PROBABILITY-BASED UNCERTAINTY EVALUATION THROUGH MARKOV CHAIN MONTE CARLO SAMPLING AND RESPONSE SURFACE TECHNOLOGIES

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ABSTRACT

Inspection of a pre-stressed concrete variable cross-section box girder bridge discovered the phenomenon of padding in expansion joint, corrosion of steel plate and local edge failure in pot type rubber bearing, and cracks of box girder. They are the main sources of structural uncertainty for structural performance evaluation, and how to quantificationally evaluate their influences on bridge performance is important. In this article, an approach using the Markov Chain Monte Carlo sampling technology and the Response Surface Method is proposed to deal with the uncertainty problem. First, a population of finite element (FE) models will be established by sampling the main uncertainty sources through the Markov Chain Monte Carlo technology. Then, the posterior probability of each FE model will be evaluated by using the measured static responses and identified structural dynamic characteristics. Especially, the second order response surface method will be used in this step to improve the computation efficiency. Through the above procedures, probability features of the defined key parameters representing structural uncertainty, including the stiffness of expansion joint, the stiffness of pot type rubber bearing and the elasticity modulus of the box girder will be estimated, which will provide valuable information for reliable structural performance evaluation.

KEYWORDS

Multiple model; MCMC sampling; response surface method; uncertainty.

INTRODUCTION

The performance of civil engineering structures under operational and environmental conditions would decrease over time and especially the degeneration of structural critical areas might lead to monolithic catastrophic failure, so the bridge structures need periodical inspection and structure health monitoring (Jang et al 2013; Gheitasi and Harris 2015).Vibration-based test is an important tool for structure health monitoring, and modal parameter identification based on vibration data have been developed for a long time. Although identified structural modal parameters reveal structural dynamic characteristics, they cannot directly support structure performance evaluation and subsequent decision making about structural maintenance and management.

Finite element modeling and updating using the monitoring results has been widely investigated. Initial finite element model generally cannot well fit field test data. Utilizing monitoring data to update the finite element model is an effective way to produce a better FE model for further structural prediction. Ching and Beck (2004) proceeded finite element model updating through Bayesian theory combining structural monitoring data. Chakraborty and Sen (2014) updated the FE model using the response surface method. Papadimitriou and Papadioti (2013) utilized the component mode synthesis technique for finite element updating. Even the finite element updating technology has been developed for a long time, its application in engineering practice is still limited. The basic reason is that the procedure of almost all finite element model updating methods is searching for an optimal model best fitting field test data, while it is very challenging to find a single optimal model really simulating the studied structure due to various kinds of uncertainty existing in stages of field test, data processing and finite element modeling (Döhler et al. 2014, Hasançebi and Dumlupınar 2013).

Even though great efforts have been performed to deal with the uncertainty problem during the finite element updating, the challenging "Non-Unique" problem still exists. Namely, multiple models with different intrinsic parameters may fit observed data well. A few researchers have made effort trying to solve the above problem. Koh and Kim (2013) firstly relegated the key parameters by the PCA method, then using K-means method to cluster the multiple models. Zhang et al. (2013) proposed the Markov chain Monte Carlo sampling method to sample the key parameters, from which a population of finite element models are generated, then all those models are weighted their posterior probabilities calculated from the Bayesian theory and the monitoring data.

The idea of using the multiple models, not the single optimum model, for structural evaluation and prediction is promising, however, the challenging problem is that it requires huge computational cost to analyze the whole population of finite element models. In this article, the response surface method (RSM) method is adopted into the multiple model method to improve its computation efficiency. From the proposed method, not only the values of the identified structural parameters and predicted structural responses, but also their probabilistic distributions are provided to deal with the uncertainty problem.

The structure of the article is as shown below. First, basic information of a pre-stressed concrete box girder bridge and its field test including bridge inspection, truck load test and ambient vibration test are described. Then, the FE modeling of the bridge and its sensitivity analyses are performed. Four key parameters including elasticity modulus of concrete, stiffness of expansion joint, stiffness of longitudinal bearing and stiffness of vertical bearing are chosen for further investigation in next step from the sensitivity analysis. Subsequently, the multiple model method using the MCMC sampling and RSM technologies are proposed, from which bridge performance evaluation under serviceability limit states is implemented. Finally, the conclusions are drawn.

BRIDGE DESCRIPTION AND FIELD TEST

Bridge Description

The studied structure is a three-span pre-stressed concrete variable cross-section continuous box girder bridge, with a 55 m main span, and two 37.5 m side spans (Figure.1a). Its typical transversal cross-section is shown in Figure.1b. Pot rubber bearings are set between bridge piers and abutments. Expansion joints are set at the box girder ends (Xu et al. 2012; Ren et al. 2007). Bridge inspection, truck load static test, and ambient vibration test were performed on the bridge to evaluate its safety conditions.

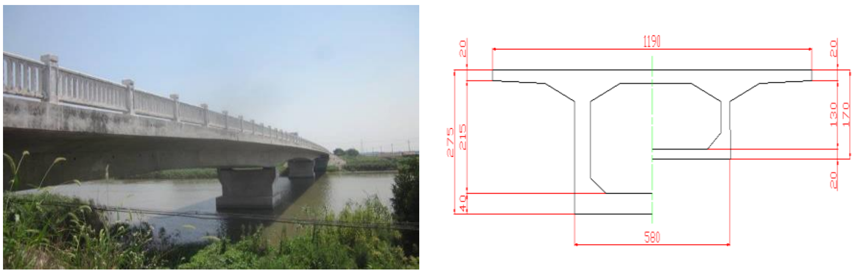


Figure 1 the studied bridge

Bridge Inspection

Regular inspections of concrete strength, rebar corrosion, chlorine ion contents, concrete resistivity, carbonation resistance of concrete, deck station, bearing station and pile foundation were performed on the bridge. Figure. 2 shows the phenomenon of blocked expansion joint, corrosion of steel plate and local edge failure in pot type rubber bearing, and cracks on box girder. Figure. 3 shows longitudinal cracks on bottom slab and diagonal cracks on the web.

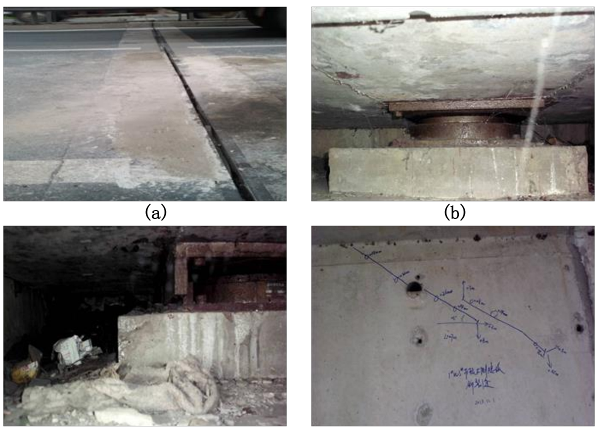

(a) (b)
(c) (d)

Figure 2 Bridge inspection results (a) blocked expansion joint (b) steel plate corrosion (c) local edge failure (d) cracks on the box girder

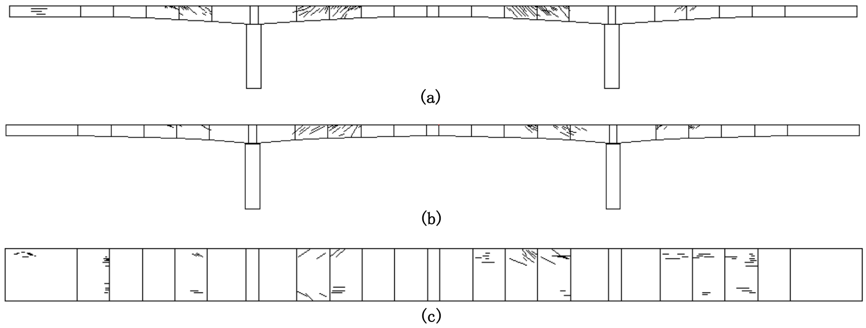


Figure 3 cracks distribution (a) upstream web (b) downstream web (c) bottom slab

Truck Load Test

Static test using four trucks were conducted on the bridge on November 2013 (Figure.4). The averaged axle loads of the test trucks are 70 KN for front axles and 280 KN for rear axles. The axle intervals of the test trucks are 360 cm for front axles and 140 cm for rear axles. Prism displacement sensors and strain sensors were deployed at the side span center, (cross-section 1-1), main span center (cross-section 2-2) and the 1/4 point of the main span (cross-section 3-3). Hence there are six load cases in three cross-sections using balance load and unbalanced load. Under the truck loads, structural deflections and strains were observed. Six load cases were implemented during the truck load test. The test cross-section of case 1 and case 2 is 1-1, case3 and case4 is 2-2, case 5 and case 6 is 3-3 as shown in Figure.4 (a). The way of truck load of case 1, case 3 and case 5 is balance load as shown in Figure.4 (c) and case 2, case 4 and case 6 is unbalance load as shown in Figure.4 (b). In addition to static tests, several running tests were executed by moving the trucks at different speeds over the bridge.

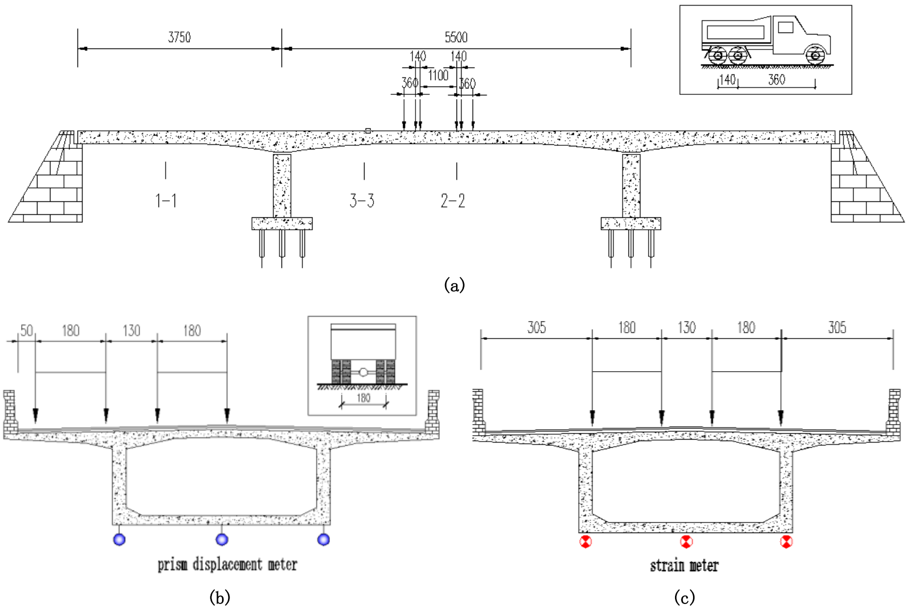


Figure 4 Truck load distribution (a) longitudinal location (b) unbalance load (c) balance load

Ambient Vibration Testing

Ambient vibration test was performed on the bridge to identify its dynamic characteristics. As shown in Figure. 5, the studied bridge was instrumented with 20 PCB 393B05 accelerometers. NI PXIe-1082 system was used for data acquisition. Figure. 6 shows two time histories of the recorded accelerations at the middle and the 1/4 point of the main span. Vibration data was processed to extract modal parameters of the bridge by the complex mode indication function (CMIF) method, which utilizes the singular value decomposition technology to enhance the uncertainty of identified results. Three modes of the bridge were identified, and the corresponding frequencies are 1.93Hz, 3.38Hz and 4.59 Hz respectively.

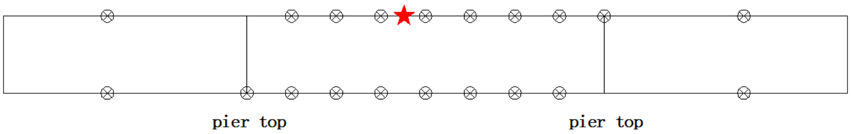
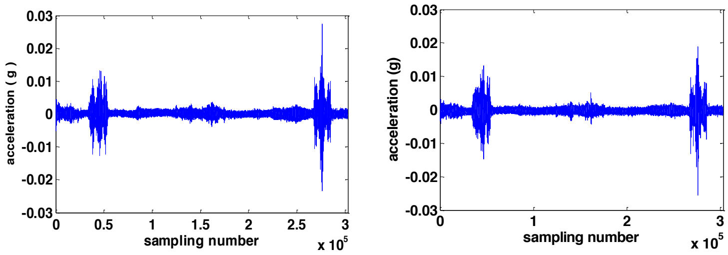


Figure 5 Instrument plan



(a) (b)

Figure 6 Typical acceleration time histories (a) middle point (b) 1/4 point of main span

FE MODELING AND SENSITIVITY ANALYSIS

Bridge FE Modeling

A linear 3D finite element model of the studied bridge was developed using large general finite element software ANSYS. Figure.7 shows an isometric view of the developed FE model, the detail cross-sections of bearing parts and mid-span of main span, and longitudinal section of 1/2 hole bridge.

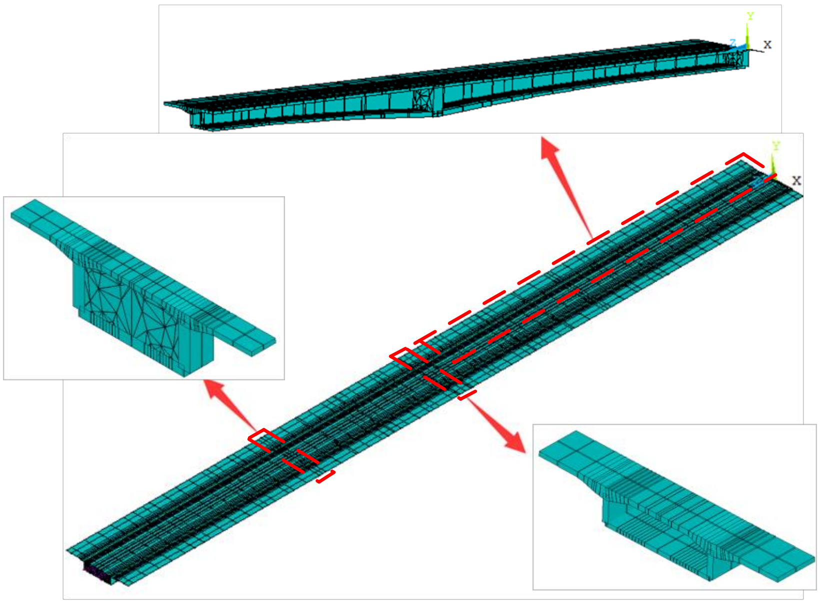


Figure 7 Finite element model of the studied bridge

A three-dimension finite element model of the studied bridge was constructed through the software ANSYS. Figure. 7 shows an isometric view of the developed FE model, the detail cross-sections of bearing parts and longitudinal cross-section of 1/2 main bridge. In the FE model, concrete is modeled by SHELL63 element which is defined by four nodes. The pre-stressed reinforced bars are modeled by LINK8 elements, which has the property of uni-axial tension-compression components with three DOFs at each node. The pre-stress is applied by falling temperature method. The cracks on the web and bottom slab (Figure.3) of the bridge deck are modeled by using a reduction coefficient to the elasticity modulus of the corresponding elements.

The boundary conditions are modeled by using linear spring elements. The expansion joint and pot rubber bearing are modeled by tension-compression linear spring (COMBIN14) elements. Linear springs are used to simulate the longitudinal, vertical and lateral support directions of reality rubber bearing. Because the stiffness of bridge pier is huge, effect of pier is not considered and the bottom of rubber bearing is fully-constrained.

The material properties are set as follows: elasticity modulus of concrete and reinforced steel bar are 3.5E4MPa, and 1.95E5 MPa respectively, and their Poisson's ratio are 0.2 and 0.3 respectively. The stiffness of the expansion joint, the longitudinal, vertical, and lateral stiffness of pot rubber bearing are set to be 5E5, 1E8, 1E11, 1E8 MPa, respectively.

The FE analysis results and field test results are compared in Table 1. It is seen that the error distribution is random and they are less than 10 percent.

Table 1 Comparison of FE analysis and field test results

Frequency	1st order	3%	displacement	Case 1	0.5%	strain	Case 1	11%
	2nd order	5%		Case 2	0.5%		Case 2	9%
	3rd order	8%		Case 3	2%		Case 3	3%
MACVALUE	1st order	0.8%		Case 4	1%		Case 4	1%
	2nd order	6%		Case 5	0.5%		Case 5	7%
	3rd order	6%		Case 6	3%		Case 6	7%

Sensitivity Analysis

Sensitivity analysis is a good way to identify the most sensitive parameters that affecting the FE analysis results. (Jung and Kim 2013). The objective function in the sensitivity analysis is defined in Eq.1 in terms of the difference between analytical and experimental results, in which the identified frequencies and mode shapes in the first three modes, observed displacements and strains in six test cases are used.

$$\mathrm{obj}=\sum_{i=1}^{3}\frac{|\omega_{ai}-\omega_{ei}|}{\omega_{ei}}+\sum_{i=1}^{3}(1-\mathrm{MAC}_i)+\sum_{i=1}^{6}\frac{|\delta_{ai}-\delta_{ei}|}{\delta_{ei}}+\sum_{i=1}^{6}\frac{|\varepsilon_{ai}-\varepsilon_{ei}|}{\varepsilon_{ei}} \tag{1}$$

$$MAC_i=\frac{|\Phi_{ai}^T\Phi_{ei}|^2}{(\Phi_{ai}^T\Phi_{ai})(\Phi_{ei}^T\Phi_{ei})} \tag{2}$$

where ω_{ai} and ω_{ei} are analytical and identified frequencies of the i th mode, respectively; Φ_{ai} and Φ_{ei} are analytical and identified modal shapes of the i th mode. The MAC value between Φ_{ai} and Φ_{ei} is defined in Eq.2. δ_{ai} and δ_{ei} are analytical and observed displacements of the i th test case; ε_{ai} and ε_{ei} are analytical and observed strains in the i th case, respectively. Weight factors of frequency, mode shape, displacement and strain are considered consistently in Eq.1 (Wang et al 2013).

9 structural parameters in finite element model representing main sources of uncertainty are selected for sensitivity analysis. They are elasticity modulus of concrete, elasticity modulus of crack1- crack4 zones (EMCZ), stiffness of expansion joint, stiffness of longitudinal, vertical and transverse bearing, respectively. Each parameter is set to 7 selected values in the sensitivity analysis. For example, the distribution of elasticity modulus of concrete is Gaussian distribution, and the 7 selected values are [3.0, 3.25, 3.4, 3.5, 3.6, 3.75, 4]$\times10^{10}$Pa , respectively. The distribution of stiffness of expansion joint is logarithmic distribution, and the 7 selected values are 0.5×10^5 , 0.5×10^6 , 0.5×10^7, 0.5×10^8 , 0.5×10^9 , 0.5×10^{10}, $0.5\times10^{11}\,\mathrm{N/m}$, respectively. Figure.8 shows the sensitive analysis results for each studied parameter. It is seen that the influence of expansion joint is the largest, followed by longitudinal bearing stiffness and vertical bearing stiffness and overall elasticity modulus. The influence of elasticity modulus in four crack regions is small, and there is no obvious influence from lateral stiffness of bearing. Therefore, stiffness of expansion joint, longitudinal and vertical bearing, and overall elasticity modulus, are selected for further study in next section.

MULTIPLE MODEL METHOD

The goal of the traditional deterministic method is to find the single FE model best matching experimental data, and then use it for structural response prediction. Due to the uncertainty existing, the predicted deterministic responses may have a big bias with actual results, so it is hard to accurately assess structural performance. In this section, a multiple model method is proposed to sample the key parameters of structure and provide the structural probabilistic information for engineers. The purpose of the multiple-model approach is not to find the single optimal model, but to generate a number of FE models by stochastic sampling. In the multiple-model method, response surface model is used to replace the FE model for computational efficiency. The Markov

Chain Monte Carlo technique is first performed to sample the key structural parameters representing main sources of uncertainty. Then a FE model population is generated using the samples, and the posterior probability of each model is evaluated by calculating the correlation between its simulation results and measurements through the Bayesian theorem. Finally, all those FE models from the stochastic sampling with their posterior probabilities are used for structural identification and performance evaluation.

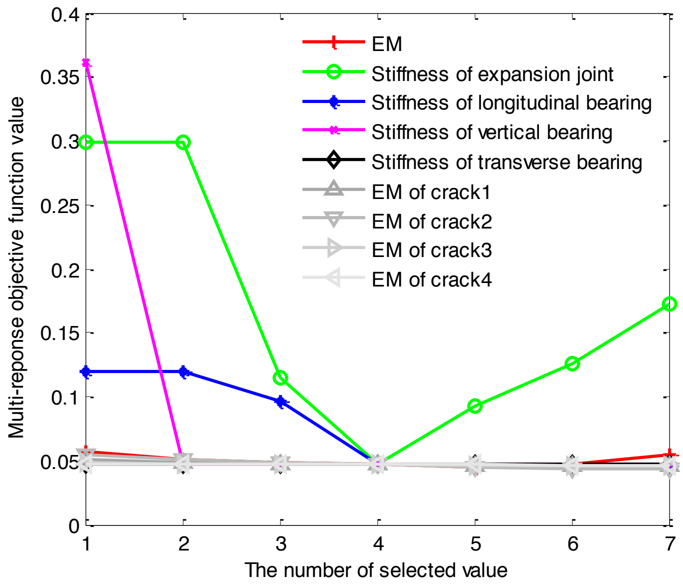


Figure 8 Sensitivity analysis result

ANSYS Batch Tool is used to operate ANSYS in background under MATLAB control in this paper, and MATLAB functions are called in ANSYS at the same time. The flow chart of the multiple models method is shown in Figure.9.

Computational speeds can be lifted dramatically with the combination of ANSYS and MATLAB. For example, running xlb.dat document once needs 20min in ANSYS Interface, but 6min in ANSYS Batch; and running self-edit function (RejectModal.m) needs 10s in MATLAB Interface, but 1s in MATLAB Command Window.

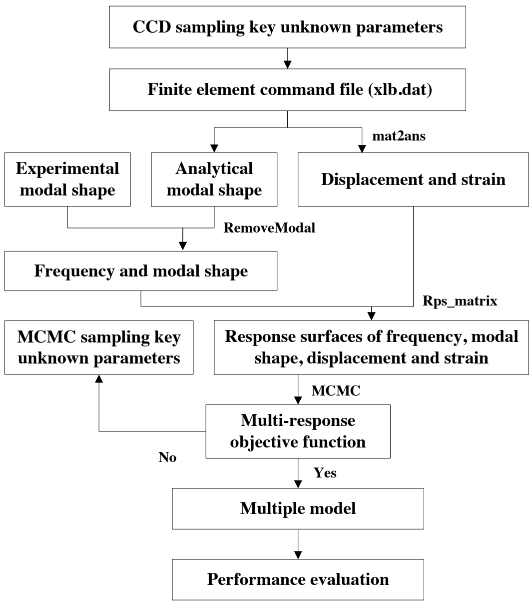


Figure 9 Flow chart of the multiple model method

MCMC Sample

Utilizing the Markov chain Monte Carlo (MCMC) sampling method can rapidly converge to the objective function $\pi(x)$ under the framework of Bayesian theory. The MCMC sample utilizing the chain sample could overcomes the impact of high dimensional parameters. The process is a recursive procedure and the steps are as follows:

1）Randomly generated initial parameters θ_0.

2）Assuming that θ_{i-1} is the k^{th} parameter of the Markov chain, candidate parameter θ_c is generated by proposal distribution $q(x)$:

$$\theta_c = q(\theta_c / \theta_{k-1}) \tag{3}$$

3）Accept probability of candidate parameter θ_c is:

$$\alpha(\theta_{k-1},\theta_c) = \min\left\{1, \frac{\pi(\theta_c)q(\theta_c/\theta_{k-1})}{\pi(\theta_{k-1})q(\theta_{k-1}/\theta_c)}\right\} \tag{4}$$

4）If θ_c is accepted, so $\theta_k = \theta_c$. Or repeat the 2 - 3 steps until θ_c is received。

5）Make k = k+1, repeat the 2-4 step until the process converges

In a multiple models framework, objective function π is elected as model probability conditioned by known structural information G; proposed distribution q is elected as the prior distribution of the key unknown parameters; the key unknown parameter θ in the studied bridge is combination of elasticity modulus, stiffness of expansion joint, bearing longitudinal stiffness and bearing vertical stiffness.

$$\pi(\theta) = P(M(\theta)|G);\ q = P(M(\theta)) = \prod_1^4 P(\theta_j) \tag{5}$$

$P(\theta_j)$ is the prior distribution of jth parameter.

According to Bayesian theory $P(M/G) = \frac{P(G/M)P(M)}{\int P(G/M)P(M)}$, Considering that $\int P(G/M)P(M)$ is constant, combining (4) and (5) we have:

$$\alpha(\theta_{i-1},\theta_c) = \min\left\{1, \frac{P(G|M_c(\theta_c))P(M_c)}{P(G|M_{i-1}(\theta_{i-1}))P(M_{i-1})}\right\} \tag{6}$$

Assuming that the error of the model belongs to normal distribution and independent from each other:

$$P(G|M_i(\theta_i)) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{obj^2}{2\sigma^2}\right) \tag{7}$$

To further improve the sampling efficiency, an extended M-H algorithm is adopted. Two novel ideas, Adaptive Metropolis (AM) and Delayed Rejection (DR), are successfully combined to improve the computational efficiency of the M-H algorithm. Please refer to the details of the reference (Haario 2006).

Response Surface Method

MCMC method could improve sampling efficiency, but it is still difficult to establish multiple models library considering the studied bridge such large scale FE model. Response surface method (RSM) is utilized in the proposed method to improve its computation efficiency. RSM is performed to establish an appropriate prediction model and to determine the regression coefficients describing the linear and quadratic contribution of each parameter and their interactions. RSM is very good at copping with the issue with implicit multi-response object function for its satisfied accuracy and good efficiency. Because the amount of sample set used to reconstruct the response surface is small (Zhao and Qiu 2013).

The second order RSM is reconstructed by Central Composite Design (CCD) method, F test of variance analysis is used to identify the parameter, and the second order polynomial response surface is as follows:

$$y = \beta_0 + \sum_{i=1} \beta_i x_i + \sum_{i=1}\sum_{j=1} \beta_{ij} x_i x_j + \sum_{i=1} \beta_i x_i^2 \tag{8}$$

Where x is structural parameter such as elasticity modulus, spring stiffness, and so on; y is response result such as frequency, displacement, strain; β is fitting coefficient of structural parameters.

RESULTS FROM MULTIPLE MODELS

First of all, four MCMC key unknown parameters of the studied bridge are needed to confirm as shown in Table 2. And then, 20 thousands samplings are implemented through MCMC sampling method and response surface technology.Figure.10 shows convergence situation of four structural parameters and probability distribution of four parameters after rejecting first 5 thousands sampling. It can be seen that results are convergent after 5 thousands sampling. So the frequency, mode shape, displacement and strain of multiple models can be get, which is made up of 15 thousands sampling. The comparison of first two order frequencies (Figure.11) and displacements of case1 (Figure.12) and case2 and strains of the case3 and case4 (Figure.13) between analytical and measuring values is list as follow. The results show correctness of multiple model through MCMC sampling and RSM.

Table 2 The convergence and distribution key structural parameters

	Elasticity modulus (MPa)	Stiffness of expansion joint (N/m)	Stiffness of longitudinal bearing (N/m)	Stiffness of vertical bearing (N/m)
MCMC initial value	2.9×10^{10}	5×10^{7}	1×10^{10}	1×10^{13}
Distribution type	normal	normal	normal	normal
Distribution interval	$[2.9\times10^{10},4.1\times10^{10}]$	$[3\times10^{7},7\times10^{7}]$	$[1\times10^{6},1\times10^{11}]$	$[1\times10^{9},1\times10^{14}]$

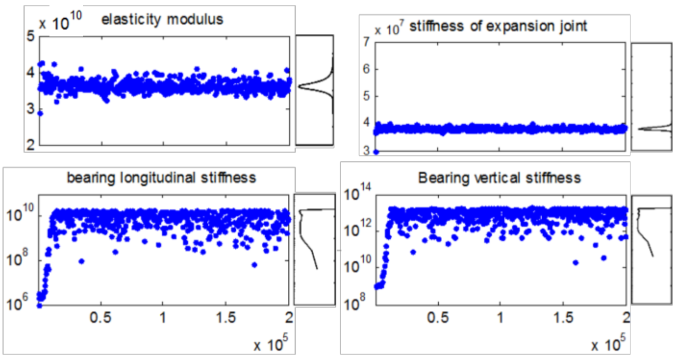


Figure.10 Multiple models and parameters probability distribution of the studied bridge

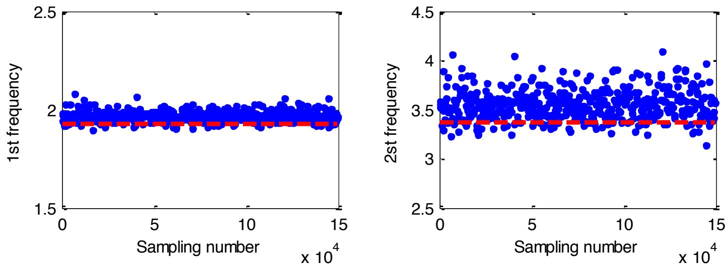


Figure.11 Comparison of analytical and experimental frequencies in the first two modes

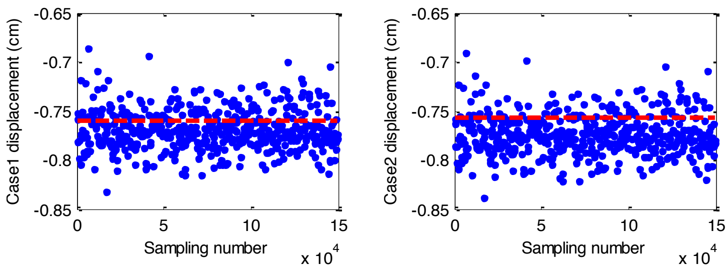


Figure 12 Comparison of analytical and experimental displacements in case1 and case2

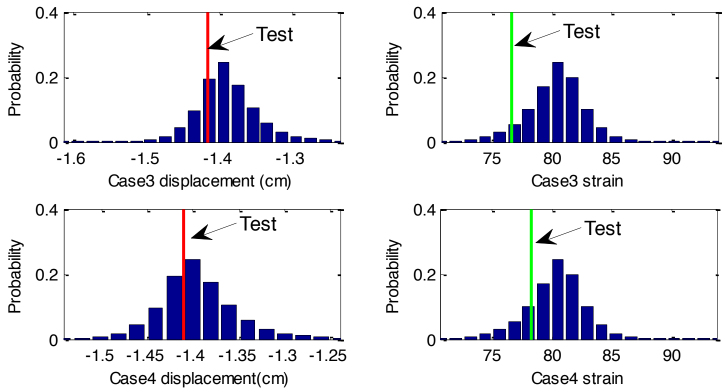


Figure 13 Comparison of analytical and experimental displacement and strain in case3 and case4

BRIDGE PERFORMANCE EVALUATION UNDER SERVICEABILITY LIMIT STATE

According to Chinese Code for Highway Bridge Loading Capacity Detection Evaluation（JTG/T J21-2011）, checking coefficient of concrete loading capacity Z1 is shown in Table 3. The deteriorate coefficient of concrete structure loading capacity ξ_e =0.01, reduction coefficient of steel cross-section ξ_s =1.0, and live load updating factor ξ_q =1.0. Through considering coefficients above, checking coefficient of concrete loading capacity Z1=1.09. So reduction effect is not considered in checking results.

Table 3 Checking coefficient of concrete loading capacity Z1

	Detection index	D_j	Weigh coefficient α_j	$D=\sum \alpha_j D_j$	Checking coefficient Z_1
Main beam	defects	3	0.4	1.8	1.11
	strength	1	0.3		
	Natural frequency	1	0.3		

Two failure modes are considered under serviceability limit state, which are appearance failure due to large main beam mid-span displacement and concrete compressive strength failure. Design load of bridge is applied in finite element model of the studied bridge. And then two response surfaces of displacement and strain of mid-span are established. Multiple model of the studied bridge established above is substituted to this two response surfaces. Probability of displacement and strain is implemented in Figure.14.

According to Chinese Code for Design of Highway Reinforced Concrete and Prestressed Concrete Bridge and Culverts（JTG D62-2004）, concrete compressive strength should not be more than $0.5f_{ck}=16.2\text{MPa}$, and displacement should not be greater than $\frac{L}{600}=9.167\text{cm}$. Figure.14 shows the bridge still has some redundancy.

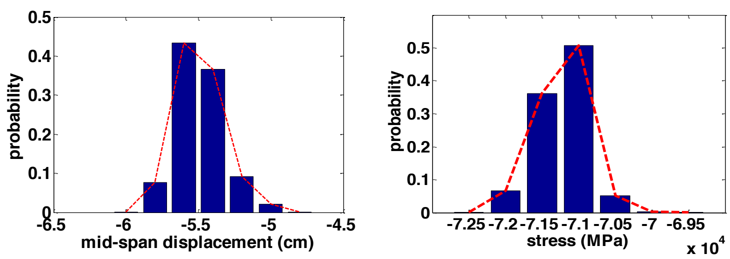


Figure.14 Bridge performance evaluation (a) mid-span displacement (b) concrete compressive stress

CONCLUSION

(1) A multiple model method through MCMC sampling and the response surface technology is proposed. The "Non-Unique" problem in the finite element updating field caused by uncertainties is solved by using a population of FE models, not a single optimum model for structural performance evaluation.

(2) Inspection of the studied bridge found the padding of expansion joint, corrosion of steel plate, local edge failure in pot type rubber bearing and cracks of box girder. However, how they influence structural performance is not clear. Sensitive analysis and the proposed multiple model method are performed to study how they influence structural performance.

(3) The results from the multiple model method are used to evaluate the load capacity of the studied bridge, which shows that the bridge still has sufficient redundancy.

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