq i$ or $i \succeq n$ and the property of completeness holds.

(Item 3: Antisymmetry) Suppose for some $n, i \in \mathcal{N}$, $n \succeq i$ and $i \succeq n$, such that $S_{ni} \geq S_{in}$ and $S_{in} \geq S_{ni}$. Since the order relation $\geq$ is antisymmetric in real numbers, it must be that $S_{ni} = S_{in}$ and $n \sim i$.

(Item 4: Transitivity) Suppose for some countries $n, i, j \in \mathcal{N}$, it were the case that $j \succeq n$ and $n \succeq i$, such that $S_{jn} \geq 0$ and $S_{ni} \geq 0$. Then by Proposition 9, it must be the case that $S_{ji} \geq 0$ and $j \succeq i$. □

If Assumptions 2 or 3 hold, Theorem 3 tells us that we can rank countries by the number of other countries they run trade surpluses with. One important implication is that there must exist at least one country that runs no manufacturing trade deficits with any other country, and at least one country that runs no manufacturing trade surpluses. This is a useful result when considering the terms-of-trade effects associated with changes in equilibrium manufacturing prices, one of the topics of the next section.

Part V

Equilibrium Analysis of Trade Liberalization Policies

This part of the thesis seeks to answer a simple question: what predictions would the Eaton and Kortum (2002) trade model make on the welfare consequences of preferential trade agreements? How would member and non-member countries be affected?

While trade agreements often include non-tariff related components, here we model preferential trade agreements solely as countries mutually agreeing to lower tariffs on imports from one another.

This part of the thesis consists of two sections. In the first section, we study the simplest possible case of trade liberalization, where one country unilaterally reduces tariffs on imports from another country in the mobile labor scenario. The bilateral or multilateral tariff reductions associated with trade agreements can be thought of as combinations of the unilateral cases, hence the focus on the simplest elemental case.

We show that when a country unilaterally decreases import tariffs, the model predicts that prices would decrease in all countries, and every country would substitute towards imports resulting in trade creation everywhere.

We also show that starting in a scenario in which there are no import tariffs and manufacturing trade is balanced for each country, changes in welfare depends only on domestic trade shares π_{nn}, and parameters α, β and θ. This result is analogous to the main result in Arkolakis, Costinot and Rodríguez-Clare (2012). Our results however also shows that other factors such as manufacturing trade balance S_n^M and import shares π_{ni}, $n \neq i$ do matter when the restrictive assumption of balanced trade are removed. In that sense, our results extends Arkolakis et al.'s findings to more general scenarios of interest.

In the second section, we consider whether global free trade constitutes a Nash equilibrium.

7 Comparative Stactics in the Mobile Labor Scenario

In this section, we study how the mobile labor equilibrium changes in the simplest possible case of trade liberalization - a unilateral tariff reduction by a country k on imports from another country l, reducing the rate of tariffs t_{kl} on imports from country l, holding all else constant.

7.1 Changes in Manufacturing Prices and Expenditure Shares

We start by considering how manufacturing prices changes with respect to the tariff rate t_{kl} and it is efficient to do so by first examining how expenditure shares π_{ni} are affected by changes to t_{kl}.

Recall that by equations 18, 19 and 20, for all $n, i \in \mathcal{N}$, we have

$$\pi_{ni} = T_i \frac{\gamma^{-\theta} d_{ni}^{-\theta} c_i^{-\theta}}{p_n^{-\theta}} \quad \text{and} \quad p_n = \gamma \left(\sum_i T_i d_{ni}^{-\theta} c_i^{-\theta} \right)^{-1/\theta}, \tag{52}$$

where $c_i = w_i^{\beta} p_i^{1-\beta}$ refers to the costs of inputs into manufacturing production.

For notational convenience we shall use $x' := \frac{\mathrm{d}x}{\mathrm{d}t_{kl}}$ to denote the derivative of some equilibrium variable x with respect to t_{kl}.

Proposition 10. *The matrix of derivatives of expenditure shares with respect to t_{kl} is*

$$\mathbf{M}\left(\pi_{kl}^{'}\right) := \theta \left[\mathbf{D}\left(\ln p_n^{'}\right) \Pi - \Pi \mathbf{D}\left(\ln c_n^{'}\right) - \frac{\pi_{kl}}{1+t_{kl}} \iota_k \iota_l^T \right]$$

Moreover,

$$\mathbf{M}\left(\ln \pi_{ni}^{'}\right) = \theta \left[\mathbf{L}\left(\ln p_n^{'}\right) \mathbf{1}_N^T - \mathbf{1}_N \mathbf{L}\left(\ln c_n^{'}\right)^T - \frac{1}{1+t_{kl}} \iota_k \iota_l^T \right]$$

Proof. Since $\mathbf{M}\left(\pi_{ni}^{'}\right) = \Pi \circ \mathbf{M}\left(\ln \pi_{ni}^{'}\right)$,[29] and since $\ln \pi_{ni} = \ln\left(T_i \gamma^{-\theta} \bar{d}_{ni}^{-\theta}\right) + \theta\left(\ln p_n - \ln c_i - \ln\left(1+t_{ni}\right)\right)$, where T_i, γ and $\bar{d}_{ni}$ are exogeneously given parameters, we have

$$\frac{d \, \ln \pi_{ni}}{d \, t_{kl}} = \theta \left[\ln p_n^{'} - \ln c_i^{'} - \ln\left(1+t_{ni}\right)^{'} \right]$$

Noting that $\ln\left(1+t_{ni}\right)^{'} = \frac{1}{1+t_{kl}}$ only if $(n,i) = (k,l)$, and 0 otherwise, we have

$$\mathbf{M}\left(\ln \pi_{ni}^{'}\right) = \theta \left[\mathbf{L}\left(\ln p_n^{'}\right) \mathbf{1}_N^T - \mathbf{1}_N \mathbf{L}\left(\ln c_n^{'}\right)^T - \frac{1}{1+t_{kl}} \iota_k \iota_l^T \right]$$

Let $[\mathbf{A}]_{ij}$ denote the element of matrix $\mathbf{A}$ on the i^{th} row and j^{th} column, and note that $\left[\Pi \circ \left(\mathbf{L}\left(\ln p_n^{'}\right) \mathbf{1}_N^T\right)\right]_{ij} = \pi_{ij} \ln p_i^{'} = \left[\mathbf{D}\left(\ln p_n^{'}\right) \Pi\right]_{ij}$ for all $(i,j) \in \mathcal{N} \times \mathcal{N}$. This implies that

$$\Pi \circ \left(\mathbf{L}\left(\ln p_n^{'}\right) \mathbf{1}_N^T \right) = \mathbf{D}\left(\ln p_n^{'}\right) \Pi$$

By similar reasoning, we have

$$\Pi \circ \left(\mathbf{1}_N \mathbf{L}\left(\ln c_n^{'}\right)^T \right) = \Pi \mathbf{D}\left(\ln c_n^{'}\right)$$

[29] The symbol $\circ$ refers to the Hadamard product, i.e. for any $m \times n$ matrices $\mathbf{A} = \mathbf{M}\left(a_{ni}\right)$ and $\mathbf{B} = \mathbf{M}\left(b_{ni}\right)$, we have $\mathbf{A} \circ \mathbf{B} = \mathbf{M}\left(a_{ni} b_{ni}\right)$.

It follows then that

$$\begin{aligned}
\mathbf{M}\left(\pi_{ni}^{'}\right) &= \Pi \circ \mathbf{M}\left(\ln \pi_{ni}^{'}\right) \\
&= \theta \left[\Pi \circ \left(\mathbf{L}\left(\ln p_{n}^{'}\right) \mathbf{1}_{N}^{T}\right) - \Pi \circ \left(\mathbf{1}_{N} \mathbf{L}\left(\ln c_{n}^{'}\right)^{T}\right) - \Pi \circ \iota_k \iota_l^T\right] \\
&= \theta \left[\mathbf{D}\left(\ln p_{n}^{'}\right) \Pi - \Pi \mathbf{D}\left(\ln c_{n}^{'}\right) - \frac{\pi_{kl}}{1+t_{kl}} \iota_k \iota_l^T\right]
\end{aligned}$$

□

Since the sum of expenditure shares of each country n is unity, i.e. $\sum_{i \in \mathcal{N}} \pi_{ni} = 1$, the sum of changes to expenditure shares of each country must be zero. This immediately gives us the derivatives of manufacturing prices with respect to t_{kl} via the next proposition.

Proposition 11. *The vector of derivatives of the logarithm each country's manufacturing goods prices with respect to the tariff rate t_{kl} is:*

$$\mathbf{L}\left(\ln p_{n}^{'}\right) = \Pi \mathbf{L}\left(\ln c_{n}^{'}\right) + \frac{\pi_{kl}}{1+t_{kl}} \iota_k$$

Proof. Since $\Pi = \mathbf{M}(\pi_{ni})$ is a row-stochastic matrix such that $\sum_{i \in \mathcal{N}} \pi_{ni} = 1$ for each $n \in \mathcal{N}$, it must be that $\frac{d}{dt_{kl}} \sum_i \pi_{ni} = \sum_i \frac{d\pi_{ni}}{dt_{kl}} = 0$. It follows directly that

$$\mathbf{M}\left(\pi_{ni}^{'}\right) \mathbf{1}_N = \mathbf{L}\left(\sum_i \frac{d\pi_{ni}}{dt_{kl}}\right) = \mathbf{0}_N$$

From Proposition (10), we have $\mathbf{M}\left(\pi_{ni}^{'}\right) = \theta \left[\mathbf{D}\left(\ln p_{n}^{'}\right) \Pi - \Pi \mathbf{D}\left(\ln c_{n}^{'}\right) - \frac{\pi_{kl}}{1+t_{kl}} \iota_k \iota_l^T\right]$, therefore

$$\begin{aligned}
\theta \left[\mathbf{D}\left(\ln p_{n}^{'}\right) \Pi - \Pi \mathbf{D}\left(\ln c_{n}^{'}\right) - \frac{\pi_{kl}}{1+t_{kl}} \iota_k \iota_l^T\right] \mathbf{1}_N &= \mathbf{0}_N \\
\Longrightarrow \mathbf{D}\left(\ln p_{n}^{'}\right) \Pi \mathbf{1}_N &= \Pi \mathbf{D}\left(\ln c_{n}^{'}\right) \mathbf{1}_N + \frac{\pi_{kl}}{1+t_{kl}} \iota_k \iota_l^T \mathbf{1}_N \\
\Longrightarrow \mathbf{L}\left(\ln p_{n}^{'}\right) &= \Pi \mathbf{L}\left(\ln c_{n}^{'}\right) + \frac{\pi_{kl}}{1+t_{kl}} \iota_k
\end{aligned}$$

□

In the mobile labor scenario, manufacturing wages are exogeneously given parameters hence $\ln w_{n}^{'} = 0$ for all $n \in \mathcal{N}$, and $\ln c_{n}^{'} = (1-\beta) \ln p_{n}^{'}$. Hence a collorary to Proposition 11 is that in the mobile labor scenario, it must be the case that

$$\begin{aligned}
\mathbf{L}(\ln p_{n}^{'}) &= (1-\beta) \Pi \cdot \mathbf{L}(\ln p_{n}^{'}) + \frac{\pi_{kl}}{1+t_{kl}} \iota_k \\
&= [\mathbf{I} - (1-\beta) \Pi]^{-1} \frac{\pi_{kl}}{1+t_{kl}} \iota_k .
\end{aligned} \tag{53}$$

From footnote 27, the inverse matrix $[\mathbf{I} - (1-\beta)\Pi]^{-1}$ exists and equals $\sum_{m=0}^{\infty} (1-\beta)^m \Pi^{(m)}$. Hence we can rewrite equation 53 as

$$\mathbf{L}(\ln p_n^{'}) = \left(\mathbf{I} + \sum_{m=1}^{\infty}(1-\beta)^m \,\Pi^{(m)}\right)\frac{\pi_{kl}}{1+t_{kl}}\iota_k. \tag{54}$$

This means that for all countries $n \neq k$,

$$\frac{d\,\ln p_n^{'}}{d\,t_{kl}} = (1-\beta)\,\frac{\pi_{kl}}{1+t_{kl}}\left(\pi_{nk} + (1-\beta)\sum_i \pi_{ni}\pi_{ik} + (1-\beta)^2\sum_i\sum_j \pi_{ni}\pi_{ij}\pi_{jk} + ...\right), \tag{55}$$

and

$$\begin{aligned}\frac{d\,\ln p_k^{'}}{d\,t_{kl}} &= (1-\beta)\,\frac{\pi_{kl}}{1+t_{kl}}\left(\pi_{kk} + (1-\beta)\sum_i \pi_{ni}\pi_{ik} + (1-\beta)^2\sum_i\sum_j \pi_{ni}\pi_{ij}\pi_{jk} + ...\right) \\ &\quad + \frac{\pi_{kl}}{1+t_{kl}}.\end{aligned} \tag{56}$$

Equations 55 and 56 provide a nice economic interpretation for how manufacturing prices evolve in response to a change to some import tariff rate t_{kl}. Consider the following very informal discussion. We can think of $\frac{\pi_{kl}}{1+t_{kl}}$ as the 'immediate' impact of the tariff change on prices in country k, the country implementing the change in the tariff rate on imports from country l. From Proposition (3), the distribution of prices of goods sold in country k does not depend on the source of the goods, hence the conditional distribution of prices of goods imported from l is identical to the general distribution of prices in k, such that the average price goods conditioned on them being sourced from l is also the unconditional mean country k manufacturing good price p_k.[30]Let p_k^0 be the price of goods in general in country k before the change in t_{kl}, then we can write p_k as

$$p_k^0 = \pi_{kl}p_k^0 + (1-\pi_{kl})\,p_k^0$$

where $\pi_{kl}p_k^0$ is the contribution to country k's price made by imports from country l. After an infinitisimal change in tariffs $d\,t_{kl}$, holding all other factors constant, the new average price of imports from l to k would be $\frac{1+t_{kl}^1}{1+t_{kl}^0}p_k^0$, where $t_{kl}^1 = t_{kl}^0 + d\,t_{kl}$ is the new tariff rate and t_{kl}^0 is the original tariff rate. Holding all other factors constant, $p_k^1 := \pi_{kl}\frac{1+t_{kl}^1}{1+t_{kl}^0}p_k^0 + (1-\pi_{kl})\,p_k^0$ is the new country k price. Hence the immediate impact on country k price due solely to the change in t_{kl} is

$$\begin{aligned}\Delta^0 \ln p_k \approx \frac{p_k^1 - p_k^0}{p_k^0} &= \frac{\pi_{kl}\left[(1+t_{kl}^1)\,p_k^0 - (1+t_{kl}^0)\,p_k^0\right]}{p_k^0\,(1+t_{kl}^0)} \\ &= \frac{\pi_{kl}}{(1+t_{kl}^0)}dt_{kl}\end{aligned}$$

[30]Recall that the measure of the set of all manufacturing goods $|\Omega| = 1$ and the measure of the set of goods k sources from any country $i \in \mathcal{N}$ is $|\Omega_{ki}| = \pi_{ki}$. The average expenditure on each good in $\Omega_{ki} = \frac{X_{ki}}{|\Omega_{ki}|} = \frac{\pi_{ki}X_k}{\pi_{ki}} = X_k$, identical to the average expenditure across all goods $\frac{X_k}{|\Omega|} = X_k$. Since the distribution of country k's quantity demanded across goods in Ω_{ki} is the same as that in Ω, this immediately implies that p_{ki}, the country k average price of goods sourced from country i must be the same as the unconditional manufacturing price in k, $p_{ki} = p_k$, at market equilibrium.

$\Delta^0 \ln p_k = \frac{\pi_{kl}}{1+t_{kl}} dt_{kl}$ could therefore be thought of as the 'zeroth' order effect, the immediate impact of the change in tariff rate t_{kl} on country k's manufacturing price index, holding all other factors such as trade shares and other countries' prices and production costs constant.

Given the use of manufacturing goods as intermediate inputs, the increase in manufacturing product prices in country k pushes up the cost of manufacturing production in the country by $\Delta^0 \ln c_k = (1-\beta)\,\Delta^0 \ln p_k = (1-\beta)\,\frac{\pi_{kl}}{1+t_{kl}} dt_{kl}$.

Examining equations 55 and 56, it should be easily seen that these immediate impacts on country k are propogated across countries through trade in manufacturing goods. For each country $n \in \mathcal{N}$ (including k), national manufacturing prices can be expressed as

$$p_n = \pi_{nk} p_{nk} + (1-\pi_{nk})\, p_n,$$

where p_{nk} is the average price of goods sourced from k, and $p_{nk} = p_n$ in the initial equilibrium. When the cost of production in country k changes, the price of goods that n sources from k changes as well. Recall that the price of a good $\omega \in \Omega_{nk}$, sourced from country k delivered to country n is $p_{nk}(\omega) = \frac{c_k d_{nk}}{z_k(\omega)}$, hence the change in the country n price of all goods $\omega \in \Omega_{nk}$ as a result of the increase in country k's production costs would be $\Delta^1 \ln p_{nk}(\omega) = \Delta^0 \ln c_k$. Since p_{nk} is the average price of all goods in Ω_{nk}, it follows immediately that the change in p_{nk} would be $\Delta^1 \ln p_{nk} = \Delta^0 \ln c_k$. The change in country n's manufacturing price would be

$$\Delta^1 p_n = \pi_{nk} \Delta p_{nk} = p_{nk} \pi_{nk} \Delta^0 \ln p_{nk}.$$

Since $p_n = p_{nk}$ at the initial equilibrium and $\Delta^1 \ln p_{nk} = \Delta^1 \ln c_k$, the proportional change in p_n would be

$$\Delta^1 \ln p_n = \frac{\Delta^1 p_n}{p_n} = \pi_{nk} \Delta^0 \ln c_k.$$

The zeroth order change in production costs in country k changes manufacturing prices in every country, which in turn changes the cost of manufacturing everywhere, triggering off a cascade of higher order changes in manufacturing prices and production costs, distributed through trade in manufacturing goods, cumulatively increasing manufacturing prices.

However, since $\lim_{m\to\infty} (1-\beta)^m \Pi^m$ is a zero-matrix, higher order effects eventually converge in magnitude to 0 and the total change in prices in each country is bounded below infinity.

As country k was the source of the initial disruption to equilibrium prices, and the only one to experience the zeroth effect on prices, intuitively we might expect the change in country k manufacturing prices to be the greatest among all the countries in $\mathcal{N}$. The next theorem shows this to be true, and that in fact, the proportional change to manufacturing *costs* in country k is greater than of *prices* in any other country.

Theorem 4. *In the mobile labor case,*

1. $\frac{d \ln p_n}{dt_{kl}} > 0$ *for each* $n \in N$.
2. $\frac{d \ln p_k}{dt_{kl}} = \max_{i \in N} \left\{ \frac{d \ln p_i}{dt_{kl}} \right\}$ *and* $\frac{d \ln c_k}{dt_{kl}} = (1-\beta)\,\frac{d \ln p_k}{dt_{kl}} \geq \frac{d \ln p_n}{dt_{kl}}$ *for each* $n \in N$.

3. *For each country $n \in N$, we have* $\frac{d \ln \pi_{nn}}{dt_{kl}} > 0$ *and* $\frac{d \ln \pi_{nk}}{dt_{kl}} < 0$.

4. *For country $i \neq l$,* $\frac{d \ln \pi_{ki}}{dt_{kl}} > 0$*, and* $\frac{d \ln \pi_{kl}}{dt_{kl}} < 0$.

Proof. (Statement 1) From equation 54 we have

$$\mathbf{L}(\ln p_n^{'}) = \left(\mathbf{I} + \sum_{m=1}^{\infty} (1-\beta)^m \, \Pi^{(m)} \right) \frac{\pi_{kl}}{1+t_{kl}} \iota_k.$$

Since expenditure shares are strictly positive, $\Pi > \mathbf{0}$ is a matrix with strictly positive elements and $\pi_{kl} > 0$. It follows immediately that for each $n \in \mathcal{N}$, the derivative $\ln p_n^{'}$ must be strictly positive as products and sums of strictly positive numbers.

(Statement 2) Expanding equation 53 gives each $\ln p_n^{'}$ as

$$\frac{d \ln p_n}{dt_{kl}} = \begin{cases} (1-\beta) \sum_{i=1}^{N} \pi_{ni} \frac{d \ln p_i}{dt_{kl}}, & \forall n \neq k \\ (1-\beta) \sum_{i=1}^{N} \pi_{ki} \frac{d \ln p_i}{dt_{kl}} + \pi_{kl}, & if \, n = k. \end{cases}$$

Since $\sum_{i=1}^{N} \pi_{ni} = 1$, the term $\sum_{i=1}^{N} \pi_{ni} \frac{d \ln p_i}{dt_{kl}}$ is the trade-weighted average of all derivatives of national log-prices. As the value of an average must be less than value of the largest constituent element, for all $n \neq k$, we have

$$\frac{d \ln p_n}{dt_{kl}} \leq (1-\beta) \max_{i \in N} \{\ln p_i\}. \quad (*)$$

We wish to show that $\frac{d \ln p_k}{dt_{kl}} = \max_{i \in N} \left\{ \frac{d \ln p_i}{dt_{kl}} \right\}$. Suppose to the contrary that $\frac{d \ln p_k}{dt_{kl}} \neq \max_{i \in N} \left\{ \frac{d \ln p_i}{dt_{kl}} \right\}$, and some country $n^* \neq k$ satisfies $\frac{d \ln p_{n^*}}{dt_{kl}} = \max_{i \in N} \left\{ \frac{d \ln p_i}{dt_{kl}} \right\}$. But it would be the case that $\frac{d \ln p_{n^*}}{dt_{kl}} \leq (1-\beta) \frac{d \ln p_{n^*}}{dt_{kl}}$, which is clearly impossible, since $(1-\beta) < 1$, and $\frac{d \ln p_{n^*}}{dt_{kl}} > 0$. Hence it must be the case that

$$\frac{d \ln p_k}{dt_{kl}} = \max_{i \in N} \left\{ \frac{d \ln p_i}{dt_{kl}} \right\}. \quad (**)$$

It follows immediately from $(*)$ and $(**)$ that

$$\frac{d \ln p_n}{dt_{kl}} \leq (1-\beta) \max_{i \in N} \{\ln p_i\} = (1-\beta) \frac{d \ln p_k}{dt_{kl}},$$

for all $n \neq k$ as required.

(Statements 3 & 4) Follows immediately from the above results on changes in national prices and the fact that for $n, i \in N$, the derivative of log-tradeshares of country $n's$ imports from country i is given as

$$\frac{d \ln \pi_{ni}}{dt_{kl}} = \theta \left(\frac{d \ln p_n}{dt_{kl}} - (1-\beta) \frac{d \ln p_i}{dt_{kl}} - \frac{d \ln (1+t_{ni})}{dt_{kl}} \right).$$

First note that Statement 3 implies that for $n \neq k$,

$$\frac{d \ln p_k}{dt_{kl}} > (1-\beta) \frac{d \ln p_k}{dt_{kl}} > \frac{d \ln p_n}{dt_{kl}} > (1-\beta) \frac{d \ln p_n}{dt_{kl}}.$$

Also note that $\frac{d\ln(1+t_{ni})}{dt_{kl}} = 0$ for all $n \neq k$ and $i \neq l$ and $\frac{1}{1+t_{kl}}$ otherwise. Then, we have:

- For all $n \in N$, $\beta \frac{d\ln p_n}{dt_{kl}} > 0$ results in $\frac{d\ln \pi_{nn}}{dt_{kl}} > 0$.
- For all $n \neq k$, $\frac{d\ln p_n}{dt_{kl}} < (1-\beta)\frac{d\ln p_k}{dt_{kl}}$ results in $\frac{d\ln \pi_{nk}}{dt_{kl}} < 0$.
- For all $i \neq l$, $\frac{d\ln p_k}{dt_{kl}} > (1-\beta)\frac{d\ln p_i}{dt_{kl}}$ results in $\frac{d\ln \pi_{ki}}{dt_{kl}} > 0$.
- Finally, since π_{ki} is increasing in t_{kl} for all $i \neq l$, and $\pi_{kl} = 1 - \sum_{i\neq l}^{N} \pi_{ki}$, it must be that π_{kl} must be decreasing in t_{kl}. That is, since $\frac{d\ln \pi_{kl}}{dt_{kl}} = -\sum_{i\neq l}^{N} \frac{\pi_{kl}}{\pi_{ki}} \frac{d\ln \pi_{ki}}{dt_{kl}}$ and $\frac{d\ln \pi_{ki}}{dt_{kl}} > 0$ for all $i \neq l$, it must be that $\frac{d\ln \pi_{kl}}{dt_{kl}} < 0$.

□

The implication of Theorem 4 is that whenever a country k unilaterally increases the tariff rate on imports from some country l, the effects are not limited to country k alone. Every country experiences an increase in the prices of manufacturing goods, although country k would experience the greatest increase. The increase in country $k's$ manufacturing price is so much greater than the rest, that even the increase in country k's manufacturing costs outweighs the increase in prices everywhere else. This is unsurprising in light of the fact that it is the increase in country k's manufacturing costs which triggers the increase in manufacturing prices elsewhere.

Since increases in manufacturing costs c_k directly affects the price of country k's exports $p_{nk}(\omega) = \frac{c_k d_{nk}}{z_k(\omega)}$ for any importing country $n \in \mathcal{N}$, the increase in c_k relative to all other countries' manufacturing costs leads to loss of country k's competitiveness in every foreign manufacturing product market. This is seen from the loss of market share as evidenced by $\ln \pi_{nk}^{'} < 0$ for every country $n \neq k$. The fact that $\ln \pi_{nk}^{'} < 0$ when $\ln p_k^{'} = \max \left\{\ln p_n^{'}\right\}_{n\in\mathcal{N}}$ suggests that in the Eaton and Kortum model, the countries' manufacturing output are gross substitutes. This result mirrors the discussion in Section III where we used the gross substitution property of national manufacturing labor forces to prove that the immobile labor equilibrium is unique.

The increase in prices everywhere also lowers the incentives for beneficial trade. The increase in national manufacturing prices lowers real manufacturing wages, depressing the production cost of domestically produced goods relative to prices of manufacturing goods in aggregate. This expands the set of locally produced goods that are price competitive in domestic markets, and every country substitutes towards domestically produced manufacturing goods, as seen from $\ln \pi_{nn}^{'} > 0$. In this sense, tariff barriers results in trade destruction (as the converse of trade creation) everywhere.

Country l, the target of country k's change in tariff policy bears the brunt of the economic damage. Not only does country l suffer from the general price increases, from $\ln \pi_{kl}^{'} < 0$ we see it also experiences a loss of market share in country k due directly to the imposition of higher tariffs on its exports.

There are some compensations for other third party countries $n \neq l$. $\ln \pi_{ki}^{'} > 0$ shows that every country besides country l - the target of the higher tariff barrier, enjoys greater market share in country k, as buyers in country k diverts purchases away from suppliers in l.

Given the loss of foreign market share and the increases in national manufacturing prices, one might question if any country k would ever wish to unilaterally increase tariffs on imports. Perhaps surprisingly, the answer is yes, especially if the global economy were initially in a state of global free trade. The analysis of the welfare implications of unilateral tariff changes is the topic of the next few sections.

7.2 Welfare Analysis in the Mobile Labor Scenario

In this section, we consider how national welfare of each country is affected by unilateral changes in import tariff rates.

In the mobile labor scenario, the only variable component of nominal national income is tariff revenue, hence $\ln Y_n^{'} = \frac{Y_n^{'}}{Y_n} = \frac{TR_n^{'}}{Y_n}$. With this in mind, taking the derivative of equation 21 for every country $n \in N$ yields the derivative of national welfare with respect to some tariff rate t_{kl} as

$$\begin{aligned} \ln W_n' &= \ln Y_n' - \alpha \ln p_n' \\ &= \tfrac{1}{Y_n}\left(TR_n' - \alpha Y_n \ln p_n'\right) \\ &= \tfrac{1}{Y_n}\left(TR_n' + (1-\beta)\, Q_n \ln p_n^{'} - X_n \ln p_n'\right), \end{aligned} \tag{57}$$

with the third equality since $\alpha Y_n = X_n - (1-\beta)\, Q_n$, by equation 22.

Since $\ln \pi_{nn}^{'} = \theta\beta \ln p_n^{'}$, we can modify equation 57 as:

$$\ln W_n^{'} = \frac{TR_n}{Y_n}\left(\ln TR_n^{'} - \ln p_n^{'}\right) - \frac{Q_n}{Y_n}\left(\frac{1}{\theta}\ln \pi_{nn}^{'}\right) + \frac{S_n^M}{Y_n}\ln p_n^{'}, \tag{58}$$

where $S_n^M = Q_n - (X_n - TR_n)$ is national manufacturing trade surplus.[31]

The first term in the right-hand-side of equation 58 refers to the welfare effect due to the change in the real value of tariff revenues.

The second term refers to the trade creation effect of tariff changes. More specifically, a decrease in the domestic share of expenditure as the result of a tariff change can be interpreted as buyers in country n shifting purchases to more efficient overseas producers, with consequently positive welfare effects.

The final term refers to direct terms-of-trade effects. It is often thought that an increase in tariff barriers on imports would negatively impact national welfare through increased prices. But when a country enjoys a manufacturing sector trade surplus, an increase in manufacturing import tariffs might actually increase national welfare *because* of increased prices. This slightly surprising result is due to the terms-of-trade effect.

Recall that when a country runs a manufacturing trade surplus such that $S_n^M > 0$, it must run a non-manufacturing trade deficit of the same size, $S_n^O = -S_n^M < 0$. Hence a country with a positive manufacturing trade surplus is a net exporter of manufacturing goods and a net importer of the non-manufacturing good. Since the non-manufacturing good is the numeraire good, increases in national manufacturing prices p_n also imply that national price of manufacturing goods increases relative to

[31] Lemma 10 in the Appendix show the same relationship for welfare changes hold in the immobile labor scenario.

that of the non-manufacturing good, and the country with a manufacturing surplus experiences an increase in real national income as the price of its exports increases against the price of its imports.

Since trade *liberalization* is associated with lower tariff rates and consequently lower manufacturing prices (from Theorem 4), countries with manufacturing trade deficits benefit from reductions in manufacturing import tariff rates, even if these reductions are implemented by some other country. Conversely, countries with manufacturing trade surpluses stand to lose through terms-of-trade effects when tariff barrier reduction measures are implemented.

Caliendo and Parro (2015) provided an extension of the Eaton and Kortum trade framework to accomodate an arbitrary J number of sectors of production, as opposed to two in the Eaton and Kortum (2002) trade model. It might be instructive to consider the connections between welfare changes in the original Eaton and Kortum (2002) model and the newer Caliendo and Parro (2015) variant.

In the Caliendo and Parro model, each of the J individual sectors use products from every other sector as intermediate inputs, and the various sectors and countries are linked together by international trade of intermediate goods. As in the mobile labor scenario, labor freely moves between sectors within each country, but like the immobile labor scenario, national wage levels are determined endogenously. Interestingly, in Caliendo and Parro, national trade deficit (aggregated across sectors) D_n is exogeneously fixed. In Caliendo and Parro, national income includes trade deficits, such that $Y_n = TR_n + w_nL_n + D_n$.

The exogeneously determined national trade deficits D_n in Caliendo and Parro is analogous to the exogeneously determined non-manufacturing income Y_n^O in the Eaton and Kortum immobile labor scenario, in the sense that D_n in Caliendo and Parro and Y_n^O in the Eaton and Kortum immobile scenario could be seem as exogeneous endowments from which demand for output from all sectors is recursively generated. In the Eaton and Kortum immobile labor scenario, this could be seen from equation 37. It is possible to derive a tensor-algebra equivalent for the Caliendo and Parro model with respect to the vector of national trade deficits $\mathbf{L}(D_n)$, and this is in fact an avenue of research we are currently pursuing for future work.

Given the discussion above, it is possible see strong parallels between a single sector version of the Caliendo and Parro model and the immobile labor Eaton and Kortum trade model. Hence it is unsurprising that tariff changes would have the same effects on national welfare in both models.

In fact, the derivative of welfare changes in the Eaton and Kortum model can be written in exactly the same form as that given in equation 16 of Caliendo and Parro (2015). For a single sector Caliendo and Parro model, equation 16 in Caliendo and Parro (2015) giving the change in national welfare following in a change in tariffs is stated as

$$\ln W_n^{'} = \frac{1}{Y_n} \sum_{i \in \mathcal{N}} \left(E_{ni} \ln c_n^{'} - M_{ni} \ln c_i^{'} \right) + \frac{1}{Y_n} \sum_{i \in \mathcal{N}} t_{ni} M_{ni} \left(\ln M_{ni}^{'} - \ln c_i^{'} \right)$$

where $E_{ni} := Q_{ni}$ is before-tariffs exports from country n to i, and $M_{ni} := Q_{in}$ is before-tariffs imports from i to n. Caliendo and Parro termed $\sum_{i \in \mathcal{N}} \left(E_{ni} \ln c_n^{'} - M_{ni} \ln c_i^{'} \right)$ the terms of trade effect as $\ln c_n^{'}$ represents changes in the world price (prices before tariffs and transportation costs) of country n's output.

The latter term $\sum_{i\in\mathcal{N}} t_{ni}M_{ni}\left(\ln M_{ni}^{'} - \ln c_{i}^{'}\right)$ was represented as a volume of trade effect. However, since $t_{ni}M_{ni} = TR_{ni}$, nominal country n tariff revenues earned on imports from i, and $\ln M_{ni}^{'} = \ln TR_{ni}^{'}$ for $n \neq k$, it would be equally valid to rewrite Caliendo and Parro's 'volume of trade effect' as $\sum_{i\in\mathcal{N}} TR_{ni}\ln\left(\frac{TR_{ni}}{c_i}\right)^{'}$, the 'change in real tariff revenues' effect.

To derive the Eaton and Kortum equivalent, substitute $\ln c_{n}^{'} = (1-\beta)\ln p_{n}^{'}$, $\ln p_{n}^{'} = \sum_{i\in\mathcal{N}}\pi_{ni}\ln c_{i}^{'}$ and $X_{ni} = Q_{in} + TR_{ni}$ into equation 57 (while equation 57 applies to the mobile labor scenario, the same result below holds in the immobile labor scenario as well) to obtain

$$\begin{aligned}\ln W_{n}^{'} &= \frac{1}{Y_n}\left(TR_{n}^{'} + Q_n \ln c_{n}^{'} - \sum_{i\in\mathcal{N}}\left(TR_{ni} + Q_{in}\right)\ln c_{i}^{'}\right)\\ &= \sum_{i\in\mathcal{N}}\left(Q_{ni}\ln c_{n}^{'} - Q_{in}\ln c_{i}^{'}\right) + \frac{1}{Y_n}\sum_{i\in\mathcal{N}} TR_{ni}(\ln TR_{ni}^{'} - \ln c_{i}^{'}),\end{aligned}$$

with the latter equality since $TR_n = \sum_{i\in\mathcal{N}} TR_{ni}$ and $Q_n = \sum_{i\in\mathcal{N}} Q_{ni}$. The equivalence between welfare changes across both models should be obvious, hence Caliendo and Parro's formulation provides another interesting way of intepreting welfare effects in the Eaton and Kortum trade model. In particular, in the next section concerning a global free trade scenario, we would consider in detail the terms-of-manufacturing-trade effect (as oppose to the 'direct' terms-of-trade effect between the manufacturing and non-manufacturing trade referred to in the discussion of equation 58).

It is also worth relating the 'trade creation' welfare term to Arkolakis, Costinot and Rodríguez-Clare's 2012 paper, "New trade models: Same old gains?". (Here after referred to as ACR.) The key result in ACR is that across a wide range of trade models, including Eaton and Kortum (2002), changes in own expenditure shares π_{nn} and trade elasticity $\frac{d\ln\left(\frac{\pi_{ni}}{\pi_{nn}}\right)}{d\ln\left(\frac{p_i}{p_n}\right)} = -\theta$ are sufficient statistics in determining national welfare consequences of external shocks, such as changes to other countries costs of production or costs of trade. This result is given as

$$\ln\left(\frac{W_n^1}{W_n^0}\right) = -\frac{1}{\beta\theta}\ln\left(\frac{\pi_{nn}^1}{\pi_{nn}^0}\right) \tag{59}$$

where $1-\beta$ refers to the cost share of intermediate goods, W_n^0 and π_{nn}^0 refer to inital national welfare and own expenditure share, and W_n^1 and π_{nn}^1 being the same variables after some exogenous shock.

Holding initial variables W_n^0 and π_{nn}^0 constant and taking derivatives of equation 59, their results can be reframed as

$$\ln W_{n}^{'} = -\frac{1}{\beta\theta}\ln\pi_{nn}^{'}. \tag{60}$$

Given the single-sector framing of the models discussed in ACR, and the assumed absence of tariff barriers, 'manufacturing' labor income βQ_n equals national income Y_n, and $TR_n = 0$. In addition, the single-sector assumption in ACR and the static nature of the equilibrium models they analyse requires that national budgets balance and no trade surplus is possible, such that $S_n^M = 0$.

Imposing the same conditions into the Eaton and Kortum welfare derivative function given in equation 58, we obtain the identical result that changes in national welfare in response to an external change in import tariffs (here interpreted as a change in costs of trade for some other country) is given by equation 60.

What equation 58 then implies is that our analysis generalizes ACR's result to scenarios involving sectoral trade imbalance and the existence of import tariffs. It shows that when these additional factors are taken into consideration, own expenditures shares and trade elasticities alone are insufficient in determining national welfare consequences of external shocks, and other issues like changes in tariff revenues and terms of trade effects remain relevant.

How then are manufacturing tariff revenues, sales revenues and expenditures influenced by changes in tariffs?

Proposition 12. *The vectors of derivatives of national manufacturing tariff revenues, product sales revenues and expenditures are given by*

$$\begin{aligned}
\mathbf{L}(TR_n^{'}) &= \mathbf{D}(\frac{TR_n}{X_n})\mathbf{L}(X_n^{'}) + \theta\left[(1-\beta)\,\mathbf{M}(Q_{ni})^{\top}\mathbf{D}(\ln p_n^{'}) - \mathbf{D}(\ln p_n^{'})\mathbf{M}(Q_{ni})^{\top}\right]\mathbf{1}_N \quad (61)\\
&\quad + (1+\theta)\,\frac{X_{kl}}{(1+t_{kl})^2}\iota_k\\
\mathbf{L}(Q_n^{'}) &= \Pi\mathbf{t}^{\top}\mathbf{L}(X_n^{'}) - \theta\left[(1-\beta)\,\mathbf{D}(\ln p_n^{'})\mathbf{M}(Q_{ni}) - \mathbf{M}(Q_{ni})\mathbf{D}(\ln p_n^{'})\right]\mathbf{1}_N \quad (62)\\
&\quad - (1+\theta)\,\frac{X_{kl}}{(1+t_{kl})^2}\iota_l\\
\mathbf{L}(X_n^{'}) &= (1-\beta)\,\mathbf{L}(Q_n^{'}) + \alpha\mathbf{L}(TR_n^{'}), \quad (63)
\end{aligned}$$

where $Q_{ni} = \frac{\pi_{in}}{1+t_{in}}X_i$ *is the manufacturing sales revenues earned by country* n *on sales to country* i.

Proof. The result for $\mathbf{L}(TR_n^{'})$ and $\mathbf{L}(Q_n^{'})$ comes from Lemma 7 in the Appendix, and noting that $\mathbf{D}(\ln c_n^{'}) = (1-\beta)\,\mathbf{D}(\ln p_n^{'})$. The result for $\mathbf{L}(X_n^{'})$ comes from the fact that in the mobile labor scenario, $\mathbf{L}(X_n) = (1-\beta)\,\mathbf{L}(Q_n) + \alpha\mathbf{L}(TR_n) + \alpha\mathbf{L}(w_n\bar{L}_n)$ and that national labor income $\mathbf{L}(w_n\bar{L}_n)$ is exogeneously fixed. □

The first terms on the RHS of the equations for $\mathbf{L}(TR_n^{'})$ and $\mathbf{L}(Q_n^{'})$ both reflect increases in tariff revenues and manufacturing revenues due to increases in national expenditures.

The second terms on the RHS of the equations reflect changes due to shifting patterns of trade or equivalently changes in expenditure shares, caused by relative changes of national manufacturing prices.

The third term in both equations refer to the direct impact[32] of the change in the tariff rate t_{kl} on manufacturing and tariff revenues.

We note in passing that the direct effect of an increase in t_{kl} on tariff revenues of k, the country implementing the tariff rate increase, is $(1+\theta)\,\frac{X_{kl}}{(1+t_{kl})^2}$. This increase in country k's tariff revenues is exactly of the same size as the decrease in the manufacturing revenues of l, the targeted country, $-(1+\theta)\,\frac{X_{kl}}{(1+t_{kl})^2}$. As $\frac{d}{dt_{kl}}\frac{t_{kl}}{1+t_{kl}} = \frac{1}{(1+t_{kl})^2} = -\frac{d}{dt_{kl}}\frac{1}{1+t_{kl}}$, we can interpret the direct impact of the tariff

[32] Direct effect here, as in the discussion surrounding changes in manufacturing prices, means the effect experienced as a direct result of the change in tariff rate t_{kl}, and not mediated through secondary channels such as changes in manufacturing costs, prices, national aggregate expenditures or expenditure shares.

increase as a transfer of country l's sales revenue to country k's tariff revenues of size $(1+\theta)\,\frac{X_{kl}}{(1+t_{kl})^2}$. This is due solely to the fact that $\frac{1}{1+t_{kl}}$, which is the proportion of X_{kl} - country k's total expenditures on imports from l earned by country l as manufacturing revenues - has decreased to the benefit of $\frac{t_{kl}}{1+t_{kl}}$, which is the proportion of X_{kl} earned by country k as tariff revenues.

In general, solving for changes to tariff revenues and hence changes to national welfare involves simultaneously solving three systems of linear functions, namely equations 61, 62 and 63. While obtaining the numerical solutions is a trivial task, it is challenging to analyze these equations to obtain meaningful insights in the general case.

However in a scenario of global free trade, a scenario in which no country imposes import tariffs, and $TR_n = 0$ for all countries $n \in \mathcal{N}$, the task of characterizing changes in tariff revenues and hence changes to national welfare becomes much simpler. This is the topic of the next section.

7.3 Global Free Trade

Definition 5. A scenario of *global free trade* is a scenario in which $t_{ni} = 0$ for all $n, i \in \mathcal{N}$, such that no tariff barriers to trade exists.

In this section, we consider whether countries have incentives to deviate from a global free trade arrangement. That is, we consider whether any country k can increase national welfare by marginally increasing tariff rate t_{kl} on imports from l, away from zero.

In the global free trade scenario, no country earns tariff revenue and $\mathbf{L}(TR_n) = \mathbf{0}_N$. In the absence of tariff wedges, national expenditures on goods equals revenues earned by suppliers, and $X_{ni} = Q_{in}$. In matrix form, we have $\mathbf{M}(Q_{ni})^\top = \mathbf{M}(X_{ni}) = \mathbf{D}(X_n)\Pi$. This greatly simplifies analysis of changes in tariff revenues in response to changes to some t_{kl}. In the mobile labor, global free trade scenario, equation 61 simplifies to[33]

$$\begin{aligned} TR_k^{'} &= X_k \pi_{kl} \\ TR_n^{'} &= 0, \forall n \neq k. \end{aligned} \tag{64}$$

Since all countries other than k do not impose import tariffs, their tariff revenues remain unchanged at 0. The change in national welfare for these countries $n \neq k$ is given as

$$\ln W_n^{'} = -\alpha \ln p_n^{'} < 0,$$

with the inequality since by Theorem 4, $\ln p_n^{'} > 0$ for every country $n \in \mathcal{N}$ as manufacturing prices

[33]In the global free trade scenario, since $t_{kl} = 0$, $\mathbf{D}(TR_n) = \mathbf{0}_{N\times N}$ and $\mathbf{M}(Q_{ni})^\top = \mathbf{D}(X_n)\Pi$, we rewrite equation 61 as

$$\mathbf{L}(TR_n^{'}) = -\theta \mathbf{D}(X_n)\left[\mathbf{I} - (1-\beta)\,\Pi\right]\mathbf{L}(\ln p_n^{'}) + (1+\theta)\,X_{kl}\iota_k.$$

Since $\mathbf{L}(\ln p_n^{'}) = \pi_{kl}\left[\mathbf{I} - (1-\beta)\,\Pi\right]^{-1}\iota_k$, and $\mathbf{D}(X_n)\iota_k = X_k\iota_k$ we have

$$\begin{aligned} \mathbf{L}(TR_n^{'}) &= -\theta\pi_{kl}\mathbf{D}(X_n)\iota_k + (1+\theta)\,X_{kl}\iota_k \\ &= X_{kl}\iota_k. \end{aligned}$$

everywhere increase in response to an increase in t_{kl}. The interpretation is that an increase in t_{kl} does not change nominal national income for any country other than k, but it does increase national manufacturing prices for these countries. Real income necessarily decreases and national welfare deteriorates as a result.

In contrast, for country k, the country deviating from the global free trade agreement, national nominal income increases due to the increase in tariff revenue, so it is not immediately apparent whether country k benefits by introducing an import tariff $t_{kl} > 0$.

By substituting equation 64 into 57, the change in national welfare for country k is given[34] by

$$\ln W_k^{'} \quad = \quad \frac{\sum_{i \in \mathcal{N}} \left(Q_{ki} \ln c_k^{'} - X_{ki} \ln c_i^{'}\right)}{Y_k}. \tag{65}$$

In equation 58, we referred to the term $\frac{S_n^M}{Y_n} \ln p_n^{'}$ as the terms-of-trade effect on national welfare. More precisely, $\frac{S_n^M}{Y_n} \ln p_n^{'}$ refers to the terms-of-trade effect specific to relative changes between prices of manufacturing and non-manufacturing goods. In equation 65, the term $\sum_{i \in \mathcal{N}} \left(Q_{ki} \ln c_k^{'} - X_{ki} \ln c_i^{'}\right)$ refers to a terms-of-trade effect for trade within the manufacturing sector. That is, 'terms-of-trade' here refers to changes in the prices of the types of manufacturing goods imported by the country k relative to its manufacturing exports.

To see this, recall that Q_{ki} refers to country k's revenues earned on manufacturing sales to country i, and X_{ki} in the global free trade scenario refers to country k's payments to suppliers in country i. From equation 4, the 'world price' net of tariffs that country k pays to suppliers in country i for a good $\omega \in \Omega_{ki}$, is $\bar{p}_{ki}(\omega) = \frac{c_i \bar{d}_{ki}}{z_i(\omega)}$, where $\bar{d}_{ki}$ is exogeneously given 'geographical trade' costs and $z_i(\omega)$ is the exogeneously given level of efficiency with which country i produces good ω.

For all countries $i \in \mathcal{N}$, $\ln \bar{p}_{ki}(\omega)^{'} = \ln c_i^{'}$. The price country k pays to foreigners for imports is influenced by solely by changes in costs of manufacturing production at the source of the imports, and $\ln c_i^{'}$ is the proportionate change in prices of country k's imports from country i. For exactly the same reason, $\ln c_k^{'}$ reflects the change in prices of country $k's$ exports.

Let $p_{ki} = \left[\int_{\omega \in \Omega_{ki}} \bar{p}_{ki} \left(\omega\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ refer to the CES aggregate price of country k's imports from country i, such that $\frac{X_{ki}}{p_{ki}}$ is the measure of real imports, and notice that $\ln p_{ki}^{'} = \ln c_i^{'}$. Then since $X_{ki} \ln c_i^{'} = \frac{X_{ki}}{p_{ki}} p_{ki}^{'}$, it is clear that$X_{ki} \ln c_i^{'}$ is the change in country k payments to foreigners in country i and similarly, $Q_{ki} \ln c_k^{'}$ is the change in payments received from foreigners in country i.

[34] Substituting equation 64 into 57, and recalling $(1-\beta) \ln p_n^{'} = \ln c_n^{'}$ for all $n \in \mathcal{N}$ gives

$$\ln W_k^{'} = \frac{X_k \pi_{kl} + Q_k \ln c_k^{'} - X_k \ln p_k^{'}}{Y_k}. \quad (*)$$

From equation 53, we have $\ln p_k^{'} = \sum_{i \in \mathcal{N}} \pi_{ki} \ln c_i^{'} + \pi_{kl} \iota_k$ in the global free trade scenario as $t_{kl} = 0$. Substituting this back into $(*)$, and letting $Q_k = \sum_{i \in \mathcal{N}} Q_{ki}$, we have

$$\ln W_k^{'} = \sum_{i \in \mathcal{N}} \frac{\left(Q_{ki} \ln c_k^{'} - X_{ki} \ln c_i^{'}\right)}{Y_k}. \quad (**)$$

This relationship in fact holds for *all* countries $n \in \mathcal{N}$ in the global free trade scenario.

The term $\sum_{i\in\mathcal{N}}\left(Q_{ki}\ln c_{k}^{'}-X_{ki}\ln c_{i}^{'}\right)$ is consequently the change in net income earned on trade by country k due to relative changes in prices of country k's manufacturing imports and exports, and hence termed the manufacturing terms-of-trade effect.

Given the discussion so far, is there any country that might benefit by deviating from global free trade? The results of Theorems 3 and 4 assert that there must always exist one country that benefits by introducing import tariffs.

Theorem 3 asserts that that must always exist some country $k\in\mathcal{N}$ that does not run a manufacturing trade deficit against any other country such that $Q_{ki}-X_{ki}\geq 0$ for all $i\in\mathcal{N}$. Suppose country k marginally increases tariff rate t_{kl} away from 0.

By Theorem 4, it must be that $\ln c_{k}^{'}>\ln c_{i}^{'}$ for all countries $i\neq k$, and we must have $Q_{ki}\ln c_{k}^{'}>X_{ki}\ln c_{i}^{'}$ for all countries $i\neq k$, and hence $\ln W_{k}^{'}>0$. A country that runs no manufacturing trade deficits would always benefit by deviating from a global free trade agreement.

But there exists an even more general result: *every* country could benefit by unilaterally increasing tariff rates away from 0.

Theorem 5. *For all countries $k\in\mathcal{N}$, in the mobile labor global free trade scenario,*

$$\frac{d\,\ln W_k}{d\,t_{kl}}=-\sum_{n\neq k}\frac{Y_n}{Y_k}\frac{d\,\ln W_n}{d\,t_{kl}}>0. \tag{66}$$

Every country can increase national welfare by unilaterally deviating from the global free trade agreement.

Proof. Transposing equation 53 in the global free trade scenario gives

$$\mathbf{L}(\ln p_{n}^{'})^{\top}=\pi_{kl}\iota_{k}^{\top}\left[\mathbf{I}-(1-\beta)\,\Pi^{\top}\right]^{-1}.\quad(*)$$

In the free trade scenario, Y_n the national income is simply $w_n\bar{L}_n$, the national labor income, such that $\mathbf{L}(Y_n)=\mathbf{L}(w_n\bar{L}_N)$. It is also the case that since $t_{ni}=0$ for all $n,i\in\mathcal{N}$, we have $\mathbf{Dt}=\mathbf{0}_{N\times N}$. Hence equation 31 can be rewritten as

$$\mathbf{L}(\alpha Y_n)=\left[\mathbf{I}-(1-\beta)\,\Pi^{\top}\right]\mathbf{L}(X_n).\quad(**)$$

Combining the pre-multiplying the LHS $(**)$ with the LHS of $(*)$, and the same of RHS of the equations, we get

$$\mathbf{L}(\ln p_{n}^{'})^{\top}\mathbf{L}(\alpha Y_n)=\pi_{kl}\iota_{k}^{\top}\mathbf{L}(X_n),$$

giving

$$\sum_{n\in\mathcal{N}}\alpha Y_n\ln p_{n}^{'}=X_{kl}=TR_{k}^{'}.\quad(***)$$

Since αY_n is nominal expenditure on consumption of manufacturing goods, we can interpret equation $(***)$ as the change in global consumption expenditure on manufacturing goods, and this increase in global manufacturing consumption is due entirely to the increase in country k's tariff income.[35]

Since

$$Y_n \ln W_n^{'} = \begin{cases} -\alpha Y_n \ln p_n^{'} & \forall n \neq k \\ TR_k^{'} - \alpha Y_n \ln p_k^{'} & otherwise, \end{cases}$$

therefore equation $(***)$ can be rewritten as

$$\sum_{n \in \mathcal{N}} Y_n \ln W_n^{'} = 0.$$

As an increase in t_{kl} decreases welfare in every country other than k, we have $\sum_{n \neq k} Y_n \ln W_n^{'} < 0$, which immediately gives us

$$Y_k \frac{d \ln W_k}{d\, t_{kl}} = -\sum_{n \neq k} Y_n \frac{d \ln W_n}{d\, t_{kl}} > 0,$$

as required. □

The basic intuition behind Theorem 5 is that deviation from the global free trade arrangement is a constant sum game. Equation 66 can be rewritten as

$$\sum_{n \in \mathcal{N}} \frac{Y_n}{\sum_{i \in \mathcal{N}} Y_i} \ln W_n^{'} = 0. \tag{67}$$

The income-weighted average of national welfare changes equals 0. Country k gains by deviating from global free trade at the expense of the other countries.

Recall that $(1-\alpha)\, Y_n$ is the real non-manufacturing consumption by any country n given national nominal income Y_n and αY_n is the country's corresponding nominal consumption of manufacturing goods. Since i) nominal incomes are unchanged for any country $n \neq k$, ii) non-manufacturing prices remain unchanged at 1 (as the non-manufacturing good is the numeraire) and iii) manufacturing prices increase in every country when country k increases import tariffs, it follows that real manufacturing consumption decreases and non-manufacturing consumption remains constant for these countries.

[35] An alternative interpretation is this:

Since $\frac{\alpha Y_n}{p_n}$ is real national manufacturing consumption for country $n \in \mathcal{N}$, the change in real manufacturing consumption for each country is

$$\left(\frac{\alpha Y_n}{p_n}\right)^{'} = \frac{\alpha}{p_n}\left(Y_n^{'} - Y_n \ln p_n^{'}\right) = \begin{cases} -\frac{\alpha}{p_n} Y_n \ln p_n^{'} & ,\forall n \neq k \\ -\frac{\alpha}{p_k}\left(Y_n \ln p_k^{'} - TR_k^{'}\right) & otherwise. \end{cases}$$

The sum of price weighted changes to real national manufacturing consumption is hence

$$\begin{aligned} \sum_{n \in \mathcal{N}} p_n \left(\frac{\alpha Y_n}{p_n}\right)^{'} &= \alpha TR_k^{'} - \sum_{n \in \mathcal{N}} \alpha Y_n \ln p_n^{'} \\ &= -(1-\alpha)\, TR_k^{'}. \end{aligned}$$

$(1-\alpha)\, TR_k^{'}$ is the increase in real non-manufacturing consumption in country k. The interpretation here is that global real manufacturing consumption falls, as labor resources are directed away from manufacturing towards non-manufacturing production in order to accommodate greater demand for non-manufacturing goods from country k.

National welfare is strictly deteriorates for these countries because of diminished real manufacturing consumption, and $\ln W_n^{'} < 0$ for all countries $n \neq k$. Then by equation 66, country k's welfare must increase.

The creation of a tariff barrier t_{kl} distorts the global allocation of manufacturing production, and increases the prices of manufacturing goods everywhere. The increase in the price of manufacturing goods results in substitution towards consumption of non-manufacturing goods. This, combined with the distortion of production allocations towards less efficient manufacturing producers, means that global real value-added (output net of intermediate goods) of manufacturing goods decreases.

To see that global real manufacturing output decreases when $\Delta t_{kl} > 0$ in the global free trade scenario, start by summing up changes in manufacturing revenues and expenditures in equations 62 and 63. Recalling that $\sum_{i\in\mathcal{N}} TR_i^{'} = TR_k^{'} = X_{kl}$, we obtain the changes in global manufacturing output and expenditure as

$$\begin{aligned}\sum_{n\in\mathcal{N}} Q_n^{'} &= \sum_{n\in\mathcal{N}} X_n^{'} - X_{kl}\\ \sum_{n\in\mathcal{N}} X_n^{'} &= (1-\beta)\sum_{i\in\mathcal{N}} Q_n^{'} + \alpha X_{kl}.\end{aligned}$$

Solving for global nominal manufacturing value-added $\beta\sum_{i\in\mathcal{N}} Q_n^{'}$ gives

$$\beta\sum_{i\in\mathcal{N}} Q_n^{'} = -(1-\alpha) X_{kl} < 0. \tag{68}$$

The increase in country k's national income via the increase in national tariff revenue $Y_k^{'} = X_{kl}$ is spent on increased nominal manufacturing consumption αX_{kl} and increased real non-manufacturing consumption $(1-\alpha) X_{kl}$.[36] Equation 68 shows that the total decrease in global manufacturing value-added equals the increase in country k's non-manufacturing consumption.

Manufacturing value-added βQ_n in each country for each country $n \in \mathcal{N}$ is manufacturing labor income $w_n L_n^M$. Total national labor force $\bar{L}_n = L_n^M + L_n^O$ is split between manufacturing and non-manufacturing sectors. Recall that in the mobile labor case, the national labor force $\bar{L}_n$ is exogeneously fixed as is the national wage rate w_n. Fixed national wages means that decreases in national manufacturing value-added is met by a proportional decrease in national labor employed in the manufacturing sector, such that $\beta Q_n^{'} = w_n \left(L_n^M\right)^{'}$. Inelastic national labor supply implies that the decrease in manufacturing employment equals the increase in non-manufacturing employments -$\left(L_n^M\right)^{'} = \left(L_n^O\right)^{'}$. Labor is the sole input to constant-returns-to-scale non-manufacturing production hence the increase in non-manufacturing production is proportional to the increase in non-manufacturing employment, with $\left(Y_n^O\right)^{'} = w_n \left(L_n^O\right)^{'}$. We then rewrite equation 68 as

$$\sum_{n\in\mathcal{N}} \left(Y_n^O\right)^{'} = (1-\alpha) X_{kl}.$$

[36] $(1-\alpha) X_{kl}$ is the increase in real country k non-manufacturing consumption since the non-manufacturing good is the numeraire good with price of 1.

Since non-manufacturing consumption remains unchanged in every country other than k, any increase in global non-manufacturing output $\sum_{n\in\mathcal{N}}\left(Y_n^O\right)^{'}$ is completely captured by country k as increased non-manufacturing consumption. The increase in country k's non-manufacturing consumption diverts labor in the global economy towards the production of non-manufacturing goods, with a consequential decrease in the global manufacturing workforce. Given the constant returns to scale technology employed in the manufacturing sector, the reduced manufacturing labor force results in decreased global real manufacturing output. This is consistent with the alternative interpretation that the decrease in global nominal manufacturing value-added combined with increased manufacturing prices everywhere mathematically necessitates lower global real manufacturing output.

Real manufacturing consumption in country k may decrease due to the higher national price of manufacturing goods, though this is mitigated in part by increased nominal manufacturing consumption expenditure αX_{kl}. But any loss in k's national welfare through lower real manufacturing consumption is more than compensated for by increased real non-manufacturing consumption $(1-\alpha)\,X_{kl}$, which in turn comes at the expense of real manufacturing consumption everywhere else. By deviating from global free trade, country k gains by a transfer of real resources from every other country to itself.

Since any country $k\in\mathcal{N}$ can increase national welfare by increasing tariffs on imports from every other country $l\in\mathcal{N}$, such that $\frac{d}{d\,t_{kl}}\ln W_k>0$, equation 65 tells us that starting from a world free trade scenario, any country can improve manufacturing sector terms-of-trade by erecting tariff barriers , as

$$\sum_{i\in\mathcal{N}}\left(Q_{ki}\ln c_k^{'}-X_{ki}\ln c_i^{'}\right)>0.$$

The interpretation of this result is the ability of national governments to exercise market power by manipulating the production costs and hence prices of their countries' exports through tariff policy. The stochastic distribution of manufacturing efficiencies ensures that for every country, there must always exist some set of manufacturing goods for which the country is most efficient producer in the world. Even in the presence of costs of international trade, there would always be some goods for which a *country* is the monopoly supplier. However, perfectly competitive markets for manufacturing goods prevents individual *firms* in any country from exercising market power, as competitive markets prevents coordination of prices or output with other firms in the same country.

Governments have the ability to compensate for this lack of coordination. By increasing import tariffs, the government in country k increases the cost of manufacturing and hence the pushing up the factory gate price of manufacturing goods produced in the country. Theorem 4 ensures that the prices of the country's imported manufacturing intermediate goods do not increase as quickly as the country's exports as $\ln c_k^{'}>\ln c_i^{'}$ for all $i\neq k$. Thus higher import tariffs increase average manufacturing value added. Furthermore, part of the increase in prices of imported goods is recovered as tariff revenues, partially off-setting any increase in expenditures on imported goods. In other words, tariff policy could serve as a means of controlling the prices of a country's exports, allowing a country to exercise market power even in the presence of perfectly competitive markets. [37]

[37] It is interesting to note that in Alvarez and Lucas (2007), Figure 5 in page 1753 demonstrates that a country can gain by raising tariff rates away from zero, with the same explanation of countries exercising market power through tariff policy.

The discussion up to this point suggests that taking other countries' tariff choices as given, there should exist some positive country k import tariff vector $(t_{k1}, ..., t_{kN}) \gg \mathbf{0}_N$ that maximizes national welfare. Given the complexity of analysing changes in national welfare when countries impose positive import tariffs rates, the analysis of countries' optimal vectors of import tariffs lies beyond the scope of this thesis, though potentially fertile ground for future research.

However, one could easily show that starting from a global free trade scenario, holding all other tariffs constant at zero, there exists an tariff rate $t^*_{kl} > 0$ that maximizes country k's national welfare. We have already established by Theorem 5 that when $t_{kl} = 0$, we have $\ln W^{'}_k > 0$ and country k can improve national welfare by increasing tariffs rate on imports from country l.

On the other hand, if t_{kl} is sufficiently high, national welfare could be increased by decreasing t_{kl}. As t_{kl} approaches ∞, imports from country l would be priced out of country k's markets, and $\pi_{kl} = 0$. Since expenditure on imports from l is non-existent, so would country k's tariff revenues. As tariff t_{kl} is lowered, imports from l increases, as would country k tariff revenues and hence national income would increase. Gaining greater access to efficient country l producers lowers manufacturing prices in country k and national manufacturing prices p_k decreases. The increase in national income and lowered manufacturing prices triggered by a decrease in t_{kl} results in greater real national income - welfare.

Then since $\ln W^{'}_k$ is obviously continuous in t_{kl}, $\lim_{t_{kl}\to 0} \ln W^{'}_k > 0$ and $\lim_{t_{kl}\to\infty} \ln W^{'}_k < 0$ there must exist some optimal tariff $t^*_{kl} \in (0, \infty)$ that maximizes country k welfare. An analytical characterization of the optimal tariff rate t^*_{kl} here would be difficult, given the role played by terms-of-trade effects in changes to national welfare. More precisely, the terms-of-trade effect resulting from changes to manufacturing prices either positively or negatively impacts national welfare depending on the direction of a country's trade balance. However, the response of national trade balance to changes in tariff policies is complex, complicating the analysis.

That aside, with Theorem 5 we could construct each country's decision whether or not to deviate from the global free trade arrangement as the classic prisoner's dilemma game, in which every country could gain by deviating from the jointly optimal outcome of global free trade, and every country loses by others' decision to do so.

From this game-theoretic perspective, it is clear that a global free trade agreement is not a Nash equilibrium and hence would quickly unravel, at least in the context of the Eaton and Kortum trade model. The typical means by which to resolve the coordination problem inherent in prisoner's dilemma type games is to establish punishment mechanisms for deviations from the desired outcome. Would it be possible to establish the global free trade arrangement as a Nash equilibrium if countries are allowed to respond to unilateral impositions of import tariffs by introducing tariffs on imports from the offending country?

But their analytical result is based on the assumption that the manufacturing labor force in this country is arbitrarily small compared to that of other countries. This assumption is required in their analysis to ensure that changes in this country has no effect on any other countries, as the arbitrarily small workforce would imply that this country's contributions to international trade flows would be inconsequential.

Theorem 5 shows that even without the assumption of a vanishingly small labor force, a country can benefit from raising tariffs away from global free trade.

To simplify the analysis, we consider this issue in the context of a *symmetrical world.*

Definition 6. A *symmetrical world* is one in which every country is identical, as are the trade costs between each pair of countries. For each country $n \in \mathcal{N}$, national technology level $T_n = \bar{T}$, national labor force $L_n = \bar{L}$ and national wage rate $w_n = \bar{w}$ for some constants $\bar{T}$, $\bar{L}$ and $\bar{w}$. For each pair countries $n, i \in \mathcal{N}$, we have trade costs $d_{ni} = \bar{d}$ for some constant $\bar{d} \geq 1$ if $n \neq i$ and $d_{nn} = 1$.

In the global free trade scenario in a symmetrical world, national labor incomes, manufacturing prices, revenues and expenditures are identical and no country runs a manufacturing trade surplus or deficit, such that for all countries $n \in \mathcal{N}$, we have $Y_n = \bar{Y}$, $p_n = \bar{p}$ and $X_n = \bar{X} = Q_n$. Since manufacturing wages and prices are identical between countries, national manufacturing expenditure shares on domestically sourced goods must be identical as well, such that $\pi_{nn} = \bar{\pi}$, and by the same reasoning expenditure shares on imports from other countries must be also identical, such that $\pi_{ni} = \frac{1-\bar{\pi}}{N-1}$ for all $n \neq i$.

Lemma 6. *Under a global free trade scenario with symmetrical countries, the following holds:*

- *for any $n, i \in N$, $\frac{d \ln c_i}{d t_{in}} = \bar{c}$;*
- *for any $n, i, j \in N$ with $i \neq j$, $\frac{d \ln c_i}{d t_{jn}} = \underline{c}$,*

for some $\bar{c} > \underline{c} > 0$.

Proof. Because $\frac{d \ln c_k}{d t_{kl}} > \frac{d \ln c_n}{d \ln t_{kl}} > 0$ for all $n \neq k$ by Theorem 4 and because all the countries and trade costs are identical the result is direct. □

Proposition 13. *Under a global free trade scenario with symmetrical countries, suppose country k deviates from the global free trade agreement by introducing import tariff $\Delta t_{kl} = \Delta t > 0$ for on imports from some country $l \in \mathcal{N} \backslash \{k\}$.*

Let $A \subseteq \mathcal{N} \backslash \{k\}$ denote the set of countries that punishes this unilateral deviation by introducing tariffs on imports from k, such that for all $n \in A$, we have $\Delta t_{nk} = \Delta t$.

Denote the total change in the logarithm of national welfare in country $i \in \mathcal{N}$ by

$$\Delta \ln W_i := \left(\frac{\mathrm{d} \ln W_i}{\mathrm{d} t_{kl}} + \sum_{n \in A} \frac{\mathrm{d} \ln W_i}{\mathrm{d} t_{nk}} \right) \Delta t.$$

Then,

$$\Delta \ln W_i = \begin{cases} \left(1 - \frac{|A|}{N-1}\right) B \Delta t & if\, i \in A \cup \{k\} \\ -\frac{|A|+1}{N-1} B \Delta t & otherwise \end{cases},$$

where $B = \frac{\bar{X}}{\bar{Y}} (1 - \bar{\pi}) (\bar{c} - \underline{c}) > 0$.

Proof. Suppose country k introduces tariffs on imports from country l such that $\Delta t_{kl} = \Delta t$ that is sufficiently small. In the mobile labor scenario, equation 57 could be written in matrix form as

$$\mathbf{L}(\ln W_n^{'}) = \mathbf{D}(Y_n)^{-1}\mathbf{D}(X_n)\left[\pi_{kl}\iota_k + \mathbf{L}(\ln c_n^{'}) - \mathbf{L}(\ln p_n^{'})\right].$$

Since $\mathbf{L}(\ln p_n^{'}) = \Pi\mathbf{L}(\ln c_n^{'}) + \pi_{kl}\iota_k$, and given the symmetrical countries assumption, $\pi_{nn} = \bar{\pi}$ and $\pi_{ni} = \frac{1-\bar{\pi}}{N-1}$ for all $n \neq i$, $Y_n = \bar{Y}$ and $Q_n = X_n = \bar{X}$ for all $n \in \mathcal{N}$, we have

$$\mathbf{L}(\ln W_n^{'}) = \mathbf{D}\Big(\frac{\bar{X}}{\bar{Y}}\Big)\left[\mathbf{I} - \Pi\right]\mathbf{L}(\ln c_n^{'}),$$

and for each country $n \in \mathcal{N}$, the change in welfare in response to some marginal increase in t_{kl} is

$$\begin{aligned}\frac{d}{d\,t_{kl}}\ln W_n &= \frac{\bar{X}}{\bar{Y}}\left(1-\bar{\pi}\right)\left(\ln c_n^{'} - \sum_{i\neq n}^{N}\frac{\ln c_i^{'}}{N-1}\right)\\ &= \begin{cases} -\frac{\bar{X}}{\bar{Y}}\left(1-\bar{\pi}\right)\left(\frac{\bar{c}-\underline{c}}{N-1}\right) & \forall n \neq k\\ \frac{\bar{X}}{\bar{Y}}\left(1-\bar{\pi}\right)\left(\bar{c}-\underline{c}\right) & n = k,\end{cases}\end{aligned}$$

with the second equality by Lemma 6.

Suppose each country $h \in A$ retaliates by introducing $\Delta t_{hk} = \Delta t = \Delta t_{kl}$. By symmetry of the countries,

$$\frac{d\ln W_h}{d\,t_{hk}} = \frac{d\ln W_k}{d\,t_{kl}} = B,$$

where $B = \frac{\bar{X}}{\bar{Y}}\left(1-\bar{\pi}\right)\left(\bar{c}-\underline{c}\right) > 0$ and

$$\frac{d\ln W_i}{d\,t_{hk}} = \frac{d\ln W_n}{d\,t_{kl}} = -\frac{B}{N-1}.$$

for all $i \neq h$ and $n \neq k$.

A country increasing tariff rates enjoys an increase in national welfare of B as a direct consequence of its actions, while all other countries suffer decreases of national welfare $-\frac{B}{N-1}$.

Suppose country $n \in \mathcal{N}\backslash A \cup \{k\}$, such that n is a bystander country maintaining no tariff barriers. Then the cumulative negative welfare effects of the $|A|+1$ tariff increases by countries $h \in A \cup \{k\}$ is

$$\begin{aligned}\Delta \ln W_n^{'} &= \left(\sum_{h\in A\cup\{k\}}\frac{\mathrm{d}\ln W_i}{\mathrm{d}t_{nk}}\right)\Delta t\\ &= -\left(\frac{|A|+1}{N-1}B\right)\Delta t.\end{aligned}$$

Countries $i \in A \cup k$ suffer $|A|$ negative effects from the $|A|$ other countries' decisions to increase tariff rates, when enjoying a positive effect from its own imposition of tariffs. Hence for all $i \in A \cup k$,

$$\begin{aligned}\Delta \ln W_i' &= \left(\frac{d \ln W_i}{d\, t_{ik}} + \frac{d \ln W_i}{d\, t_{kl}} + \sum_{h \in A \setminus i} \frac{\mathrm{d} \ln W_i}{\mathrm{d} t_{nk}} \right) \Delta t \\ &= \left(1 - \frac{|A|}{N-1} \right) B.\end{aligned}$$

□

Proposition 13 implies that deviating from global free trade is a weakly dominant strategy even if the punishment mechanism is implemented. Every country enjoys increases in welfare from unilaterally deviating from a global free trade agreement whenever retaliation is not universal,[38] i.e. if $A \subset \mathcal{N} \setminus \{k\}$. If retaliation is universal, i.e. if $A = \mathcal{N} \setminus \{k\}$, deviating from the global free trade arrangement does not make the deviating country worse off.

When a set A is a singleton, we immediately obtain the following result as a corollary to Proposition 13. It simply states that when country k and country l increase tariffs on imports from each other, these two countries gain, while the remaining countries loses.

Corollary 1. *Under a global free trade scenario with symmetrical countries, when* $\Delta t_{kl} = \Delta t_{lk} = \Delta t > 0$ *for* Δt *sufficiently small, then* $\Delta \ln W_k > 0$*,* $\Delta \ln W_l > 0$*, and* $\Delta \ln W_n < 0$ *for* $n \in N \setminus \{k, l\}$.

Corollary 1 states that there is always a pair of countries who are better off not joining a global free trade network.

Within a partial equilibrium framework under the assumption of symmetrical countries, Furusawa and Konishi (2007) demonstrated that global free trade is a stable network in the sense of Jackson and Wolinsky (1996). In their definition, a *pairwise stable network* is a network in which no country has an incentive to cut a link with another and for any unlinked pair of countries, at least one of them has no incentive to form a link with the other.

A bilateral free trade agreement between country n and i can be thought as a link, which is an unordered pair of two countries. A global free trade agreement, as conceptualized as the collection of bilateral free trade agreements between all possible pairs of countries, would therefore be represented from the network perspective as a network in which every country is linked with every other country.

Since Corollary 1 indicates at least 2 countries would find it welfare enhancing to abandon their bilateral free trade agreement, i.e. to extinguish the link between their respective nodes, in contrast to

[38] Proposition 13 also implies that the design of the punishment mechanism for unilateral deviation from global free trade is important in determining the effectiveness of the punishment mechanism in maintaining global free trade as a stable outcome. Once again, suppose country k imposes tariffs on imports from some country l. If the rules of the free trade agreement only allows the country transgressed upon to retaliate, then $\Delta \ln W_k > 0$ (as in Corollary 1) and country k is still better off by deviating. However, if collective punishment is allowed, that is, if every country is allowed to impose tariffs on imports from the transgressing country k, then $\Delta \ln W_k = 0$ and country k does not have an incentive to deviate.

But the effectiveness of the punishment mechanism is fragile. Even if collective punishment is permitted, it could be easily shown that country k might find it best to simply impose tariffs on imports from every country i.e. $\Delta t_{ki} > 0$ for all $i \in \mathcal{N} \setminus k$, since the eventual punishment would be the same, that every country imposing tariffs on imports from k. It is in fact the case that punishment, no matter whether individual or collective, would not be effective in maintaining the global free trade arrangement.

Furusawa and Konishi, we find that a global free trade scenario does not represent a pairwise stable network, at least within the context of the Eaton and Kortum trade model.

Analytical results for the immobile labor scenario, under similar assumptions of global free trade and symmetrical countries are presented in the Appendix.

8 Numerical Simulations

In this section, we perform numerical simulations to examine how the fundamental variables of the model such as tariff rates, the presence of intermediate goods, the relative level of technology and labor forces, affect the welfare consequences of preferential trade agreements.

The numerical simulations in this section are aimed to develop a better understanding of *direction* of changes to welfare, prices and trade patterns, brought on by the implementation of preferential trade agreements, particularly as an avenue to apply and discuss our theoretical results obtained previously in the thesis.

It must be noted that the results here are not intended as a guide to the consequences of real world examples of trade agreements, as the baseline scenario implemented in the numerical simulations here is strictly hypothetical, with many parameters chosen for computational convenience.

8.1 Simulation Method

The conventional method to conduct comparative statics used in Eaton and Kortum (2002) requires the full set of model parameters: the values of α, β, θ, the matrix of geographical distances $\mathbf{M}\left(\bar{d}_{ni}\right)$, the vector of manufacturing technology levels $\mathbf{L}\left(T_n\right)$, the vector of total labor forces in each nation $\mathbf{L}(\bar{L}_n)$, and the vector of wages in each nation $\mathbf{L}(w_n)$.

In this section we use a different method that can be conducted with less information on parameters and does not require computing a fixed point as a solution to the system of equations. This method only takes bilateral trade and tariff data as exogenous information in addition to the values of α, β, and θ. In this method, we first obtain the trade share matrix $\mathbf{M}(\pi_{ni})$. By equation 53, we obtain the changes in the prices for all the countries and then the changes in the trade shares with Proposition 10. Simultaneously solving equations 61, 62, and 63, we obtain the changes in tariffs, manufacturing revenues, and expenditures, which together with the changes in prices give us the changes in national welfare by equation 57. We find the new equilibrium by numerically integrating the changes over the tariff changes.

We consider a model with three countries. The baseline scenario for our counterfactural exercise in one in which [39] each country imposes the same tariff rate $\bar{t}$, on the manufacturing imports from all other countries. We define a bilateral PTA as being formed when two countries agree to mutually remove tariffs on the imports from each other, and examine how national welfare and other equilibrium variables change for both member and non-member countries following the formation of PTAs.

[39]Our program is not confined to three countries, but can be applied to many countries, such that $n > 3$. However, it is desirable to keep the analysis as simple as possible.

In the benchmark scenario, all the countries are identical and trade costs are symmetrical. In other scenarios, we make changes to initial tariff rates, the cost share of intermediate goods, the relative technological levels and labor forces to see how welfare consequences for members and nonmembers are influenced by each country's characteristics.

8.2 The Benchmark Case

Table 1 provides the baseline levels of the parameters we use for the simulations. Dekle, Eaton and Kortum (2007) suggest that α usually ranges from 0.25 to 0.5. We use 0.25. Following Dekle *et al.* (2007) and Cheong and Takayama (2013), we use $\beta = 0.28$. Given that γ only influences the absolute value of manufacturing prices, but not the relative manufacturing prices or any other endogenously determined variables, we simply set $\gamma = 1$.[40] We use $\theta = 8.21$ based on the estimations in Cheong and Takayama (2013), for which Eaton and Kortum (2002) obtain a similar value.[41] The two parameters, $w_n = 2.35$ and $\bar{L}_n = 247.14$, are chosen for our convenience so that wages, prices, and hence trade shares in equilibrium are the same for the mobile and immobile labor cases in the baseline setting.[42]

$N = 3$	$d_{ni} = 1.2 \; \forall i \neq n$	$\alpha = 0.25$	$\beta = 0.28$	$\gamma = 1$
$t_{ni} = 0.05 \; \forall i \neq n$	$\theta = 8.21$	$T_n = 1$	$w_n = 2.35$	$\bar{L}_n = 247.14$

Table 1: List of Parameters

Table 2 shows the changes in the equilibrium values in the benchmark case. We present the changes in manufacturing trade deficits for country n, denoted by D_n, and the domestic trade share, π_{nn} in absolute terms, and other results in % terms. In a symmetrical case, D_n is initially zero, and so the

[40]Following Eaton and Kortum (2002), we chose $\gamma = 1$ for computational simplicity since the value of σ is inconsequential when considering relative changes in prices, trade shares and national welfare. Quoting from footnote 18 of Eaton and Kortum (2002): "While our framework allows for the possibility of inelastic demand ($\sigma \leq 1$), we must restrict $\sigma < 1 + \theta$ in order to have a well defined price index. *As long as this restriction is satisfied, the parameter σ can be ignored, since it appears only in the constant term (common across countries) of the price index*."(emphasis added)

[41]Simonovska and Waugh (2014) suggest that a value for θ of approximately 4 might be more appropriate. Numerical simulation of the benchmark case were performed with $\theta = 4$. It was found that while the direction of changes in equilibrium variables remains consistent with our chosen baseline scenario, the magnitude of the changes were different. In particular, compared with our baseline settings, when using $\theta = 4$ we found that the magnitude of welfare changes were increased, while the magnitude of changes in trade flows (e.g. changes in own trade shares π_{nn}) were reduced.

These observations could be explained by recognizing thatθ is a measure of elasticity of trade, the responsiveness of trade flows to changes in trade costs. In the context of changes to tariff rates, θ indicates the ease with which countries can shift to lower cost suppliers when the imports from a particular country are made more expensive due to heightened tariff barriers. It is hence no surprise that the size of change in trade flows was decreased when θ was reduced from 8.21 (as in our baseline settings) to 4, with changes in own trade shares $\Delta\pi_{nn}$ for member countries dropping from -0.04 to -0.02.

When buyers find it difficult to substitute towards lower cost suppliers in response to higher import tariffs, the welfare effects of tariff changes would be amplified. Correspondingly, we naturally would associate lower values of θ with greater absolute sizes of changes in welfare associated with changes in tariff regimes. When θ was decreased from 8.21 to 4, it was found that the welfare decrease suffered by PTA member countries jumped from -0.41 basis points to -10.36 basis points.

As this exercise shows, accurate estimates of θ would be critical in the empirical application of the Eaton and Kortum model.

[42]In this section, we only consider the case of mobile labor, because the purpose of the numerical simulations in this section was to illustrate and discuss the theoretical results we have obtained for the mobile labor scenario. Numerical simulation results and commentary for the immobile labor scenario are included in the appendix.

ratio of changes is not well defined. In addition, π_{nn} is the *proportion* of expenditure on domestic goods, and so absolute changes in π_{nn} are more appropriate.

As $Y_n = w_n\bar{L}_n + TR_n$, and labor income $w_n\bar{L}_n$ is fixed, the change in tariff revenues affects national welfare. As tariffs are eliminated with a PTA, members lose tariff revenues, and so welfare decreases when tariff rates are sufficiently high, as shown in Table 2. The reduction of tariff revenues for members is around 50%, which may appear large. With the 3-country setting, initially, each country collects tariffs on imports from two countries. After the formation of a PTA, member counties now obtain tariff revenues from only one country (the nonmember country), and tariff revenues fall by approximately 50%.

As asserted in Theorem 4, the change in price is larger for member countries and thus own trade shares decrease more. In this specific benchmark case, the nonmember country gains. As Theorem 4 also states, own trade share in the nonmember country decreases and import expenditure increases, which in turn increases tariff revenues. Combined with the lower prices, increased tariff revenues improve nonmember country's welfare.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta L_n$	$\%\Delta p_n$	ΔD_n	$\Delta \pi_{nn}$
Member	-0.4 $\times 10^{-2}$	-54.22	5.17	-2.04	-5.02	-0.04
Non-Member	0.20	0.54	-7.14	-0.78	10.05	-0.01

Table 2: The Benchmark Case (unit: % except for D_n and π_{nn})

8.3 Tariffs

In this subsection, we examine how the welfare consequences of a PTA are affected by a higher pre-PTA tariff. We set the initial tariff rates before the PTA as 50%, instead of 5%. In this scenario, as shown in 3, member countries are better off compared to the benchmark case. The decrease in price is larger and so does the own trade share. Even though the tariff revenues decrease more, member countries gain due to the substantial decrease in prices. From equation 58, the gains from sourcing efficiencies, which represent the second term, dominate the losses from tariff revenues, which represent the first term.

In this scenario, the nonmember also gains. It is worth noting that the proportional increase of tariff revenues of the nonmember country is much larger compared to the one in the benchmark case. This is because countries' initial import shares from others are very small due to the high initial tariff rates, which generate very low initial tariff revenues, and consequently a very high proportional change for a small absolute change of tariff revenues.

8.4 Intermediate Goods

By Proposition 11, particularly as described in equations (55) and (56), the indirect effects on prices of a PTA are transmitted through trade in intermediate goods. Here, we study how the presence of

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta L_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
Member	1.74	-73.52	2.93	-7.94	-2.71	-0.17
Non-Member	0.35	52.75	-4.52	-0.40	5.43	-0.01

Table 3: Changes for a Tariff Rate of 50% (unit: % except for D_n and π_{nn})

intermediate goods affects welfare consequences of a PTA. In order to do so, we set the cost share of intermediate goods $1-\beta = .01$ so that intermediate goods play a negligible role in production. 4 demonstrate that prices in nonmember countries are largely unaffected by the formation of a PTA as the indirect effects on prices vanish and nonmember country's welfare gains become negligible. The only remaining effects are the direct effects of tariff changes on member countries.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta L_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
Member	0.02	-52.78	1.07	-0.68	-0.91	-0.04
Non-Member	0.05×10^{-2}	0.03	-1.25	-0.02×10^{-1}	1.82	-0.99×10^{-4}

Table 4: Changes for Intermediate Goods, $\beta = .99$ (unit: % except for D_n and π_{nn})

8.5 Technology

In this subsection, we consider how a country's relative technological level affects the welfare consequences of a PTA. A country's technological level determines the competitiveness of its products in markets and a country i with a higher technology level has a greater market share π_{ni} everywhere. Here, we assume that one of the three countries has a higher technological level. Then, we consider the following two scenarios. First, one country forms a PTA with the higher technological level country. Second, the country forms a PTA with a country with the same technological level as itself.

Table 5 and Table 6 present the results of the first scenario and second scenario, respectively. Here, we assume $T_1 = 1.5$ and $T_2 = T_3 = 1$. Country 2 gains by forming a PTA with the similarly developed country, while conversely loses if the other member country has a higher level of technology. The primary factor driving this result is the change in tariff revenues. Country 2 imports substantially more from the higher technological level country, country 1 than country 3. If it forms a PTA with country 1, it loses the larger part of its tariff revenues. On the other hand, forming a PTA with the less competitive country has only a smaller effect on tariff revenues.

Our finding does not support the "natural trading partners hypothesis" (see Krugman (1991), Summers (1991) and Wonnacott and Lutz (1989)). As pointed out in Bhagwati (1993), the point of the natural trading partners hypothesis is that a trading partner with which a country trades more would also be a natural candidate as PTA partner, as such a PTA is less likely to be trade diverting. On the other hand, Krishna (2003) uses U.S. trade data to empirically study this hypothesis, and find no evidence to support it. In our analysis, from country 2's point of view, higher technology country 1 which naturally trades more with country 2 (compared to country 3 with a lower level of technology), is less desirable as a PTA partner.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta L_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	0.07×10^{-1}	-53.85	4.08	-1.91	-4.60	-0.03
2	-0.02	-59.71	5.72	-2.28	-4.82	-0.04
3	0.21	0.28	-7.32	-0.83	9.42	-0.01

Table 5: Changes in the Technological Level when $T_1 = 1.5$ (unit: % except for D_n and π_{nn})

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta L_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	0.17	1.04	-6.58	-0.65	10.87	-0.01
2 and 3	-0.04×10^{-1}	-48.94	5.84	-1.91	-5.44	-0.03

Table 6: Changes in the Technological Level when $N = 3$ and $T_1 = 1.5$ (unit: % except for D_n and π_{nn})

8.6 Trade Deficits

From equation 58, the initial level of trade deficits, $D_n = X_n - Q_n - TR_n$ is an important determinant of welfare effects upon a PTA. In this subsection, we consider how each country's trade deficits affect welfare consequences of a PTA. As D_n is endogenous in the model, we assume that one country (country 1) has a greater national labor force and thus this country has a higher demand in manufacturing products than others. This country consequently runs trade deficits, while the other countries run trade surplus before a PTA.

Table 7 presents the results. Unlike in other tables, in this table, we present D_n instead of ΔD_n. A positive D_n means that country n is running trade deficits before a PTA. The changes in prices and own trade shares for the two member countries are the same. However, country 1 with trade deficits gains after a PTA, while the other member country (country 2) loses.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta L_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	0.06	-55.33	1.76	-2.04	6109.32	-0.04
2	-0.07	-52.76	9.30	-2.04	-3054.66	-0.04
3	0.19	-0.26	-7.80	-0.78	-3054.66	-0.01

Table 7: Changes for National Labor Forces when $N = 3$ and $\bar{L}_1 = 2\bar{L}_2$ (unit: % except for D_n and π_{nn})

Part VI

Concluding Remarks

In this thesis we have provided an overview of the Eaton and Kortum (2002) model of international trade including some recent literature underpinning the stochastic specification of production technology used in the model.

We have provided proofs of existence of the general equilibria in the Eaton and Kortum trade model, and the sufficient conditions for these equilibria to be unique.

The endogenously determined trade balance was also considered and a ‘nice’ analytical result was obtained for the pattern of trade surpluses and deficits that would occur in the equilibrium of the Eaton and Kortum trade framework.

Using the Eaton and Kortum (2002) general equilibrium model, we then proceeded to consider how PTAs (through the prism of elemental cases of unilateral trade liberalization) affects both nonmember and member countries’ trade flows and welfare. Our main findings are summarized as follows. First, when one country decreases tariffs on imports from another country, the price level decreases in every country. Second, such trade liberalization by any number of countries leads to trade creation everywhere as the domestic expenditure shares for all countries decrease. Third, some nonmember countries may gain from a PTA. Finally, global free trade is not stable.

Various extensions are possible from here. Several recent extensions and applications of the Eaton-Kortum model of international trade assume perfect substitutability of labor between different industrial sectors (for example, Caliendo and Parro, 2015). In this thesis, we have focused on the case where labor is mobile between sectors within a nation, although as our results on the immobile labor scenario in the appendix also show that different assumptions of labor mobility result in some differences of the outcomes. It might be interesting to extend our analysis to models with multiple manufacturing sectors between which labor mobility is neither completely mobile nor immobile. Modelling sectoral labor forces imperfect substitutes would allow for endogenously determined differentials in wages between sectors with each country. This would be useful in analyzing changes in national income inequality arising from trade liberalization policies.

Also, as the Eaton and Kortum (2002) model excludes capital flows, another interest avenue of future research might involve modeling the non-manufacturing good as financial assets, the trade of which generates international capital flows. However, since demand for financial assets are determined at least in part by expectations of future consumption, an extension of the Eaton and Kortum trade framework to accommodate multiple time periods might be required. The flipside of future consumption is of course current investment. It would also be interesting to see it might be possible to incorporate investment into technology augmenting capital goods.

Another extension would be to study the process of coalition formation. In this thesis, we simply considered the case of global free trade, in a sense, a grand coalition of all trading nations. Our analysis of national welfare consequences of individual trade agreements, viewed as stepping stones towards a global free trade agreement, could useful in analyzing complicated sequential trade negotiations.

Finally, as alluded to in the discussion after Proposition 6, the triangular inequality assumption for trade costs in the context of the Eaton and Kortum trade framework, precludes the possibility of entrepots. In empirical studies employing the Eaton and Kortum trade framework, this often results in entrepots such as Singapore being omitted from the data set used, or being amalgamated into arguably unnatural economic units with other larger countries.[43] It might be interesting to see if entrepots might be admitted in the analysis by relaxing the triangular inequality assumption and allowing for the possibility of trans-shipment. For example, the realized price of some good ω in country n might then be specified as $p_n(\omega) = \min\left\{\min_{i\in\mathcal{N}}\left\{p_{ii}(\omega)d_{ni}\right\}, \min_{i,j\in\mathcal{N}}\left\{p_{ii}(\omega)d_{nj}d_{ji}\right\}\right\}$, with the entrepot country j potentially earning revenues $p_{ii}(\omega)d_{ji}\left(d_{nj}-1\right)$.

[43] For example, in Alvarez and Lucas (2007), Singapore was combined with Malaysia and the Philippines into one single economic 'country'.

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Part VII

Appendix I: Immobile Labor Scenario Numerical Simulation

9 Immobile Labor Scenario Simulation Method

Our procedure to compute an equilibrium is as follows. We denote each variable under mobile and immobile labor cases by superscript $\mathcal{M}$ and $\mathcal{I}$, respectively. In order to ensure comparability across mobile and immobile labor cases, we need to ensure that prices, expenditure shares, national income and national welfare are equal across the mobile and immobile labor cases in the starting baseline scenario. In this way, we can make sure that the starting economic situation is the same between the two cases and compare how the labor mobility assumptions could affect welfare consequences of an PTA.

We take as exogenously given parameters, the values of α, β, θ, the matrix of geographical distances $\mathbf{M}\left(\bar{d}_{ni}\right)$, the vector of manufacturing technology levels $\mathbf{L}\left(T_n\right)$ and the vector of total labor forces in each nation $\boldsymbol{L}$ For the purposes of immobile labor scenario, we also take the vectors of non-manufacturing income $\mathbf{L}\left(Y_n^O\right)$ and manufacturing labor force $\mathbf{L}\left(L_n^{\mathcal{I}}\right)$ as exogenously given.

The matrix of tariffs $\mathbf{M}\left(t_{ni}\right)$ is determined by the PTA combinations. For the baseline scenario in which PTA's do no exist, $t_{ni}=\bar{t}$ for all $n \neq i$, and 0 otherwise. For cases in which any country k forms a bilateral PTA with another country l, $t_{kl}=t_{lk}=0$, with all other tariff rates remaining the same as in the baseline scenario.

To ensure comparability between the cases of mobile and immobile labor, we set wages in the mobile labor case to equal equilibrium manufacturing wages in the baseline scenario of the immobile labor so that $w_n^{\mathcal{M}}=w_n^{\mathcal{I}}$ holds for every n. This ensures that equilibrium manufacturing prices and expenditure shares are exactly the same between the mobile and immobile labor cases in the baseline scenario, and $p_n^{\mathcal{M}}=p_n^{\mathcal{I}}$ and $\pi_{ni}^{\mathcal{M}}=\pi_{ni}^{\mathcal{I}}$ hold (denoted by p_n and π_{ni}, respectively).

The welfare consequences of PTA differs between the mobile and immobile labor cases. The key points that we stress here are:

- in the case of mobile labor, technological level is more relevant than other factors;
- in the case of immobile labor, manufacturing labor forces and nonmanufacturing income are also important factors;
- in the case of immobile labor, there is a positive relationship between trade balance and welfare consequences.

9.1 Numerical Simulations

We numerically calculate the changes in the equilibrium variables, across the nations, and also compare differences in outcomes under the immobile labor cases with the ones in the mobile labor case. By using parameters in Table 1, we choose $L_n = 60$ and $Y_n^O = 440$ so that wages, prices and hence trade shares in equilibrium are the same between the mobile and immobile cases in the baseline scenario. The last two sections are particularly for the case of immobile labor.

9.2 Simulation Results

9.2.1 Mobile and Immobile Labor Cases

Consider the welfare changes associated with the establishment of a PTA under the two different assumptions in labor mobility. In the case of mobile labor, as $1 = \frac{w_n \bar{L}_n}{Y_n} + \frac{TR_n}{Y_n}$, the change in welfare is brought by the change in tariff revenues. In the case of immobile labor, it has more complicated channels and because $1 = \frac{Y_n^O}{Y_n} + \beta \frac{Q_n}{Y_n} + \frac{TR_n}{Y_n}$, both tariff revenues and changes in production become important in determining welfare consequences. Notice that if $\frac{Y_n^O}{Y_n}$ is larger, $\beta \frac{Q_n}{Y_n} + \frac{TR_n}{Y_n}$ is smaller.

In the mobile labor scenario, members lose tariff revenues and so welfare decreases when tariff rates are moderate. When tariff rates are very high, even members gain, because efficiency dominates tariff revenues. Nonmembers always gain due to the positive externality of lower prices. In the immobile labor case, the effects are opposite in that members gain and nonmember lose through wage changes. Wages increases for members and decreases for nonmembers. The following table summarizes these points.

The following table is the outcome in the immobile labor case of the benchmark scenario in Table 2.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta \pi_{nn}$
Member	0.13	-51.04	2.12	-0.41	-0.10	-0.04
Non-Member	-0.07	-1.76	-0.12	0.09	0.21	0.04×10^{-1}

Table 8: Immobile Labor Case when $N = 3$ (unit: 0.01%)

The following table is the outcome in the immobile labor case of the scenario in Table 3.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta \pi_{nn}$
Member	1.80	-63.26	2.16	-6.02	-1.04	-0.17
Non-Member	0.09	11.98	-2.16	-2.26	2.09	-0.02×10^{-1}

Table 9: When $N = 3$ (unit: 0.01%)

The following table is the outcome in the immobile labor case of the scenario in Table 4.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
Member	0.04	-51.23	0.54	-0.19	0.04	-0.04
Non-Member	-0.03	-2.59	0.11	0.20	-0.09	0.06×10^{-1}

Table 10: Immobile Labor Case when $N = 3$ and $\beta = .99$ (unit: 0.01%)

9.2.2 Technology

The following table is the outcome in the immobile labor case of the scenarios in Table 5 and Table 6, respectively.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	0.16	-50.48	1.97	-0.27	-0.42	-0.04
2	0.10	-56.90	2.29	-0.59	0.22	-0.05
3	-0.07	-1.70	-0.14	0.09	0.21	0.04×10^{-1}

Table 11: Immobile Labor Case when $N = 3$ and $T_1 = 2$ (unit: 0.01%)

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	-0.07	-1.90	-0.09	0.10	0.19	0.04×10^{-1}
2 and 3	0.12	-45.70	2.13	-0.37	-0.11	-0.04

Table 12: Immobile Labor Case when $N = 3$ and $T_1 = 2$ (unit: 0.01%)

9.2.3 Nonmanufacturing Income and Trade Imbalance

If a country has a relatively small nonmanufacturing sector, then in the immobile labor case the country tends to gain more by joining the PTA, while losing more if excluded. In Table 17 and Table 13, country 1 has a smaller nonmanufacturing income $Y_1^O = 220$. The intuition is that country 1 relies heavily on manufacturing for a national income. Thus its manufacturing exports is more important and joining the PTA increases its international market shares. However, if excluded, trade is diverted to the other countries.

It is also understandable that country 2 gains more by forming a PTA with country 3, rather than with country 1. Country 3 has a larger economy and the effects on prices and wages are of a larger scale when country 2 and 3 form a PTA. Table 17 demonstrate the differences between the two scenarios.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	0.33	-49.99	2.47	-0.19	-0.72	-0.05
2	0.02	-55.23	1.76	-0.61	0.57	-0.04
3	-0.07	-1.70	-0.11	0.11	0.16	0.04×10^{-1}

Table 13: Immobile Labor Case when $PTA = \{1, 2\}$ and $Y_1^O = 220$ (unit: 0.01%)

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	-0.09	-2.09	-0.11	0.11	0.16	0.04×10^{-1}
2 and 3	0.11	-47.66	2.19	-0.34	-0.10	-0.04

Table 14: Immobile Labor Case when $PTA = \{2, 3\}$ and $Y_1^O = 220$ (unit: 0.01%)

$n = 1$	$D_n = -34.23$
$n \neq 1$	$D_n = 17.12$

Table 15: Baseline Manufacturing Trade Imbalance when $N = 3$ and $Y_1^O = 220$

9.2.4 Manufacturing Labor Forces

If a country has a smaller manufacturing labor force (L_n), then in the immobile labor case the country gains less by joining the PTA, while other members gain more. In Table 16, country 1 has a smaller manufacturing labor force $L_1 = 30$. Country 1 manufacturing wages tend to be higher because of the relative scarcity in labor, and hence country 1 is less competitive in the international market. Lowering trade barriers results in greater imports and lower exports.

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	0.04	-52.42	2.52	-0.71	0.55	-0.05
2	0.17	-37.58	1.84	-0.02	-0.77	-0.03
3	-0.07	-2.00	-0.07	0.14	0.16	0.04×10^{-1}

Table 16: Immobile Labor Case when $PTA = \{1, 2\}$ and $L_1 = 30$ (unit: 0.01%)

Country n	$\%\Delta W_n$	$\%\Delta TR_n$	$\%\Delta w_n$	$\%\Delta p_n$	ΔD_n	$\Delta\pi_{nn}$
1	-0.07	-1.42	-0.20	0.07	0.20	0.04×10^{-1}
2 and 3	0.14	-63.01	2.10	-0.49	-0.08	-0.05

Table 17: Immobile Labor Case when $PTA = \{2,3\}$ and $L_1 = 30$ (unit: 0.01%)

$n = 1$	$D_n = 35.92$
$n \neq 1$	$D_n = -17.93$

Table 18: Baseline Manufacturing Trade Imbalance when $N = 3$ and $L_1 = 30$

Part VIII

Appendix II: Immobile Labor Comparative Statics

In this appendix, we provide some results on unilateral deviations from global free trade in the immobile labor scenario, specifically, the welfare effects of an imposition of tariff rate $t_{kl} > 0$. This result is presented in Proposition 15.

Due to the complexity[44] of equilibrium changes in the immobile labor scenario, we only provide results for a scenario in which some country k unilaterally deviates from global free trade in a symmetrical world, a world analogous to the one defined in Definition 6 of the main text. Due to the limited results obtained in the immobile labor scenario, commentary would be kept to a minimum.

10 Derivatives in the General Case

The general case here refers to the Eaton and Kortum (2002) general equilibrium model in general, in the sense that results in this section applies to both the mobile and immobile labor scenarios.

1. Let $\Pi\mathbf{t} := \mathbf{M}(\frac{\pi_{ni}}{1+t_{ni}})$. Then

$$\begin{aligned} \frac{d}{dt_{kl}}\Pi &:= \mathbf{M}\Big(\frac{d}{dt_{kl}}\pi_{ni}\Big) = -\theta\left[\Pi\mathbf{D}(\ln c_n^{'}) - \mathbf{D}(\ln p_n^{'})\Pi\right] - \theta\frac{\pi_{kl}}{1+t_{kl}}\iota_k\iota_l^\top \\ \frac{d}{dt_{kl}}\Pi\mathbf{t} &:= \mathbf{M}\Big(\frac{d}{dt_{kl}}\frac{\pi_{ni}}{1+t_{ni}}\Big) = -\theta\left[\Pi\mathbf{t}\mathbf{D}(\ln c_n^{'}) - \mathbf{D}(\ln p_n^{'})\Pi\mathbf{t}\right] - (1+\theta)\frac{\pi_{kl}}{(1+t_{kl})^2}\iota_k\iota_l^\top \end{aligned}$$

Proof. The basic concept is to take the log-derivatives. Notice that $\pi_{ni} = \frac{T_i c_i^{-\theta}\bar{d}_{ni}^{-\theta}(1+t_{ni})^{-\theta}}{p_n^{-\theta}}$, and that T_i and $\bar{d}_{ni}$ are fixed parameters, and $\frac{dt_{ni}}{dt_{kl}} = 0$ if $(k,l) = (n,i)$ and 0 otherwise, hence

[44] As an illustration of the complexities involved, this entire Appendix, with at least 10 lemmas and propositions, is constructed merely to present the relative welfare effects in Proposition 15.

$$
\begin{aligned}
\frac{d}{dt_{kl}}\frac{\pi_{ni}}{1+t_{ni}} &= \frac{\pi_{ni}}{1+t_{ni}}\left(\frac{d}{dt_{kl}}\ln\pi_{ni} - \frac{d}{t_{kl}}\ln\left(1+t_{ni}\right)\right) \\
&= \frac{\pi_{ni}}{1+t_{ni}}\frac{d}{dt_{kl}}\left(\ln\left(T_i\bar{d}_{ni}^{-\theta}\right) - \theta\left(\ln c_i - \ln p_n\right) - (1+\theta)\ln\left(1+t_{ni}\right)\right) \\
&= \begin{cases} -\theta\frac{\pi_{ni}}{1+t_{ni}}\left(\ln c_i^{'} - \ln p_n^{'}\right) & for\ (n,i)\neq(k,l) \\ -\theta\frac{\pi_{kl}}{1+t_{kl}}\left(\ln c_l^{'} - \ln p_k - (1+\theta)\frac{1}{1+t_{kl}}\right) & otherwise. \end{cases}
\end{aligned}
$$

Consolidated into matrix form, we get the expression as presented in the statement of the lemma above. The derivation for $\frac{d}{dt_{kl}}\Pi$ is found in Proposition 10. □

Since $Q_n = \sum_{i\in\mathcal{N}}\frac{\pi_{in}}{1+t_{in}}X_i$, it is always the case that $\mathbf{L}(Q_n) = \Pi\mathbf{t}^\top\mathbf{L}(X_n)$. Since $TR_n = X_n\sum_{i\in\mathcal{N}}\frac{t_{ni}}{1+t_{ni}}\pi_{ni} = X_n\sum_{i\in\mathcal{N}}\left(1-\frac{\pi_{ni}}{1+t_{ni}}\right)$, collected into matrix form, we have $\mathbf{L}(TR_n) = \mathbf{D}(X_n)\left[\mathbf{I}-\Pi\mathbf{t}\right]\mathbf{1}_N$. This provides us with the next result on the derivatives of tariff revenues and manufacturing revenues.

Lemma 7. *Since $\mathbf{L}(TR_n) = \mathbf{D}(X_n)\left[\mathbf{I}-\Pi\mathbf{t}\right]\mathbf{1}_N$, we have*

$$
\mathbf{L}(TR_n^{'}) = \mathbf{D}(\frac{TR_n}{X_n})\mathbf{L}(X_n^{'}) + \theta\left[\mathbf{M}(Q_{ni}^r)^\top\mathbf{D}(\ln c_n^{'}) - \mathbf{D}(\ln p_n^{'})\mathbf{M}(Q_{ni}^r)^\top\right]\mathbf{1}_N + (1+\theta)\frac{X_{kl}}{\left(1+t_{kl}\right)^2}\iota_k. \tag{69}
$$

Since $\mathbf{L}(Q_n^r) = \Pi\mathbf{t}^\top\mathbf{L}(X_n)$, we have

$$
\mathbf{L}(Q_n^{r'}) = \Pi\mathbf{t}^\top\mathbf{L}(X_n^{'}) - \theta\left[\mathbf{D}(\ln c_n^{'})\mathbf{M}(Q_{ni}) - \mathbf{M}(Q_{ni})\mathbf{D}(\ln p_n^{'})\right]\mathbf{1}_N - (1+\theta)\frac{X_{kl}}{\left(1+t_{kl}\right)^2}\iota_l. \tag{70}
$$

Notice that the country increasing the tariffs, k, enjoys an immediate increase to tariffs, $(1+\theta)\frac{X_{kl}}{(1+t_{kl})^2}$ equivalent to the loss of revenue suffered by country l, whose exports on are being acted against.

Proof. Since $\mathbf{L}(TR_n) = \mathbf{D}(X_n)\left[\mathbf{I}-\Pi\mathbf{t}\right]\mathbf{1}_N$, taking derivatives w.r.t. to t_{kl}, we have

$$
\begin{aligned}
\mathbf{L}(TR_n^{'}) &= \left(\frac{d}{dt_{kl}}\mathbf{D}(X_n)\right)\left[\mathbf{I}-\Pi\mathbf{t}\right]\mathbf{1}_N - \mathbf{D}(X_n)\left(\frac{d}{dt_{kl}}\Pi\mathbf{t}\right)\mathbf{1}_N \\
&= \mathbf{D}(\ln X_n^{'})\mathbf{D}\left(X_n\right)\left[\mathbf{I}-\Pi\mathbf{t}\right]\mathbf{1}_N \\
&\quad +\mathbf{D}(X_n)\left[\theta\left[\Pi\mathbf{t}\mathbf{D}(\ln c_n^{'}) - \mathbf{D}(\ln p_n^{'})\Pi\mathbf{t}\right] + (1+\theta)\frac{\pi_{kl}}{\left(1+t_{kl}\right)^2}\iota_k\iota_l^\top\right]\mathbf{1}_N \\
&= \mathbf{L}(\frac{TR_n}{X_n})\mathbf{L}(X^{'}) + \theta\left[\mathbf{M}(Q_{ni}^r)^\top\mathbf{D}(\ln c_n^{'}) - \mathbf{D}(\ln p_n^{'})\mathbf{M}(Q_{ni}^r)^\top\right]\mathbf{1}_N \\
&\quad + (1+\theta)\frac{X_{kl}}{\left(1+t_{kl}\right)^2}\iota_k,
\end{aligned}
$$

with the second equality since

$$\begin{aligned}
\mathbf{D}(\ln X_n^{'})\mathbf{D}\left(X_n\right)\left[\mathbf{I}-\Pi\mathbf{t}\right]\mathbf{1}_N &= \mathbf{D}(\ln X_n^{'})\mathbf{L}(TR_n) \\
&= \mathbf{D}(TR_n)\mathbf{L}(\ln X_n^{'}) \\
&= \mathbf{D}(\frac{TR_n}{X_n})\mathbf{L}(X_n^{'}).
\end{aligned}$$

The proof for the expression for $\mathbf{L}(\frac{d}{dt_{kl}}Q_n^r)$ is similar, noticing that

$$\frac{d}{dt_{kl}}\Pi\mathbf{t}^\top = \left(\frac{d}{dt_{kl}}\Pi\mathbf{t}\right)^\top .$$

□

11 Derivatives in Immobile Labor Scenario

In the immobile labor scenario, we have $\mathbf{L}(X_n) = \chi\mathbf{L}(Q_n) + \alpha\mathbf{L}(TR_n) + \alpha\mathbf{L}(Y_n^O)$ where $\chi := 1-\beta\left(1-\alpha\right)$, and Y_n^O is exogeneously determined non-manufacturing income.[45] Taking derivatives with respect to t_{kl}, we have $\mathbf{L}(X_n^{'}) = \alpha\mathbf{L}(TR_n^{'}) + \chi\mathbf{L}(Q_n^{r'})$.

Lemma 8. *The vector of derivatives of national expenditure on manufacturing goods is given as:*

$$\begin{aligned}
\left[\mathbf{I}-\alpha\mathbf{D}\left(\frac{TR_n}{X_n}\right)-\chi\Pi\mathbf{t}^\top\right]\mathbf{L}(X_n^{'}) &= \left(\chi\left[\frac{d}{dt_{kl}}\Pi\mathbf{t}^\top\right]\mathbf{D}\left(X_n\right)-\alpha\mathbf{D}(X_n)\left[\frac{d}{dt_{kl}}\Pi\mathbf{t}\right]\right)\mathbf{1}_N \\
&= \alpha\theta\left[\mathbf{M}(Q_{ni})^\top\mathbf{D}(\ln c_n^{'})-\mathbf{D}(\ln p_n^{'})\mathbf{M}(Q_{ni})^\top\right]\mathbf{1}_N \\
&\quad +\chi\theta\left[\mathbf{M}(Q_{ni})\mathbf{D}(\ln p_n^{'})-\mathbf{D}(\ln c_n^{'})\mathbf{M}(Q_{ni})\right]\mathbf{1}_N \\
&\quad +\left(1+\theta\right)\frac{X_{kl}}{\left(1+t_{kl}\right)^2}\left(\alpha\iota_k-\chi\iota_l\right).
\end{aligned}$$

Proof. The result follows immediately from substituting expressions for $\mathbf{L}(TR_n^{'})$ and $\mathbf{L}(Q_n^{'})$ from Lemma 7 into $\mathbf{L}(X_n^{'}) = \alpha\mathbf{L}(TR_n^{'}) + \chi\mathbf{L}(Q_n^{r'})$. □

Lemma 9. *Since manufacturing wages is $\mathbf{L}(w_n) = \beta\mathbf{D}(L_n^M)^{-1}\mathbf{L}(Q_n)$ where β is the labor share of income in the manufacturing sector and L_n^M is the fixed national manufacturing labor force, we have*

$$\mathbf{L}(\ln w_n^{'}) = \mathbf{L}(\ln Q_n^{'}).$$

And since $\beta\mathbf{L}(\ln w_n^{'}) + (1-\beta)\,\mathbf{L}(\ln p_n^{'}) = \mathbf{L}(\ln c_n^{'})$ and $\mathbf{L}(\ln p_n^{'}) = \Pi\mathbf{L}(\ln c_n^{'}) + \frac{\pi_{kl}}{1+t_{kl}}\iota_k$, it follows immediately that

$$\beta\mathbf{L}(\ln w_n^{'}) = \left[\mathbf{I}-(1-\beta)\,\Pi\right]\mathbf{L}(\ln c_n^{'}) - (1-\beta)\,\frac{\pi_{kl}}{1+t_{kl}}\iota_k.$$

[45] An alternative interpretation of Y_n^O would be that Y_n^O represents country n's before-trade stock of financial assets. Since the non-manufacturing good is defined as the numeraire good, and trade in the non-manufacturing good is costless, this characterization fits well.

Lemma 10. *Let* $W_n := \frac{Y_n^O + w_n L_n + TR_n}{p_n^\alpha}$ *denote country* n*'s aggregate national welfare. The derivatives of log-welfare with respect to tariff rate* t_{kl} *is given as*

$$\ln W_n' = \frac{1}{Y_n}\left[\beta Q_n\left(\ln w_n' - \ln p_n'\right) + TR_n\left(\ln TR_n' - \ln p_n'\right) + S_n^M \ln p_n'\right], \tag{71}$$

where $S_n^M := Q_n - (X_n - TR_n)$ *is national manufacturing trade surplus.*

Notice that national welfare is increasing in manufacturing real wages, ${}^{w_n}/_{p_n}$ *and increasing in real tariff revenues* ${}^{TR_n}/_{p_n}$.

Proof. $\ln W_n = \ln Y_n - \alpha \ln p_n$ where $Y_n = Y_n^O + TR_n + \beta Q_n$ is national income. Hence

$$\ln W_n' = \frac{1}{Y_n}\left(Y_n' + \alpha Y_n \ln p_n'\right).$$

Since α is the share of national income spent on consumption of manufacturing goods, total national manufacturing expenditure less expenditure on manufacturing intermediate goods equals αY_n, such that $\alpha Y_n = X_n - (1-\beta) Q_n$.

Since national non-manufacturing income Y_n^O is fixed by assumption, we have $Y_n' = TR_n' + \beta Q_n'$.

Substituting the two expression, $\alpha Y_n = X_n - (1-\beta) Q_n$ and $Y_n' = TR_n' + \beta Q_n'$ into the expression for $\ln W_n'$ and some arrangement of the terms gives the required equation. □

Comparing Lemma 10 and equation 58, bearing in mind that $\beta\left(\ln w_n' - \ln p_n'\right) = -\frac{1}{\theta}\ln \pi_{nn}'$ in the immobile labor scenario, we see that in the Eaton and Kortum trade model in general, national welfare is governed by the same three factors, changes in own trade shares as a measure of efficiency of production allocation, changes in real value of tariff revenues, and terms of trade effects between manufacturing and non-manufacturing sectors.

12 Global Free Trade in the Immobile Labor Scenario

In this section, we obtain results for the immobile labor scenario of the Eaton and Kortum trade model, under the assumption of global free trade, as defined in Definition 5.

Lemma 11. *In the immobile labor scenario, under the assumption of global free trade,*

1. $\Pi \mathbf{t} = \Pi$ *such that,*
2. $\mathbf{M}(Q_{ni})^\top = \mathbf{D}(X_n)\Pi$ *and* $\mathbf{M}(Q_{ni}) = \Pi^\top \mathbf{D}(X_n)$*;*
3. $\mathbf{L}(TR_n') = X_{kl}\iota_k$*;*
4. $\mathbf{L}(Q_n') = \Pi^\top \mathbf{L}(X_n') - \theta \mathbf{D}(Q_n)\left[\mathbf{I} - \Psi\Pi\right]\mathbf{L}(\ln c_n') + \mathbf{D}(Q_n)\left[\theta \pi_{kl}\Psi\iota_k - (1+\theta)\psi_{lk}\iota_l\right]$*, where* $\Psi = \mathbf{M}(\psi_{ni}) = \mathbf{D}(Q_n)^{-1}\mathbf{M}(Q_{ni})$ *and* $\psi_{ni} := \frac{Q_{ni}}{Q_n}$ *is the share of country* n*'s manufacturing output sold to country* i*. Term* Ψ *the export share matrix.*

Proof. As $t_{ni}=0$ and $TR_n=0$ for all n,i and $\mathbf{M}(Q_{ni})^\top=\mathbf{D}(X_n)\Pi$, we have

$$\begin{aligned}\mathbf{L}(TR_n^{'}) &= \theta\mathbf{D}(X_n)\left[\Pi\mathbf{D}(\ln c_n^{'})-\mathbf{D}(\ln p_n^{'})\Pi\right]\mathbf{1}_N+(1+\theta)\,X_{kl}\iota_k\\ &= \theta\mathbf{D}(X_n)\left[\Pi\mathbf{L}(\ln c_n^{'})-\mathbf{L}(\ln p_n^{'})\right]+(1+\theta)\,X_{kl}\iota_k\\ &= \theta\mathbf{D}(X_n)\left[-\pi_{kl}\iota_k\right]+(1+\theta)\,X_{kl}\iota_k\\ &= X_{kl}\iota_k.\end{aligned}$$

The remainder of the results comes from substituting $\Pi\mathbf{t}=\Pi$ and $\Psi=\mathbf{D}(Q_n)^{-1}\mathbf{M}(Q_{ni})$ into equation 70. □

Lemma 12. *In the immobile labor scenario, under the assumption of global free trade,*

$$[\mathbf{I}-\mathbf{B}]\,\mathbf{L}(\ln c_n^{'})=\frac{\pi_{kl}}{1+\beta\theta}\left((1-\beta)\,\iota_k+\eta\Psi\iota_k-\beta\,(1+\theta)\,\frac{X_k}{Q_l}\iota_l\right),$$

where $\mathbf{B}=\frac{1}{1+\beta\theta}\left[(1-\beta)\,\Pi+\theta\beta\Psi\Pi+\chi\Psi\mathbf{D}\left(\frac{Q_n}{X_n}\right)[\mathbf{I}-(1-\beta)\,\Pi]\right]$, *and* $\eta=\beta\,(\alpha+\theta)-\chi\,(1-\beta)\,\frac{Q_k^r}{X_k}$.

Proof. Under the world free trade scenario, the following statements hold

$$\begin{aligned}\mathbf{L}(Q_n^{'}) &= \Pi^\top\mathbf{L}(X_n^{'})-\theta\mathbf{D}(Q_n^r)\,[\mathbf{I}-\Psi\Pi]\,\mathbf{L}(\ln c_n^{'})\\ &\quad+\mathbf{D}(Q_n)\,[\theta\pi_{kl}\Psi\iota_k-(1+\theta)\,\psi_{lk}\iota_l]\\ \mathbf{L}(X_n^{'}) &= \chi\mathbf{L}(Q_n^{'})+\alpha\pi_{kl}\mathbf{D}(X_n)\iota_k\\ \mathbf{L}(\ln Q_n^{'}) &= \frac{1}{\beta}\Big([\mathbf{I}-(1-\beta)\,\Pi]\,\mathbf{L}(\ln c_n^{'})-(1-\beta)\,\pi_{kl}\iota_k\Big)\end{aligned}$$

Substituting the second statement into the first and some rearrangement yields

$$\begin{aligned}\left[\mathbf{I}-\chi\Pi^\top\right]\mathbf{D}(Q_n^r)\mathbf{L}(\ln Q_n^{'}) &= \mathbf{D}(Q_n^r)\,[\pi_{kl}\,(\alpha+\theta)\,\Psi\iota_k-(1+\theta)\,\psi_{lk}\iota_l]\\ &\quad-\theta\mathbf{D}(Q_n^r)\,[\mathbf{I}-\Psi\Pi]\,\mathbf{L}(\ln c_n^{'}).\end{aligned}$$

Substituting $\mathbf{L}(\ln Q_n^{'})=\mathbf{L}(\ln w_n^{'})=\frac{1}{\beta}\left([\mathbf{I}-(1-\beta)\,\Pi]\,\mathbf{L}(\ln c_n^{'})-(1-\beta)\,\pi_{kl}\iota_k\right)$ into the above and rearranging yields the slightly unwieldy expression

$$\begin{aligned}\mathbf{A}\mathbf{L}(\ln c_n^{'}) &= \mathbf{D}(Q_n^r)\,[\pi_{kl}\beta\,(\alpha+\theta)\,\Psi\iota_k-\beta\,(1+\theta)\,\psi_{lk}\iota_l]\\ &\quad+(1-\beta)\,\pi_{kl}\left[\mathbf{I}-\chi\Pi^\top\right]\mathbf{D}(Q_n^r)\iota_k,\end{aligned}$$

where $\mathbf{A}:=\left[\mathbf{I}-\chi\Pi^\top\right]\mathbf{D}(Q_n^r)\,[\mathbf{I}-(1-\beta)\,\Pi]+\beta\theta\mathbf{D}(Q_n^r)\,[\mathbf{I}-\Psi\Pi]$.

Consider the term $\left[\mathbf{I}-\chi\Pi^\top\right]\mathbf{D}(Q_n^r)$. Since $\Pi^\top\mathbf{D}(X_n)=\mathbf{M}(Q_{ni})=\mathbf{D}(Q_n^r)\Psi$, implying that $\Pi^\top=$

$\mathbf{D}(Q_n^r)\Psi\mathbf{D}(X_n)^{-1}$, it follows that

$$\begin{aligned}\left[\mathbf{I}-\chi\Pi^{\top}\right]\mathbf{D}(Q_n^r) &= \left[\mathbf{D}(Q_n)-\chi\mathbf{D}(Q_n^r)\Psi\mathbf{D}(\frac{Q_n^r}{X_n})\right] \\ &= \mathbf{D}(Q_n)\left[\mathbf{I}-\Psi\mathbf{D}\left(\frac{\chi Q_n^r}{X_n}\right)\right],\end{aligned}$$

and we can rewrite the matrix $\mathbf{A}$ as

$$\mathbf{A}=\mathbf{D}(Q_n)\left(\left[\mathbf{I}-\Psi\mathbf{D}\left(\frac{\chi Q_n^r}{X_n}\right)\right]\left[\mathbf{I}-(1-\beta)\,\Pi\right]+\beta\theta\left[\mathbf{I}-\Psi\Pi\right]\right),$$

and the expression defining $\mathbf{L}(\ln c_n^{'})$ as

$$\begin{aligned}\left(\left[\mathbf{I}-\Psi\mathbf{D}\left(\frac{\chi Q_n^r}{X_n}\right)\right]\left[\mathbf{I}-(1-\beta)\,\Pi\right]+\beta\theta\left[\mathbf{I}-\Psi\Pi\right]\right)\mathbf{L}(\ln c_n^{'}) &= \left[\pi_{kl}\beta\left(\alpha+\theta\right)\Psi\iota_k-\beta\left(1+\theta\right)\psi_{lk}\iota_l\right] \\ &\quad+(1-\beta)\,\pi_{kl}\left[\mathbf{I}-\Psi\mathbf{D}\left(\frac{\chi Q_n^r}{X_n}\right)\right]\iota_k,\end{aligned}$$

as we pre-multiply both sides of the equation by $\mathbf{D}(Q_n^r)^{-1}$. Rearranging the coefficient of $\mathbf{L}(\ln c_n^{'})$ on the left hand side of the equation above yields,

$$\left[\mathbf{I}-\mathbf{B}\right]\mathbf{L}(\ln c_n^{'})=\frac{\pi_{kl}}{1+\beta\theta}\left((1-\beta)\,\iota_k+\eta\Psi\iota_k-\beta\left(1+\theta\right)\frac{X_k}{Q_l}\iota_l\right)$$

where $\mathbf{B}=\frac{1}{1+\beta\theta}\left[(1-\beta)\,\Pi+\theta\beta\Psi\Pi+\chi\Psi\mathbf{D}\left(\frac{Q_n}{X_n}\right)\left[\mathbf{I}-(1-\beta)\,\Pi\right]\right]$, and $\eta=\beta\left(\alpha+\theta\right)-\chi\left(1-\beta\right)\frac{Q_k^r}{X_k}$.
□

Unlike in the mobile labor scenario, the changes in immobile labor equilibrium arising from a change in tariff rate t_{kl} remains complicated, even with the assumption of global free trade. We shall simplify the analysis even further by assuming a world with symmetrical countries.

13 Welfare Implications of Trade Liberalization

The analysis in the immobile labor scenario is complex. We provide some intuitive results by first considering the simplest possible scenario, one in no country imposes a tariff on imports from any other country, and in which every country is identical in every regard and the distances between each pair of countries is exactly the same. Call this the symmetrical global economy scenario under immobile labor assumptions.

Definition 7. In the *symmetrical global economy* in the immobile labor scenario, it is assumed that

- Every country is identical, such that for each country $n \in \mathcal{N}$,

$$\begin{aligned} T_n &= \bar{T} \\ Y_n^O &= \bar{Y}^O \\ L_n^M &= \bar{L}, \end{aligned}$$

  where $\bar{T}$, $\bar{Y}^O$ and $\bar{L}$ are exogeneously given constants.

- Manufacturing trade costs between each pair of countries is identical. For each pair of countries $n, i \in \mathcal{N}$ with $n \neq i$,

$$d_{ni} = \bar{d} \geq 1$$

Combining the symmetrical world and global free trade assumptions, it must also be the case that $t_{ni} = 0$ for all $n, i \in \mathcal{N}$.

Since every country is identical to every other country in every way, it must be the case that equilibrium prices and wages are the same across all countries such that for all $n, i \in \mathcal{N}$,

$$\begin{aligned} w_n &= w_i = \bar{w} \\ p_n &= p_i = \bar{p}, \end{aligned}$$

where $\bar{w}$ and $\bar{p}$ are some endogenously determined numbers.

It must also be the case that each country's manufacturing output and expenditure must be the same as all others, such that for all n, i,

$$\begin{aligned} X_n &= X_i = \bar{X} \\ Q_n &= Q_i = \bar{Q}. \end{aligned}$$

Under the symmetrical world assumption, no country can run a trade surplus such that $Q_n = X_n$ for every country n. If any country runs a manufacturing trade surplus, then by the symmetry of each country's circumstances, every country must run a trade surplus, which is an impossibility, given that global manufacturing trade must balance. Hence

$$\bar{X} = \bar{Q}$$

or equivalently in vector form,

$$\mathbf{L}(X_n) = \bar{X}\mathbf{1}_N = \mathbf{L}(Q_n).$$

Lemma 13. *In the immobile labor scenario, under the assumptions of global free trade and symmetrical world, for every country n, we have*

$$\pi_{nn} = \bar{\pi} := \frac{1}{1 + (N-1)\,\delta},$$

where $\delta := \bar{d}^{-\theta}$*. For every pair of different countries* $n \neq i$*,*

$$\pi_{ni} = \delta\bar{\pi}.$$

Let $\Pi^2_{ni} := \sum_{j\in\mathcal{N}} \pi_{nj}\pi_{ji}$ *refer to the element of the matrix* Π^2 *on the* n^{th} *row and* i^{th} *column.*
It is the case that

$$\begin{aligned}
\Pi^2_{nn} &:= \sum_{j=1}^{N} \pi_{nj}\pi_{jn} = \bar{\pi}^2\left[1+(N-1)\,\delta^2\right] \\
\Pi^2_{ni} &:= \sum_{j=1}^{N} \pi_{nj}\pi_{ji} = \Pi^2_{nn} - \bar{\pi}^2\,(1-\delta)^2\,,
\end{aligned}$$

noting that the diagonal elements of Π^2 *are identical to one another, and that the off-diagonal elements of* Π^2 *are also identical to each other. Denote* $\bar{\pi}_2 := \bar{\pi}^2\left[1+(N-1)\,\delta^2\right]$*, such that for all* n*, we have* $\Pi^2_{nn} = \bar{\pi}_2$*.*
Finally, it is the case that the manufacturing expenditure shares matrix is symmetric and is identical to the manufacturing export share matrix, such that

$$\Pi = \Pi^\top = \Psi.$$

Proof. For each pair of countries n, i, by the definition of manufacturing expenditure shares we have

$$\begin{aligned}
\pi_{ni} &= \frac{T_i\left(w_i^\beta p_i^{(1-\beta)}\right)^{-\theta} d_{ni}^{-\theta}}{p_n^{-\theta}}. \\
\pi_{nn} &= T_n\left(\frac{w_n}{p_n}\right)^{-\theta\beta}.
\end{aligned}$$

Under the symmetrical global assumption, we have $T_i = \bar{T}$, $w_n = w_i = \bar{w}$ and $p_n = p_i = \bar{p}$, and $d_{ni} = \bar{d}$, hence

$$\begin{aligned}
\pi_{nn} &= \bar{T}\left(\frac{\bar{w}}{\bar{p}}\right)^{-\theta\beta} \\
\pi_{ni} &= \bar{T}\frac{\left(\bar{w}^\beta \bar{p}^{1-\beta}\right)^{-\theta}}{\bar{p}^{-\theta}}\bar{d}^{-\theta} \\
&= \bar{T}\left(\frac{\bar{w}}{\bar{p}}\right)^{-\theta\beta}\bar{d}^{-\theta} \\
&= \pi_{nn}\bar{d}^{-\theta}.
\end{aligned}$$

Since $\pi_{nn} + \sum_{i \neq n}^{N} \pi_{ni} = 1$, and there are $N-1$ countries which are not country n, we have

$$\begin{aligned}
\pi_{nn} + \sum_{i \neq n}^{N} \pi_{nn} \bar{d}^{-\theta} &= 1 \\
\implies \pi_{nn} \left(1 + (N-1)\, \bar{d}^{-\theta}\right) &= 1 \\
\implies \pi_{nn} &= \frac{1}{1 + (N-1)\, \bar{d}^{-\theta}}.
\end{aligned}$$

Define $\bar{\pi} := \frac{1}{1+(N-1)\bar{d}^{-\theta}}$. Then as $\pi_{ni} = \bar{\pi}\delta = \pi_{in}$ for any pair of different countries n, i, the trade share matrix Π must be symmetric such that $\Pi = \Pi^{\top}$.

Next, consider the elements of the matrix product, Π^2, and let $\delta := \bar{d}^{-\theta}$.

$$\begin{aligned}
\Pi^2_{nn} &= \sum_{j=1}^{N} \pi_{nj} \pi_{jn} \\
&= \pi^2_{nn} + \sum_{j \neq n}^{N} \pi_{nj} \pi_{jn} \\
&= \bar{\pi}^2 + (N-1)\, \bar{\pi}^2 \delta^2 \\
&= \bar{\pi}^2 \left[1 + (N-1)\, \delta^2\right].
\end{aligned}$$

For the off-diagonal elements of Π^2, we have

$$\begin{aligned}
\Pi^2_{ni} &= \sum_{j=1}^{N} \pi_{nj} \pi_{ji} \\
&= \pi_{nn} \pi_{ni} + \pi_{ni} \pi_{ii} + \sum_{j \neq n,i}^{N} \pi_{nj} \pi_{ji} \\
&= 2\bar{\pi}^2 \delta + (N-2)\, \bar{\pi}^2 \delta^2 \\
&= \bar{\pi}^2 \left[1 + (N-1)\, \delta^2 - \left(1 - 2\delta + \delta^2\right)\right] \\
&= \Pi^2_{nn} - \bar{\pi}^2 \left(1 - \delta\right)^2.
\end{aligned}$$

Finally, we consider the matrix of manufacturing export shares, $\Psi = \mathbf{M}(\frac{Q^r_{ni}}{Q^r_n})$. In general the definition of manufacturing export shares gives $\Psi = \mathbf{D}(Q_n)^{-1} \Pi \mathbf{t}^{\top} \mathbf{D}(X_n)$. Under the free-trade assumption, we have $\Pi\mathbf{t} = \mathbf{M}(\frac{\pi_{ni}}{1+t_{ni}}) = \mathbf{M}\left(\frac{\pi_{ni}}{1+0}\right) = \Pi$, and under the symmetrical world assumption, we have $\mathbf{D}(Q_n) = \mathbf{D}(X_n) = \bar{X}\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. We have also just shown that $\Pi = \Pi^{\top}$ under the symmetrical world assumption, hence

$$\begin{aligned}
\Psi &= \mathbf{D}(Q_n)^{-1} \Pi^{\top} \mathbf{D}(X_n) \\
&= \left[\frac{1}{\bar{X}} \mathbf{I}\right] \Pi \left[\bar{X}\mathbf{I}\right] \\
&= \Pi.
\end{aligned}$$

□

Lemma 14. *In the symmetrical world scenario, it is the case that for all n, i,*

$$\bar{\pi} := \pi_{nn} > \Pi^2_{nn} > \Pi^2_{ni} > \pi_{ni} = \bar{\pi}\delta.$$

Proof. We first show that $\Pi^2_{ni} > \pi_{ni}$ for all n, i. Since $\pi_{nn} = \bar{\pi}$ and $\pi_{ni} = \bar{\pi}\delta$ for all n, i, and $\bar{\pi} + (N-1)\,\bar{\pi}\delta = 1$, we have

$$\begin{aligned}
\Pi^2_{ni} &= \sum_{j=1}^{N} \pi_{nj}\pi_{ji} \\
&= 2\bar{\pi}^2\delta + (N-2)\,\bar{\pi}^2\delta^2 \\
&= \bar{\pi}\delta\left[1 + \bar{\pi}\,(1-\delta)\right] \\
&> \bar{\pi}\delta,
\end{aligned}$$

hence $\Pi^2_{ni} > \pi_{ni}$ for all n, i.
Then since $\bar{\pi} = [1 - (N-1)\,\bar{\pi}\delta]$ and $\Pi^2_{nn} = \left[1 - \sum_{i\neq n}^{N} \Pi^2_{ni}\right]$, it follows immediately from $\Pi^2_{ni} > \bar{\pi}\delta$ that $\bar{\pi} > \Pi^2_{nn}$ for all n. □

Lemma 15. *Define $\mathbf{L}(z_n) := [\mathbf{I} - \mathbf{B}]^{-1}\,\iota_k$, where $\mathbf{B}$ is given in Lemma 12. Then it is the case that:*

- $z_n = z_m$ for all $n, m \neq k$, and
- $z_k = \max_{i\in N} z_i$.
- Finally, it is the case that for any $n \neq k$,

$$\beta\,(1-\chi)\,z_n = \left([(1-\beta) + \chi]\,\pi_{nk} + [\beta\theta - \chi\,(1-\beta)]\,\Pi^2_{nk}\right)(z_k - z_n)\,.$$

Proof. As defined previously, $\mathbf{B} = \lambda_0\Pi + \lambda_1\Pi^2$, where $\lambda_0 = \frac{(1-\beta)+\chi}{1+\beta\theta}$ and $\lambda_1 = \frac{\beta\theta - \chi(1-\beta)}{1+\beta\theta}$. By the definition of $\mathbf{L}(z_n)$, we have

$$\left[\mathbf{I} - \lambda_0\Pi - \lambda_1\Pi^2\right]\mathbf{L}(z_n) = \iota_k.$$

For any $n \neq k$, we have

$$z_n - \lambda_0\sum_i \pi_{ni}z_i - \lambda_1\sum_i \Pi^2_{ni}z_i = 0$$

implying that

$$z_n - \lambda_0\left(\pi_{nn}z_n + \sum_{i\neq n}\pi_{ni}z_i\right) - \lambda_1\left(\Pi^2_{nn}z_n + \sum_{i\neq n}\Pi^2_{ni}z_i\right) = 0.$$

Take any other $m \neq n, k$, and notice that since for all m, n, i, we have $\pi_{nn} = \pi_{mm} = \bar{\pi}$, $\Pi^2_{nn} =$

$\Pi^2_{mm} = \bar{\pi}_2$, $\pi_{ni} = \pi_{mi} = \bar{\pi}\delta$ and $\Pi^2_{mi} = \Pi^2_{ni} = \bar{\pi}_2 - \bar{\pi}^2 (1-\delta)^2$, we have

$$\begin{aligned} z_m - z_n &= \lambda_0 \left(\pi_{mm} z_m - \pi_{nn} z_n + \sum_{i \neq m} \pi_{mi} z_i - \sum_{i \neq n} \pi_{ni} z_i \right) \\ &\quad + \lambda_1 \left(\Pi^2_{mm} z_m - \Pi^2_{nn} z_n + \sum_{i \neq m} \Pi^2_{mi} z_i - \sum_{i \neq n} \Pi^2_{ni} z_i \right) \\ &= \left(\lambda_0 \bar{\pi} (1-\delta) + \lambda_1 \bar{\pi}^2 (1-\delta)^2 \right) (z_m - z_n) \end{aligned}$$

which implies that

$$(1 - \bar{\pi}(1-\delta) [\lambda_0 + \lambda_1 \bar{\pi}(1-\delta)]) (z_m - z_n) = 0.$$

Since $(1 - \bar{\pi}(1-\delta) [\lambda_0 + \lambda_1 \bar{\pi}(1-\delta)]) \neq 0$ as $\bar{\pi}(1-\delta) \in (0,1)$, , we must have $z_m - z_n = 0$ or equivalently, $z_m = z_n$.

Next, we show that $z_k > z_n$ for all $n \neq k$. $[\mathbf{I} - \mathbf{B}]\, \mathbf{L}(z_n) = \iota_k$ implies that

$$z_k - \lambda_0 \left(\pi_{kk} z_k + \sum_{i \neq k} \pi_{ki} z_i \right) - \lambda_1 \left(\Pi^2_{kk} z_k + \sum_{i \neq k} \Pi^2_{ki} z_i \right) = 1.$$

Since for any $i, n \neq k$ we have $\pi_{ki} = \pi_{kn}$ and $z_n = z_i$, such that $\sum_{i \neq k} \pi_{ki} z_i = (1 - \pi_{kk}) z_n$ for some arbitrary $n \neq k$, we can write the above as

$$z_k - \lambda_0 (\pi_{kk} z_k + (1 - \pi_{kk}) z_n) - \lambda_1 \left(\Pi^2_{kk} z_k + \left(1 - \Pi^2_{kk}\right) z_n \right) = 1.$$

Then, given that we can write for any country n,

$$z_n - \lambda_0 (\pi_{nk} z_k + (1 - \pi_{nk}) z_n) - \lambda_1 \left(\Pi^2_{nk} z_k + \left(1 - \Pi^2_{nk}\right) z_n \right) = 0,$$

taking the difference of the two equations above gives us

$$(z_k - z_n) - \lambda_0 (\pi_{kk} - \pi_{nk}) (z_k - z_n) - \lambda_1 \left(\Pi^2_{kk} - \Pi^2_{nk} \right) (z_k - z_n) = 1$$

or equivalently,

$$\left(1 - \lambda_0 \bar{\pi}(1-\delta) - \lambda_1 (\bar{\pi}(1-\delta))^2 \right) (z_k - z_n) = 1,$$

noting that $\bar{\pi}(1-\delta) \in (0,1)$.

It remains to be shown that $1 - \lambda_0 \bar{\pi}(1-\delta) - \lambda_1 (\bar{\pi}(1-\delta))^2 > 0$.

If $\lambda_1 > 0$, then it is immediately obvious that $\lambda_1 (\bar{\pi}(1-\delta))^2 < \lambda_1 \bar{\pi}(1-\delta)$ which then implies that

$$\begin{aligned} 1 - \lambda_0 \bar{\pi}(1-\delta) - \lambda_1 (\bar{\pi}(1-\delta))^2 &> 1 - (\lambda_0 + \lambda_1) \bar{\pi}(1-\delta) \\ &> 0 \end{aligned}$$

since $(\lambda_0 + \lambda_1) = 1 - \frac{\beta(1-\chi)}{1+\beta\theta} \in (0,1)$.

Consider the case when $\lambda_1 < 0$. By contradiction, suppose that there exists $w \in (0,1)$, such that

$$w^2\lambda_1 + w\lambda_0 - 1 \geq 0$$

or equivalently,

$$-w^2 |\lambda_1| \geq 1 - w\lambda_0.$$

Since $w \in (0,1)$ implies that $w^2 < w$, it follows that

$$\begin{aligned} -w|\lambda_1| > -w^2|\lambda_1| &\geq 1 - w\lambda_0 \\ \implies 0 &> 1 - w\lambda_0 + w|\lambda_1| \\ &= 1 - w(\lambda_0 + \lambda_1) \\ &= 1 - w\left(1 - \frac{\beta(1-\chi)}{1+\beta\theta}\right) \\ &= 1 - w + \frac{\beta(1-\chi)}{1+\beta\theta}. \end{aligned}$$

But this implies that

$$1 - w < -\frac{\beta(1-\chi)}{1+\beta} < 0$$

which then implies that $w > 1$ which is a contradiction, since $w \in (0,1)$ by hypothesis.

Hence it is always the case that $1 - \lambda_0\bar{\pi}(1-\delta) - \lambda_1(\bar{\pi}(1-\delta))^2 > 0$.

Then since $\left(1 - \lambda_0\bar{\pi}(1-\delta) - \lambda_1(\bar{\pi}(1-\delta))^2\right) > 0$, we have $z_k - z_n > 0$ for any arbitrary $n \neq k$ as required, and we have shown that $z_k = \max_n z_n$.

We now show that final result of the lemma. Since for any $n, i \neq k$, it is the case that $z_n = z_i$, then $\sum_{i\neq k}^N \pi_{ni} z_i = z_n \sum_{i\neq k}^N \pi_{ni} = z_n(1-\pi_{nk})$ and similarly, $\sum_{i\neq k}^N \Pi_{ni}^2 z_i = (1-\Pi_{nk}^2)z_n$. Therefore for any $n \neq k$, we can write the n^{th} row of $[\mathbf{I}-\mathbf{B}]\,\mathbf{L}(z_n) = \iota_k$ as

$$z_n - \lambda_0(\pi_{nk}z_k + (1-\pi_{nk})z_n) - \lambda_1\left(\Pi_{nk}^2 z_k + \left(1-\Pi_{nk}^2\right)z_n\right) = 0,$$

or equivalently,

$$(1-(\lambda_0+\lambda_1))z_n = \left(\lambda_0\pi_{nk} + \lambda_1\Pi_{nk}^2\right)(z_k - z_n).$$

Since $1-(\lambda_0+\lambda_1) = \frac{\beta(1-\chi)}{1+\beta\theta}$, dividing both sides of the equation by $1+\beta\theta$ above yields the required result. □

Lemma 16. *Define for any $l \in \mathcal{N}$, $\mathbf{L}(z(l)_n) := [\mathbf{I}-\mathbf{B}]^{-1}\iota_l$. It is the case that the l^{th} element of the vector,*

$$z(l)_l = z_k$$

and all other elements $n \neq k$ of the vector

$$z(l)_n = z_n.$$

Let $\mathbf{L}(g_n) := [\mathbf{I} - \mathbf{B}]^{-1}\,\Pi\iota_k$. *Then it is the case that*

$$\begin{aligned} g_k &= z_n + \bar{\pi}\left(z_k - z_n\right) \\ g_n &= z_n + \bar{\pi}\delta\left(z_k - z_n\right), \end{aligned}$$

for all $n \neq k$.

Since $\bar{\pi} \in (0,1)$ *and* $\delta \in (0,1)$, *it follows immediately that*

$$z_k > g_k > g_n > z_n,$$

for all $n \neq k$.

Proof. Define $\mathbf{L}(z(i)_n) := [\mathbf{I} - \mathbf{B}]^{-1}\,\iota_i$. It is clear that $\mathbf{L}(z_n)$ as defined in Lemma 15 is identical to $\mathbf{L}(z(k)_n)$.

Let $\mathbf{P}_{ij} := \mathbf{I} - (\iota_i - \iota_j)(\iota_i - \iota_j)^\top$ refer to the permutation matrix that interchanges the i^{th} and j^{th} rows of any conformable matrix that it pre-multiplies, and interchanges the i^{th} and j^{th} columns of matrices it post-multiplies. Since $[\mathbf{I} - \mathbf{B}]$ has diagonal elements sharing a single constant value and off-diagonal elements that similarly are identical to one another, it is easily verified that $\mathbf{P}_{ij}[\mathbf{I} - \mathbf{B}] = [\mathbf{I} - \mathbf{B}]\,\mathbf{P}_{ij}$.

It is then the case that $\mathbf{L}(z(i)_n) = \mathbf{P}_{ij}\mathbf{L}(z(j)_n)$, as

$$[\mathbf{I} - \mathbf{B}]\,\mathbf{P}_{ij}\mathbf{L}(z(i)_n) = \mathbf{P}_{ij}[\mathbf{I} - \mathbf{B}]\,\mathbf{L}(z(i)_n) \;\;=\;\; \mathbf{P}_{ij}\iota_i = \iota_j.$$

Pre-multiplying both sides with $[\mathbf{I} - \mathbf{B}]^{-1}$ immediately gives the result

$$\mathbf{P}_{ij}\mathbf{L}(z(i)_n) = [\mathbf{I} - \mathbf{B}]^{-1}\,\iota_j = \mathbf{L}(z(j)_n).$$

This means that for any arbitrary country i, the vectors $\mathbf{L}(z(k)_n)$ and $\mathbf{L}(z(i)_n)$ are identical, except with the i^{th} and k^{th} elements of each vector interchanged, such that $z(k)_k = z(i)_i$ and $z(k)_i = z(i)_k$, and for all other $n \neq k, i$, we have $z(k)_n = z(i)_n$ for any $n \neq j, i$.

Since $z(k)_i = z_i$, $z(k)_n = z_n$ and for all $n, i \neq k$, $z_i = z_n$, and $z(k)_k = z_k$, we have shown that the first part of the lemma holds.

We now consider the second part of the lemma concerning $\mathbf{L}(g_n) := [\mathbf{I} - \mathbf{B}]^{-1}\,\Pi\iota_k$. Since $\Pi\iota_k = \Pi_{*k}$, the k^{th} column of the trade expenditure share matrix, we can write

$$\begin{aligned} \Pi\iota_k &= \pi_{kk}\iota_k + \sum_{i\neq k}^{N}\pi_{ik}\iota_i \\ &= \bar{\pi}\iota_k + \bar{\pi}\delta\sum_{i\neq k}^{N}\iota_i. \end{aligned}$$

Then

$$\begin{aligned}
\mathbf{L}(g_n) &= [\mathbf{I}-\mathbf{B}]^{-1}\Pi\iota_k \\
&= \bar{\pi}[\mathbf{I}-\mathbf{B}]^{-1}\iota_k + \bar{\pi}\delta\sum_{i\neq k}^{N}[\mathbf{I}-\mathbf{B}]^{-1}\iota_i \\
&= \bar{\pi}\mathbf{L}(z(k)_n) + \bar{\pi}\delta\sum_{i\neq k}^{N}\mathbf{L}(z(i)_n).
\end{aligned}$$

This implies that for the k^{th} element of $\mathbf{L}(g_n)$,

$$\begin{aligned}
g_k &= \bar{\pi}z(k)_k + \bar{\pi}\delta\sum_{i\neq k}^{N}z(i)_k \\
&= \bar{\pi}z(k)_k + \bar{\pi}\delta(N-1)z_n \\
&= \bar{\pi}z_k + (1-\bar{\pi})z_n,
\end{aligned}$$

with the second equality since for all $i \neq k$, $z(i)_k = z(k)_i = z_i$, and $z_n = z_i$ for any $i, n \neq k$, and the third equality since $\pi_{kk} + \sum_{i\neq k}^{N}\pi_{ki} = 1$, $\pi_{kk} = \bar{\pi}$ and $\pi_{ki} = \bar{\pi}\delta$ for all $i \neq k$, together implies that $\bar{\pi} + (N-1)\bar{\pi}\delta = 1$ or equivalently, $(N-1)\bar{\pi}\delta = (1-\bar{\pi})$.

For any $j \neq k$, for similar reasons, we have

$$\begin{aligned}
g_j &= \bar{\pi}z(k)_j + \bar{\pi}\delta z(j)_j + \bar{\pi}\delta\sum_{i\neq n,k}^{N}z(i)_j \\
&= \bar{\pi}z_n + \bar{\pi}\delta z_k + (N-2)\bar{\pi}\delta z_n + \bar{\pi}\delta z_n - \bar{\pi}\delta z_n \\
&= (N-1)\bar{\pi}\delta z_n + \bar{\pi}(1-\delta)z_n + \bar{\pi}\delta z_k \\
&= (1-\bar{\pi})z_n + \bar{\pi}z_n + \bar{\pi}\delta(z_k - z_n) \\
&= z_n + \bar{\pi}\delta(z_k - z_n),
\end{aligned}$$

as required. □

Proposition 14. *In the symmetrical world scenario with global free trade and immobile labor,*

$$\ln c_k^{'} > 0 > \ln c_n^{'} > \ln c_l^{'}.$$

Proof. In the symmetrical world scenario, since $\Psi = \Pi = \Pi^{\top}$, and Π is a matrix with diagonal elements of $\bar{\pi} = \frac{1}{1+(N-1)\delta}$ and off-diagonal elements of $\bar{\pi}\delta$, we can write the vector of log-cost derivatives w.r.t to t_{kl} as

$$\mathbf{L}(\ln c_n^{'}) = \frac{\bar{\pi}\delta}{1+\beta\theta}[\mathbf{I}-\mathbf{B}]^{-1}\left[(1-\beta)\iota_k + \eta\Pi\iota_k - \beta(1+\theta)\iota_l\right].$$

This can be rewritten as

$$\mathbf{L}(\ln c_n^{'}) = \frac{\bar{\pi}\delta}{1+\beta\theta}\left[(1-\beta)\left[\mathbf{L}(z(k)_n - \mathbf{L}(g_n)\right] + \beta\left(\chi+\theta\right)\left[\mathbf{L}(g_n) - \mathbf{L}(z(l)_n)\right] - \beta\left(1-\chi\right)\mathbf{L}(z(l)_n)\right],$$

where $\mathbf{L}(z(i)_n) := [\mathbf{I}-\mathbf{B}]^{-1}\iota_i$ for any $i \in N$, and $\mathbf{L}(g_n) := [\mathbf{I}-\mathbf{B}]^{-1}\Pi\iota_k$.

By Lemma 16, let $z^* := z(k)_k = z(l)_l$ and $\hat{z} := z(i)_n$ for any $n \neq i$, noting that $z^* > g_k > g_n > \hat{z}$ for all $n \neq k$. The change in manufacturing cost for country k, the country imposing the tariff, is given as

$$\ln c_k^{'} = \frac{\bar{\pi}\delta}{1+\beta\theta}\left[(1-\beta)\left[z^* - g_k\right] + \beta\left(\chi+\theta\right)\left[g_k - \hat{z}\right] - \beta\left(1-\chi\right)\hat{z}\right].$$

The change in cost for country l, is written as

$$\ln c_l^{'} = \frac{\bar{\pi}\delta}{1+\beta\theta}\left[(1-\beta)\left[\hat{z} - g_n\right] + \beta\left(\chi+\theta\right)\left[g_n - \hat{z}\right] - \beta\left(1-\chi\right)\hat{z}\right],$$

and the change in cost for all other countries $n \neq k, l$ is given as

$$\ln c_n^{'} = \frac{\bar{\pi}\delta}{1+\beta\theta}\left[(1-\beta)\left[\hat{z} - g_n\right] + \beta\left(\chi+\theta\right)\left[g_n - z^*\right] - \beta\left(1-\chi\right)z^*\right].$$

Since it is the case that

$$z^* > g_k = \hat{z} + \bar{\pi}\left(z^* - \hat{z}\right) > g_n = \hat{z} + \bar{\pi}\delta\left(z^* - \hat{z}\right) > \hat{z},$$

we have

$$\ln c_k^{'} > \ln c_n^{'} > \ln c_l^{'}.$$

And since

$$\begin{aligned} \beta\left(1-\chi\right)z_n &= \left[\left((1-\beta)+\chi\right)\pi_{nk} + \left(\beta\left(\chi+\theta\right)-\chi\right)\Pi_{nk}^2\right]\left(z^* - \hat{z}\right) \\ z^* - g_k &= \left(1-\bar{\pi}\right)\left(z^* - \hat{z}\right) \\ g_k - \hat{z} &= \bar{\pi}\left(z^* - \hat{z}\right), \end{aligned}$$

we have

$$\ln c_k^{'} = \frac{\bar{\pi}\delta}{1+\beta\theta}\left[(1-\beta)\left(1-\bar{\pi}-\pi_{nk}\right) + \beta\left(\chi+\theta\right)\left(\bar{\pi} - \Pi_{nk}^2\right) + \chi\left(\Pi_{nk}^2 - \pi_{nk}\right)\right].$$

Since $\pi_{nk} = \bar{\pi}\delta$, and $1-\bar{\pi} = (N-1)\,\bar{\pi}\delta$, we have $1-\bar{\pi}-\pi_{nk} = (N-2)\,\bar{\pi}\delta \geq 0$ since the nature of a international trade model implies more than one country and the number of countries $N \geq 2$. By the results of Lemma 14, it must be the case that $\bar{\pi} > \Pi_{nk}^2 > \pi_{nk}$. Hence it must be the case that

$$\ln c_k^{'} > 0.$$

Since $\pi_{nk} = \bar{\pi}\delta$ and $\Pi^2_{nk} = \bar{\pi}\delta + \bar{\pi}^2\delta(1-\delta)$, we can write $\beta(1-\chi)\hat{z}$ as

$$\beta(1-\chi)\hat{z} = \left([(1-\beta)+\beta(\chi+\theta)]\bar{\pi}\delta + [\beta(\chi+\theta)-\chi]\bar{\pi}^2\delta(1-\delta)\right)(z^* - \hat{z}),$$

which when substituted into the expression for $\ln c_n^{'}$ combined with the fact that $g_n - \hat{z} = \bar{\pi}\delta(z^* - \hat{z})$ gives

$$\ln c_n^{'} = \frac{\bar{\pi}\delta}{1+\beta\theta}\left[-(1-\beta)\bar{\pi}\delta(2-\chi\bar{\pi}(1-\delta)) - \beta\theta\bar{\pi}^2\delta(1-\delta)\right](z^* - z_n).$$

Since $\chi\bar{\pi}(1-\delta) \in (0,1)$ implies that $2-\chi\bar{\pi}(1-\delta) > 0$, it must be that

$$\ln c_n^{'} < 0.$$

Hence

$$\ln c_k^{'} > 0 > \ln c_n^{'} > \ln c_l^{'}$$

where k is the country imposing tariffs, t_{kl}, on imports from country l, and $n \neq k, l$ is any arbitrary third party country. □

13.1 Welfare in Symmetrical Scenario with Global Free Trade

Lemma 17. *In a scenario with global free trade (and not necessarily symmetrical scenario),*

$$\mathbf{L}(\ln W_n^{'}) = \mathbf{D}(Y_n)^{-1}\left[\mathbf{D}(Q_n)\mathbf{L}(\ln c_n^{'}) - \mathbf{D}(X_n)\Pi\mathbf{L}(\ln c_n^{'})\right],$$

or equivalently, for all countries $n \in N$,

$$\ln W_n^{'} = \frac{1}{Y_n}\left[Q_n \ln c_n^{'} - X_n \sum_i \pi_{ni} \ln c_i^{'}\right].$$

Proof. In general, for any arbitrary country $n \in N$, changes in national welfare can be expressed as $\ln W_n^{'} = \frac{1}{Y_n}\left[Q_n^r \beta\left(\ln w_n^{'} - \ln p_n^{'}\right) + TR_n^{'} - (X_n - Q_n^r)\ln p_n^{'}\right]$. Since $\ln c_n^{'} = \beta \ln w_n^{'} + (1-\beta)\ln p_n^{'}$ implies that $\ln c_n^{'} = \beta\left(\ln w_n^{'} - \ln p_n^{'}\right)$, we have

$$\ln W_n^{'} = \frac{1}{Y_n}\left[Q_n^r \ln c_n^{'} + TR_n^{'} - X_n \ln p_n^{'}\right].$$

In matrix form, we can write

$$\mathbf{L}(\ln W_n^{'}) = \mathbf{D}(Y_n)^{-1}\left[\mathbf{D}(Q_n)\mathbf{L}(\ln c_n^{'}) + \mathbf{L}(TR_n^{'}) - \mathbf{D}(X_n)\mathbf{L}(\ln p_n^{'})\right].$$

In the world free trade scenario, $\mathbf{L}(TR_n^{'}) = X_k \pi_{kl}\iota_k$ and $\mathbf{D}(X_n)\mathbf{L}(\ln p_n^{'}) = \mathbf{D}(X_n)\Pi\mathbf{L}(\ln c_n^{'}) + X_k\pi_{kl}\iota_k$, in which case we can immediately write

$$\mathbf{L}(\ln W_n^{'}) = \mathbf{D}(Y_n)^{-1}\left[\mathbf{D}(Q_n)\mathbf{L}(\ln c_n^{'}) - \mathbf{D}(X_n)\Pi\mathbf{L}(\ln c_n^{'})\right].$$

$\square$

Proposition 15. *In the symmetrical scenario, under global free trade arrangements,*

$$\ln W_k^{'} > 0 > \ln W_n^{'} > \ln W_l^{'}$$

for $n \neq k, l$*.*

Proof. Under the symmetrical world assumption, under world free trade arrangements, the vector of log-welfare derivatives with respect to t_{kl} reduces to the simple expression

$$\mathbf{L}(\ln W_n^{'}) = \frac{\bar{X}}{\bar{Y}} \left[\mathbf{I} - \Pi\right] \mathbf{L}(\ln c_n^{'}),$$

where $\bar{X}$ and $\bar{Y}$ are the common national manufacturing expenditures and national incomes shared by all countries.

For country k,

$$\begin{aligned} \ln W_k^{'} &= \frac{\bar{X}}{\bar{Y}} \left(\ln c_k^{'} - \sum_{i=1}^{N} \pi_{ki} \ln c_i^{'} \right) \\ &= \frac{\bar{X}}{\bar{Y}} \left(\sum_{i=1}^{N} \pi_{ki} \left(\ln c_k^{'} - \ln c_i^{'} \right) \right). \end{aligned}$$

Since $\ln c_k^{'} > \ln c_n^{'}$ for all $n \neq k$, we must have $\ln W_k^{'} > 0$.

For all countries $n \neq k, l$,

$$\ln W_n^{'} = \frac{\bar{X}}{\bar{Y}} \left(\sum_{i=1}^{N} \pi_{ni} \left(\ln c_n^{'} - \ln c_i^{'} \right) \right).$$

Since $\ln c_n^{'} = \ln c_i^{'}$ for all countries $n, i \neq k, l$, we have $\sum_{i\neq k,l}^{N} \pi_{ni} \left(\ln c_n^{'} - \ln c_i^{'} \right) = 0$. Furthermore, $\pi_{nk} = \pi_{nl} = \bar{\pi}\delta$ so we can write

$$\ln W_n^{'} = \frac{\bar{X}}{\bar{Y}} \bar{\pi}\delta \left(\ln c_l^{'} - \ln c_k^{'} \right) < 0$$

since $\ln c_k^{'} = \max_i \ln c_i^{'} > \min_i \ln c_i^{'} = \ln c_l^{'}$.

Finally, for country l,

$$\begin{aligned} \ln W_l^{'} &= \frac{\bar{X}}{\bar{Y}} \left(\sum_{i} \pi_{li} \left(\ln c_l^{'} - \ln c_i^{'} \right) \right) \\ &= \frac{\bar{X}}{\bar{Y}} \bar{\pi}\delta \left(\ln c_l^{'} - \ln c_k^{'} \right) + \frac{\bar{X}}{\bar{Y}} \sum_{i\neq k,l} \left(\ln c_l^{'} - \ln c_i^{'} \right). \end{aligned}$$

Since $\frac{\bar{X}}{\bar{Y}} \bar{\pi}\delta \left(\ln c_l^{'} - \ln c_k^{'} \right) = \ln W_n^{'}$ for any arbitrary $n \neq k, l$ and $\sum_{i\neq k,l} \left(\ln c_l^{'} - \ln c_i^{'} \right) < 0$ as $\ln c_l^{'} =$

$\min_i \ln c_i^{'}$, it immediately follows that

$$\ln W_l^{'} < \ln W_n^{'}$$

for all $n \neq l$. $\square$

There is a difference in welfare consequences between the mobile and immobile labor scenario. In the mobile labor scenario, $\ln W_n^{'} = \ln W_l^{'}$ for all $n \neq k$. In the immobile labor case, changes in t_{kl} affects the target of tariff policy t_{kl}, the country l more negatively than third-parties, $n \neq k, l$, due to the effects of reduced demand on country l's output and consequent decreases in national manufacturing wages. Further analysis is merited to disentangle the mechanisms by which manufacturing wages are determined in the immobile labor scenario.