# TENSION ESTIMATES IN CABLE STAYED BRIDGES

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## ABSTRACT

A benchmark problem on an existing cable-stayed bridge was recently proposed. Recorded signals are available for standard working conditions and for special events (typhoons!). In this contribution, the authors report their attempt to detect significant variations in the cable tension during these extreme events.

## KEYWORDS

Cable-stayed bridge, cable tension, catastrophic events, structural health monitoring, typhoon.

## INTRODUCTION

The presence of complex boundary conditions usually imposes difficulties in estimating the cable forces in cable-stayed bridges when using conventional model-based force identification methodologies (Yan *et al* 2015). Multiple models have been exploited to set practical approximate formulations and empirical explicit expressions in order to estimate forces in cables, for instance Ren *et al*. (2005), Nam and Nghia (2011) and Fang and Wang (2012), among the others. The aforementioned model-based methods are applicable to cables with pure pinned or clamped end conditions. However, for existing in-service bridges, the cables are generally protected by steel tubes close to the deck and pylon anchorage zones.

The structural modal properties, for instance frequencies, damping ratios, and mode shapes, are employed for dynamic analysis, for estimating the tension in cables, for detecting the structural damage, and so forth.

The modal identification of the bridge and cable tension estimation are usually achieved by the usage of the ambient acceleration data (Yun *et al*. 2014). Moreover, when typhoon condition are considered, significant variations in the stay-cables can be observed.

A benchmark problem on an existing cable-stayed bridge was recently proposed and it focused on the modal identification (Ni *et al*. 2015; Casciati *et al*. 2015). Particularly, during a typhoon, some frequencies that during service conditions are not detected, were excited. Here-hence, the authors want to verify whether the cables tension varies during these particular events.

Furthermore, recorded signals in different working conditions, both standard and special events (typhoons) are available. In this work, the authors report their attempt to detect significant variations in the cable tension during these extreme events, by updating and calibrating properly a finite element model developed in the MSC® Marc Mentat environment (MSC 2014).

## THE TING KAU BRIDGE

The Ting Kau Bridge (TKB) is a three-towers cable-stayed bridge situated in Hong Kong., whose total length is 1177m. In Figure 1, an ambient framework of the bridge is shown. The bridge spans from the Tuen Mun Road to the Tsing Yi Island (Bergermann and Schlaich 1996). It is composed by four spans, of which two are considered as main, while the remaining two are the side-ones. So, the two main spans are 448m and 475m long respectively, whereas the two side-spans are both 127m long. The general layout of the bridge is illustrated in Figure 2.

Figure 1 Location in aerial view

The bridge is divided into two carriageways of width 18.8m, and there are three slender single-leg towers between them with 170m, 198m, 158m height, respectively. Every 4.5m there is the presence of two steel girders along the edges of the deck with steel crossgirders, and a concrete slab on the top of each carriageway. Moreover, a 5.2m gap is present in each carriageway, and it is linked to the others by connecting crossgirders every 13.5m. The deck is supported by 384 stay cables in four cable planes.

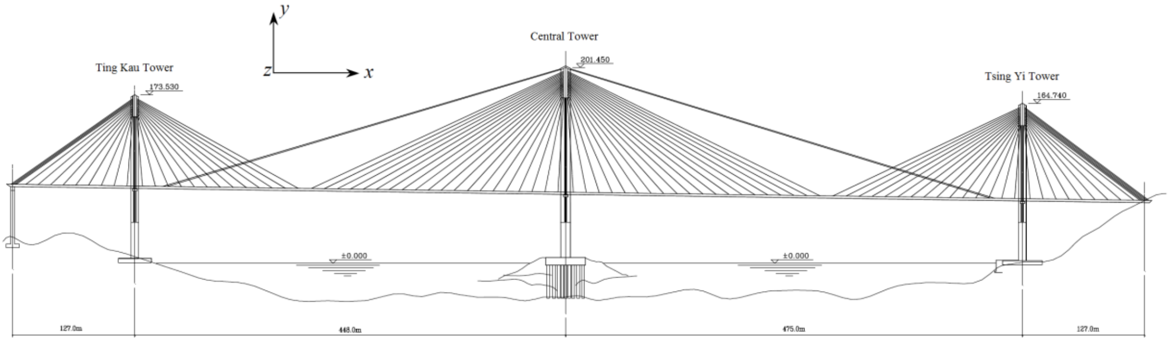

Figure 2 General layout of the Ting Kau Bridge

A sole characteristic of the bridge consists in the arrangement of the three single-leg towers, strengthened by longitudinal and transverse cables, whose function is stabilizing.

Then, 8 longitudinal stabilizing cables used to diagonally connect the top of the central tower to the Ting Kau and Tsing Yi towers, whose length reaches 465m, are installed. Whereas, 64 cables are employed to strengthen the three towers in the lateral direction (Ni *et al*. 2015).

Within a long-term structural health monitoring system conceived by the Hong Kong SAR Government Highways Department, during the construction of the bridge and also after its completion in 1999 (Wong 2004; Ko and Ni 2005), more than 230 sensors have been placed on the TKB.

On the bridge, accelerometers, anemometers, strain gauges, temperature sensors, GPS, and weigh-in-motion sensors are deployed (Wong 2007; Ni *et al*. 2011). A total of 24 uniaxial, 20 biaxial, 1 triaxial accelerometers are installed on the deck of the two main spans and two-side ones, the longitudinal stabilizing cables, the top of the three towers, and the base of the central tower. They form a total of 67 accelerometers and they monitor the dynamic response of the bridge.

In this work, only the accelerometers placed on the deck are considered. Within the benchmark problem, such number was lowered from 24 to 16 sensors, but maintaining the collection of the same modal information. Figure 3 and Figure 4 illustrate the old and the new sensors deployment (Casciati *et al*. 2015).

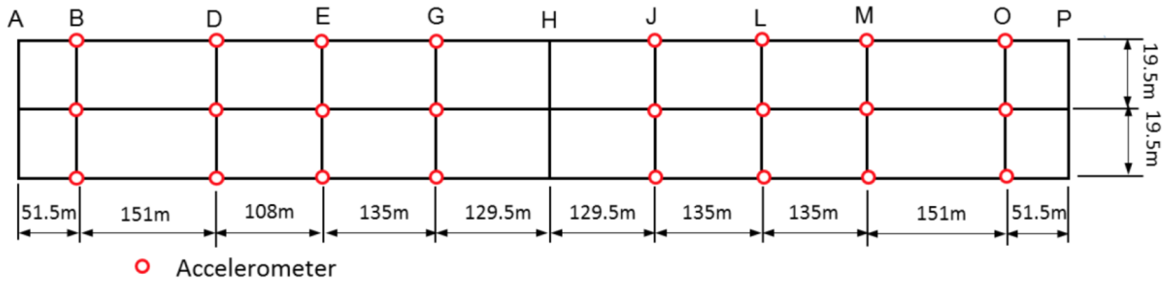

Figure 3 Actual deployment of accelerometers at the bridge deck

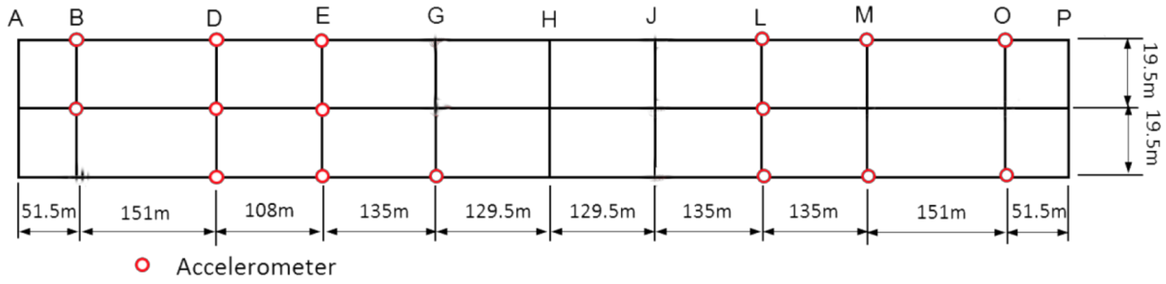

Figure 4 New proposed deployment of accelerometers at the bridge deck

In each section from *A* to *P* in Figure 3 and Figure 4, two accelerometers are installed on the east and west side of the longitudinal steel girders. They measure the vertical acceleration, while a further accelerometer is installed on the central crossgirder and measures the transverse acceleration. The set sampling frequency is equal to 25.6Hz.

## GOVERNING RELATIONS

Let consider an inclined cable under tension force *T* (Figure 5). First, a coordinates system is defined: the *x*-axis along the cable chord and the *y*-axis along the perpendicular direction (Nam e Nghia 2011). The cable has a mass per unit length denoted as *m*, a chord of length *l*, a finite bending stiffness *EJ* and it is inclined at an angle $\alpha$ to the horizontal ( $0 \le \alpha \le \pi/2$ ).

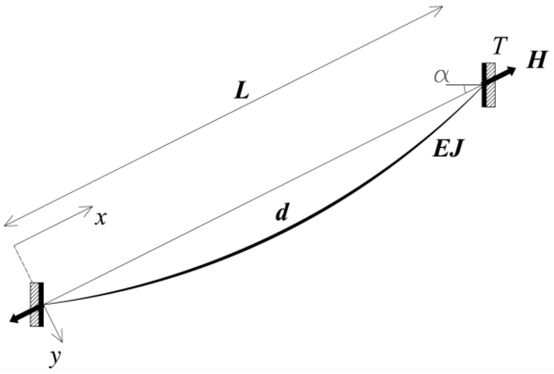

Figure 5 A model of an inclined cable

The dynamic equation of the cable in plane motion $v(x,t)$, in *y*-direction, is expressed as (Zui *et al*. 1996; Fujino and Hoang 2008):

$$H\frac{\partial^2 v}{\partial x^2}-m\frac{\partial^2 v}{\partial t^2}+h\frac{\partial^2 v}{\partial x^2}-EJ\frac{\partial^4 v}{\partial x^4}=0 \qquad (1)$$

where $H = T_h/\cos\alpha$ denotes the chord tension, and $T_h$ is the horizontal component of the cable tension, *h* is the additional tension in the cable caused by the motion. Eq. (1) assumes that the cable tension *T* is sufficiently large so the static profile of the cable can be described by the parabola (Irvine 1981):

$$y = 4d\frac{x}{l}\left(1 - \frac{x}{l}\right) \tag{2}$$

where $d = mgl^2 \cos\alpha / 8H$ is the sag at mid-span.

Eq. (1) corresponds to the most general case of a cable where both the sag and the flexure in the cable are considered.

## ESTIMATION OF CABLE TENSION

The cable tension can be estimated by its relation with the wave number. By considering proper simplifying assumptions of a small flexural rigidity parameter, the cable tension can be estimated in a simpler way.

Consider the flexural rigidity parameter $\varepsilon$ (Hoang and Fujino 2007) and the wave number $\beta$ :

$$\varepsilon = \frac{EJ}{Hl^2} \tag{3}$$

$$\beta = \omega\sqrt{m/H} \tag{4}$$

Some characteristic equations can be derived for anti-symmetric case vibration node, where the sag will generate no additional cable tension:

$$\tan\left(\frac{1}{2}\beta_{0n} l\right) = \beta_{\varepsilon n}, \qquad n = 2, 4, \ldots \tag{5}$$

$$\tan\left(\frac{1}{2}\beta_{0n} l\right) = \frac{\beta_{\lambda n}}{1 - \beta_{\varepsilon n}\beta_{\lambda n}}, \qquad n = 1, 3, \ldots \tag{6}$$

where $\beta_{0n}$ is the wave number of an individual node $n$, $\beta_{\varepsilon n} = \sqrt{\varepsilon}\beta_{0n} l$ and $\beta_{\lambda n} = 1/2\,\beta_{0n} l - \left(4/\lambda^2\right)\left(1/2\,\beta_{0n} l\right)^3$.

Eqs. (5) and (6) are logical extensions of the wave number of odd and even vibration mode of a sag cable proposed by Irvine and Caughey (1974), considering the flexural rigidity of the cable.

## THE PROBLEM STATEMENT

### *The Finite Element Model*

In this work, the bridge is modeled by the MSC® Marc Mentat software. The pre-processor of the software is very powerful and a sophisticated model can be built in order to investigate some issues.

The deck is discretized as a series of beam elements. The deck slab is made in reinforced concrete, while the cross beams and the cross girders are constituted by steel. For this reason, a particular attention is dedicated to the difference between the reinforced concrete elements and the steel ones, in order to preserve the real behavior of the structure. The towers, whose height changes from one to another one, are modeled by beam-type elements. The assigned material is also here reinforced concrete. Each tower is fixed. Finally, the 384 cables are modeled as beam-type elements where the shear forces and the bending moment are suppressed in order to simulate the behavior of a truss cable. The material of each cable is steel.

The realized model is composed by 2295 nodes and 4638 elements. Figure 6 illustrates the finite element model built in the Marc Mentat environment.

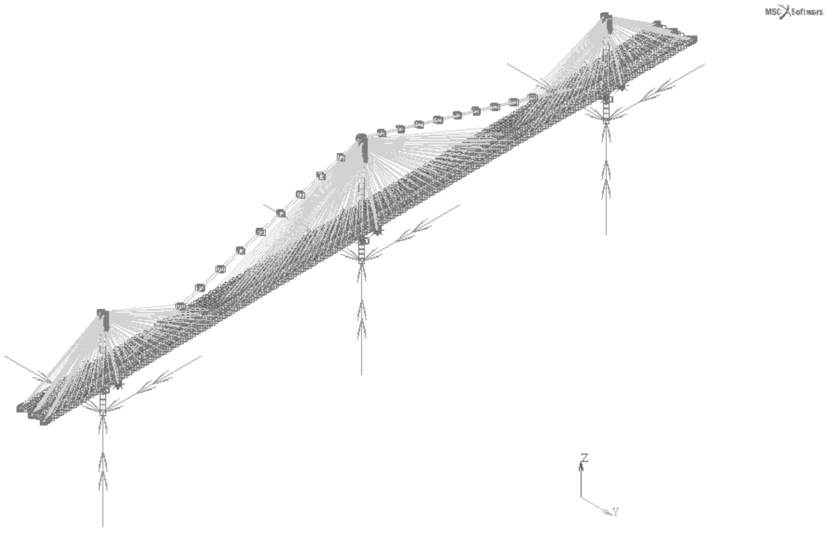

Figure 6 Finite element model of the bridge implemented in MSC® Marc Mentat software

***Analysis Results***

The results of the analysis reported in this section aim to detect significant variations in cable tension during typhoon event.

First, the model has been tested only by a simply gravity load applied on the each element. Hence, the modal features of the bridge are properly detected. Then, a further step is to test the tension in the cables in order to verify if there are some discrepancies when a typhoon occurs.

For each cable, a proper tension were provided and the finite element model is calibrated for replaying the behavior of the bridge in its current conditions.

Indeed, the attempt is focused on the detection of significant variations in the cable tension during an extreme event, such a typhoon.

For sake of exemplification, the first ten eigenvalues recorded during a typhoon event (named Maggie) are reported in Table 1.

Table 1 Identified modal frequencies of the first ten modes under typhoon condition

| Mode No. | Frequency (Hz) |
|---|---|
| 1 | 0.165 |
| 2 | 0.227 |
| 3 | 0.262 |
| 4 | 0.290 |
| 5 | 0.301 |
| 6 | 0.322 |
| 7 | 0.361 |
| 8 | 0.372 |
| 9 | 0.385 |
| 10 | 0.395 |

After the calibration of the finite element model, the frequencies are reported in Table 2. The way toward a cable tension variation estimation from these results is currently in progress.

Table 2 Identified modal frequencies of the first ten modes in MSC Marc Mentat environment

| Mode No. | Frequency (Hz) |
|---|---|
| 1 | 0.105 |
| 2 | 0.121 |
| 3 | 0.168 |
| 4 | 0.169 |
| 5 | 0.175 |
| 6 | 0.258 |
| 7 | 0.264 |
| 8 | 0.279 |
| 9 | 0.291 |
| 10 | 0.345 |

## CONCLUSIONS

A benchmark problem on an existing cable-stayed bridge was recently proposed and it focused on the modal identification. Recorded signals for standard conditions and special events, for instance typhoons, are available. In this work, the authors verify that the cables tension is the same during these particular events and the modal features (e.g., frequencies and mode shapes) are detected.

Furthermore, the authors report their attempt to detect significant variations in the cable tension during these extreme events, by updating and calibrating properly a finite element model developed in the MSC® Marc Mentat environment.

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