STRUCTURAL OPTIMIZATION WITH UNCERTAINTY AND ITS RELATION TO PERFORMANCE BASED DESIGN

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ABSTRACT

Structural optimization plays certain role from concept development, numerical algorithm to practical solution in the performance and life-cycle based structural engineering. This presentation briefly reviews the history of structural optimization and its application in civil engineering. Structural topology optimization and surrogate model-based optimization approach together with metaheuristic algorithms is discussed in more detail. The relation of structural optimization with performance based and life-cycle based structural design is illustrated through some of our research work on reliability-based design optimization and damage-reduction optimum deign of structural system. These works provide some optimization methodology, design concept and numerical algorithms, which may facilitate the performance based and life-cycle based structural engineering.

KEYWORDS

Structural optimization with uncertainty, performance-based design, topology optimization, reliability-based design optimization, damage-reduction optimum deign.

INTRODUCTION

History of structural optimization can be traced back to the early work by Michell (1904). In 1950-1960s, a surge of study on structural optimization was observed in the literatures. Optimum designs of elastic truss, frame, grillage and structural element such as beam, column and plate attracted a great attention in structural engineering community and were solved by analytic approach and variational method (Rozvany and Prager, 1979; Rozvany 1989). It is interesting to note that many well-known researchers of structural optimization such as Prager and Rozvany have the background of civil engineering.

Since 1960’s the advent and fast development of modern computer together with the finite element method and mathematical programming pushed the numerical method of structural optimization to the central stage of modern structural optimization. In late 1970’s and 1980’s there were two major numerical approaches: both Optimality Criterion approach and Mathematical Programming were the hot research topics. The approximate concept (Schmidt 1976) and sequential approximate programming such as SLP (Sequential Linear Programming), SQP (Sequential Quadratic Programming), SCP (Sequential Convex Programming) (Fleury

1986) and MMA(Method of Moving Asymptotic) (Svanberg 1987) were developed. As these methods are gradient-based optimization algorithm, an efficient sensitivity analysis is critical (Arora 2004). In FEA (Finite Element Analysis) context, the DSA (Direct Sensitivity Analysis), Adjoint Sensitivity Analysis and SAM (Semi-Analytic sensitivity Method) (Cheng and Liu 1987; Pedersen et al. 1989) were developed for structural size and shape optimization. Soon after, the specific software was developed for the numerical solution of large scale structural optimization problems. Application of structural optimization becomes more frequent in areas such as aeronautic and aerospace industries. Serious attempts, including teaching structural optimization in classroom and introducing students the new technology, were also made to expand its application in civil engineering (Hernandez et al. 2015).

In theoretical aspect, several progresses were seen in 1980's. Study on the bimodal optimum design of clamped column (Olhoff 1977) opened the way of the non-differential sensitivity analysis of multiple eigenvalues. It was later found that many optimum structures have multiple buckling loads or vibration frequencies. Optimum design of thin solid elastic plate for minimum compliance (Cheng and Olhoff 1981) pointed out the need of including microstructure in the optimization formulation, which was recognized as pioneering work of the modern topology optimization.

Nevertheless, a widespread application of structural optimization remains quite challenging in civil engineering. There are many reasons. First of all, design problem in civil engineering usually involves a very large number of constraints and design variables. Many design constraints are from the design code provision and are mostly not given in the form of algebraic or differential equations. The values of design variables are often discrete and limited within the list given by industrial standard. Structural weight is only a part of design objective, sometimes less important than the cost of construction, manufacture and attached facility in the building. The mathematical formulation of structural optimization in civil engineering is much more complicated than that in the other engineering field. Moreover, it is often difficult to isolate one structural part from the whole structural system. Voices from civil engineering community continuously press the structural optimization community to tackle real engineering problems instead of simple academic examples. New benchmark optimization problems were proposed and discussed in recent literatures (Alimoradi et al. 2010, Mueller et al. 2012).). On the other hand, there were already a number of well-developed softwares in civil engineering providing optimization function for specific type of structures. For example, the member size optimization of tall buildings is a well-established field of application for large scale optimization algorithms. In such cases where the initial cost of construction is high, the economic advantage of optimal engineering design is obvious. Research in this area has been vigorously explored since the early 1990's.

In recent years, two important developments in structural optimization open a new perspective to its application in the area of civil engineering. One is the fast development of structural topology optimization, which will be elaborated in the next section. Another is the surrogate model based optimization approach together with the emerging meta-heuristic algorithms, made possible by ever increasing computational power available in daily design activities.

The surrogate model based optimization approach relies on the results of physical experiment or expensive numerical simulation for the given structural system at a number of sample points. With a well-designed DOE (Design Of Experiment), such data can help construct approximate mapping between the interesting input parameters and the structural response output, i.e. a surrogate model. Then numerical optimization techniques are applied to find the optimum designs of the surrogate model, which is rough approximation of optimum design of the original optimization problem. For this approach to help practical engineering design, it has following general requirements:

Designer has reliable software as a black box for the problem analysis

Designer manages to formulate the optimization problem, that is, to identify a reasonable set of critical design variables and important structural response to be constrained or included in objective function. This can be obtained from analysis software or the user self-coded algorithm.

The emerging meta-heuristic algorithms (Kaveh and Talatahari2009; Saka and Geem 2013), such as GA (Genetic Algorithm), PSO (Particle Swarm Optimization), colony optimization, chaos optimization algorithm (Yang et al. 2007; Yang et al. 2014) and tabu search etc., are strong complementary methods of gradient-based optimization. They have provided alternative tools to engineering designers, in particular for design optimization problems under design code provisions and discrete design variables encountered in engineering practice. However, the efficiency of meta-heuristic algorithm remains to be further improved. It is often seen 5000 calls of structural analysis is carried out for an optimum solution by the meta-heuristic algorithm. It is interesting to remember that structural optimality criterion people strived for obtaining optimum design by 10 calls of structural analysis in 1980s.

CONTINUUM STRUCTURAL TOPOLOGY OPTIMIZATION AND BUILDABILITY

Classical structural topology optimization originates from the study of minimum weight design of truss structure under stress constraints (Michell 1904). The classical Michell truss theory was then further elaborated and a set of elegant analytic solutions was obtained under various boundary and external load conditions (Rozvany 1989). One important result is that under single static load case and symmetric stress constraints (i.e., the strength limit for tension and compression are the same), the corresponding optimal truss structure is statically determined. It is also found that if the parameters involved in the optimization problems being scaled appropriately, this optimal design also coincides with the optimal design for minimum compliance and the optimal design of rigid-plastic truss structures under available volume constraint. When this classical truss topology optimization under stress constraints and multiple load cases is solved by ground approach (Sved and Ginos 1968), however, it is surprising to discover that global optimal topology cannot be reached by continuous gradient-based optimization algorithms. This so-called singular optimum was now understood as caused by the possible discontinuity of the stress constraint function when topology change occurs (Cheng and Jiang 1992) and could be solved by relaxing the stress constraint in mathematically rational ways (Cheng and Guo 1997).

Modern continuum structural topology optimization was initiated by Bendsoe and Kikuchi in 1988. Since topology optimization provides an effective approach for finding optimum connection of continuum structure, topology optimization techniques are quickly becoming recognized as a powerful tool for conceptual design in various engineering areas. Several methods of structural topology optimization are available. Homogenization method (Bendsoe and Kikuchi 1988) assumes the structure to be optimized consists of material with microstructure of given configuration, in which microstructure parameters are chosen as the design variable. Meanwhile, size optimization techniques are applied to obtain optimum microstructural parameter, which determines material distribution and structural topology. The macrostructure effective coefficients corresponding to the material microstructure is obtained by mathematical asymptotic homogenization method. Many research work focuses on minimum structural compliance design under the given material volume constraints, which generates the most efficient force transmission path. Though this method results in structural topology in which the intermediate density can be related to porous material, the method is time consuming. SIMP (solid isotropic material with penalty) (Zhou and Rozvany 1991) is the most popular method to speed up and simplify this general approach. In SIMP, an artificial relation between material density and Young modulus is assumed instead, and a penalty is imposed on the artificial relation to push the element density either zero or one. Since the implementation of this approach is straightforward, the method is widely applied to multiphysical topology design problems. Other feasible methods include the Evolution method (Xie and Steven 1993) and the level set method (Wang et al. 2003).

Even with the great success of SIMP, numerical implementation of topology optimization has encountered many problems. Most common ones are treatment of grey elements, checkerboard design, and mesh-dependent solution in minimum structural compliance design. Appearance of hinge and disconnect design in optimum compliant mechanism design is also difficult. Similarly, singular optimum needs to be treated carefully in stress-constrained and frequency-constrained optimum topology design. Furthermore, many 3D topology optimization results are simply difficult to visualize, let alone manufacture. From the viewpoint of engineering application, these problems all are part of the manufacturability or buildability, critical to turn topology optimization results into reality application. A solution with many grey elements or checkerboard areas is difficult to use for extracting manufacturable design. Therefore, linear density filter and sensitivity filter were developed to reduce the checkerboard solution but led to higher number of grey element. The nonlinear Heaviside filter removes the grey element and yields black-white final design. Moreover, the parametric nonlinear Heaviside filter (Xu et al. 2010) further stabilizes the iteration history. To control the minimum and maximum length or special feature in the final topology design, the projection method and robust design approach was also studied. Recently, Guo and his co-workers (2014) proposed morphable component method that has great potential to control structural features and addresses constructability, extending further the level-set based concepts. Nevertheless, manufacturability is still a great challenge that is further complicated by the requirement of special manufacturing process or new manufacturing techniques.

In the field of civil engineering, topology optimization already finds good usage in conceptual high-rise building design by systematically exploring the structural design space for efficient, novel structural layouts (Bobby et al. 2015). The lateral bracing systems are under study for its optimum topology (Stromberg et al. 2012). Manufacturing constraints such as pattern gradation and repetition are studied to facilitate the conceptual design for buildings, i.e. layout optimization (Stromberg 2011). There are also the growing interests in the architectural community to make topology optimization as a means of generating aesthetic and efficient structural forms (Beghini et al. 2014), as well as searching for design methodologies that facilitate weight reduction of concrete structures while maintaining the required load-carrying capacity. Overall, expansion of structural topology optimization is necessary for its wider application in civil engineering. For example, the existing structural topology optimization formulation mostly addresses the linear elastic and deterministic problem. However, topology optimization problem with consideration of uncertainty, structural non-linear and dynamic response needs further study.

RELIABILITY-BASED STRUCTURAL OPTIMIZATION AND SEQUENTIAL APPROXIMATE APPROACH

Traditional structural optimization commonly deals with deterministic optimization. However, there are many inherent uncertainties in the real-world environment of civil engineering structures. If uncertainties are neglected there is a possibility that the final optimum designs may perform unsatisfactorily. Commonly referred to as RBDO (reliability-based design optimization) in structural optimum design, it has several formulations to address inherent uncertainty (Cheng et al. 1998). When the reliability index is applied to describe the probabilistic constraint, the so-called RIA (reliability index approach) for RBDO is stated as:

$$\begin{aligned} &\min \quad \operatorname{cost}(\mathbf{X}) \\ &\text{s.t.} \quad \beta_j^{accept} \le \beta_j(\mathbf{X}) \quad \left(j = 1,\ldots,N_p\right) \\ &\qquad \mathbf{X}^L \le \mathbf{X} \le \mathbf{X}^U \end{aligned} \tag{1}$$

In the following, we take one formulation as an example to review these approaches. In general, the solution of this formulation needs the simultaneous solution of an optimization problem and calculation of the reliability index, which involves another optimization problem. Many solution approaches have been developed in the literature. In general, RBDO approaches are divided into three categories: double loop approaches, single loop approaches and decoupled approaches based on how the two optimization problems are handled. Methods based on decoupling the optimization loops for the reliability analysis allow the adoption of efficient deterministic optimization algorithms to be applied after results of the reliability analysis being available. This kind of decoupling method has received considerable discussion (Royset et al. 2001; Du and Chen 2004). Instead of introducing them in details, here we concentrate on the sequential approximate programming approach.

Sequential approximate programming (SAP) approach in structural optimization developed in 1980s' and solved the general nonlinear mathematical programming. For simplicity of discussion, we show the approach by optimization problem with only one inequality constraint,

$$\begin{aligned} &\text{To find } \mathbf{X} \\ &\min f(\mathbf{X}) \\ &\text{s.t.} \quad g(\mathbf{X}) \le 0 \end{aligned} \tag{2}$$

The sequential approximate programming solves the problem by solving a sequence of sub-optimization problems, that is,

$$\begin{aligned} &\text{for } k=1,2,\ldots \\ &\text{To find } \mathbf{X}^k \\ &\min f^k(\mathbf{X}) \\ &\text{s.t.} \quad g^k(\mathbf{X}) \le 0 \end{aligned} \tag{3}$$

The objective and constraint function in Eq. 3 is the approximation of their corresponding one in Eq. 2 and their approximation should be improved with the iterations. For example, $f^k(\mathbf{X})$ is chosen as the linear approximation in the vicinity of $\mathbf{X}^{k-1}$ obtained in the previous (k-1)th iteration. Quadratic approximation and diagonal quadratic approximation are examples. By introducing the carefully constructed transformation of design variable and objective or constraint function, many variant algorithms were developed in the literature, and some of them are very efficient. For example, the method of moving asymptotic (MMA) is mostly often applied in structural topology optimization.

The success of SAP drives us to propose a sequential approximate programming for RBDO, which solves a sequence of approximate programming as

$$\begin{aligned}
&\text{for } k = 1,2,\ldots \\
&\min \quad \mathrm{cost}^k(\mathbf{X}) \\
&\text{s.t.} \quad \beta_j^{accept} \le \beta_j^k(\mathbf{X}) \quad \left(j = 1,\ldots,N_p\right) \\
&\qquad \mathbf{X}^{\mathrm{L}} \le \mathbf{X}^{\mathrm{L}k} \le \mathbf{X} \le \mathbf{X}^{\mathrm{U}k} \le \mathbf{X}^{\mathrm{U}}
\end{aligned} \tag{4}$$

The question here is how to construct the approximate functions of the reliability index constraint function in RDBO. The commonly-used linear approximation requires reliability index value and its sensitivity at the (k-1)th iteration, which is difficult to obtain. Instead, we use approximate reliability index and its sensitivity at (k-1)th iteration to construct the approximate reliability index constraint function.

$$\beta^k(\mathbf{X}) \approx \hat{\beta}\left(\mathbf{X}^{k-1}\right) + \left(\nabla_{\mathbf{u}}\hat{\beta}\left(\mathbf{X}^{k-1}\right)\right)^T\left(\mathbf{X} - \mathbf{X}^{k-1}\right) \tag{5}$$

$$\hat{\beta}\left(\mathbf{X}^{k-1}\right) = \frac{g\left(\mathbf{X}^{k-1},\mathbf{u}^{k-1}\right) - \left(\nabla_{\mathbf{u}} g\left(\mathbf{X}^{k-1},\mathbf{u}^{k-1}\right)\right)^T \mathbf{u}^{k-1}}{\left\|\nabla_{\mathbf{u}} g\left(\mathbf{X}^{k-1},\mathbf{u}^{k-1}\right)\right\|} \tag{6}$$

where **u** is the most probable failure point (MPFP) of the limit state surface in u-space. The (k-1)th estimation of MPFP $\mathbf{u}^{k\text{-}1}$ for the design $\mathbf{X}^{k\text{-}1}$ is reserved and substituted into Eq. 6 to obtain a updated estimation $\mathbf{u}^k$ for the design $\mathbf{X}^k$, which will be used in the next iteration.

$$\mathbf{u}^k = -\hat{\beta}\left(\mathbf{X}^{k-1}\right)\frac{\nabla_{\mathbf{u}} g\left(\mathbf{X}^{k-1},\mathbf{u}^{k-1}\right)}{\left\|\nabla_{\mathbf{u}} g\left(\mathbf{X}^{k-1},\mathbf{u}^{k-1}\right)\right\|} \tag{7}$$

$$\nabla_{\mathbf{u}}\hat{\beta}\left(\mathbf{X}^{k-1}\right) = \frac{\nabla_{\mathbf{u}} g\left(\mathbf{X}^{k-1},\mathbf{u}^{k-1}\right)}{\left\|\nabla_{\mathbf{u}} g\left(\mathbf{X}^{k-1},\mathbf{u}^{k-1}\right)\right\|} \tag{8}$$

Due to the space limitation, more detailed description is referred to (Cheng et al. 2006; Yi and Cheng 2008). The above method can be applied to performance measure approach (PMA), in which the performance measure is applied to describe the probabilistic constraint. From numerical tests of several structural examples, its efficiency were reported in literatures such as Aoues and Chateauneuf (2010). The basic idea of SAP is also applied to solve size and topology optimization with reliability constraints based on probablity and multi-ellipsoid convex model hybrid model (Kang and Luo 2010). to minimize the economic cost under many ncertainty factors in an efficient manner. Here, the multi-ellipsoid convex modeling technique is developed as a powerful tool to cope with bounded uncertainty arising from different sources (Kang and Luo 2009). When the random parameters are known for engineering system, the reliability-based design optimization based on classic probability theory performs well for this type of problem (Liu et al. 2014). The performance of these algorithms, including accuracy, efficiency and robustness, are the key of the RBDO. It is worthy to note that Yang and his colleagues (Yi and Yang 2009; Yang and Xiao 2013) investigated the essential reasons of iterative failure of some widely used algorithms in reliability analysis and design optimization and so on, such as FORM (first order reliability method), PMA, SAP with PMA based probabilistic optimization, SORA (sequential optimization and reliability assessment) based on the theory of nonlinear dynamics, and discovered the chaotic dynamics mechanism of period oscillation, bifurcation and chaos of iterative solutions. Further, they suggested the stability transformation method (STM) of convergence control for these iterative algorithms from the new perspective of chaotic control. To enhance the efficiency and performance of RBDO with STM for convergence control, Li and Meng *et al*. developed the modified STM through relaxing the iterative step size of radial direction and adaptively determining the control factor of STM with respect to PMA for probabilistic constraint estimation in RBDO (Li et al. 2015).

PERFORMANCE BASED STRUCTURAL DESIGN AND DAMAGE-REDUCTION DESIGN

In later 1990's, I learned the concept of performance-based seismic engineering and its importance to the future engineering design. Performance-based engineering (PBE) is a general methodology that allows designers to conceive and assess the performance of complex structural systems subject to various hazards by rigorously taking into account the pertinent uncertainties (Cornell et al. 2002; Bozorgnia and Bertero 2004; Fischinger 2014). With the PBE approach, the designer can define the performance objective for the structural system during its desired/expected design life, and take advantage of specific criteria and methods for verifying that the agreed performance objectives are met. To realize performance-based structural design the optimization methodology is one of the most important ingredients from the concept formulation of PBE to numerical algorithm. Many works have been accomplished in performance-based earthquake engineering over the past two decades. The related processes have been established that facilitate probabilistic seismic hazard analysis, evaluation of relevant engineering demand parameters through advanced modeling and nonlinear response history analysis, quantification of damage measures and associated repair/replacement costs at the component level, and aggregation of losses for structural and nonstructural systems. The outcome of performance-based earthquake engineering is a probabilistic assessment of direct economic loss and collapse failure due to earthquake action.

There are two different formulations for optimization considering the balance of cost and benefit. One formulation is written as (Cheng and Li 2000)

$$\begin{aligned}
&\text{Find } \mathbf{X} \\
&\min\ W(\mathbf{X}) = C(\mathbf{X}) + P_{f_s}(\mathbf{X})C_{f_s} \\
&\text{s.t. } P_{f_s}(\mathbf{X}) \le \left[P_{f_s}\right] \\
&\quad g_j(\mathbf{X}) \le 0,\ j=1,2,...,m
\end{aligned} \tag{9}$$

System reliability in this formulation introduces additional computational complexity to RBDO, especially when the limit-states have statistical dependence. To apply this formulation, we need to know the system reliability and the system failure cost, which are difficult to obtain. Many researchers (Royset et al. 2001; Liang et al. 2007) made important contribution for the numerical algorithms of RBDO with system reliability. Furthermore, minimizing the system reliability often leads to simultaneous failure design, i.e. all failure modes have the same failure probability, or, a uniform strength design. However, in a system life cycle, various hazard and various limit state caused different damage and different economic loss. Therefore, in many cases, it is more rational to solve the following optimization formulation,

$$\begin{aligned}
&\text{Find } \mathbf{X} \\
&\min\ W(\mathbf{X}) = C(\mathbf{X}) + \sum_{i=1}^{n_p} P_{f_i}(\mathbf{X})C_{f_i} \\
&\text{s.t. } P_{f_i}(\mathbf{X}) \le \left[P_{f_i}\right],\quad i = 1,2,...,n_p \\
&\quad g_j(\mathbf{X}) \le 0,\ j = 1,2,...,m
\end{aligned} \tag{10}$$

It is interesting to note that under this formulation, the optimum design of Eq. 10 has different failure probability for different failure modes. For low failure cost component or limit state, we could allow high failure probability. This observation leads to the concept of damage-reduction-based seismic design, or fuse design of structural system. Here, the structural system is either physically or functionally designed as two parts, the main-function part and the damage reduction part (Li and Cheng 1998). The main-function part satisfies the serviceability requirements of the structural system. The damage-reduction part composes of several damage reduction elements, which work under hazard loads to ensure the safety of the main-function part, and further maintain the serviceability of the structural system by specific damage-reduction techniques or even by failure of damage-reduction elements. The formulation of damage-reduction optimum design for seismic high-rise structures is presented in (Li and Cheng 2003), in which damage-reduction design examples of RC frames are examined. The optimal design results show that several measures of structural seismic performance, including the life-cycle cost, severe earthquake action and the story-drift reliability index of the weakest story, can be improved by damage-reduction design compared with conventional design. Finally, it is pointed out that, the idea of damage-reduction structure with sacrificing the secondary damage-reduction part or components such as energy dissipated damper, supporting or link beam is fairly similar to that of earthquake resilient structure (Fischinger 2014). Optimum design of resilient structure is a promising direction to performance based structural engineering.

FINAL REMARKS

Performance based and life-cycle based structural engineering covers a broad area of research activities and technical advancement. Structural optimization provides theory and algorithm from concept development, numerical algorithm to practical solution in PLSE. Abundant publications on these topics are available, and author is sorry for being not able to cover many other important works herein.

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