# A NEW METHOD FOR FORMULATING CRACK SPACING MODELS OF RC TIES

G.Kaklauskas[1], R.Ramanauskas[2], R.Jakubovskis[3], V.Gribniak[4], M.Juknys[5]
Department of Bridges and Special Structures, Vilnius Gediminas Technical University, Lithuania
[1] VGTU, Sauletekis av.11 LT-10223 Vilnius, Lithuania, Email: gintaris.kaklauskas@vgtu.lt
[2] VGTU, Sauletekis av.11 LT-10223 Vilnius, Lithuania, Email: regimantas.ramanauskas@vgtu.lt
[3] VGTU, Sauletekis av.11 LT-10223 Vilnius, Lithuania, Email: ronaldas.jakubovskis@vgtu.lt
[4] VGTU, Sauletekis av.11 LT-10223 Vilnius, Lithuania, Email: viktor.gribniak@vgtu.lt
[5] VGTU, Sauletekis av.11 LT-10223 Vilnius, Lithuania, Email: mantas.juknys@vgtu.lt

## ABSTRACT

Cracking of concrete is one of the most complicated phenomena in reinforced concrete analysis and is one of the key aspects governing serviceability analysis of RC structures. Current methods for investigating cracking rely on empirical approaches that give unreliable results with errors of multiple times the real value. A new non-empirical method based on the combination of the stress-transfer and the smeared approaches is proposed for deriving crack spacing models. The stress-transfer approach governs the strain distribution of the reinforcement between the consecutive cracks whereas the smeared approach allows for the estimation of the mean strain of the element. The suggested method introduces the concept of damage zones: the bond in the area adjacent to the normal cracks is considered to be fully damaged, thus bond behaviour is non-uniform in the segment between cracks. Crack spacing models were derived using the load-strain analysis method presented in the Eurocode 2 and were shown to give results that are in good agreement with the crack spacing values taken from available experimental data.

## KEYWORDS

Crack spacing, stress-transfer approach, smeared crack, RC tie, compatible model

## INTRODUCTION

In general practice, design of reinforced concrete (RC) structures is often based on the assumption of perfect bond between concrete and reinforcement, i.e. no physical slip is allowed. This simplification might be reasonable in load capacity analysis of structures with proper reinforcement detailing and anchorage; however it becomes unreasonable when serviceability (cracking and deformation) of RC structures is considered. Neglecting bond-slip leads to significant inaccuracies in the assessment of serviceability performance of structures (Oehlers *et al*. 2012). Existing serviceability models are conflicting with each other, thus crack analyses are commonly carried out separately to general deformation analyses. Previous studies have shown that deflection predictions by different code techniques may vary up to 100%, whereas variability of crack width predictions was of even higher order (Gribniak *et al*. 2013, Balazs *et al*. 2013).

Cracking is considered to be one of the most complicated issues in the research of RC structures. Some of the key aspects affecting the cracking behaviour are the interaction between concrete and reinforcement, crack propagation along the boundary of larger aggregate particles at micro level, stress relief and stress redistribution after every new crack appears. The mentioned factors contribute to the highly non-linear nature of concrete and partly explain why cracks are of various orientations, shapes and lengths (normal, diagonal or longitudinal as well as major and secondary cracks). Cracks make the stress-strain state in the tensile zone highly unpredictable and it is known that classical theories often provide inaccurate crack width and crack spacing predictions.

In general structural design practice there are two prevailing approaches in crack analysis of RC structures that enable the estimation of maximum allowable crack widths in serviceability checks:

1. The classical and most common approach in the design codes, which is based on the bond between reinforcement and concrete (Beeby 2005). Where the crack width is estimated using a generalized equation:

$$w = k \cdot \frac{\emptyset_s}{\rho} \cdot \varepsilon \qquad (1)$$

where $k$ is an empirical coefficient, $\varnothing_s$ is the bar diameter, $\rho$ is the reinforcement ratio, $\varepsilon$ is mean strain value of tension reinforcement between cracks. Bond stresses are assumed to be constant when deriving Eq.1 in order to simplify the reinforcement-concrete interaction; however, this simplification introduces empirical parameters.

2. Concrete cover approach, in which crack spacing is directly related only to the concrete cover of the reinforcement (Broms 1965):

$$w = k_1 \cdot c \tag{2}$$

where $k_1$ is the empirical coefficient and $c$ is the concrete cover. These models have to be calibrated to specific experimental elements and therefore may be inaccurate for other scenarios of reinforcement arrangements.

There are variations of both approaches, which mostly alter the empirical coefficient values (Borosnyoi and Balazs 2005). Despite the extensive experimental and analytical investigations, errors in crack width predictions are still significantly scattered and yield unreliable results in the structural analysis of RC structures (Perez 2013).

This paper proposes a new methodology for deriving crack spacing models for RC ties at the stage of stabilised cracking. The study advances a new concept of combining the stress-transfer and the smeared crack approaches. The former approach governs the strain distribution of the reinforcement between the consecutive cracks whereas the latter takes the advantage of knowledge about the mean strain of the member. An important part of the modelling approach is the introduction of the so called *damage zones* located in the vicinity of the major cracks. It has been assumed that the bond between concrete and reinforcement bar is damaged in these zones and the bond-slip behaviour differs from the areas outside the damage zones. Based on the experimental strain profiles of RC ties, it has been found that the length of the damage zone depends on the diameter of the bar and load level. The proposed new method presented in this paper relies on these findings to derive crack spacing models.

## BEHAVIOUR OF A REINFORCED CONCRETE TIE

A RC element tie shown in Figure 1 is used to illustrate the interaction of cracking, deformation and bond behaviour in RC structures. Such elements are often chosen due to their simplicity and reasonably good representation of the stress and strain states of the tensile zone in RC structures (Fantilli 2007).

During the initial loading stage (OA), the deformation behaviour of the tensile member is almost linearly elastic, as shown in Figure 1(a). Composite action and compatibility of reinforcement and concrete strains are attained in the element with discontinuities due to slip occurring only in small regions at the loaded ends (refer to Figure 2(a)). Bond stresses are directly related to slip and therefore develop only in regions with non-zero slippage. The bond stresses increase together with the value of slip, whereas at the ends of segments the bond stresses diminish until they reach zero at the location of the cracks, where the slip changes direction and the absolute value of it is the largest. This effect can be explained by the presence of localized concrete damage near the crack plane, which significantly reduces the bond action (Ruiz 2007).

With increasing load, strains in the reinforcement as well as in the concrete grow until a certain limit of concrete cracking is reached. The first crack appears in the section where stresses in concrete transferred through bond action reach its tensile strength. This causes an immediate redistribution of stresses in the cracked section: concrete stresses and strains drop to zero at the location of the crack, thus the entire tensile force is transferred only through the reinforcement bar. Further away from the crack, part of the force is transferred to the concrete through the bond action. Stresses and strains in the concrete increase over distance until the tensile strength of the concrete is reached and a new crack may form again. The distance, required to reach the tensile strength of the concrete is often called the *transfer length* and is the most influencing factor in cracking analysis (Beeby 2005). Distribution of strains, slip and bond stresses after formation of the first cracks is shown schematically in Figure 1b. The cracking phenomena also cause a sudden drop in stiffness of an element which is evident from the force versus average strain diagram.

After the first crack has occurred, any further increase of the load creates new cracks, and the RC element proceeds to the crack formation phase (stage 2 and part AB in the diagram). It should be noted that the local effect of bond deterioration near the crack plane becomes more significant with increasing number of cracks. After the crack formation stage, the so called stabilized cracking phase is reached and the RC member becomes separated into a number of concrete blocks (see Figure 1(c)). During this stage the tensile stresses are still transferred to the concrete over the reinforcement due to the bond action but the length of these blocks is insufficient to allow the stresses in the concrete to reach the tensile strength. The length of all blocks falls into a specific interval $l_{tr} \le l_{cr} \le 2l_{tr}$, where $l_{tr}$ is the transfer length. From available published data (Bigaj 1999; Borosnyoi and Balazs 2005) it was determined that an average block length (crack spacing) could be in the range of $1.3l_{tr}$-$1.5l_{tr}$. The concept of stabilized cracking is controversial, formation of new cracks in this phase is very limited and has an insignificant influence on the final crack pattern (Perez 2013) The deformation behaviour with a usually stable number of cracks continues until the reinforcement yields (stage 3 and part BC in the diagram).

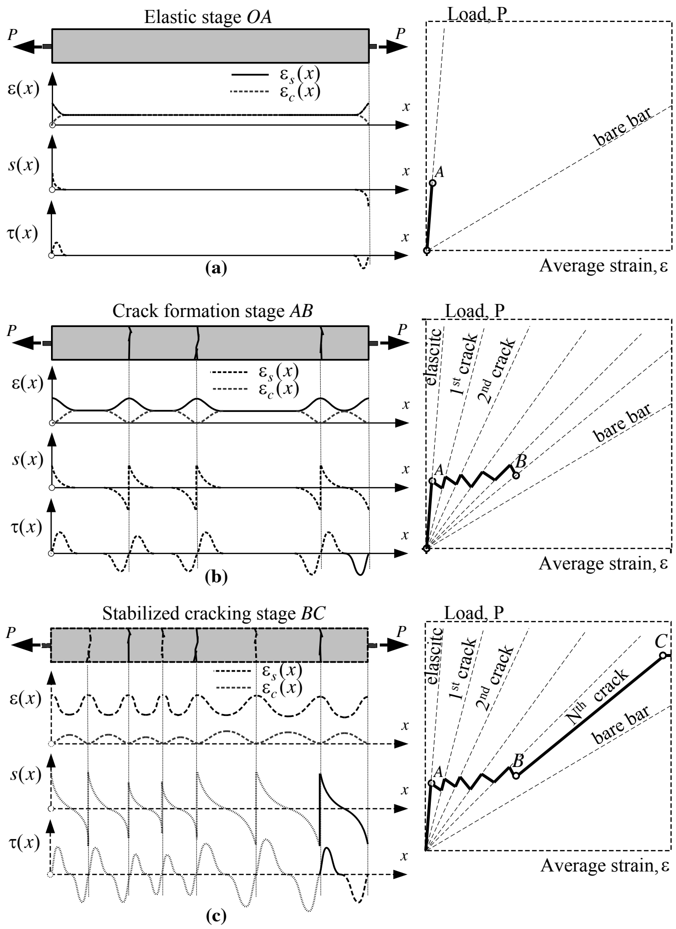


Figure 1 Behaviour of a RC tie: a) the elastic, b) crack formation and c) stabilized cracking stages

The behaviour of a RC tie between two consecutive cracks has been experimentally investigated by Houde (1974) Scott and Gill (1987) and Kankam (1997). Figure 2(a) shows the reinforcement strain distribution along a 200mm length bar at various load stages as reported by Kankam (1997). It can be observed that the variation of the reinforcement strains in the middle part of the bar can be well approximated by a first order (linear) polynomial equation for higher load levels and a second order (parabolic) polynomial for lower load levels:

$$\varepsilon_s(x) = \varepsilon_0 + a_1 x \tag{3}$$

$$\varepsilon_s(x) = \varepsilon_0 + a_1 x^2 \tag{4}$$

where $a_1$ is the shape function coefficient and $\varepsilon_0$ is the minimum reinforcement strain at the centre between cracks.

The use of a linear approximation Eq. (3) yields a reasonably good match with experimental results, particularly in the final loading stages, with coefficient of determination in the range of 0.9-0.95. Approximation of the experimental strain curves by a second order polynomial Eq. (4) also provided results with good agreement, with coefficient of determination ranging from 0.92 to 0.95 at earlier loading stages and from 0.88 to 0.9 for the later loading stages as can be seen from Figure 2(b). A number of other experimental results were also analysed (Houde 1974, Kankam 1997), proving that linear or parabolic approximation of strain curves may be used in the mid part of an element.

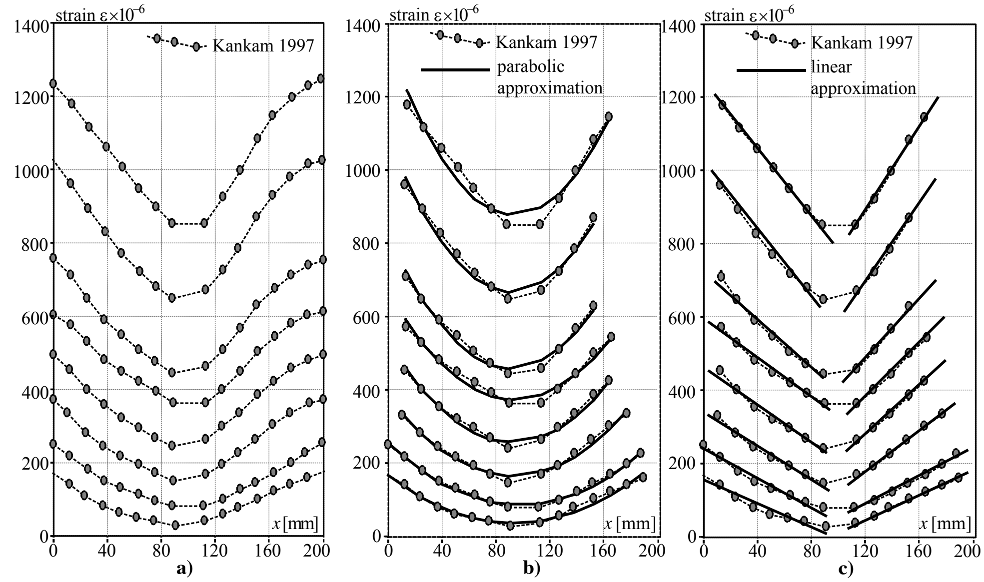


Figure 2 Steel strain distribution between cracks: a) experimental results of Kankam (1997); b) approximation of the results with second-order polynomial; c) approximation of the results with first-order polynomial

## DERIVATION OF CRACK SPACING MODELS

This section describes the procedure for deriving the crack spacing model in detail. The suggested approach is developed for the stabilized cracking stage and relies on the assumption that the bond behaviour in the central area of the block between adjacent cracks is different from that near the normal cracks. The concrete close to the cracks is considered to be damaged and therefore the strains follow a different law in the vicinity of the cracks. The change in the strain distribution implies that the bond-slip relationships in the middle section between cracks and in the areas close to the cracks are different. This effect is clearly visible in the strain diagrams presented in Figure 2(a). This locally damaged concrete zone is further defined as the damage zone and the length of this zone is denoted by $l_d$. To keep the naming consistent, the central part of the concrete block between these damage zones is called the effective zone and is further denoted by $l_{ef}$. To simplify the proposed concept, it is assumed that the bond in the damage zones is fully damaged. This implies that the bond stresses in this zone are equal to zero and thus the reinforcement strains are constant and equal to the strains at the location of the crack. The distance between cracks can be expressed through the lengths of the damage and effective zones:

$$l_{cr} = l_{ef} + 2l_d \tag{5}$$

Tests results of Houde (1973) were further investigated to provide a quantitative expression for the damage zone (refer to Figure 3(a)). In this case, the effective zone is assumed to follow a parabolic law while the damage zones have fully degraded bond. The lengths of each zone were estimated by equating the integrals of the experimental and fitted curves with an additional assumption that the minimum strains $\varepsilon_0$ of the curves coincide. The evolution of the damage zone with increasing load is obtained from this approach. The same procedure was applied to additional tests of Kankam (1997) and is summarized in Figure 3(b). A simple linear equation is suggested from the analysis of the experimental results to evaluate the damage zone length in relation to the load level:

$$l_d = 1000\varepsilon_s\phi_s \tag{6}$$

where $\phi_s$ is the bar diameter and $\varepsilon_s$ is the reinforcement strains at the location of the crack.

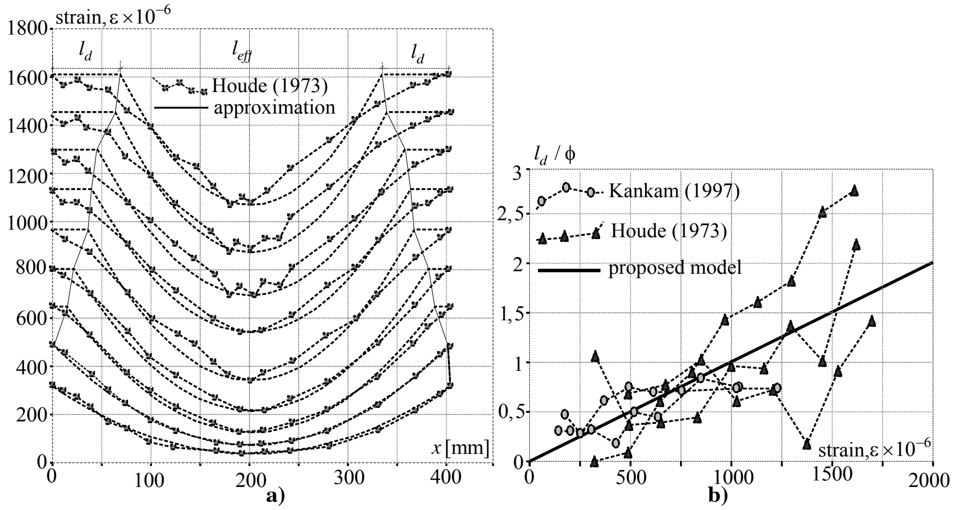


Figure 3 Effective and damage zones: a) distribution of steel strains; b) development of the damage zone

The other condition is to maintain identical bond-slip relationship for the effective area without the reinforcement ratio as a variable parameter. A way to achieve this is by applying the same reinforcement strain function $\varepsilon(x)$ for all the RC ties. The same strain and slip rate can be ensured by keeping the minimal reinforcement strains $\varepsilon_0$ identical in the middle section of each RC tie. An important aspect in this concept is the independence of the final crack spacing from concrete compressive strength, which had been shown to be the case by Farra (1993) and is also confirmed in the current study.

The described technique of calculating the effective and damage zones enables the derivation of crack spacing models, consistent with the assumed average strain model. All the equations are presented with symmetry conditions, for one half of the crack spacing length. It is important to note that the suggested method depends on the reference element. The reinforcement bar diameter and reinforcement ratio values of this chosen element are hence referred to as reference values. Another key reference value is the crack spacing $l_{cr}$ of the chosen element. The benefit of this method is that in order to develop a crack spacing model only a single data point is required, i.e. the reference value $l_{cr}$, which can be obtained either from experimental data or estimated by empirical or numerical methods. This ensures the simplicity of this algorithm.

***Procedure for the linear shape function***

1. For a given load $P_i$, with reference diameter $Ø_{ref}$ and reference reinforcement ratio $\rho_{ref}$, the average deformation $\varepsilon_{mi}$ of an RC element is calculated according to the Eurocode 2 (or other design codes). The maximum reinforcement steel strains $\varepsilon_i$ are also known for the load level and are considered to be constant over the damage zone length.

2. The average distance between cracks $l_{cr}$ is assumed to be known, thus the effective and damage zone lengths can be determined from Eq. (5) and (6).

3. Substituting the known values into Eq. (3) provides the shape function of the reinforcement strains over the effective length:

$$\varepsilon_0 + a_1(0.5 l_{ef}) = \varepsilon_i \tag{7}$$

4. Since the minimum strains and the shape function coefficient are unknown, an additional equation is required. The average and maximum deformation values $\varepsilon_{mi}$ and $\varepsilon_i$ have been already determined, along with the lengths of the effective and damage zones, thus the minimum deformation $\varepsilon_0$ and coefficient $a_1$ can also be found by integrating strains over the crack spacing length. Since the shape function only applies to the strains over the effective zone length and the maximum deformation in the damage zones is constant, the integral can be expressed as:

$$\varepsilon_i \cdot l_d + \int_0^{0.5l_{ef}} \varepsilon_s(x) = \varepsilon_{mi} \cdot 0.5l_{cr} \quad (8)$$

5. The shape function coefficient and the minimum strain can now be obtained by solving the following system of equations:

$$\begin{cases} \varepsilon_0 + a_1(0.5l_{ef}) = \varepsilon_i \\ a_1 \cdot \frac{(0.5l_{ef})^2}{2} + \varepsilon_0(0.5l_{ef}) + \varepsilon_i l_d = \varepsilon_{mi} \cdot (0.5l_{cr}) \end{cases} \quad (9)$$

The obtained coefficient $a_1$ value is constant for the same reinforcement diameter regardless of the reinforcement ratio.

6. After the shape coefficient and the minimum strain are determined, bond stresses can be expressed by differentiating the strain shape function:

$$\tau(x) = \frac{E_s\phi_s}{4}\frac{d\varepsilon_s}{dx} = a_1\frac{E_s\phi_s}{4} \quad (10)$$

Note: the bond stresses for a linear shape function are constant.

7. Following the previously discussed condition of fixing the minimum strains for all other RC ties and ensuring identical bond, the shape function coefficient for different reinforcement diameters can thus be evaluated from the bond equation:

$$a_1 = \frac{4\tau}{E_s\phi_s} \quad (11)$$

8. After obtaining the shape function coefficient, the length of the effective zone and thus the crack spacing are obtained by rearranging the initial shape function expression Eq. (7) :

$$l_{ef} = \frac{\varepsilon_i - \varepsilon_0}{0.5a_1} \quad (12)$$

Since the load level is known, the damage zone length and the crack spacing are easily found from Eqs. (5) and (6). Having the shape function coefficients for chosen reinforcement diameters, the crack spacing values can be found for any other reinforcement ratio value of interest to the designer.

***Procedure for the parabolic shape function***

The procedure for the parabolic shape function requires slight changes and further steps to obtain a crack spacing model. The initial steps and assumptions are the same as those described for the linear case. The difference is the nature of the shape function, for which the initial steps produces the following system of equation in order to find the coefficient and minimum strain values:

$$\begin{cases} \varepsilon_0 + a_1(0.5l_{ef})^2 = \varepsilon_i \\ a_1 \cdot \frac{(0.5l_{ef})^3}{3} + \varepsilon_0(0.5l_{ef}) + \varepsilon_i l_d = \varepsilon_{mi} \cdot (0.5l_{cr}) \end{cases} \quad (13)$$

1. The bond stresses are not constant for the second order polynomial shape function and are equal to:

$$\tau(x) = \frac{E_s\phi_s}{4}\frac{d\varepsilon_s}{dx} = 2a_1x\frac{E_s\phi_s}{4} \quad (14)$$

2. The main difference between the procedures for the linear and parabolic shape functions is that the latter depends on the distance from the centre of the investigated section. In this case it is not possible to find the coefficients for other bar diameters directly and further steps are necessary. To keep the bond identical between investigated RC ties, the distribution of slip between reinforcement and concrete over the effective length is required. This distribution is found by integrating the reinforcement strain shape function. For simplicity, concrete strains are neglected:

$$s(x) = \int \varepsilon_s(x)dx = \varepsilon_0 x + \frac{1}{3}a_1x^3 \quad (15)$$

3. Since both the bond and slip distributions vary with distance, a unique bond-slip relationship $\tau(s)$ can be derived by equating Eqs. (14) and (15). The steps for a parabolic shape function are summarized in Figure 4.

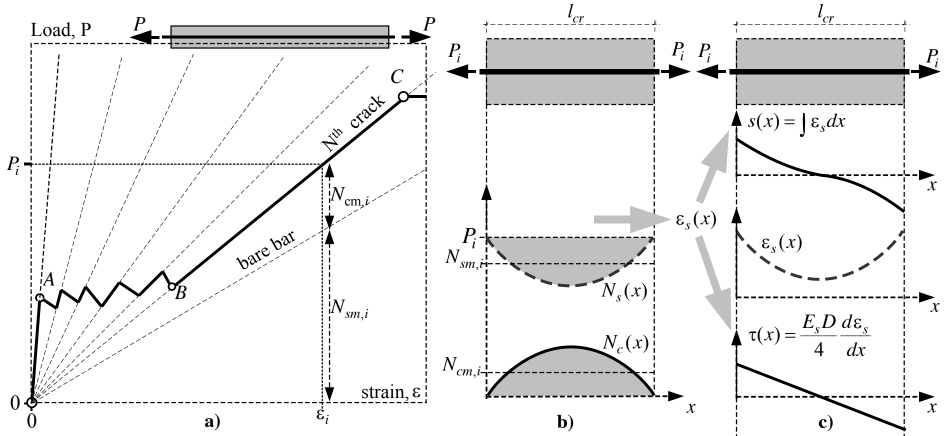


Figure 4 The proposed bond modelling method: a) average concrete and steel forces; b) shape functions; c) calculation of slip and bond stress distribution

4. In contrast to the linear shape function, the coefficients and the effective, damage zone and thus crack spacing lengths are found numerically by using the stress transfer approach and applying the obtained bond-slip relationship $\tau(s)$. With the obtained shape function coefficients for other RC ties, same as for the linear case, average distance between cracks $l_{cr}$ can now be obtained.

## RESULTS OF CRACK SPACING ANALYSIS

Reinforced concrete ties with identical material properties were further investigated using the linear and parabolic strain shape functions following the procedures described above. The concrete section was taken as a square with side dimensions determined from the investigated reinforcement diameters and ratios. The reference element for all further analyses was a 100mm by 100mm cross-section RC element, with Ø14mm reinforcement, 1.54% reinforcement ratio and crack spacing $l_{cr}$ = 161.2mm, which is the mean crack spacing value from available experimental data for a bar diameter of 14mm. Concrete compressive strength was taken as 28MPa and the tensile strength was 2MPa. The load level was taken as a pseudo service load equal to 300MPa induced stress in the reinforcement, which has a yield strength of 500MPa. The external force $P_i$ was assumed to be applied directly to the reinforcement bar. A comparison of the distribution of strains obtained for several selected reinforcement diameters and reinforcement ratios is presented in Figure 5. The different behaviour is clearly visible between the linear and parabolic shape functions. One important aspect to note is the different minimum reinforcement strains. The constant deformations represent the damage zone, for which the assumption of a fully damaged bond was made.

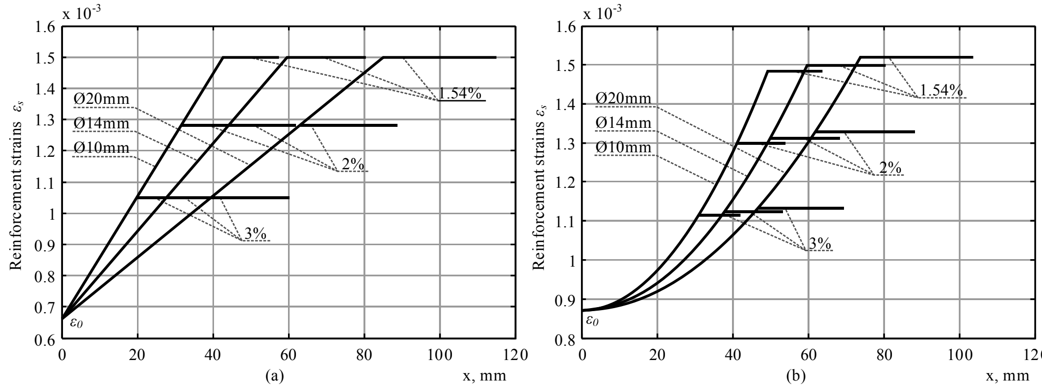


Figure 5 Distribution of strains for the a) linear and b) parabolic strain shape functions

The average distances between cracks obtained by the linear and second order polynomial models are presented in Figure 6. The results are plotted against available published experimental data. The experimental results in the literature exhibit considerable scattering, and the experimental mean of this data is shown for comparison purposes. The numerical analyses results represent a RC tie of 100mm by 100mm dimensions with varying reinforcement ratio. A reinforcement bar diameter $Ø_{ref}$ =14mm and reinforcement ratio $\rho_{ref}$ = 1.54% was chosen as the reference case for which the crack spacing value was assumed to be known and equal to $l_{cr}$ = 161.2mm. Both proposed approaches yield favourable results but with a slightly different behaviour. The parabolic model coincides very well with the mean experimental data for larger reinforcement diameters and is acceptable for smaller diameters. The linear model behaviour is noticeably different and does not coincide as well as the parabolic one but it has the advantage of being a simpler approach, which is easier to apply for predicting cracks.

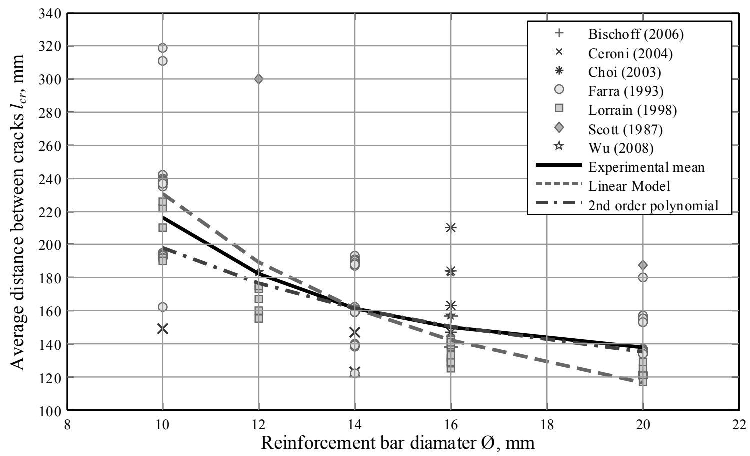


Figure 6 Average distance between cracks obtained by the linear and parabolic models compared against experimental results

Crack spacing models were derived for both shape function cases. Following the steps described above, two equations were derived for the linear and parabolic shape functions, respectively, to estimate the crack spacing of RC ties:

$$l_{cr,linear} = 17.8Ø \cdot \rho^{-1.026} + e^{0.454\rho} \quad (16)$$

$$l_{cr,parabolic} = 10.55Ø \cdot \rho^{-0.6} + e^{4.16\rho^{-0.181}} \quad (17)$$

It should be noted that the derived models are valid for the specific chosen reference element, changing the reference element properties would provide a different crack spacing model.

## PARAMETRIC STUDY

A parametric study was carried out in order to demonstrate the influence of concrete tensile strength and load level on the average crack spacing. The basic properties and parameters are kept the same as in the previous section. In the study, both the linear and parabolic models were investigated. The concrete tensile strength $f_{ct}$ was fixed at 2MPa when varying the load level, while a load level of 300MPa was used as the base when exploring the impact of varying tensile strength. The findings clearly show that both of the tested variables influence the crack spacing but the tensile strength of the concrete does not have a significant impact on the results when compared to varying the load level. The effect of different load levels is more pronounced and is more visible for reinforcement bars of larger diameter. In contrast to the curves obtained by varying the tensile strength of concrete, where a larger spread is visible with smaller diameter bars. It is important to note, that certain combinations of variables like material properties, load levels, average strains and the initial assumption of crack spacing for the reference element lead to cases where results cannot be obtained due to the inability to find coefficients for the shape functions that are physically viable. This is the case with concrete tensile strength greater than 2.5MPa and 3.0MPa for linear and parabolic models respectively. Figure 7 summarizes the findings of the parametric study.

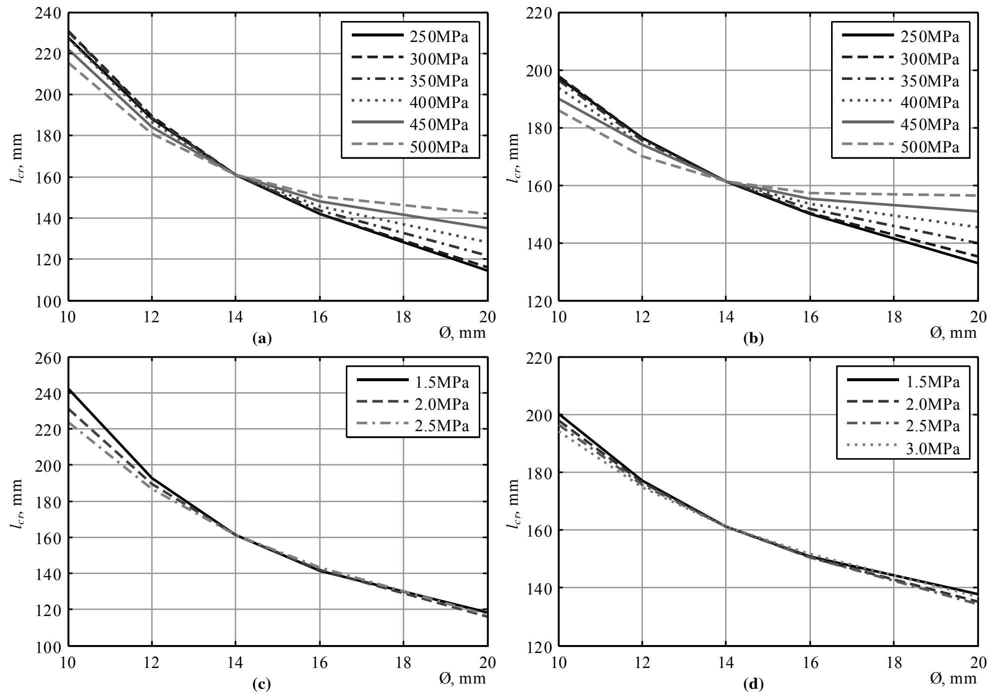


Figure 7 Results of parametric analysis for linear and polynomial models: a) linear model with a varying load level, b) parabolic model with varying load, c) linear model with varying concrete tensile strength, and d) parabolic model with varying concrete tensile strength

## CONCLUSIONS

A new method for developing crack spacing models based on compatibility of the smeared crack and stress transfer approaches has been proposed. The concept relies on the average deformations of the element at a chosen loading level within the stabilized cracking stage, which can be obtained by any smeared crack approach, while further calculations rely on the stress transfer concept to attain the deformation shape function. The new approach has many benefits that are important to the field of structural engineering. The proposed approach combines bond-slip relationship, cracking and deformation models to develop the crack spacing model. The suggested method is transparent, particularly for the linear shape function case which requires few steps and consists of simple calculations that also conveys the physical meaning in a clear way. Very little initial data is required, essentially parameters of only one key element with a reference diameter, reinforcement ratio and crack spacing are needed in order to derive the crack spacing model for RC ties with other geometrical and reinforcement configurations. This method allows the user to develop unique models based on custom requirements, by introducing additional assumptions such as changing the shape functions and the damage zone models. Therefore the proposed concept can act as a very flexible base platform for further crack spacing investigations of RC ties and can be potentially modified and applied to elements with unconventional reinforcement.


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