NUMERICAL STUDY ON LDL^T DECOMPOSITION-BASED DAMAGE LOCATING VECTOR METHOD FOR TRUSS STRUCTURES

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ABSTRACT

Real-time structural health monitoring is very important for truss structures especially those having large-spans. In recent years, many methods have been proposed for damage monitoring of truss structures. However, damage sensitivity of these methods is still required to be improved. In this work an efficient damage localization technique for truss structures is proposed, which is based on the LDL^T decomposition of the flexibility difference matrix and the Damage Locating Vectors (DLV) method. Compared with the present Stochastic DLV (SDLV) method, the proposed method is modified in two ways. First of all, the way of calculating the damage locating vectors is modified by using LDL^T decomposition instead of Singular Value Decomposition. Secondly, in order to compute the flexibility, the mass matrix which is obtained from the finite element model is used to mass-normalize mode shapes identified from ambient excitations. As a result, the proposed LDL^T-DLV method has a higher sensitivity to damage for different types of truss members. The effectiveness of the proposed LDL^T-DLV method is validated with the numerical example of a laboratory-scale Bailey truss bridge.

KEYWORDS

Damage localization, damage localization, LDL^T-DLV method, truss structure, structural health monitoring.

INTRODUCTION

Monitoring-based structural condition assessment has gained in popularity in the past years, because it can provide abundant information on the structural condition through different kinds of sensors (Ni et al 2012). Vibration-based damage detection is a challenging and important research topic. The fundamental elements of damage detection algorithms are damage localization, damage quantification, and the effects of local damage on the global performance (Dorvash et al 2014). The interest to monitor a structure for purpose of damage detection at an early stage is prevailing in the fields of civil, mechanical and aerospace engineering (Li et al 2013).
Lots of vibration-based damage detection methods (Ni et al 2001; Lei et al 2013; Min et al 2015; Blachowski et al 2015; Cao et al 2015; An et al 2015) have been developed in the past decades. Among these methods, the flexibility-based technique (Pandey and Biswas 1994; An and Ou 2013a) has attracted considerable attention. It is worth to mention that the DLV method (Bernal 2002) is one of the most important flexibility-based methods for damage localization of truss structures. Gao (2005) verified this method using experimental data from a laboratorial truss model. However, the DLV method requires the knowledge of input excitations, which limits its applications. To address this problem, Bernal (2006) proposed the Stochastic DLV (SDLV) method which allows applying the DLV method without the knowledge input excitations. Nagayama et al. (2009) conducted a study based on the SDLV method on a truss structure, in which a lower cord is replaced with an element having a 52.7% cross section loss to simulate the damage. An et al. (2014) conducted a study based on the SDLV method to further investigate the influence of different formulations of the observation matrix on the accuracy of damage detection results. These studies presented great progress, but challenges remain; for example, the damage sensitivity is still required to be improved.
To improve the damage sensitivity of the present DLV method, this paper proposes a damage localization method based on the DLV method and the LDL^T decomposition of the flexibility difference before and after damage. To validate the proposed damage localization method, numerical validation of a laboratory-scale simply-supported steel-truss bridge has been conducted. Some initial conclusions are summarized.

THE PROPOSED LDL^T-DLV METHOD

This section proposes the LDL^T-DLV method based on the LDL^T decomposition, DLV method (Bernal, 2002) and the SDLV method (Bernal, 2006). However, we introduce two modifications. First of all we modify the way of calculating damage locating vectors by using LDL^T decomposition instead of Singular Value Decomposition. Secondly, we use the mass matrix of the structure under consideration to mass-normalize mode shapes identified from ambient excitations.

Change in Flexibility

The flexibility matrix $\mathbf{F}$ of any structural system can be expressed by its modal parameters (natural frequencies and mode shapes)

$$\mathbf{F} = \mathbf{K}^{-1} = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \boldsymbol{\varphi}_i \boldsymbol{\varphi}_i^T \tag{1}$$

where the input vector $\boldsymbol{\varphi}_i$ is the *i*-th mass-normalized mode shape; ω_i is the *i*-th modal frequency, and *n* is the number of degrees of freedom of the structure. In this paper, the mass-normalized mode shapes are obtained using the analytical mass matrix derived by the FE model updating.
After the mode shapes and modal frequencies are identified, the change-of-flexibility matrix, denoted by $\mathbf{F}_\Delta$, can be computed as

$$\mathbf{F}_\Delta = \mathbf{F}_\mathrm{H} - \mathbf{F}_\mathrm{D} \tag{2}$$

where $\mathbf{F}_\mathrm{H}$ and $\mathbf{F}_\mathrm{D}$ are the flexibility matrices of the undamaged and the damaged structure, respectively.

LDL^T Decomposition of $\mathbf{F}_\Delta$

We can determine LU matrix decomposition of $\mathbf{F}_\Delta$. However, since $\mathbf{F}_\Delta$ is symmetric indefinite matrix LU decomposition takes the form of LDL^T decomposition, which is special form of Cholesky decomposition. Moreover LDL^T has better than LU interpretation from damage detection point of view.
So substituting LDL^T for difference in flexibility, we get

$$\mathbf{F}_\Delta = \mathbf{LDL}^\mathrm{T} \tag{3}$$

where $\mathbf{L}$: lower unit triangular matrix, $\mathbf{D}$: diagonal matrix of the form.
In the typical task for damage detection some of values on the diagonal $d_{i,i}$ are equal or close to zero. This means that the original matrix $\mathbf{F}_\Delta$ is rank deficient and close to zero values in diagonal of $\mathbf{D}$ matrix indicate null space of $\mathbf{F}_\Delta$.
So, to find number of independent Damage Locating Vectors (or dimension of the null space of difference in flexibility) we select those columns of $\mathbf{L}^{-\mathrm{T}}$ which corresponds to close to zero values in $\mathbf{D}$ matrix. If we order these values at the end of $\mathbf{D}$ matrix we can write it formally as

$$\mathbf{F}_\Delta \mathbf{L}^{-\mathrm{T}} = \mathbf{L}\begin{bmatrix}\mathbf{D_1} & \mathbf{0}\end{bmatrix} \tag{4}$$

In order to select the vectors which belong to the null space, the following approach is proposed:

1) Calculate the $\mathbf{L}$ and $\mathbf{D}$ matrices from $\mathbf{F}_\Delta$ using a LDL^T decomposition (for example with the ldl() function in MATLAB).
2) Then, calculate the inverse matrix of transpose of lower unit triangular matrix $\mathbf{L}^\mathrm{T}$.
3) Determine auxiliary matrix $\mathbf{V} = \mathbf{LD}$.
4) Calculate norm of individual columns of $\mathbf{V}$ and then remove those columns of $\mathbf{L}^{-T}$, which correspond to the values of norm of $\mathbf{V}$ greater than some tolerance value τ these columns of $\mathbf{L}^{-T}$ are not belonging to null space of $\mathbf{F}_\Delta$, so they are not DLVs.
5) Denote the number of remaining columns of $\mathbf{L}^{-\mathrm{T}}$ as *p*,

$$p = \dim(\mathbf{L}^{-\mathrm{T}} : \frac{\|\mathbf{V}_i\|}{\|\mathbf{V}_i\|_{\max}} < \tau) \tag{5}$$

6) Finally, the present columns of $\mathbf{L}^{-\mathrm{T}}$ refer to damage locating vectors

Normalized Accumulated Stress

All the p damage locating vectors are applied as static forces to the FE model. The accumulated stress index is defined as the characterizing stress in the j-th element normalized by the largest stress over all measured elements,

$$\sigma_j = \sum_{i=1}^{p} \left| \frac{\sigma_{ij}}{\sigma_{ij,\max}} \right| \tag{6}$$

where σ_{ij} : the stress in the j-th element caused by the i-th DLV.

Finally, the normalized accumulated stress index NSI is introduced, and the elements whose accumulated stress index satisfies the following inequality are marked as damaged:

$$NSI = \frac{\sigma_j}{\sigma_{j,\max}} \leq b \tag{7}$$

where p: the number of the Damage Locating Vectors, b: the threshold, whose recommended value is 0.10.

NUMERICAL VALIDATION

Research Object

The truss model considered in this paper, i.e. the DUT Truss Benchmark Model, was previously presented in detail in earlier studies by An and Ou (2013b), so it is only briefly described herein. Its span, width, and height are 8 m, 0.56 m, and 0.9 m, respectively. The process of FE modeling of this structure has been introduced in the earlier study (An and Ou 2013b). The FE model (Figure 1) consisting of 312 beam elements and 108 nodes has been implemented in the MATLAB environment. Attempts have been made to update the initial FE model, and as a result of this procedure, the first two vertical frequencies (19.1 and 52.5 Hz) of the numerical model have been adjusted to the experimental results (18.4 and 53.3 Hz).

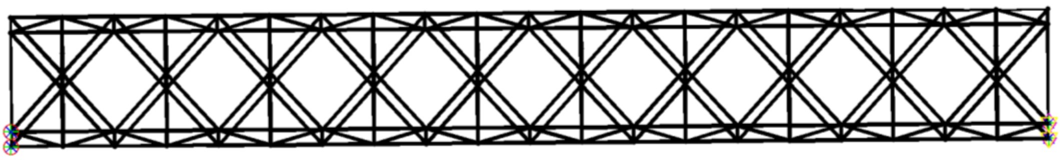

Figure 1 Finite element model of the steel truss model

Damage Cases

The substructure in the rectangle line is selected as the object for investigation. Four numerical damage cases (Table 1) are implemented to identify the corresponding damaged truss members. The sampling frequency of the measurements is 500 Hz.

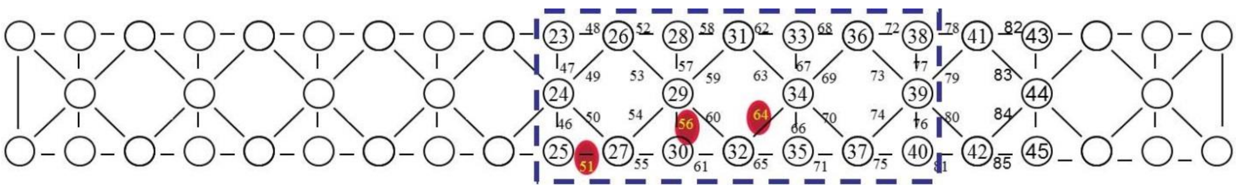

Figure 2 The substructure under investigation

Table 1 Numerical damage cases

Damage case		Stiffness reduction
Single damage cases	1	Member 51#: 20% stiffness reduction
	2	Member 56#: 30% stiffness reduction
	3	Member 64#: 20% stiffness reduction
Multiple damage cases	4	Member 51, 56 and 64: 30%, 35% and 20% stiffness reduction, respectively

Simulations of Damage Localization

In this section, the same accelerations are used in the same damage case to compare the results based on two methods, i.e. the SDLV method, and the LDL^T-DLV method. The truss is excited using a band-limited white noise up to 150 Hz in the vertical direction at node 47. A band-limited white noise with a 5% noise level is added to simulate the measurement noise.

Damage cases 1~3 are single damage cases which simulate damage of a lower chord, a vertical truss member and a lower diagonal truss member, respectively. Damage cases 4 is a multiple damage case which simulate the damage of all damaged members in damage cases 1~3 simultaneously. Nodes 23~40 in Figure 2 are selected for measurements. In accordance with the SDLV method (Bernal, 2006) and the proposed LDL^T-DLV method, it can be seen from Figure 3 that results based on the proposed LDL^T-DLV method are better than those using the SDLV method.

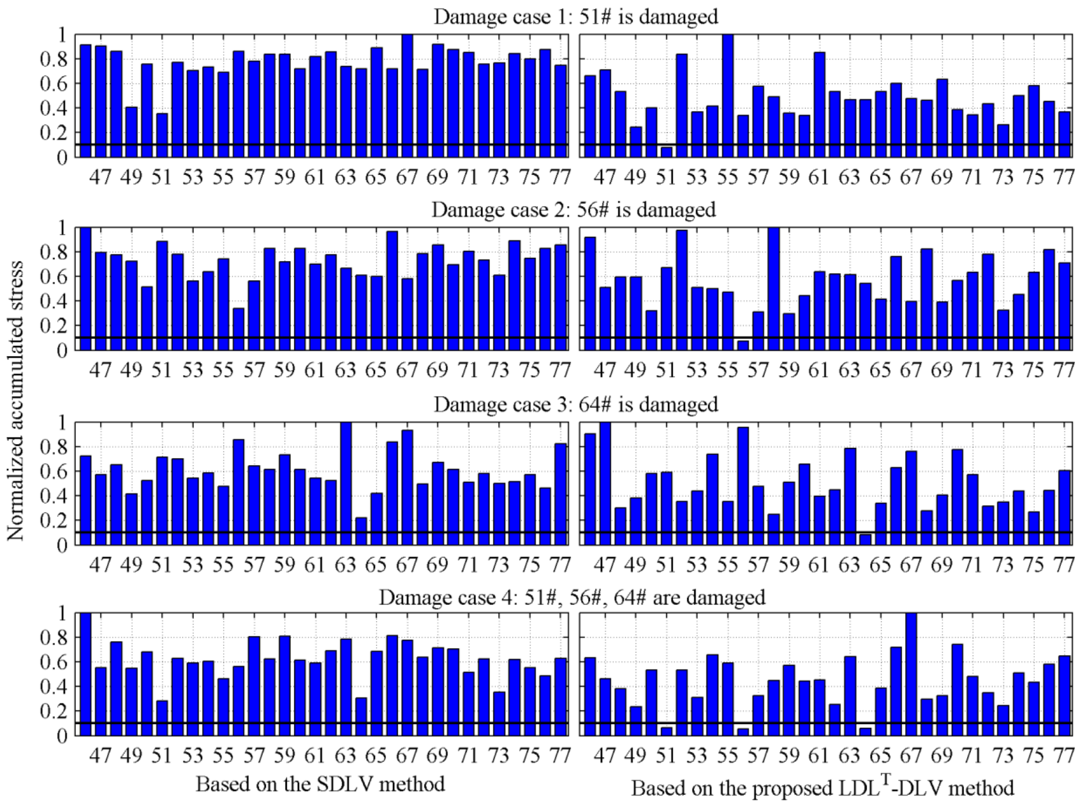

Figure 3 Damage localization results for damage cases 1~4 (5% noise) based on the SDLV method and the LDL^T-DLV method

CONCLUSIONS

This paper proposes a damage localization method for truss structures. The proposed method has been validated using a Bailey bridge benchmark model. The conclusions are summarized as follows:

(1) All the numerical damage localization results indicate that the proposed LDL^T-DLV method can be successfully used in the damage localization of truss structures. Therefore, it can be used in real-time structural health monitoring of truss structures under ambient excitation.

(2) The proposed method has a higher sensitivity to small damage compared with the SDLV method.

(3) Future research will consider the experimental validation of the proposed method based on the DUT Truss Benchmark Model.

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