LONG-TERM DETERIORATION EFFECTS ON THE BUCKLING STRENGTH OF METALLIC BRIDGE GIRDERS

Pam Billy Fom, Boulent Imam and Marios K. Chryssanthopoulos
Department of Civil and Environmental Engineering, University of Surrey,
Guildford, Surrey, GU2 7XH, United Kingdom
p.fom@surrey.ac.uk

ABSTRACT

Bridges are an essential part of the transport infrastructure. A considerable number of these bridges are metallic, in many cases exceeding 100 years of age having suffered deterioration from environmental attack such as atmospheric corrosion. In order for infrastructural managers to make informed decision in terms of life-cycle cost perspective, reliable prediction of the remaining strength and service life of deteriorating bridges is essential. Deterioration models have been developed over the years to predict long-term material loss under different atmospheric conditions and environments. The aim of this paper is to quantify the effects of long-term deterioration, based on these models, on the remaining strength of metallic bridge girders, comprising of a number of plates. To obtain a useful insight into this problem, the finite element method is employed. In this paper, different plate elements, of varying slenderness and boundary conditions and representative of real bridge configurations, are analysed under different deterioration scenarios, brought about through material loss at different locations of the element. The effects of various parameters such as the degree/severity of material loss and the corrosion pattern (uniform versus non-uniform) on the buckling strength of the plates are quantified through both linear eigenvalue and non-linear analyses. The results of this study show that critical buckling strength of web panels may significantly drop at higher percentages of corrosion degradation and patterns, with the failure mode likely to change with increased deterioration. Differences between the critical buckling stresses obtained from the linear and non-linear analyses are presented.

KEYWORDS

Long-term deterioration, buckling strength, metallic bridge, girders, finite element analysis.

INTRODUCTION

Since the start of civilisation bridges for roads, railways, walkways and canals have been built, therefore it is impossible to conceive today's world without bridges (Biezma and Schanack 2007). One of the main construction materials for these bridges is steel. Most metallic bridges are made of carbon and low alloy steel (i.e. steel with carbon content in the range of 0.12-2.0% as the main alloying constituent) due to the competitive initial cost advantage of this material over others like aluminium and stainless steel products (Gardner et al. 2007). Though the later material tends to be gaining recognition recently owing to it its whole-life-cycle cost advantage over time but a large stock of existing metallic bridges are made of carbon steel. Every bridge exists in an environment, and in this environment of existence these bridges suffer from atmospheric corrosion due to exposure to the environment. The long-term deterioration of these metallic bridges is triggered by this exposure condition which is defined by environmental and atmospheric parameters like relative humidity, temperature, pollutants and time of wetness. These parameters expedite the deterioration process including the rate of corrosion (Kallias and Imam 2013). Giving the global challenge of climate change on the environment, it is thought that the environment of existence of these bridges is also changing accordingly which may, in turn, influence the deterioration rates.

Corrosion is one of the important factors that reduce structural capacity during service life (Saad-Eldeen *et al*. 2013). Corrosion can cause three basic changes in steel: loss of material, reduction of section parameters and building of corrosion products (Cruz *et al*. 2006), as shown in Figure 1. Usually, at the design stage, a corrosion thickness allowance is added to compensate for the potential thickness reduction due to corrosion over service life of the structure (Saad-Eldeen *et al*. 2013). Protective coating are also applied to metallic surfaces to prevent or slow down corrosion. However, successful history of steel bridges has not and is not without catastrophes (Biezma and Schanack 2007).

Structural design involves description of structural behaviour and strength prediction. Therefore, the assessment of strength, stiffness and stability requires the evaluation of the structural capacity of the structural components that makes up the structural system. For bridges, plate girders are an important structural component because of their ability in load resistance. Plate girders are mainly made of plates and are provided when hot-rolled sections show insufficient strength or are not economical for design purposes (Cruz *et al*. 2006). Modern plate girders are often made by welding plates together whereas, in older bridges, riveted built-up girders was the main way of constructing them. Corrosion can appear in different forms. Uniform corrosion manifests itself through a uniform thickness reduction across a large area over a structural member. On the other hand, more localised corrosion can appear in terms of pitting corrosion or pit holes. Corrosion can also be non-uniform showing different severity and extent over a structural member. Field experience has shown that the bottom parts of plate-girders can be more susceptible to corrosion losses due to the accumulation of dirt and water over time.

The aim of this paper is to investigate the long-term deterioration effects on buckling strength of metallic bridges using numerical analysis. A range of plate elements, with varying slenderness, representing elements used as part of bridge girders are analysed through linear and non-linear buckling analysis under different deterioration scenarios assumed through material loss at different locations of the element. The effects of various parameters such as the degree/severity of material loss and the corrosion pattern (uniform versus non-uniform) on the buckling strength of the plates are quantified.

Figure 1 Corroded railway plate girder Bronx- Edenwald, USA.

Corrosion Damage Assessment

(BS EN ISO 9223 2012) classifies atmospheric corrosivity into two categories: classification based on corrosivity determination by corrosion rate measurement on standard coupons, and classification based on estimates of corrosivity based on environmental information. This paper considers corrosion damage assessment based on environmental information considering expected potential changes in the climate over the design life span of the bridge, i.e. 120 years. The dose-response function relative to carbon steel was used as presented in equation (1) below.

$$r_{corr} = 1.77{P_d}^{0.52}.\exp(0.020RH + f_{st}) + 0.102{S_d}^{0.62}.\exp(0.033RH + 0.040T) \quad (1)$$

where f_{st}=0.150(T-10) when T≤10ºC otherwise -0.054(T-10), T=Temperature in ºC, r_{corr}=1st year corrosion rate of metal in μ_m/year, RH=relative humidity in %, P_d=SO_2 deposition in $mgm^{-2}day^{-1}$ and S_d=Cl^- deposition in $mgm^{-2}day^{-1}$.

Equation (1) estimates the first year corrosion while equation (2) below, given in (BS EN ISO 9224 2012) estimates corrosion attack (D) beyond the first year.

$$D = r_{corr} t^{b} \tag{2}$$

where t=exposure time in years, b=metal-environment-specific time exponent usually <1.

Equation (2) predicts corrosion loss up to twenty years followed by corrosion product layer protection which slows down the rate of corrosion after the first twenty years. The relationship is exponential within the first twenty years after which it becomes linear. To capture behaviour after the first twenty years Equation (2) is differentiated with respect to time to give the rate of corrosion.

$$\frac{dD}{dt} = b r_{corr} t^{b-1} \tag{3}$$

The corrosion attack from year 20 onwards is given by

$$D(t > 20) = r_{corr}\left(20^{b} + b(20^{b-1})(t - 20)\right) \tag{4}$$

Results from equation (4) were used as a guide for the parametric studies presented in this paper.

PLATE GIRDERS

Plate girders are used in both road and railway bridges, sometimes in buildings when high loads and/or large spans are envisaged. They are made of steel plates which are usually welded or bolted/riveted together to form an I-section (Clarke and Coverman 1987). One advantage of plate girders is that they can be built to any size to meet with design requirement but, their geometrical proportions are usually governed by web buckling. Stiffeners are used to strengthen the web for increased allowable buckling stress. For long span construction where rolled beams are not adequate, generally the choice is between the use of compound beams or plate girders. In modern bridges where continuous construction is used to reduce maximum moments, plate girders may not be required until the span exceeds about 25m. Generally, these built-up standard beams are more economical when they are below 25m span. But currently, plate girders spanning 60-100m are in use owing to the introduction of automatic welding techniques and the development of steels which allow the use of normal procedure for welding thick plates thus reducing fabrication cost. Box girder decks are also made of slender plates welded together. Older bridges are made up of a number of plate elements and/or angles riveted together to form I-sections or other cross-sectional types used for example in truss bridges as top/bottom chords, hangers and diagonal members.

BUCKLING BEHAVIOUR OF PLATE GIRDERS

Buckling is a nonlinear phenomenon whereby a structure cannot any further take up load with its original geometry and so it changes its shape in order to find an alternative equilibrium position (Sosa *et al*. 2006). The way in which buckling occurs depends on how the plate is loaded, its geometrical and material properties as well as the boundary conditions. If the plate is loaded such that most of its strain energy is in the form of membrane compression, and if there exist an opportunity that this stored-up membrane energy can be converted into bending energy, buckling occurs (Bushnell 1981). Usually, plate membrane stiffness is by far greater than the bending stiffness and a thin plate can absorb most of the membrane energy without deforming too much, but for it to absorb an equivalent amount of bending strain energy it must deform much more (Bushnell 1981). So generally, when large deformations exist it converts the membrane energy into bending energy which results in buckling.

FINITE ELEMENT MODELLING
Preamble

Numerical methods, mainly finite element analysis (FEA), remain the most advanced tool in rational structural analysis which makes it possible to predict the strength of complex structures more accurately when compared to classical theoretical methods (Ok *et al*. 2007; Saad-Eldeen *et al*. 2013). Therefore, this section deals with the aspect of linear and nonlinear FEA used to evaluate the typical response of uncorroded and corroded plate girders. As mentioned earlier, plate girders can be built to any size but the geometrical proportion is being governed by web buckling. This paper presents results of corrosion degraded web plates under compressive uniaxial loads.

Problem Description

Three different models differentiated by their slenderness (width to thickness) (b/t) ratio were analysed. The models were designated as slender, intermediate and stocky with slendernesses equal to 107, 71 and 28 respectively. Generally, the models had an aspect ratio of 2 based on the plate dimensions of 856mm × 428mm and thicknesses of 4, 6 and 15.3mm respectively. The material is assumed to be isotropic elastic with young modulus (E) of 206,000N/mm^2, Poisson's ratio (ν) of 0.3, yield stress (σ) of 275N/mm^2 and ultimate strength of 485N/mm^2. The material stress-strain law adopted herein followed the 2nd distribution on Figure 5. Figure 5 defines the frequently used material constitutive law in numerical studies, and these are generally assumed to represent the actual behaviour of structural steel (Boissonnade and Somja 2012).The boundary conditions on the model are simply supported therefore a buckling coefficient (K) of 4 is used.

Methodology

The problem at hand is a stability issue and nonlinear stability studies will require two types of analyses: linear eigenvalue analysis and postbuckling or collapse analysis. The eigenvalue analysis is a required step for the more general collapse or load-displacement response. The general purpose nonlinear finite element commercial code ABAQUS/Standard is used for the analysis (Simulia 2014). Since corrosion degradation leads to section thickness loss, one of the nonlinear behaviours expected is geometric nonlinearity which may lead to large displacement effects. So to activate the large-displacement formulation capability, ABAQUS requires that in the *STEP module, the general static Riks procedure type should be engaged.

Including nonlinear effects in an ABAQUS simulation involves defining either geometric, boundary or material nonlinearities. Geometric nonlinearity include all nonlinear geometric effects due to large deflections, rotations, deformations, preloading and load stiffness. If the mentioned effects are not significant, the predicted model response will be as if the *Nlgeom in the step module were set to off. Boundary nonlinearity involves defining contact interactions between bodies. While modelling material nonlinearity involves defining material properties like plasticity and hyperelasticity. Results in this paper were computed taking into account both geometrical as well as material non-linearity in order to capture the elasto-plastic behaviour of the analysis.

FE MODEL VALIDATION

Eigenvalue Analysis

To validate the FE models used in this study, two versions of each model i.e. slender, intermediate and stocky plates were used: first plate to conduct the eigenvalue buckling analysis and the second to carry out the Riks control analysis.

The eigenvalue buckling analysis was used to obtain estimates of critical load (limit load) at which the response of the structure will bifurcate, assuming that the response prior to bifurcation is essentially linear. The aim was to investigate singularities in a linear perturbation of the structure's stiffness matrix thereby understanding the structure's maximum sustainable load. The eigenvalue analysis was also useful in providing guidance about the finite element mesh design, element type selection and the appropriate boundary conditions to be used.

Results from the linear eigenvalue FEA were compared to buckling theory through

$$\sigma_{cr} = \frac{K\pi^2 E t^2}{12(1-\vartheta^2)b^2} \tag{4}$$

where σ_{cr} =critical buckling stress, k=buckling coefficient, E=modulus of elasticity, t=thickness, b=width, ϑ=Poisson's ratio

The buckling mode shapes were also investigated to see how consistent they are with typical buckling modes, as shown in Figure (2).

Table 1 presents results from these analyses. As it can be seen, very good agreement between the FE results and the theoretical values is obtained validating the way the plate was modelled in the FE software and the number of elements used as a mesh.

Riks Control Analysis

The next phase of the validation process was performing the full nonlinear load-displacement analysis on the same models (As-new) to investigate whether the eigenvalue buckling prediction already obtained was accurate and at the same time check whether the model is imperfection sensitive. The Riks control analysis is one that overcomes the limitations of both the load and displacement control analyses. The load control analysis can only provide response up to the limit load giving solver limitation of matrix inversion, while the displacement control takes the response beyond the limit load to capture elasto-plastic behaviour in the direction of positive displacement however, limited by snap-back response. However, the Riks analysis is able to provide response past these limitations using a nonphysical variable called the arc-length which enables the postbuckling and collapse behaviour to be captured. Therefore, in this analysis both geometric and material nonlinearities were factored in to improve model fidelity. Table (2) presents results from the nonlinear analysis while Figures (3) and (4) show the load-displacement response for the as-new plates. As can be seen, non-linearity, both material as well as geometric, plays an important role in reducing the critical buckling strength of a plate. This shows the importance of the need to carry out non-linear analysis as it demonstrates that by running solely linear eigenvalue analyses, this will not provide a realistic representation of the buckling strength reduction of plates of these proportions.

Table 1 Linear eigenvalue critical buckling stress

Plate ID	Slenderness (b/t) ratio	Critical buckling stress (N/mm^2)		% Difference	Number of Elements	Element type
		Theoretical	Finite Element			
Slender	107	65.1	63.0	-3.2	7040	C3D20R
Intermediate	71	146.4	141.4	-3.4	7040	C3D20R
Stocky	28	952.0	903.1	-5.1	6400	C3D20R

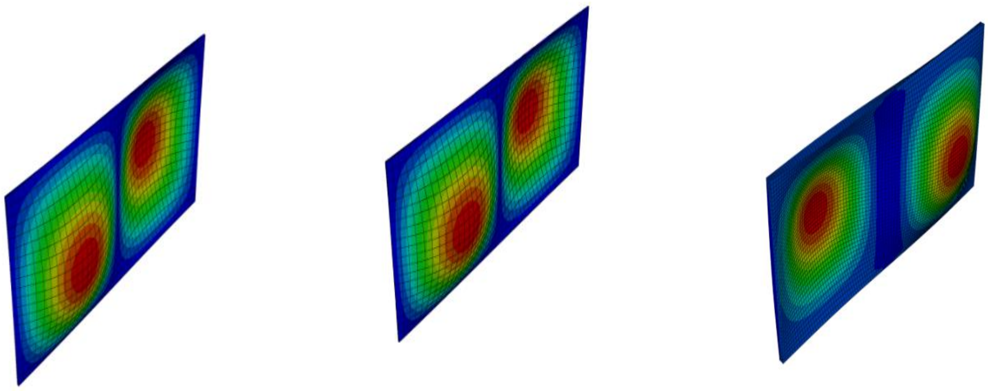

Figure 2 FE mode shapes for slender, intermediate and stocky plates respectively

Table 2: Comparison of linear and nonlinear critical buckling stress

Model ID	Slenderness (b/t)	Critical buckling stress (N/mm^2)			
		Theoretical	Linear eigenvalue	Nonlinear value	% difference between linear & nonlinear
Slender	107	65.1	63.0	42.5	-32.6
Intermediate	71	141.4	141.4	94.3	-33.3
Stocky	28	952.0	903.1	280.0	-68.9

PARAMETRIC STUDIES

In other to assess the long-term reduction of buckling strength due to corrosion effects, parametric analyses were carried out on corroded models, see Figure (7) for geometric parameters. Plate section thickness was considered to be lost at 10%, 30%, 50%, 70% and 90% of the original thickness (t) of each plate model of slender, intermediate and stocky. The models were considered under uniform corrosion scenario as well as non-uniform corrosion scenario. The models were loaded in uniaxial compression as a uniform pressure. In order to capture the non-uniform corrosion patterns on the FE models, solid elements have been used for the analysis, due to the inability of shell elements to model such patterns. The static, Riks algorithm procedure was followed for a geometric and material nonlinear analysis with 10% initial geometric imperfection, following the first eigenmode, as a function of the plate thickness in order to initiate buckling. As validated, the mesh design sensitivity was between 20-25 seeds generating between 6000-9000 elements. The Kirchoff theory for thin plates is assumed considering large displacements and small strains.

For the non-uniform corrosion scenario, two parametric conditions were considered. The first one was the ratio between corroded depths (b_c) over whole depth (b) i.e. b_c/b (see Figure 7) is varied in percentages of 10%, 30%, 50%, 70%, 90% and 100% while the thickness loss (t_c) is kept constant at 30% in one occasion and 50% in another. The second condition is where the aforementioned condition was reversed at similar percentages. Tables (3) & (4) present results from the uniform corrosion scenario whereas Table (5) shows the results from the non-uniform corrosion scenario.

DISCUSSION OF RESULTS

Combining results of the rate/degree of corrosion as well as the type, the following predictions were made on the elastic and post-buckling critical buckling stress of the web panel. From the results presented in Tables 3, 4 and 5, it can be seen that the critical buckling stress of web panel drops with increasing thickness and corrosion depth damage. A 30% loss of plate thickness can be seen to result in about 50% decrease of the buckling strength for the slender and intermediate plates. Even a 10% loss in thickness may result in 20% decrease of strength, demonstrating the fact that even low levels of corrosion loss can have a significant effect on buckling strength. It is observed that the behaviour of the slender and intermediate plates are similar in Figure (6) and both failed due to buckling, whereas, the stocky plate show yielding failure behaviour. Yielding failure is ductile in nature, therefore it provides warning before collapse (Kayser and Nowak 1989). Of course, the stocky plate's estimated critical strength is greater than the material strength hence the yield failure. But as it is damaged by corrosion an increase in slenderness (b/t) ratio is observed over time (see Table (3)) and once its slenderness approaches the intermediate to slender regime, its behaviour changes accordingly leading it to fail due to buckling.

Corrosion loss is also seen to reduce the geometrical section properties because of the reduction in section area. This happens in a nonlinear fashion because such properties like second moment of area (I) or radius of gyration (r) are related to the square or cubic dimensions (Kayser and Nowak 1989). Table (3) shows that stiffness is lost at almost the same percentage with the deterioration, hence the loss in buckling capacity as these geometrical stiffness functions are affected. Again, the shear resistance of the web may be affected because of the corrosion degradation and the web is known to majorly sustain the shear force. Because of this web function, it is usually designed to operate at elastic nonbuckling stress levels (Kayser and Nowak 1989). Therefore, if the failure mechanism changes from yielding to buckling as seen in Tables (3) & (4) due to section loss for the stocky plate, shear capacity might degenerate rapidly more so that the section classification might change. Hence, this demonstrates the need for further investigating in detail shear behaviour as well to understand more deeply the instability phenomena.

The bearing capacity of a plate may be affected by corrosion. Bearing forces are primarily resisted by the web, immediately above the support (Kayser and Nowak 1989). This is why webs are generally reinforced by stiffeners especially above intermediate supports. At the time of construction a girder may not require a stiffener but when affected by corrosion over time, stiffeners are very necessary to maintain structural integrity. This is what a nonlinear analysis like this aimed to achieve: to detect the critical web region and what strength reduction is possible.

Table 5 shows the critical buckling stresses obtained for the non-uniform corrosion scenarios, assuming that only part of the plate deteriorates, as shown in Figure (7). Such tables may be extremely useful for the purposes of bridge element assessments, where following inspections and identification of corrosion patterns, bridge managers can estimate the potential loss of buckling strength through them. With additional analyses capturing the entire range of thickness and depth corrosion, as well as considering different plate dimensions (and therefore slendernesses), a number of spectra mapping buckling strength loss under different scenarios can be obtained.

The corrosion scenarios investigated in this paper have been done so in an incremental manner (see Tables (3) to (5)) to review the full range of variation of the buckling strength with respect to corrosion. Use of Equations (1) to (4) can aid towards predicting the effects of climate change on buckling strength through investigating the influence of changes in the environmental parameters and/or atmospheric pollutants over time. By investigating different scenarios of changes in environmental conditions, the potential increase in corrosion loss can be quantified through these equations and use of Tables similar to (3), (4) and (5) will determine the reduction in the buckling strength brought about by the former.

Table 3 Critical buckling stress for uniform corrosion scenario at 10% initial imperfection

Corroded thickness (t_c) %	Critical buckling stress (N/mm^2), b/t & I in (10^{-5} or $*10^{-6}$(m^4))								
	Slender	b/t	I	Intermediate	b/t	I	Stocky	b/t	I
0.0t	42.5	107	2.61	94.3	71	3.92	280.0	28	9.99
0.1t	34.2	119	2.35	77.5	79	3.53	276.3	31	8.99
0.3t	20.9	153	1.83	47.1	102	2.74	275.0	40	6.99
0.5t	11.3	214	1.31	24.2	143	1.96	159.1	56	4.99
0.7t	4.1	357	7.84*	8.86	238	1.18	55.8	93	2.99
0.9t	0.46	1070	2.61*	0.97	713	3.92*	6.45	280	9.80*
t	0.00			0.00			0.00		

Table 4 Reduction factors for uniform corrosion scenario

Corroded thickness (t_c) %	Normalised critical buckling stress		
	Slender	Intermediate	Stocky
0.0t	1	1	1
0.1t	0.81	0.82	0.99
0.3t	0.49	0.49	0.98
0.5t	0.27	0.26	0.57
0.7t	0.09	0.09	0.19
0.9t	0.01	0.01	0.02
t	0.00	0.00	0.00

Table 5 Critical buckling stress for non-uniform corrosion scenario

Depth reduction (b_c/b) %	Critical stress (N/mm^2) Slender		Critical stress (N/mm^2) Intermediate		Critical stress (N/mm^2) Stocky	
	0.3t	0.5t	0.3t	0.5t	0.3t	0.5t
0.1b	45.0	45.8	102.9	87.5	273.7	273.0
0.3b	36.3	31.9	85.0	60.0	268.4	262.4
0.5b	32.1	26.4	71.1	51.5	267.8	259.9
0.7b	29.9	21.9	65.5	43.1	250.0	254.4
0.9b	25.0	14.8	55.0	33.6	242.1	186.4
b	20.6	11.4	48.6	24.3	165.8	159.1

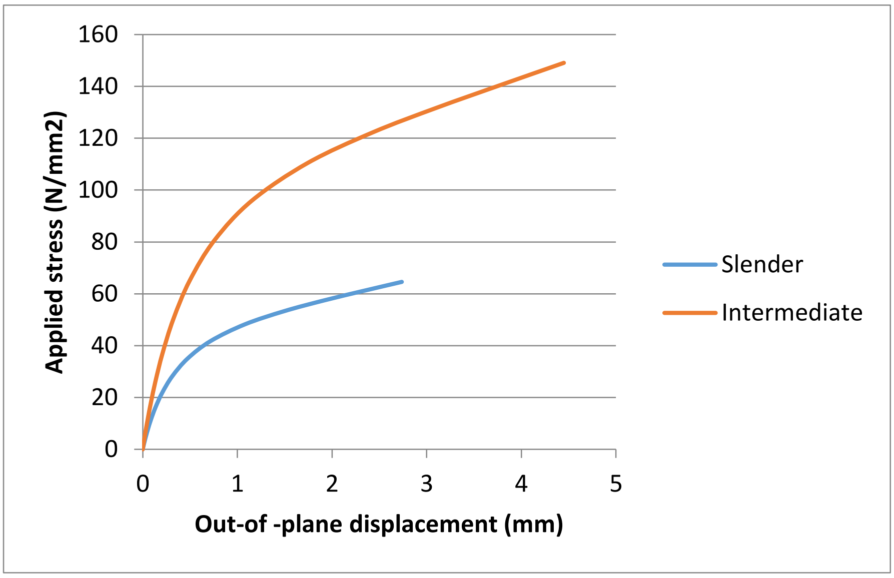

Figure 3 Load-displacement response for slender and intermediate plates

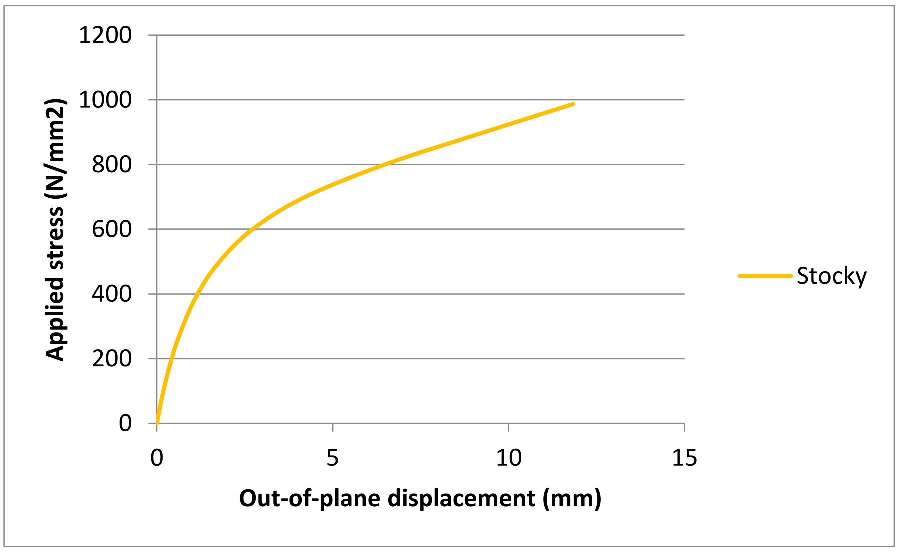

Figure 4 Load-displacement response for stocky plate

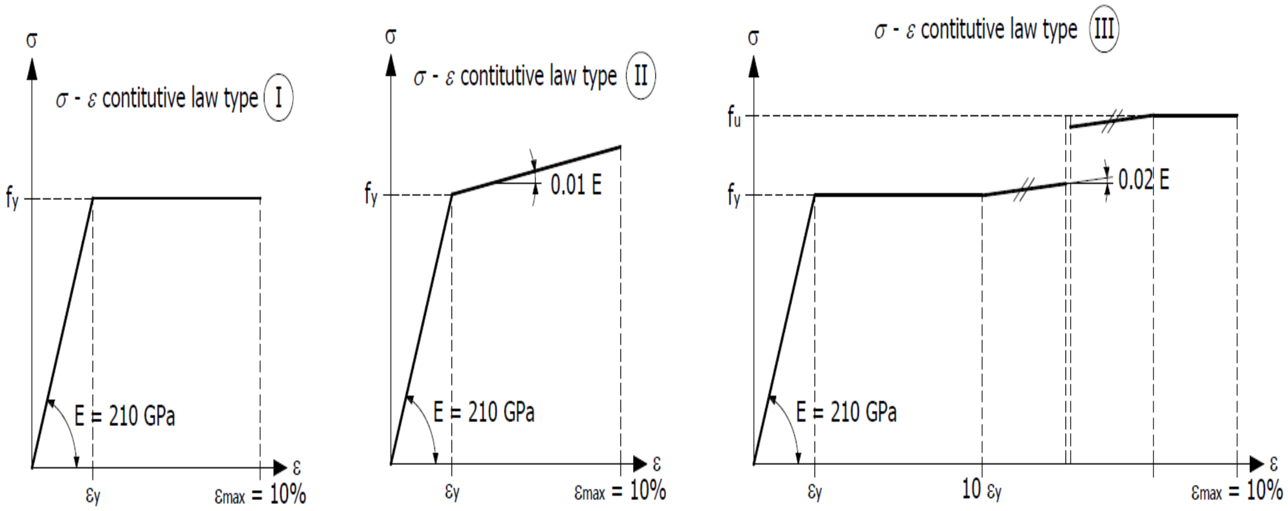

Figure 5 Typical steel material constitutive law

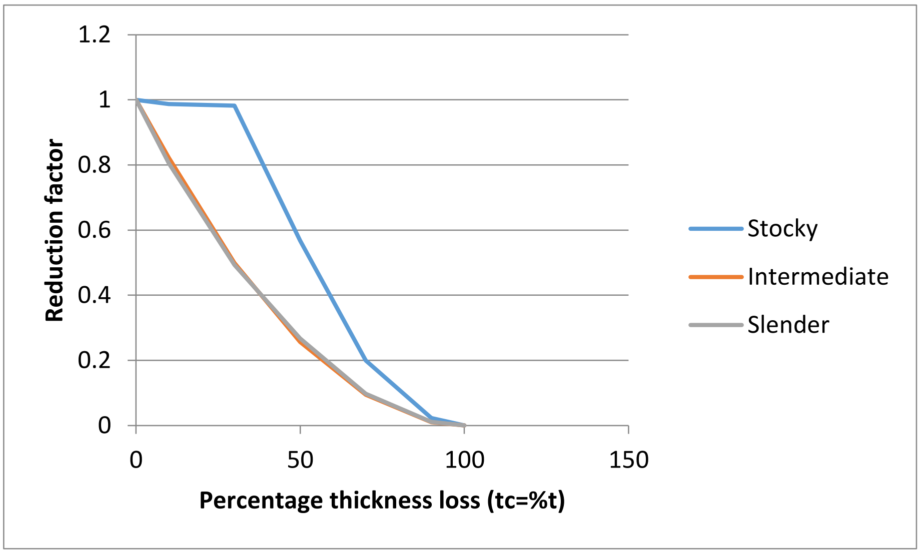

Figure 6 Reduction in buckling capacity of plates due to uniform corrosion

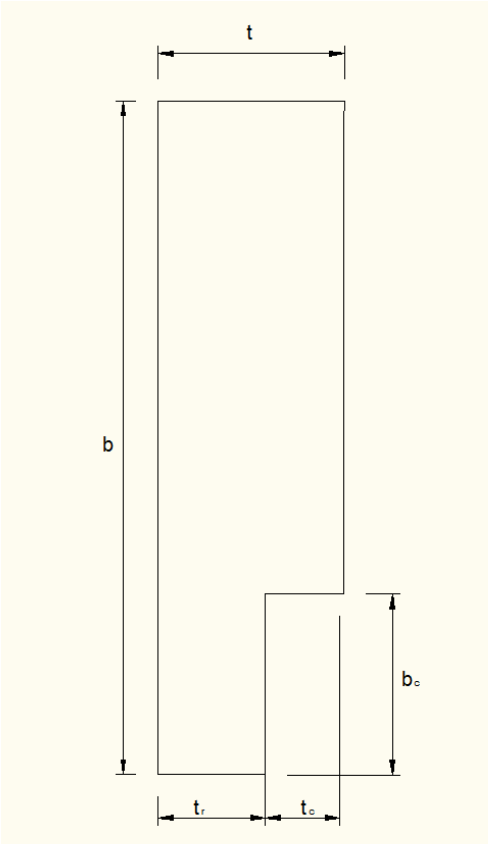

Figure 7 Geometric parameters of a non-uniformly corroded web plate

CONCLUSIONS

The long-term deterioration effects on buckling strength due to structural exposure to changing environmental climate was investigated parametrically by combining information about the structure's location and rate of corrosion with FE analysis approach. Corrosion which results from such exposure can have a significant effect on compression strength because of the possibility of buckling. Plate elements made of carbon steel, typically representative of elements found on existing metallic bridges, subjected to uniaxial compression load, were studied in this paper.

Numerical analysis results illustrate that critical buckling strength of web panels significantly reduce at higher corrosion degradation. For example, 30% thickness loss in uniform corrosion can lead to a 50% strength loss in slender and intermediate plates but only 2% for the stocky plate. According to the obtained results the behaviour of slender and intermediate plates is similar and both fail in buckling, while the stocky plate illustrates yielding failure behaviour. However, when affected by corrosion this ductile yielding behaviour can be compromised because of corrosion effect on geometrical properties.

The obtained results were verified by well-established Euler buckling plate equations theoretically and the compatibility was good. Sensitivity analysis show that C3D20R quadratic solid elements seeded to between 20-25 seeds generating up to 6000-9000 elements on models allows for results convergence at a reduced computer running time because of the element reduced integration algorithm.

In the future this research will consider the effects of roughness, pitting and corrosion shape irregularity on strength. By investigating these scenarios, together with further parametric analyses capturing the entire range of corrosion losses both under uniform and non-uniform patterns, spectra mapping buckling strength loss on plates different proportions can be established. These can be extremely useful for the purposes of bridge element assessments where following bridge inspections, identification of potential corrosion can lead to a quick estimate of buckling strength reduction. A further extension on the effect of corrosion on the lateral torsional buckling resistance of built-up sections will also be considered.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of the Petroleum Technology Development Fund Nigeria.

REFERENCES

Biezma, Maria Victoria and Frank Schanack. 2007. "Collapse of Steel Bridges." Journal of Performance of Constructed Facilities 21 (5): 398-405.

Boissonnade, N. and H. Somja. 2012. "Influence of Imperfections in FEM Modeling of Lateral Torsional Buckling." Proceedings of the annual stability conference, structural stability research council, Texas

BS EN ISO 9223. 2012. Corrosion of Metals and Alloys- Corrosivity of Atmospheres- Classification, Determination and Estimation. UK: BSI Standards Publication limited.

BS EN ISO 9224. 2012. Corrosion of Metals and Alloys- Corrosivity of Atmospheres- Guiding Values for the Corrosivity Categories. UK: BSI Standards Publication limited.

Bushnell, David. 1981. "Buckling of Shells-Pitfall for Designers." AIAA Journal 19 (9): 1183-1226.

Clarke, Antony Bryan and Sidney H. Coverman. 1987. "Structural Steelwork--Limit State Design." Chapman and Hall, 11 New Fetter Lane, London EC 4 P 4 EE, UK, 1987.

Cruz, Paulo JS, Lúcio Lourenço, Hélder Quintela, and Manuel Filipe Santos. 2006. "Influence of Web Thickness Reduction in the Shear Resistance of Non-Prismatic Tapered Plate Girders." 6th International symposium for steel bridges, Prague

Gardner, Leroy, Rachel Bethan Cruise, Cui Ping Sok, K. Krishnan, and J. Ministro Dos Santos. 2007. "Life-Cycle Costing of Metallic Structures." Proceedings of the ICE-Engineering Sustainability 160 (4): 167-177.

Kallias, Alexandros N. and Boulent M. Imam. 2013. "Effect of Climate Change on the Deterioration of Steel Bridges."International Association for Bridge and Structural Engineering (IABSE) Symposium Report, Volume 99, Number 6, pp. 1731-1738..

Kayser, Jack R. and Andrzej S. Nowak. 1989. "Capacity Loss due to Corrosion in Steel-Girder Bridges." Journal of Structural Engineering 115 (6): 1525-1537.

Ok, D., Y. Pu, and A. Incecik. 2007. "Computation of Ultimate Strength of Locally Corroded Unstiffened Plates under Uniaxial Compression." Marine Structures 20: 100-114.

Saad-Eldeen, S., Y. Garbatov, and C. Guedes Soares. 2013. "Effect of Corrosion Severity on the Ultimate Strength of a Steel Box Girder." Engineering Structures 49: 560-571.

Simulia, DCS. 2014. "ABAQUS 6.14 Analysis User's Manual: Getting Started with Abaqus Interactive Edition." Abaqus 6.14 Documentation: 22.2.

Sosa, Eduardo M., Luis A. Godoy, and James GA Croll. 2006. "Computation of Lower-Bound Elastic Buckling Loads using General-Purpose Finite Element Codes." *Computers & Structures* 84 (29): 1934-1945.