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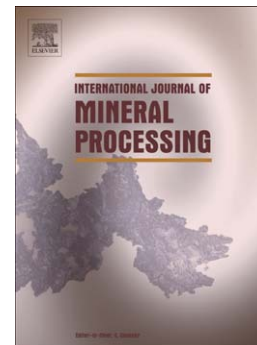
A review of the applications of the JK size-dependent breakage model

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## **A review of the applications of the JK size-dependent breakage model Part 1: Ore and coal breakage characterisation**

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### **ABSTRACT**

It has been 10 years since the JK size-dependent breakage model was developed (Shi and Kojovic, 2007). This paper reviews 20 applications of the model for the mineral and coal industries in the past 10 years to encourage its use in future applications. The review is divided into three parts: Part 1 for ore and coal breakage characterisation, Part 2 for assessment of material strength and energy requirement for size reduction, and Part 3 for modelling comminution equipment.

Part 1 of the review covers the model structure, its applications in high energy single impact data reduction, low energy incremental breakage modelling, reduced breakage testing method, a new impact breakage testing method using mixed particles in a wide size range, fine particle breakage characterisation (JKFBC), and a multi-component breakage model for coal grinding. It has been proved through these applications that the JK size-dependent breakage model is a useful tool for ore and coal breakage characterisation.

Keywords: Breakage characterisation; Size effect; Modelling; Energy efficiency.

## 1. Introduction

It is well known that the crack density of larger particles is much greater than that for smaller particles (Krajcinovic, 1996; Tavares and King, 1998). In view of this, bigger particles tend to be weaker and easier to break than smaller particles. The coarse particles drop weight test (DWT) data (Banini, 2000) and the fine particle breakage characterisation (JKFBC) data (Shi and Xie, 2015) have confirmed this trend. The graph (Fig. 1) of energy input requirement in relation to particle size (Hukki, 1962) provides further evidence to support the size effect on comminution energy requirement. As particle size decreases, it exerts more resistance to breakage, and hence requires more energy input. As shown in Fig. 1, the energy input requirement indicated by Kick, Bond, Rittinger increases in a power law fashion with an exponent changing from 1 to 4 for a decreased particle size.

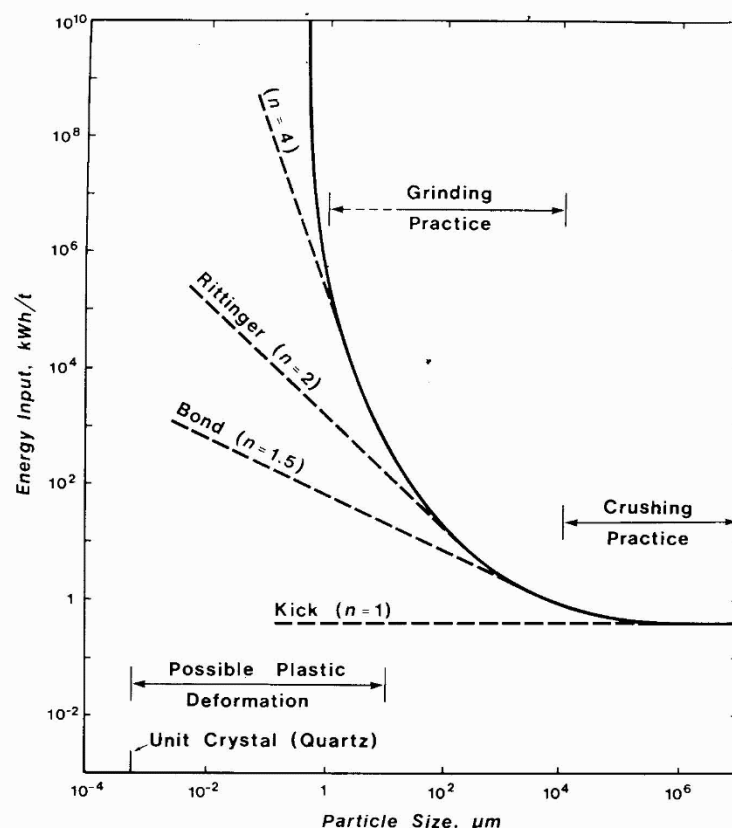


Fig. 1. Energy input requirement in relation to particle size (Hukki, 1962)

To describe the degree of breakage in relation to particle size effect and specific energy input, a size-dependent breakage model was developed in 2006, and published in 2007 (Shi and Kojovic, 2007). In the past 10 years, the model has been applied in consulting work for data reduction for the JKRBT (JK Rotary Breakage Tester, Shi et al. 2009) tests, breakage characterisation, assessment of particle strength, estimation of energy requirement for size reduction, and equipment modelling. These applications cover ore, coal, coke and cement. The equipment modelled with the JK size-dependent breakage model includes crusher, SAG mill, ball mill, HPGR, ball-race mill (HGI mill), and vertical spindle mill. The application of the model has also been extended from conventional mechanical breakage to non-conventional breakage, such as high voltage pulse fragmentation. In all these applications, it has been proved that the JK size-dependent breakage model provides a useful tool for the mineral and coal industries.

As the publications related to the application of the JK size-dependent breakage model are scattered in various journals, conferences, students' theses and technical reports, the objective of this review paper is to gather the major applications, mainly from the present author's work, in one place to demonstrate how this model can be applied for the mineral and coal industries and to inspire potential applications in the future. This is particularly important in the current energy-sensitive world, since the JK size-dependent breakage model has the potential to improve breakage characterisation and equipment modelling by taking into account the effect of particle size on the energy and size reduction relationship. The review is divided into three parts: Part 1 for ore and coal breakage characterisation, Part 2 for assessment of particle strength and energy requirement for size reduction, and Part 3 for modelling comminution devices. This is Part 1 of the review.

## 2. The JK size-dependent breakage model

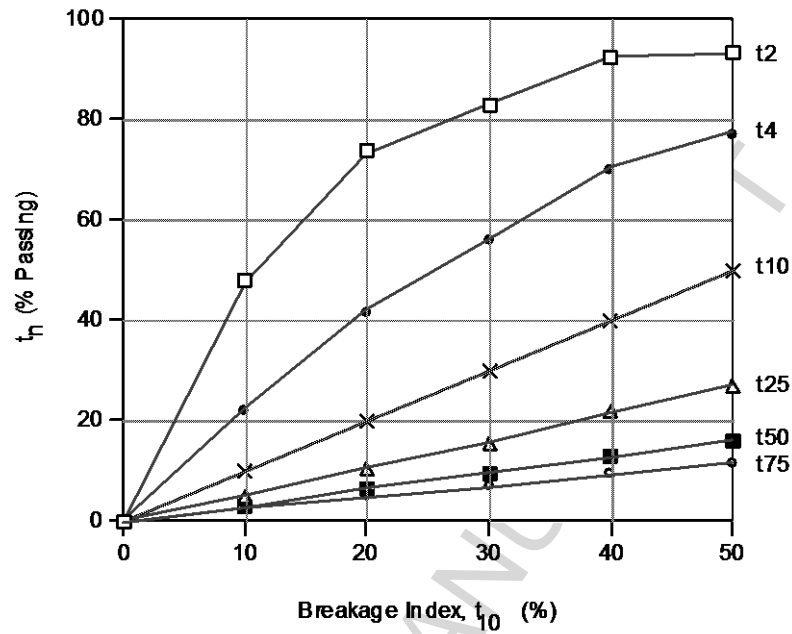
Combining the two approaches from Rumpf's similarity consideration (Rumpf, 1973) and the Weibull statistics of fracture mechanics (Weibull, 1951), Vogel and Peukert (2003, 2004) developed a breakage probability model. This model takes the following form (Eq. (1)):

$$S = 1 - \exp \left\{ - f_{mat} \cdot x \cdot k (W_{m,kin} - W_{m,min}) \right\} \quad (1)$$

where  $f_{mat}$  ( $\text{kg J}^{-1} \text{m}^{-1}$ ) is the material breakage property,  $x$  (m) the initial particle size,  $k$  the successive number of impacts with the single impact energy,  $W_{m,kin}$  ( $\text{J kg}^{-1}$ ) the mass-specific kinetic energy,  $W_{m,min}$  ( $\text{J kg}^{-1}$ ) the threshold energy below which breakage does not occur.

In modelling a breakage event, knowing whether or not a particle will break at all (breakage probability) is necessary, but it is only part of the process. It is more important to know the particle progeny size distribution following breakage. However it is also more difficult to model the breakage distribution function than the breakage probability, since the breakage probability in the Vogel and Peukert work can be simply represented with a single number, while the breakage distribution function is a matrix.

At the JKMRC (Julius Kruttschnitt Mineral Research Centre), there is a long history in modelling breakage distribution using a single parameter,  $t_{10}$ , the cumulative percentage of product passing  $1/10^{\text{th}}$  of the initial feed size. For a given feed size, the larger  $t_{10}$  value represents the finer product. Narayanan and Whiten (1988) found that this parameter is uniquely related to other points on a family of size distribution curves,  $t_n$ , defined as the cumulative percentage of product passing a given fraction of the initial size,  $x/n$ . Fig. 2 depicts the  $t_n$ -family curves for a range of ore types (Narayanan, 1985; Narayanan and Whiten (1988)). The  $t_n$ -family curves have been independently confirmed using different materials over a wide range of fracture energies by Pauw and Maré (1988) and by King and Bourgeois (1993).



**Fig. 2. Determination of size distribution parameter  $t_n$  from the breakage index  $t_{10}$  (Narayanan and Whiten (1988))**

As long as the  $t_{10}$  breakage index can be predicted, the whole size distribution of a product can be determined from the  $t_n$ -family curves. The JKMRRC has been using a breakage model (Eq. (2)) (Napier-Munn et al., 1996) for a long time (named JK breakage model in this review paper):

$$t_{10} = A(1 - e^{-b \cdot E_{cs}}) \quad (2)$$

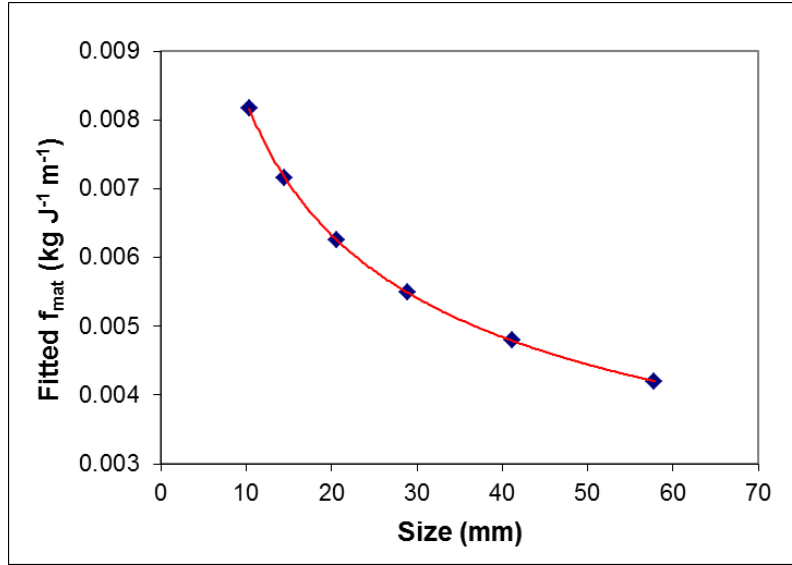
where  $E_{cs}$  ( $\text{kWh t}^{-1}$ ) is mass specific energy,  $A$  and  $b$  are the ore impact breakage parameters, which can be fitted to ore breakage testing data. The JK breakage model does not take into consideration the particle size effect on the breakage index  $t_{10}$ ; instead it uses one set of  $A$ ,  $b$  parameters to represent all particle sizes. Apparently bias will be introduced by using the size-averaged model parameters (Banini, 2000).

Since particle size ( $x$ ) is incorporated in the Vogel-Peukert breakage probability model, it was anticipated that the particle size effect on breakage can be accommodated using this more fundamental breakage model structure. The Vogel-Peukert breakage probability model was therefore revised to describe the breakage index  $t_{10}$  (Eq. (3)) by Shi and Kojovic (2007):

$$t_{10} = M \{1 - \exp[-f_{mat} \cdot x \cdot k(E_s - E_{min})]\} \quad (3)$$

where  $M$  (%) represents the maximum  $t_{10}$  for a material subject to breakage,  $E_s$  ( $\text{J kg}^{-1}$ ) the mass-specific energy, and  $E_{min}$  ( $\text{J kg}^{-1}$ ) the energy threshold (the units of the other variables are the same as in Eq. (1)). Vogel and Peukert (2004) assumed that the product of  $x$  and  $E_{min}$  is a constant, which can be treated as a model parameter and fitted to the breakage testing data. In a standard DWT or JKRBT test, the specific energy range is from  $0.1 \text{ kWh t}^{-1}$  ( $360 \text{ J kg}^{-1}$ ) to  $2.5 \text{ kWh t}^{-1}$  ( $9000 \text{ J kg}^{-1}$ ). A typical energy threshold for a basalt rock is approximately  $40 \text{ J kg}^{-1}$  (Larbi-Bram et al., 2010). Therefore the energy threshold  $E_{min}$  in the breakage tests with high impact specific energy may be ignored without significant bias, and it can be set to  $E_{min} = 0$  in Eq. (3). This is useful to reduce the number of model parameters when the testing data points are limited. However, in low energy incremental breakage, the energy threshold  $E_{min}$  cannot be set to zero.

In validating the revised breakage model (Eq. (3)), Shi and Kojovic found that the effect of particle size on breakage was not adequately represented by the term  $x$  alone in Eq. (3). They found that if the material property parameter  $f_{mat}$  is fitted to individual feed size fractions, the fitted  $f_{mat}$  values are closely related to particle size  $d$ . Fig. 3 shows an example of the fitted  $f_{mat}$  values for various particle sizes using basalt rock DWT data published in the literature (Banini, 2000).



**Fig. 3. Example of the fitted breakage model parameter  $f_{mat}$  in relation to particle size  $d$**

Numerous DWT and JKRBT test data have confirmed this trend. An equation to describe the parameter  $f_{mat}$  in relation to particle size was therefore established (Eq. (4)):

$$f_{mat} = p \cdot d^{-q} \quad (4a)$$

or

$$f_{mat} = p \cdot x^{-q} \quad (4b)$$

Note that the unit of particle size  $d$  in Eq. (4a) is mm, while  $x$  in Eqs. (1), (3) and (4b) is m. For mineral engineering application, the mass specific energy often uses  $\text{kWh t}^{-1}$  ( $1 \text{ kWh t}^{-1} = 3600 \text{ J kg}^{-1}$ ). By substituting Eq. (4a) into Eq. (3) and making unit conversion:

$$t_{10} = M \left\{ 1 - \exp \left[ -3.6 p \cdot d^{(1-q)} \cdot k (E_{cs} - E_o) \right] \right\} \quad (5a)$$

or substituting Eq. (4b) into Eq. (3):

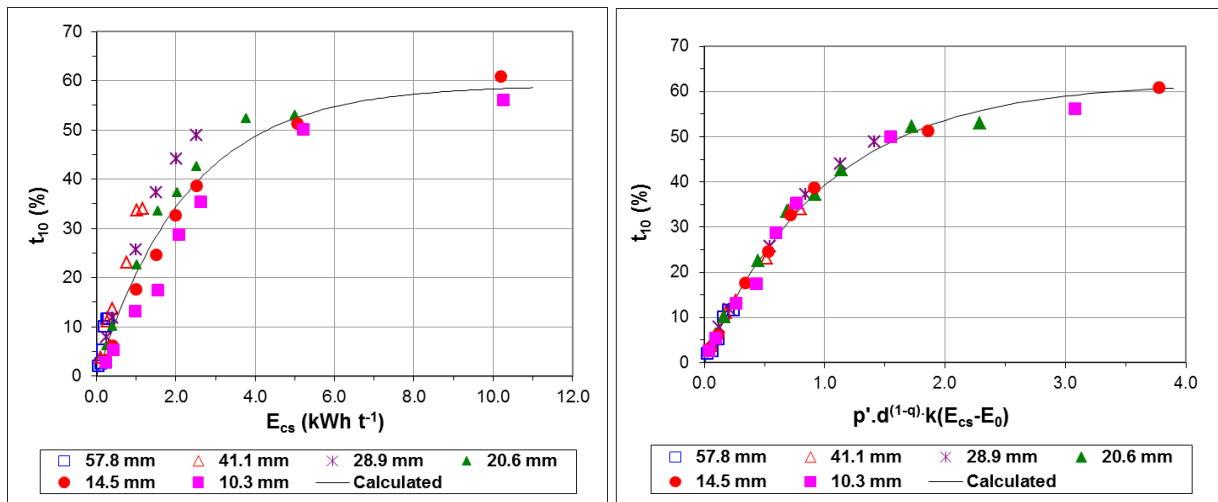
$$t_{10} = M \left\{ 1 - \exp \left[ -3600 p \cdot x^{(1-q)} \cdot k (E_{cs} - E_o) \right] \right\} \quad (5b)$$



where  $E_{cs}$  ( $\text{kWh t}^{-1}$ ) is mass specific energy (same as in Eq. (2)), and  $E_o$  ( $\text{kWh t}^{-1}$ ) is energy threshold. The three model parameters,  $M$ ,  $p$  and  $q$ , in Eqs. (5a) or (5b) can be determined simultaneously from one set of breakage test data. Note that the fitted parameter  $p$  in Eqs. (5a) and (5b) has different values.

Parameter  $q$  in Eq. (5) describes the effect of particle size on breakage. It often takes a value between 1 and zero in ore breakage characterisation. When  $q = 1$ , the exponent  $(1-q)$  of particle size is zero, and  $d^{(1-q)}$  (or  $x^{(1-q)}$ ) is unity, indicating that the particle size effect on breakage is diminished. When parameter  $q$  approaches zero, the exponent  $(1-q)$  of particle size takes a value of 1, indicating a strong particle size effect on breakage. Parameter  $q$  can take a negative value to give the exponent  $(1-q)$  larger than 1, indicating a very strong particle size effect on breakage. The negative parameter  $q$  values were sometimes found in fine grinding of coal samples using the JKFBBC. Parameter  $q$  may in some rare cases take a value larger than 1 to give the size exponent  $(1-q)$  a negative value, indicating that larger particles are more competent. The advantage of using parameters  $M$ ,  $p$ ,  $q$ , instead of the parameters  $A$ ,  $b$  in the JK breakage model (Eq. (2)), is that the effect of particle size on the breakage response can be directly quantified using the breakage testing results. Eq. (5a) or (5b) is therefore called the JK size-dependent breakage model in this review paper, in order to be identified from the traditional JK breakage model.

Fig. 4a shows the JK breakage model (Eq. (2)) fitted to one set of basalt rock DWT data (Banini, 2000). A significant size effect on the breakage index  $t_{10}$  is apparent, with larger particles producing a higher  $t_{10}$  at the same specific energy input. Fig. 4b gives the result using Eq. (5a) to fit the same dataset, where  $p'$  in the x-axis title is equal to  $3.6p$ . All particles in various sizes fall on the same trend line, indicating that the particle size effect on breakage can be well modelled using Eq. (5).



a. JK breakage model (Eq. (2))

b. JK size-dependent breakage model (Eq. (5a))

**Fig. 4. Comparison of model fitting results for the same basalt rock DWT data (Shi and Kojovic, 2007)**

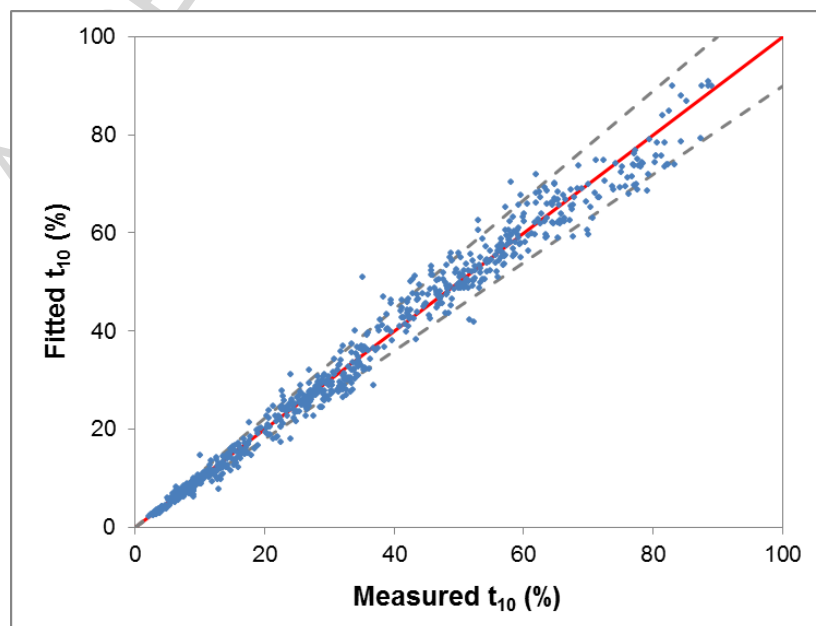
A method that is based on the JK size-dependent breakage model to predict breakage properties of a particulate material when subjected to impact was patented by the JKMRC in 2006. Part of the work was published in Shi and Kojovic (2007). The JK size-dependent breakage model was kept for internal use within the JKMRC and JKTech until 2015 when the complete model was published (Shi et al., 2015, online in November 2014). In the past 10 years, the JK size-dependent breakage model has found wide applications. Part 1 of this review paper presents its use for ore and coal breakage characterisation.

### 3. Data reduction for high energy single impact breakage

One of the most common applications of the JK size-dependent breakage model is for breakage testing data reduction. In a standard DWT, five narrowly sized particles are tested, each with three different specific energy levels, resulting in 15 product samples. Each product is sieved to determine the breakage index  $t_{10}$ . In a standard JKRBT test, four size fractions are tested at three specific energy levels for each size. As all the variables shown in Eq. (5) ( $t_{10}$ ,  $d$ ,  $E_{cs}$ ) are available from the DWT and JKRBT test data, the JK size-dependent breakage model can be applied directly to fit the three model parameters  $M$ ,  $p$  and  $q$  to the breakage testing data. Since the standard breakage tests are performed in a single-particle, single

impact mode, parameter  $k$  in Eq. (5) is unity.  $E_0$  in Eq. (5) is set to zero as  $E_0$  is small compared with the specific energy input in the high energy impact tests.

The JK size-dependent breakage model usually fits the breakage testing well, as shown in Fig. 4b. In a JKRBT validation project involving Anglo Platinum, Barrick Gold, BHP Billiton, JKMRC, Rio Tinto and Teck, 68 sets of standard JKRBT test data were collected using eight JKRBT units deployed worldwide (Shi et al., 2009). Fig. 5 presents the measured  $t_{10}$  values in comparison with the fitted  $t_{10}$  by the JK size-dependent breakage model. There are 795 data points in the plot covering a very wide  $t_{10}$  range from 2.1 to 89.0. A statistical t-test on paired comparisons between the measured  $t_{10}$  and fitted  $t_{10}$  was performed. With a mean difference of -0.16 between the measured and fitted  $t_{10}$  and a degree of freedom 794, the standard deviation of the difference is 2.86. This gives a t-value of -1.07, indicating that the difference is not significant at the 95% confidence level. The statistical analysis of the 795 pairs of  $t_{10}$  values confirms that the JK size-dependent breakage model can work well for high energy single impact breakage characterisation. The JK size-dependent breakage model has been coded in software for commercial service in JKRBT data reduction.



**Fig. 5. Comparison of the measured  $t_{10}$  values of 68 sets of standard JKRBT test data versus the fitted  $t_{10}$  values using the JK size-dependent breakage model, the dotted lines representing  $\pm 10\%$  boundaries**

Fig. 5 also shows that a number of  $t_{10}$  data points predicted by the JK size-dependent breakage model exceeds the  $\pm 10\%$  boundaries of the measured data. It was observed that this was caused by ore variation, the particle shape in particular. The tested material may contain some flaky or elongated particles. These particles would receive different impact specific energies in the JKRB tests (due to the effective rotation radius, refer to Shi et al. 2009), or in the DWT tests (due to the particle mass), compared with the cubic-shape particles in the same screen size fraction. As a result, the predicted  $t_{10}$  values using the size-averaged specific energy may depart from the measured  $t_{10}$ .

After the breakage model for  $t_{10}$  (Eq. (3)) was published by Shi and Kojovic (2007), Nadolski et al. (2014) presented a revised version of this model as shown in Eq. (6), in which  $E_{min}$  was set to zero, and particle size  $x$  was replaced with the square root of  $x$ . Eq. (6) was used for piston compression data reduction. They found that Eq. (6) can well represent the particle size effect and gave a better fit in multi-layer particle compression tests.

$$t_{10} = M \left\{ 1 - \exp \left[ - f_{mat.} \cdot x^{0.5} \cdot k (E_{cs} - E_{min}) \right] \right\} \quad (6)$$

Davaanyam (2015) further modified Eq. (6) by changing the exponent 0.5 to a more flexible exponent  $n$  (Eq. (7)):

$$t_{10} = M \left[ 1 - \exp \left( - f_{mat.} \cdot x^n \cdot E_{sp} \right) \right] \quad (7)$$

Davaanyam (2015), Davaanyam et al. (2015) and Nadolski et al. (2015) showed that Eq. (7) fits HPGR data well. Fig. 6 gives one example of the data fitting result. Coincidentally, Eqs. (6) and (7) are actually the same equation as the JK size-dependent breakage model (Eq. (5)) developed 10 years ago. When the parameter  $q$  in Eq. (5b) takes the value 0.5,  $1-q = 0.5$ ,

Eq. (6) becomes Eq. (5b). Similarly, when  $q = 1-n$ , which leads to  $1-q = n$ , Eq. (7) becomes Eq. (5b) for  $E_{min} = 0$ . These independent tests further prove the validity of the JK size-dependent breakage model for high energy breakage characterisation.

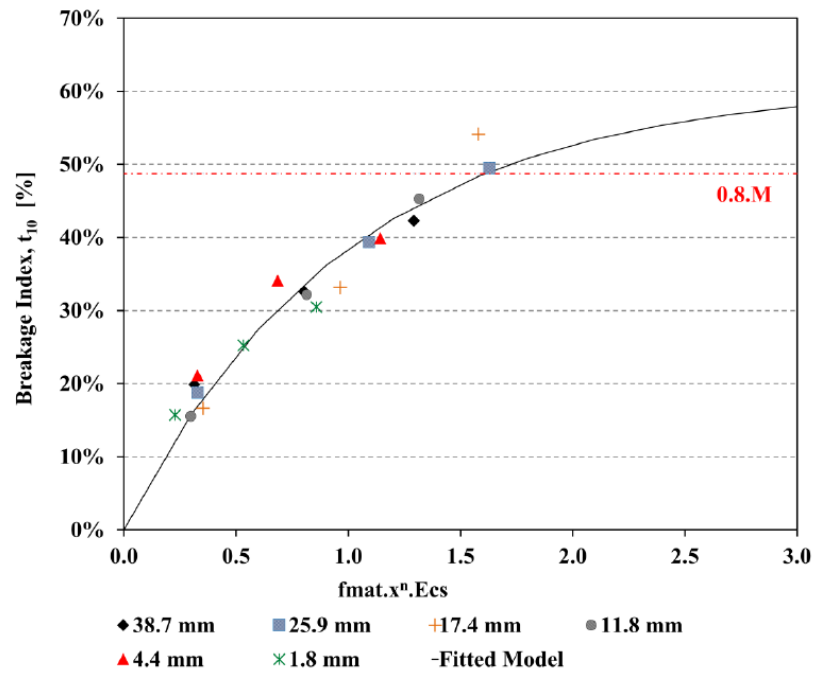


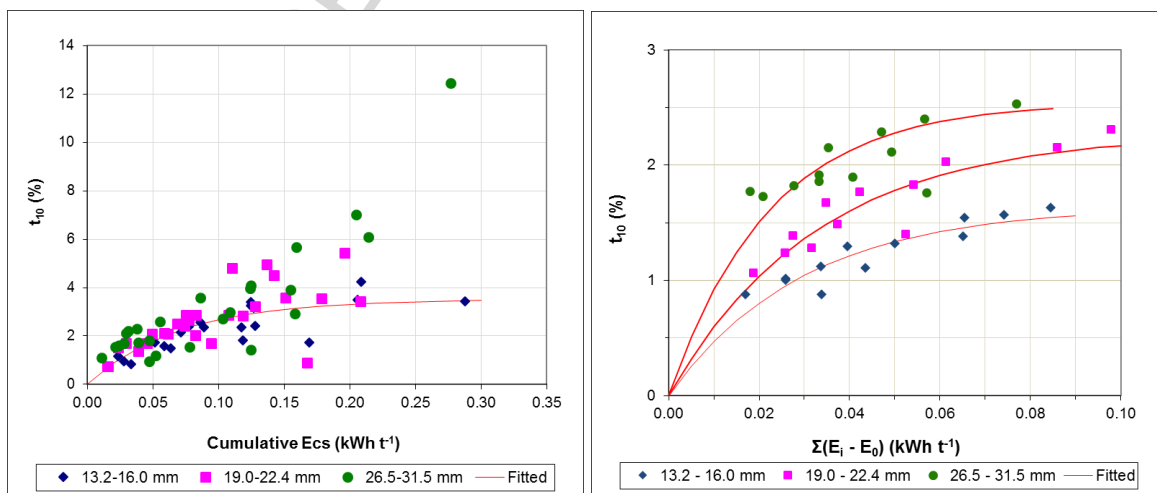
Fig. 6. Energy breakage model (Eq. (6) fitted to the piston compression data of New Afton ore sample (Nadolski et al. (2015))

#### 4. Low energy incremental breakage

DEM simulations of the impact energy distribution pattern in an AG mill operation have revealed that small energy impacts take place at a much higher frequency than high energy impacts (Djordjevic et al, 2004). The broken particles in the mill may result from receiving a single impact of high energy, or multiple impacts of low energies. The JK size-dependent breakage model applying to single impact breakage data at relatively high energy levels has already been validated. To develop a consolidated model for AG/SAG milling and to predict the repetitive impact breakage for DEM simulations, it is necessary to model incremental breakage with low impact energies.

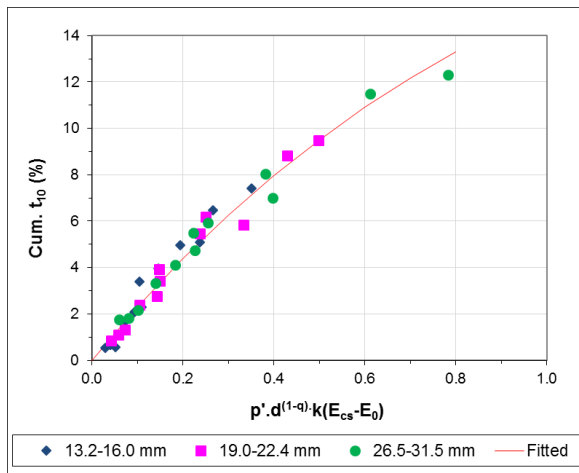
Two sets of published incremental breakage testing data (Whyte, 2005; Pauw and Maré, 1988) were collected and used to validate the JK size-dependent breakage model for incremental breakage characterisation. Whyte conducted a number of repetitive impact tests with the DWT installed at the JKMRC. Three particle sizes were tested: 26.5–31.5 mm, 19.0–22.4 mm and 13.2–16.0 mm. It was estimated by Whyte that the approximate minimum energy required to cause breakage was  $0.08 \text{ kWh t}^{-1}$ . The experiment was designed with 30%, 40% and 50% of the minimum energy of  $0.08 \text{ kWh t}^{-1}$  respectively. In each test, 100 particles were used. Each particle was impacted up to ten times until it broke. After each particle breakage event, the progeny particles were placed in ten hit-classes, in which all progenies subject to the same number of hits at the same single impact energy from the same original size fraction were combined.

It was observed in Whyte's thesis (2005) that the plot of  $t_{10}-E_{cs}$  for the incremental breakage tests was very scattered (Fig. 7a). This may be attributed to an insufficient number of particles being used to determine the product  $t_{10}$  in some Hit-classes – only one particle was available for sieving to determine the  $t_{10}$  in some classes.



a. Eq. (2) fits Whyte data (Whyte, 2005)

b. Eq. (2) fits individual size of regrouped data



c. Eq. (5a) fits regrouped cumulative  $t_{10}$  data

**Fig. 7: Comparison of the model fitting results using Whyte (2005) incremental impact breakage data (Shi, 2006)**

Whyte's data were re-organised in four groups, in which some Hit-classes were combined to increase the number of particles in the groups: 1 Hit, 2 Hits, 3-4 Hits and 5-10 Hits. Note that the measured  $t_{10}$  values in the combined groups decrease as a result of the increased feed mass in each Hit-class. The grouped data were used to fit a modified version of Eq. (2), with  $E_{cs}$  in Eq. (2) was replaced with  $\sum(E_i - E_0)$ , where  $E_i$  is mass specific energy in  $i^{\text{th}}$  impact,  $E_0$  is energy threshold, and parameter  $b$  in Eq. (2) was replaced with  $b'$ . The modified version of Eq. (2) was fitted to each size fraction (keeping constant  $E_0 = 0.01 \text{ kWh t}^{-1}$ ), and generated three sets of  $A$  and  $b'$  parameters. The fitting results are shown in Fig. 7b and the fitted model parameters are listed in Table 1 (Morrison et al., 2007).

**Table 1: Size-specific parameter values of the modified Eq. (2) fitted to Whyte regrouped data (Morrison et al., 2007)**

Size (mm)	$A$	$b'$	$Axb'$
13.2-16	1.64	33.70	55.27
19.0-22.4	2.28	30.35	69.20
26.5-31.5	2.55	44.87	114.42

The term  $Axb'$  is an indicator of ore resistance to breakage, a larger  $Axb'$  value indicating less resistance to breakage. Table 1 shows that the  $Axb'$  values increase with particle size, which confirms the particle size effect on breakage. However, the fitted  $b'$  values for the individual size fractions varies irregularly. This gives rise to difficulties in estimating the model parameters for the untreated size fractions from the calibrated size-specific model parameters in simulations or in DEM.

The JK size-dependent breakage model was employed to fit the Whyte's regrouped data with one set of  $M$ ,  $p$ ,  $q$  parameters to describe the size effect. The term  $xE_0$  was treated as the fourth parameter (in addition to  $M$ ,  $p$ ,  $q$ ) in the fitting. Note that the cumulative  $t_{10}$  values and the cumulative net specific energy from the Hit-classes were used in the fitting. The result is given in Fig. 7c for comparison (Shi, 2006). Obviously, the fitting quality has been significantly improved by using the JK size-dependent breakage model to fit the regrouped data. In particular the particle size effect on the incremental breakage result has been well represented, as the particles in the three feed sizes are falling on one trend line in Fig. 7c in comparison with Fig. 7b.

Pauw and Maré (1988) published a paper on incremental breakage. They conducted single particle incremental impact breakage tests with a drop weight test device for four feed sizes at 10 levels of impact energy. The paper includes the testing details together with the raw product sizing results. A sample of the AG mill feed in a gold mine was used for the tests. Only particles that did not contain gold reef were selected, so that they would be homogeneous in terms of composition and mineral structure. Four particle sizes were tested: 9.5–12.7 mm, 6.7–9.5 mm, 4.75–6.7 mm and 3.35–4.75 mm, but the paper does not give the sizing result for the 3.35–4.75 mm test.



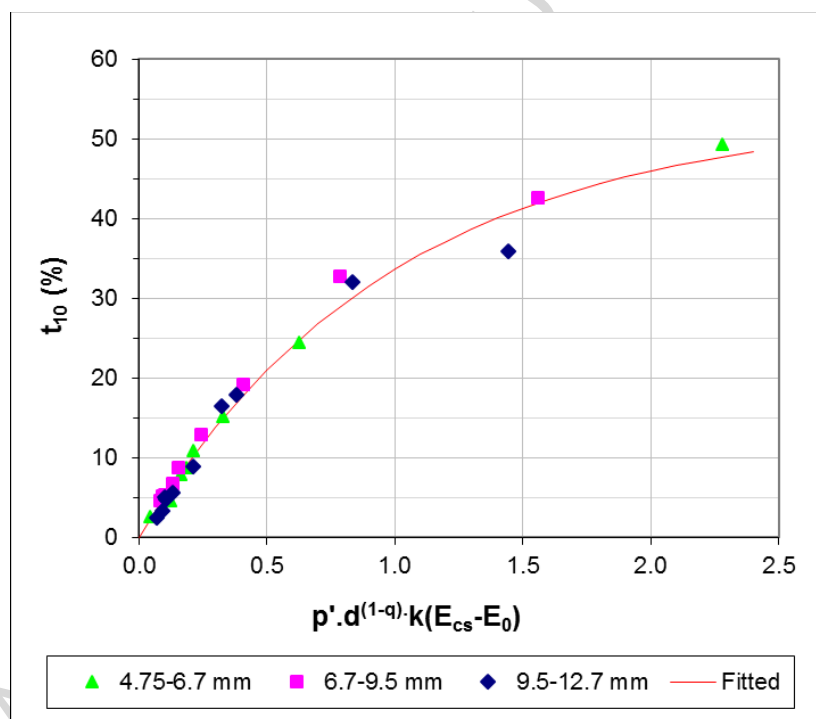
The tests were conducted in a different way to Whyte's. Particles from each size fraction were separated into approximately 10 different samples, each subjected to a given impact energy. The number of particles and the total mass of each sample were recorded. Particles in each sample were subjected to the same impact energy. After each impact, the particle was checked against the lower limit of the screen aperture. Any product that was still too large was subjected to a further impact at the same energy level until it broke to a size smaller than the screen aperture. The total number of impacts per sample was recorded and all the products in a given sample were combined for sizing. This procedure provides sufficient particles in the product for size analysis, and hence the size distribution data are deemed to be reliable. However, the information on how the progeny properties vary with each impact is not available since the products from different number of impacts were not collected separately. Values of  $t_{10}$  were calculated from the raw data. Table 2 presents the re-calculated incremental breakage data used for the model validation.

**Table 2: Re-calculated incremental breakage data from Pauw and Maré (1988)**

Sample	No. impact	No. Particle	Sample wt (g)	$E_k$ (J)	Specific $E_s$ ( $J\ kg^{-1}$ )	Average k	$t_{10}$ (%)
<i>Size 4.75 – 6.7 mm</i>							
3.92 J	65	65	17.1	3.924	14907	1.0	49.26
1.18 J	61	61	17.1	1.177	4197	1.0	24.50
0.59 J	66	63	17.2	0.589	2158	1.0	15.22
0.39 J	68	63	17.3	0.392	1427	1.1	10.87
0.29 J	80	63	17.1	0.294	1085	1.3	8.71
0.196 J	114	68	17.2	0.196	777	1.7	7.83
0.147 J	136	53	17.0	0.147	458	2.6	4.54
0.098 J	217	56	17.0	0.098	322	3.9	4.84
0.049 J	690	57	16.3	0.049	171	12.1	2.65
<i>Size 6.7 – 9.5 mm</i>							
7.85 J	49	49	37.4	7.848	10275	1.0	42.62
3.92 J	53	52	39.7	3.924	5140	1.0	32.81
1.96 J	55	52	38.9	1.962	2626	1.1	19.10
0.98 J	71	64	40.9	0.981	1535	1.1	12.87
0.59 J	87	62	44.6	0.589	819	1.4	8.73
0.49 J	70	43	33.5	0.491	631	1.6	6.70
0.29 J	130	61	42.9	0.294	418	2.1	5.35
0.20 J	198	53	39.5	0.196	263	3.7	5.13
0.15 J	276	55	38.0	0.147	213	5.0	4.66
0.098 J	700	9	6.2	0.098	142	77.8	N/A
<i>Size 9.5 – 12.7 mm</i>							
17.66 J	25	25	46.2	17.658	9555	1.0	35.88
9.81 J	28	27	49.3	9.810	5373	1.0	32.04
4.91 J	26	25	49.7	4.905	2467	1.0	17.83
3.92 J	27	24	48.0	3.924	1962	1.1	16.45
1.96 J	38	26	49.8	1.962	1024	1.5	8.83
1.47 J	31	22	47.5	1.472	682	1.4	5.63
0.98 J	43	21	49.4	0.981	417	2.0	4.74
0.74 J	55	27	50.1	0.736	397	2.0	5.00
0.44 J	98	25	48.3	0.441	228	3.9	3.26
0.29 J	163	23	47.9	0.294	141	7.1	2.48
0.15 J	742	9	15.1	0.147	88	82.4	N/A

Fig. 8 shows the results of the JK size-dependent breakage model fitting to the re-calculated Pauw and Maré's data. As the sizing data for the very high numbers of repeated impacts (k

value around 80, Table 2) were not published in the paper, those two points (marked with N/A in the last column) are not included. The JK size-dependent breakage model works well for the incremental breakage results. The merit of this modelling approach is that the calibrated model can predict the product size distributions for a given feed particle size, the number of incremental impacts, and the specific energy input of each impact. The model can be used for ore degradation simulations and size reduction simulations in a grinding mill when the particles are subjected to low energy attrition/abrasion breakage.

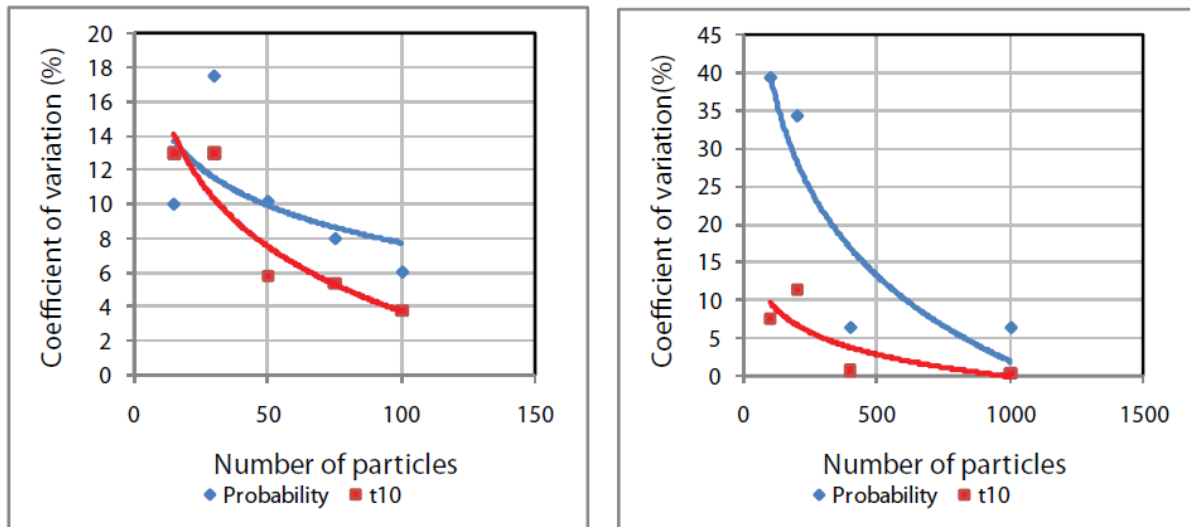


**Fig. 8. The JK size-dependent breakage model fitted to the re-calculated Pauw and Maré's data presented in Table 2 (Shi, 2006)**

Larbi-Bram (2010) in his Ph.D. work conducted a detailed experimental study on incremental breakage, with a particular emphasis on its characterisation for AG/SAG mill modelling. The investigation was carried out by using gravity dropping particles, JKRBT low energy repetitive impacts and tumbling particles with a very low charge loading in two laboratory scale mills (1.1 m and 0.6 m in diameter respectively). A high speed video camera and an ordinary digital camera were installed to record particle motion for impact velocity, and thus the specific energy calculations. Three major breakage criteria, breakage

probability, breakage index ( $t_{10}$ ) and energy threshold in the body breakage and surface breakage modes, were studied. The JK size-dependent breakage model was used extensively to process the high energy single impact and the low energy incremental breakage data, with modifications on one of the model parameters, the  $M$  in Eq. (5a), by accounting for the specific energy effect on the parameter. The Vogel-Peukert breakage probability model (Eq. (1)) was modified by incorporating Eq. (4) to replace the  $f_{mat}$  parameter in Eq. (1). The Vogel-Peukert equation then becomes a size-dependent breakage probability model. The two size-dependent breakage models (for probability and  $t_{10}$ ) were used by Larbi-Bram to process the experimental data.

Larbi-Bram et al. (2010) also proposed a guideline on the minimum number of particles required in the breakage characterisation tests (Fig. 9). For a high energy impact test with specific energy larger than  $0.25 \text{ kWh t}^{-1}$ , a minimum of 30 particles per test is recommended for  $t_{10}$  and 50 particles for breakage probability, preferably 100 particles if available, to achieve a CoV (Coefficient of Variation defined as percentage of standard deviation divided by the mean  $t_{10}$  or probability) better than 10%. On the other hand, for low energy impact tests at less than  $0.1 \text{ kWh t}^{-1}$ , more than 600 particles are required to achieve a CoV better than 10% for breakage probability. For reference, Vogel and Peukert (2004) used 2500 particles per test in the size range of 2-3 mm to determine ore breakage probability. Any incremental breakage data acquired with less than the minimum number of particles will result in unreliable characterisation.



a. Impact energy larger than  $0.25 \text{ kWh t}^{-1}$       b. Impact energy smaller than  $0.1 \text{ kWh t}^{-1}$

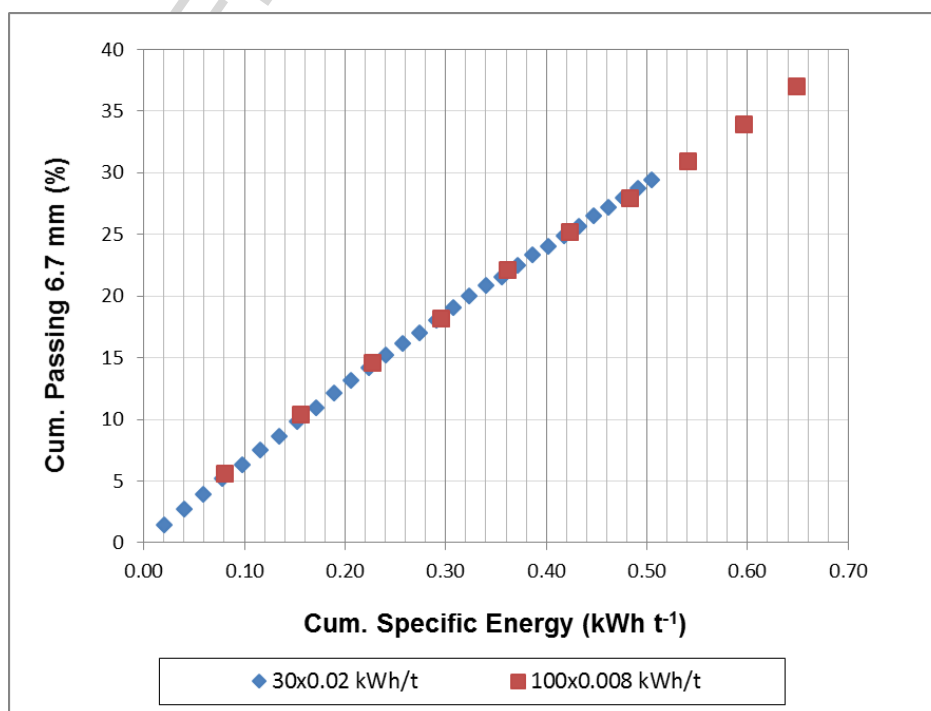
**Fig. 9. Effect of number of particles on the CoV of measured breakage probability and  $t_{10}$  (Larbi-Bram, et al., 2010)**

Larbi-Bram et al. (2010) showed evidence that using ore parameters derived from the standard DWT test at high energy single impacts over-predicted the breakage indices observed in the laboratory mill testing the same ore, which predominantly resulted from low energy repetitive impacts that were typically found in AG/SAG mills. However, for the low energy incremental breakage, regardless of the breakage achieved by the JKRBT, gravity dropping or tumbling, the plots of the cumulative  $t_{10}$  values and the cumulative specific energy input are on the same trend line for the same ore. This supports the view that ore breakage in a tumbling mill can be characterised with JKRBT low energy incremental breakage tests followed by application of the JK size-dependent breakage model to derive ore-specific characteristic parameters.

The same trend was observed in a coke incremental breakage result (Shi, 2012). A coke sample in the size range of 26.5-45 mm was tested. A closing screen with 6.7 mm aperture was used for the incremental breakage tests. After each cycle of impact test, the JKRBT product was sieved on the 6.7 mm screen. The oversize particles were then subjected to the next cycle of breakage with the same specific energy level as before. Two low specific

energy levels were used for the incremental breakage:  $0.02 \text{ kWh t}^{-1}$  and  $0.008 \text{ kWh t}^{-1}$ . For the  $0.02 \text{ kWh t}^{-1}$  test, 30 cycles of impact were performed. For the  $0.008 \text{ kWh t}^{-1}$  test, particles were subjected to 10 impacts as one cycle, then sieved to return the oversize to the JKRBT for another 10-impact cycle. A total of 100 impacts (10 cycles x 10-impact) were undertaken. Approximately 100 particles per energy test were used for the incremental JKRBT tests. The amount of -6.7 mm material in each cycle was used as a fineness indicator.

The cumulative percentage passing 6.7 mm was plotted against the cumulative specific energy (Fig. 10). It is shown that the two incremental breakage methods produce identical breakage results. This indicates that for the low energy breakage event, the breakage result is decided by the cumulative specific energy for a given coke, regardless of the energy level at each impact. This observation also implies that the energy threshold of the coke sample in the 26.5-45 mm size may be small, which has a negligible effect on the incremental breakage results.

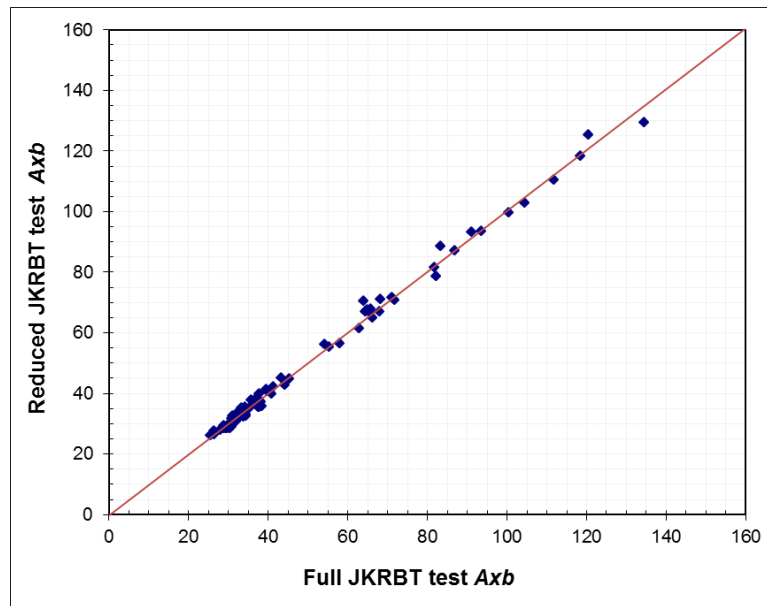


**Fig. 10.** The JKRBT incremental breakage results using 30 cycles of  $0.02 \text{ kWh t}^{-1}$  in comparison with 10x10-impact cycles of  $0.008 \text{ kWh t}^{-1}$  for a coke sample (Shi, 2012)

## 5. A reduced breakage test

As shown in Fig. 4b all measurement points from different particle sizes subjected to various specific energy input levels fall on the same trend line, which implies that this characteristic curve can be determined using less breakage tests. The merits of using reduced breakage tests to determine breakage characteristic parameters are obvious, including the reduced testing time, cost and the amount of sample required. The last one is particular important where the availability of the testing sample is limited.

Extensive effort was made to test various combinations of size and energy settings for the DWT by the present author 10 years ago. It was proved statistically that using five sets of DWTs, together with the JK size-dependent breakage model, provides a similar ore competence indicator, the  $Axb$  values, to that determined using the standard 15 sets of DWT for each ore sample. A similar exercise was performed using the 68 sets of the JKRBT database to derive a reduced JKRBT testing method, in which only five tests at various energy and size combinations were consistently used to determine the model characteristic parameters. The three characteristic parameters ( $M$ ,  $p$ ,  $q$ ) of the JK size-dependent breakage model were then employed to predict the  $t_{10}$  values for the standard DWT size at the nominal energy levels. The JK breakage model (Eq. (2)) was used to fit the calculated  $t_{10}$  data points to determine the DWT equivalent model parameters  $A$  and  $b$ . Fig. 11 shows the  $Axb$  values determined by the reduced JKRBT tests (five tests per sample), versus those determined by the standard JKRBT tests (12 tests per sample). A statistical t-test on paired comparisons confirms that the  $Axb$  values determined by the five JKRBT tests per sample are not statistically different to those determined by the 12 JKRBT tests per sample (Shi et al., 2013).



**Fig. 11. Comparison of the  $Axb$  values determined by the full 12 JKRBT tests with those determined by the five JKRBT tests per sample, for 68 sets of various ore data (Shi et al., 2013)**

In the reduced breakage testing method, the effect of particle size on breakage results is measured and mechanistically modelled. This approach is advantageous to other reduced breakage characterisation methods, in which one feed size is tested and the size effect is estimated from a database or calibrated with an additional full DWT.

Similar to the application in developing the reduced breakage test, the JK size-dependent breakage mode has found another application to process the incomplete DWT data. In a standard DWT test, the top particle size tested is 53-63 mm. Due to the ore availability limitation, sometimes the top size results were found to be missing in the DWT data. The  $Axb$  value determined without the 53-63 mm testing data would be different to those from the standard DWT data. In such case the JK size-dependent breakage model can be applied to establish the characteristic  $M$ ,  $p$ ,  $q$  parameters from the incomplete DWT test data. The calibrated test model is then used to predict the missing  $t_{10}$  values for the coarse particle size. The predicted  $t_{10}$  values for the missing coarse particles and the measured  $t_{10}$  values for the other size fractions are used together to fit Eq. (2) to determine the  $A$  and  $b$  parameters. This method has been applied in treating incomplete DWT testing data.



## 6. Breakage tests using mixed particles in a wide size range

In the standard impact breakage characterisation tests, particles in narrowly sized fractions are tested at various specific energy levels. For example, the standard JKMRC DWT uses five size fractions: 13.2–16 mm, 19–22.4 mm, 26.5–31.5 mm, 37.5–45 mm, and 53–63 mm, and the JKRBT uses four of the five DWT testing size fractions (without 53-63 mm). Despite significantly reducing the time taken to break particles in the JKRBT test, it is still time consuming to prepare these narrowly sized particles (over 360 pieces for one characterisation test), and to carry out size analysis on the 15 DWT or 12 JKRBT products. In addition, only particles in the specified size fractions are used for breakage characterisation. In some cases the ability to supply sufficient particles in the desired size fractions, in order to complete one characterisation test, may become a problem. These limitations restrict their application when dealing with massive ore samples such as geo-metallurgical testing, ore pre-weakening evaluation and online ore competence measurement for comminution circuits. The JK size-dependent breakage model enables a new ore breakage characterisation method to be developed using the JKRBT to test mixed particles in a wide size range (namely the Wide-size JKRBT tests for short) to overcome the limitations in the existing ore breakage characterisation methods (Zuo and Shi, 2016).

In the new testing procedure, the received feed sample is sieved to determine the size distribution. Particles in the typical JKRBT testing size range (13.2-45 mm) are combined, and tested with the JKRBT. The mixed feed size range can be further extended to 5.6-45 mm. Since three model parameters ( $M$ ,  $p$ ,  $q$ ) in the JK size-dependent model for high energy impact test are required to be calibrated, a minimum of three JKRBT tests, and preferably four tests, should be conducted. This can be arranged as two feed size distributions tested with two energy levels for each ore. It does not require an equal number of particles in each size fraction, such as 30 particles per size for the traditional DWT or JKRBT tests. Instead, it uses all particles in the desired size range. This is particularly

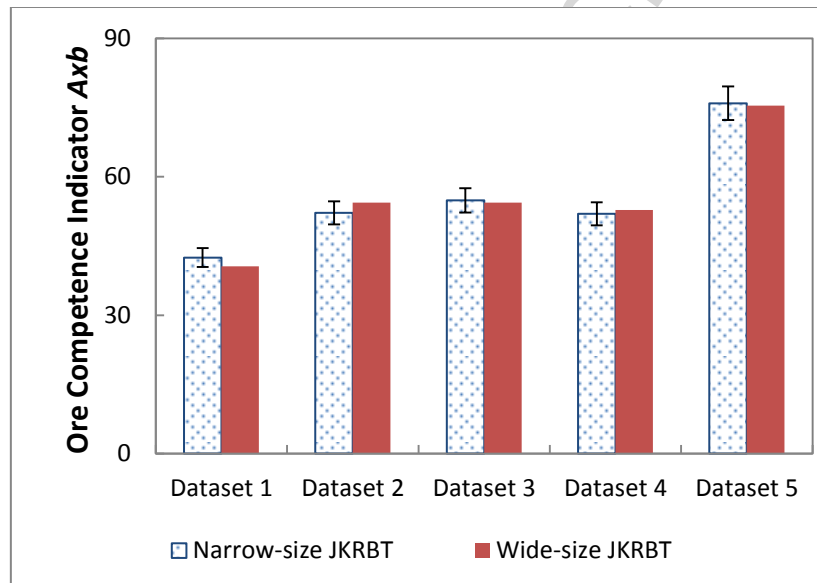
useful when the sample availability is limited. The actual specific energy used for each particle size fraction can be calculated in the data reduction stage according to the JKRBT rotor speed and the particle size using an equation presented in Shi et al. (2009). The three or four JKRBT products for each ore sample are sieved separately to determine the  $t_{10}$  values.

The general approach taken in the data reduction procedure is summarised below (Zuo and Shi, 2016):

- Divide the wide-size feed into several virtual narrow size fractions by simulation;
- Use the size-dependent breakage model with initially guessed values of  $M$ ,  $p$ ,  $q$  parameters to calculate  $t_{10}$  values for each virtual narrow size of the feed;
- Use the calculated  $t_{10}$  value and the  $t_n$ -family curves to calculate the product size distribution matrix resulting from breakage of each narrow size fraction of the feed;
- Sum up the product size distribution matrices using the feed proportion to form a combined product size distribution;
- Estimate the weighted error between the calculated and the measured product size distributions;
- Repeat the procedures for the three or four products at various impact specific energy levels and calculate the total error (sum of squares, SSQ);
- Adjust the model parameters  $M$ ,  $p$ ,  $q$  iteratively until the SSQ converges to a minimum.

Five sets of JKRBT data were used to validate the Wide-size breakage characterisation method. Fig. 12 shows the  $Axb$  values determined from the Wide-size JKRBT tests in comparison with the traditional narrow-size method for all the five datasets. The error bars display  $\pm$  one SD (standard deviation) of the narrow-size JKRBT tests. The SD was calculated by a CoV (Coefficient of Variation, defined as a ratio of SD to  $Axb$ ) value of 0.048

from a JKRBT validation research project (Shi and Kojovic, 2011). The preliminary investigation demonstrates that the Wide-size JKRBT characterisation method in conjunction with the JK size-dependent breakage model can achieve similar characterisation results as the traditional narrow-size JKRBT method. Further validation is underway.



**Fig. 12. Ore competence indicator  $A_{xb}$  values determined by the Wide-size JKRBT tests using the JK size-dependent model in comparison with that determined by the traditional narrow-size JKRBT tests, the error bars representing  $\pm$  one SD (Zuo and Shi, 2016)**

Since the Wide-size JKRBT breakage characterisation method uses mixed particles in a wide size range, one of the potential applications is for online ore competence measurement. Despite the fact that the JKRBT can do rapid ore breakage characterisation, the preparation of narrowly sized particles for the standard JKRBT test is one of the major barriers preventing its implementation for online breakage characterisation.

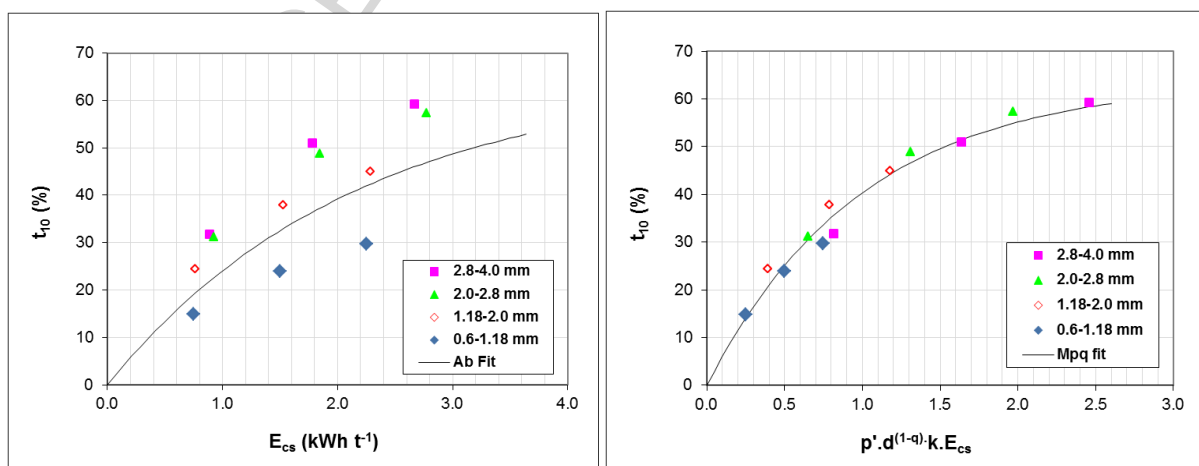
## 7. Fine particle breakage characterisation

The standard DWT and JKRBT tests use particles coarser than 13.2 mm. In secondary grinding ball mills or tertiary grinding stirred mills, the majority of the feed particles are smaller than 5 mm. It is questionable whether the breakage characteristic parameters derived from the DWT or JKRBT can represent the properties of the prevailing ore particles in ball mills or stirred mills. In particular the particle strength (indicated by the required energy) increases in a power law fashion with an exponent changing from 2 to 4 for particle size smaller than 1 mm (refer to Fig. 1). Because of the lack of a suitable breakage characterisation device to test the <math>-5\text{ mm}</math> material, the existing ball mill model developed by Whiten (1974) and implemented in the commercial software JKSimMet uses a default breakage appearance function.

A device developed at the JKMRRC for coal breakage characterisation (Shi and Zuo, 2014) has been used for fine ore particle breakage characterisation (Shi and Xie, 2015, 2016). The test rig is named the JKFBFC (JK Fine-particle Breakage Characteriser). The JKFBFC was modified from the standard Hardgrove Grinding Index (HGI) mill that is popularly used in the coal industry to measure coal grindability. A precision torquemeter was installed in the HGI mill to record energy utilisation during grinding. A computer interface system was employed to log the torque measurement data.

For each JKFBFC test, three to five narrowly sized particles in the 0.6-4.75 mm feed are tested, each with three grinding time periods. A constant volume (60 ml, approximately 40 to 50 g depending on solids density) of particles is used for each test. The ore sample is placed in a stationary grinding bowl, in which eight steel balls can run in a circular path. A ball race with a gravity load of 284 N is placed on top of the balls. The ground product is sieved to establish its size distribution and to determine the  $t_{10}$  values. The recorded mill grinding torque in each test is downloaded, and the input net specific energy is calculated.

The JK size-dependent breakage model was employed for the JKFCB data reduction. The energy threshold parameter,  $E_0$ , in Eq. (5) was set to zero compared with the grinding energy input levels. The total net energy (gross energy – non-load energy) recorded by the torque meter was used to calculate the specific energy as represented by the term  $KE_{cs}$  in Eq. (5). This is another application of the model, different to the aforementioned characterisations where impact breakage takes place. In the JKFCB test, size reduction is achieved mainly by means of compression bed grinding breakage. Fig. 13 shows an example of a copper ore ground in the JKFCB (Shi and Xie, 2015). The particle size effect is very strong, with larger particles achieving significantly higher  $t_{10}$  values at the same specific energy input (Fig. 13a). The size-averaged characteristic parameters (Eq. (2)) would give a large bias in the JKFCB application. Fig. 13b shows that all data from the four tested sizes fall on the same trend line predicted by the JK size-dependent breakage model, which confirms that the JK size-dependent breakage model can well represent the compression bed breakage data, in addition to the single impact breakage data in the aforementioned sections.



a. JK breakage model (Eq. (2))

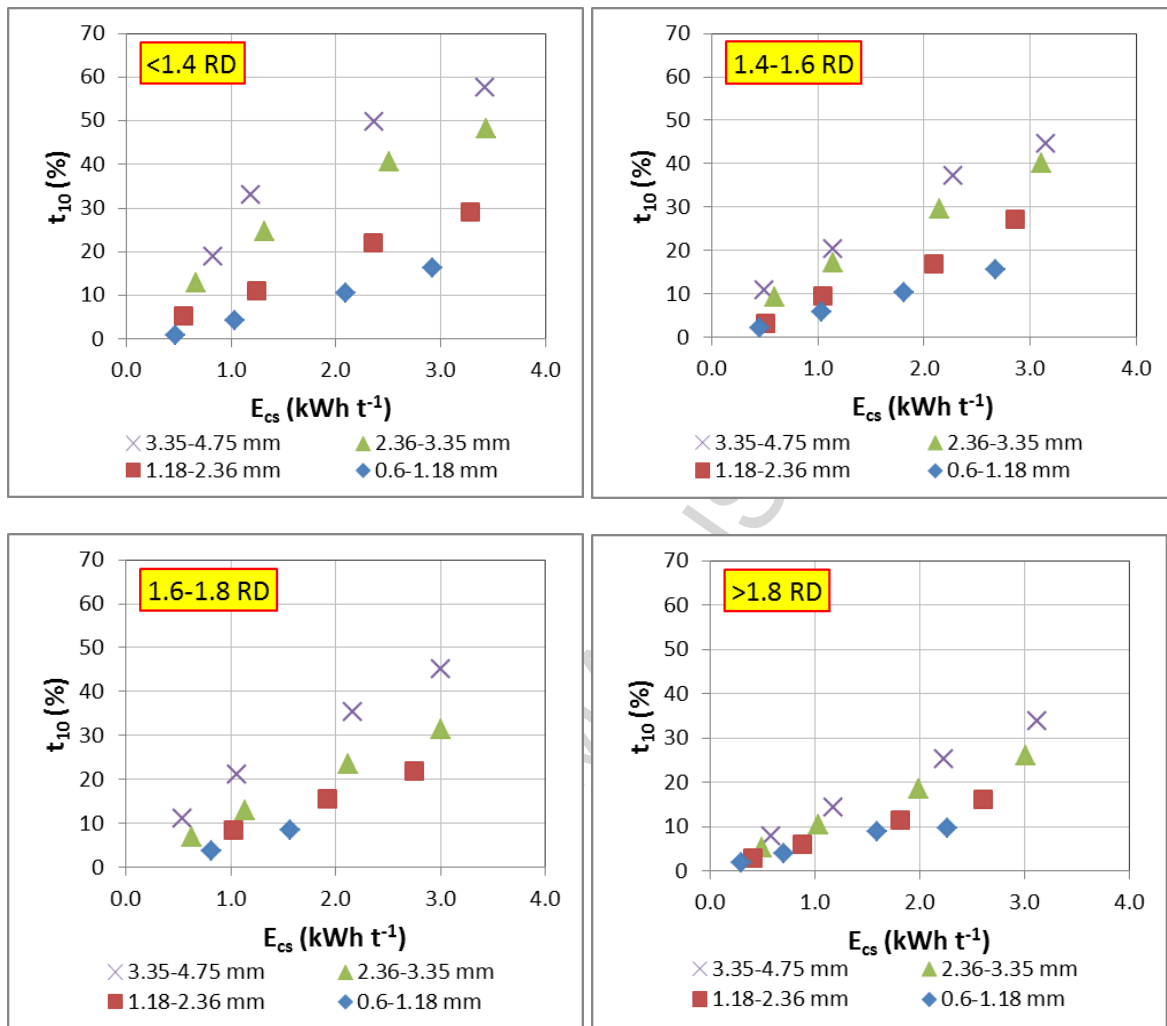
b. JK size-dependent breakage model (Eq. (5a))

**Fig. 13. Comparison of two breakage models fitted to the JKFCB test data for a copper ore sample (Shi and Xie, 2015)**

A number of JKFCB tests on various ore samples have now been conducted. The JK size-dependent breakage model was applied to characterise their breakage parameters. In all cases, the model fits the data well. The JKFCB and the JK size-dependent breakage model have been employed as a new breakage characterisation method to supply the fine ore characteristic parameters for a specific energy-based ball mill model, which will be presented in Part 3 of this review.

## 8. Multi-component characterisation for coal grinding

In order to study the influence of particle size and density on size reduction, an experiment was conducted using the JKFCB mill to grind two coal samples collected from an Australian power station and a Chinese power station. The Australian coal has an HGI of 52, and the Chinese coal has HGI of 80, which is softer than the Australian coal sample. The coal samples were sized and then fractionated by float-sink testing. The Australian coal sample was collected by the JKMRRC, and the Chinese coal sample was collected and processed by JKMRRC's research partner, China University of Mining and Technology, before transported to JKMRRC for the JKFCB tests. The JKFCB tests were performed based on particle density and size. About 60 tests (nominally 4 RDs x 4 sizes x 4 energies) for each coal were undertaken. The breakage index  $t_{10}$  was determined from the product size distribution. The  $t_{10}$  values of the Australian coal were plotted against specific energy for the four feed sizes in each density group (Fig. 14). The Chinese coal exhibited similar trends. It is obvious that larger particles produce a higher  $t_{10}$  when subjected to the same specific energy. This size effect is similar to the trend observed in impact breakage of ores. The size effect is more pronounced in the low density group. As density increases, the size effect becomes less pronounced.



**Fig. 14. Particle size effect on coal breakage for the four density groups, using the Australian coal sample ground in the JKFCB (Shi and Zuo, 2014)**

Fig. 15 demonstrates the effect of particle density on coal grinding. High density material appears harder to grind at the same specific energy level. The trend is much more pronounced in the large particle size fractions than in the smaller size fractions. The density effect decreases rapidly as the particle size decreases. For the smallest size fraction of 0.6-1.18 mm tested, the density effect disappears completely, with all four density groups falling on the same trend line.

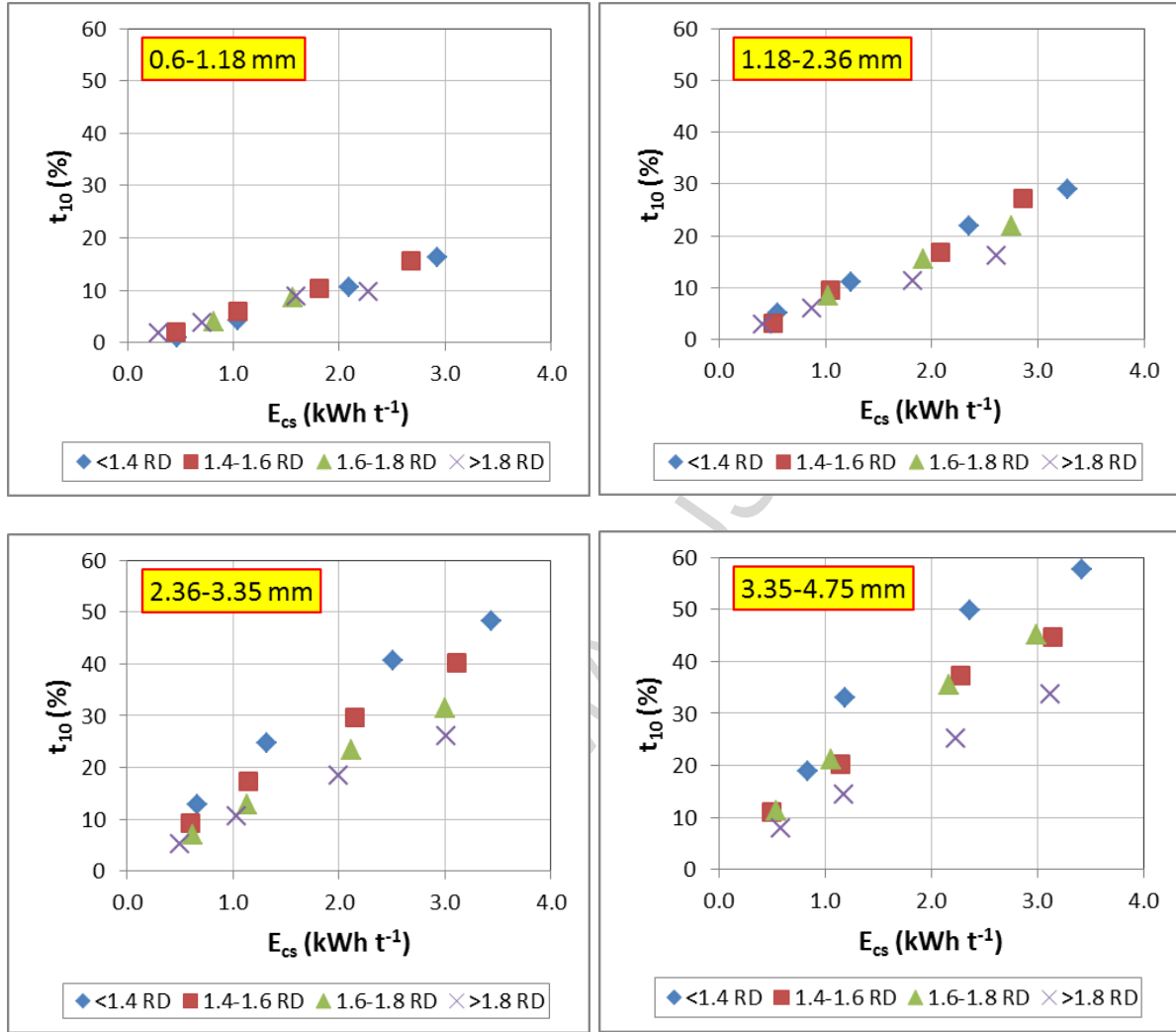


Fig. 15. Particle density effect on coal breakage, using the Australian coal sample ground in the JKFCB (Shi and Zuo, 2014)

The JK size-dependent breakage model was revised for the multi-component breakage data. The model takes the following form (Shi, 2014a):

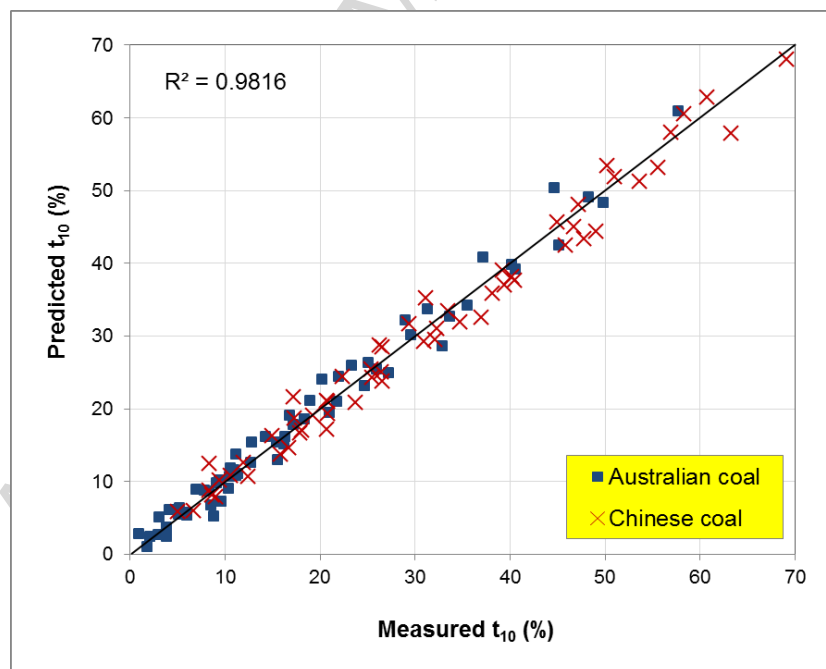
$$t_{10} = \frac{M}{(RD/RD_{min})^c} \cdot \{1 - \exp[-3600p \cdot x^{(1-q)} \cdot kE_{cs}]\} \quad (8)$$

where  $t_{10}$ ,  $M$ ,  $p$ ,  $q$  and  $d$  have been defined in Eqs. (3), (4) and (5),  $RD$  is the relative density of the particle,  $RD_{min}$  is the minimum relative density of the sample ( $RD_{min} = 1.25$  for coal),  $c$  is a parameter determining the trend line position, and  $kE_{cs}$  ( $\text{kWh t}^{-1}$ ) is the total net mass-specific energy input in the JKFCB. When parameter  $c = 0$ , the denominator in Eq. (8)



equals to unity, and the multi-component model reduces to the JK single component size-dependent breakage model.

The model incorporates four parameters,  $M$ ,  $c$ ,  $p$  and  $q$ , which can be calibrated by fitting the model to the measured data. Fig. 16 shows the fitting results for both the Australian coal and the Chinese coal. There are 120 data points in the plot, with only four model parameters fitted for each coal. Despite the very wide range of changes in the  $t_{10}$  data (from 0.9% to 69.1%), the multi-component breakage model can well describe the 3D breakage results, ie. the size reduction in relation to the effects of particle size, coal density and energy input.



**Fig. 16. The multi-component breakage model fitted to the JKFBC data of the Australian and Chinese coal samples (Shi, 2014a)**

In analysing the correlation between the fitted JK size-dependent breakage model parameters and the HGI values of the two coal samples, Shi (2014b) realised that the calibrated JK size-dependent breakage model can be used to predict coal grindability. Details of this work will be presented in Part 2 of this review paper.

## 9. Conclusion

The JK size-dependent breakage model (Shi and Kojovic, 2007) has found wide applications for ore and coal breakage characterisation since it was developed. These applications include high energy single impact characterisation, low energy incremental breakage modelling, breakage characterisation using a reduced number of breakage tests, using mixed particles in a wide size range, fine particle breakage characterisation by JKFBBC, and multi-component breakage model for coal. With access to this model, researchers and engineers working in the mineral and coal industries are now in a better position to conduct accurate ore and coal breakage characterisations than 10 years ago.

## Acknowledgement

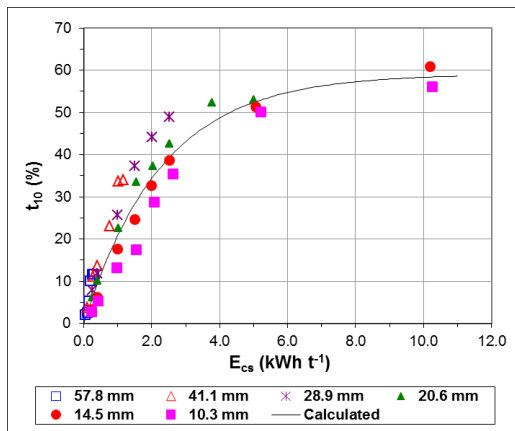
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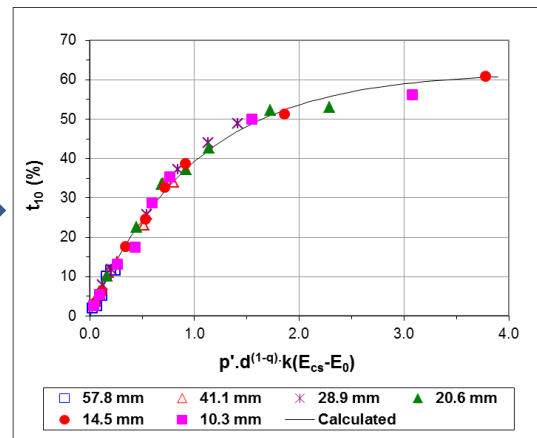
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## Graphical abstract – Part 1



JK breakage model



JK size-dependent breakage model

### Part 1 Research Highlights

- Structure and mathematical description of the JK size-dependent breakage model
- Applications of this model in the past 10 years presented in three parts review
- Part 1 for ore and coal breakage characterisation with single and incremental impacts
- New breakage testing methods developed based on the size-dependent model
- A multi-component breakage model (energy-size-density) developed for coal grinding