FATIGUE LIFE PREDICTION OF RC BEAMS STRENGTHENED WITH EXTERNALLY BONDED FRP SHEETS

Wen-wei Wang* and Hui Huang
Department of Bridge Engineering, School of transportation,
Southeast University, China 210096
*Email: wangwenwei@seu.edu.cn

ABSTRACT

Based on the failed criterion of steel reinforcement fracture, an analytical model is presented for predicting the fatigue life of reinforced concrete (RC) beam strengthened with fiber reinforced polymer (FRP) sheets. In this model, the load cycle is divided into some loading blocks evenly and the stress amplitude of the tensile steel reinforcement is thought to be invariable in each loading block. Considering the degradation of material performance, including concrete creep, the stress amplitude of the tensile steel reinforcement is obtained by using the traditional sectional analysis method. Therefore, the fatigue life of the strengthened beam is carried out by using the well-known Palmgren-Miner rule. The reliability of the proposed analytical model is validated through comparisons with previous test results reported by the relative research groups. The compared results show that the proposed models can predict the fatigue life of the strengthened beam with an acceptable degree of accuracy.

KEYWORDS

FRP sheets, strengthening, RC beam, steel rupture, fatigue life.

INTRODUCTION

Due to the repetitive cyclic loading, many reinforced concrete structures, such as bridges, are needed to be rehabilitated so as to satisfy its serviceability. Up to now, some strengthening methods, like externally bonding steel plate, external prestressing strand and enlarging member section, have been developed for strengthening those deficient/damaged reinforced concrete structures. However, one potential solution is via the bonding of FRP sheets due to the merits of FRP materials, such as high strength-to-weight ratio, non-corrosive property and easy of handling (Teng *et al*. 2002; Rougier *et al*. 2007; EIsayed *et al*. 2007).

The research works, including theoretical analysis and experimental program, of FRP strengthened RC beams subjected to the fatigue load have been well established (Barnes *et al*. 1999; Papakonstantinou *et al*. 2001; Heffernan *et al*. 2004; Quattlebaum *et al*. 2005; Toutanji *et al*. 2006; Yu *et al*. 2011; Xie *et al*. 2012). However, limited models have been proposed for predicting the fatigue life of FRP strengthened RC beams. In 2001, El-Tawil *et al*. presented an analytical model to compute the static and fatigue responses of FRP strengthened RC beams. This model was constructed with the fiber section method and taken into the consideration of fatigue damage of the concrete. In 2011, Ferrier *et al*. used the sectional analytical approach to analysis the increased deflection of FRP strengthened RC beams under fatigue loading. When refers to the finite-element analysis, some researchers (Zhang *et al*. 2008; Loo *et al*. 2012) used the finite-element software to model the debonding behavior between concrete and steel/FRP sheets for strengthened beams under fatigue loading.

Generally, the fatigue failure process of the FRP sheet strengthened RC beams could be divided into three distinct stages (as seen in Figure 1): 1) Crack propagation stage. During this stage, flexural and shear cracks appeared in the pure moment and moment-shear regions of the beams and some of these rapidly developed into the main cracks. It was demonstrated that this first stage takes up no more than 10% of the total fatigue load cycles; 2) Damage accumulation stage. After the first stage, the changes in observable fatigue damage become minimal for a long period of time. The increments in the number of cracks and developments of the maximum crack length and width were all stable basically. This second stage takes up more than 90% of the total fatigue life, and little degeneration of the flexural stiffness is observed; 3) Failure stage. After substantial fatigue damage accumulation, the tensile steel reinforcement ruptured at a certain main cracked section (i.e. maximum bending moment section). Then, the tensile force carried by the steel reinforcement was transferred to the FRP sheets, which led to the debonding or rupture of the FRP material. The strengthened RC beams lost their fatigue capacities with vanish of the tensile materials (as seen in Figure 2). This final stage lasted a relatively short time.

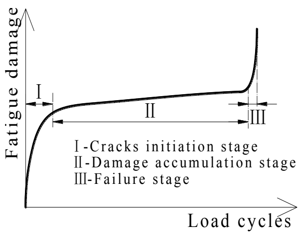

Figure 1 Fatigue failure process

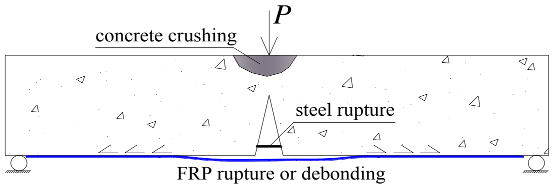

Figure 2 Typical fatigue failure mode

As above mentioned, the rupture of tensile steel reinforcement at the main cracked section was the controlling failure mode for the FRP sheets strengthened RC beams under fatigue loading. Therefore, the fatigue life of such strengthened members can be determined according to the fatigue life of tensile steel reinforcement. In this paper, an analytical model for predicting the fatigue life of FRP sheets strengthened RC beams was proposed based on the Palmgren-Miner rule (Miner 1945) and the sectional analysis method. The FRP debonding induced slippage between FRP and concrete was ignored in this analytical model, since the relative slippage can be restricted by the mechanical interlocking and friction at the debonding area (Iwashita *et al*. 2007).

PREDICTED MODEL OF FATIGUE LIFE

Failure Criterion

According to the Palmgren-Miner rule, applying n_0 cycles with a stress amplitude σ_{s0} and corresponding fatigue life endurance N_0, is equivalent to consuming n_0/N_0 of the fatigue resistance (Schijve 2009). This assumption can be applied to any subsequent block of load cycles until happening of the tensile steel reinforcement rupture. When the tensile steel reinforcement experiences more than one block of load cycles, the total consumed fatigue resistance can be written as:

$$D = \sum \frac{n_i}{N_i} \tag{1}$$

where D is the consumed fatigue resistance ($D \leq 1$); n_i is the specified number of repetitions for the specified stress amplitude σ_{si}; N_i is the corresponding number of repetitions to failure for the stress amplitude σ_{si}. The relationship between N_i and σ_{si} for ribbed and smooth steel reinforcement is given as (BS5400 1978):

$$N_i \sigma_{si}^m = K_0 \Delta^d \tag{2}$$

where m is the inverse slope of the mean-line $\log\sigma_{si}$-$\log N_i$; K_0 is the constant term relating to the mean-line of the statistical analysis results; Δ is the reciprocal of the anti-log of the standard deviation of $\log N_i$; d is the number of standard deviations below the mean-line. The values of these terms with the mean-line relationship are shown in Table 1.

Table 1 Parameters for Eq. (2)

Parameter	m	K_0	$\triangle$	d
Ribbed steel reinforcement	4	2.34×10^{15}	0.657	0
Smooth steel reinforcement	3.5	1.08×10^{14}	0.625	0

Using the determined fatigue failure criterion of tensile steel reinforcement, the fatigue life of FRP strengthened RC beams can be predicted by the summation of the corresponding fatigue load cycles of each stress amplitude until the rupture failure of tensile steel reinforcement occurs (i.e. D=1):

$$N_p = \sum n_i \tag{3}$$

where N_p is the predicted fatigue life.

Determining Stress Amplitudes of Tensile Steel Reinforcement

In order to use the aforementioned Palmgren-Miner rule to predict the fatigue life of a FRP strengthened RC beams, the variable stress amplitudes of tensile steel and the corresponding number of load cycles for the specified stress amplitude should be determined at first. As such, the discretized method was adopted to divide the whole fatigue loading process into many constant loading blocks and the sectional analysis method was used to calculate the stress amplitude corresponding to each loading block.

Discretizing of the variable stress amplitudes

The stress amplitude of the tensile steel reinforcement changed continuously with increasing load cycles due to the generation and propagation of flexural and shear cracks and the deterioration of the material performance (ACI Committee 215 1997), as shown by the dotted line in Figure 3. It is obvious that a strong nonlinear relationship exists between the stress amplitude in the tensile steel reinforcement and the number of load cycles. For simplicity, the discretized method was adopted here to divide the fatigue loading process into many constant loading blocks (i.e. each block with the same number of load cycles), and the stress amplitude was assumed to be unchanged within each specific loading block. It is noting form Figure3 that there is a large gap between the supposed stress amplitude and actual one in the first few loading blocks (i.e. crack propagation stage) when ignoring the gradual development of flexural cracks and this gap will diminish quickly with increasing load cycles. Therefore, this large gap can be neglected because the crack propagation stage experiences a short period of time relative to the total fatigue life.

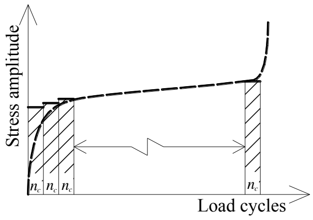

Figure 3 Discretization of steel stress amplitude

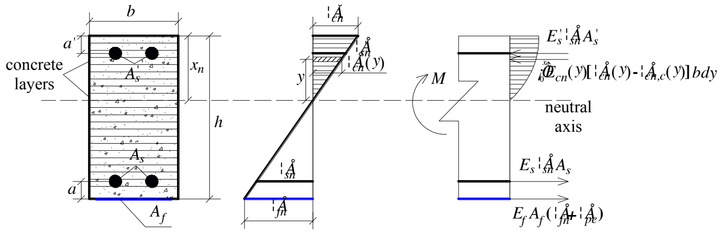

Figure 4 Strain-stress distributions

Calculating the stress amplitude of each loading block

Before determine the stress amplitude of each loading block using the sectional analysis method, the following assumptions should be noted: 1) Plane sections are considered to remain plane during the fatigue loading. This assumption is reasonable because an approximately linear strain distribution along the beam height was experimentally observed during the fatigue loading (Shahawy *et al*. 1999); 2) No bond-slip is assumed between concrete and other component materials (i.e. steel reinforcement and FRP); and 3) Due to the low tensile strength of concrete, the tension role of concrete is ignored in the calculation.

A cracked section of a strengthened beam is shown in Figure 4. The concrete portion in the top of the beam section can be conceptually divided into many thin layers along the depth direction. Then, based on the sectional equilibriums of external and internal forces and moments, the following equations can be expressed:

$$P = E_s\varepsilon_{sn}A_s + E_f(\varepsilon_{fn} + \varepsilon_{pi})A_f - \int_0^{x_n} E_{cn}(y)[\varepsilon_{cn}(y) - \varepsilon_{cn,c}(y)]bdy - E_s'\varepsilon_{sn}'A_s' \quad (4)$$

$$M = E_s\varepsilon_{sn}A_s(h - x_n - a) + E_f(\varepsilon_{fn} + \varepsilon_{pi})A_f(h - x_n) + \int_0^{x_n} E_{cn}(y)[\varepsilon_{cn}(y) - \varepsilon_{cn,c}(y)]bydy + E_s'\varepsilon_{sn}'A_s'(x_n - a') \quad (5)$$

where P is the axial force (for a simply supported beam: P=0); M is bending moment induced by external actions; x_n is the depth of the compression zone for concrete at the n^{th} cycle; E_s', E_s and E_f are the elastic modulus of compressive steel reinforcement, tensile steel reinforcement and FRP, respectively; $E_{cn}(y)$ is the effective elastic modulus of the specified concrete layer at the n^{th} cycle; ε_{sn}' and ε_{sn} are the longitudinal strains at the centroid of the compressive steel reinforcement and tensile steel reinforcement, respectively; ε_{fn} is the FRP strain caused by the fatigue load; ε_{pe} is the pre-strain of the FRP sheets if there is a prestress; $\varepsilon_{cn}(y)$ and $\varepsilon_{cn,c}(y)$ are the total strain and the creep strain of the specified concrete layer at the n^{th} cycle; A_s', A_s and A_f are the cross sectional areas of the compressive steel reinforcement, tensile steel reinforcement and FRP, respectively; b is the beam width; a' is the distance from the center of the compressive steel reinforcement to the top surface; a is the distance from the center of the tensile steel reinforcement to the subsurface; y is the distance between the centroid of the specified concrete layer and the neutral axis.

Using an iterative approach and combining Eq. 4 and Eq. 5, the stress of the tensile steel reinforcement σ_{sn} can be obtained after determining the concrete layer stress $\sigma_{cn}(y)$, the compressive steel reinforcement stress σ_{sn}' and the FRP stress σ_{fn}. When the upper limit of the fatigue load acts on the strengthened beam, the maximum stress of the tensile steel reinforcement $\sigma_{sn,\max}$ can be calculated by substituting the corresponding maximum moment $M_{\max}$ into Eq. 5. Similarly, the minimum stress of the tensile steel reinforcement $\sigma_{sn,\min}$ is corresponding to the minimum moment $M_{\min}$ induced by the lower limit of the fatigue load. Therefore, the stress amplitude of the tensile steel reinforcement can be determined according to the following equation:

$$\sigma_{si} = \sigma_{sn,\max} - \sigma_{sn,\min} \tag{6}$$

where $\sigma_{sn,\max}$ and $\sigma_{sn,\min}$ are the maximum and minimum stresses generated in the tensile steel reinforcement, respectively.

Time-dependent Constitutive Relationships of Component Materials

To obtain the maximum and minimum stresses of the tensile steel reinforcement accurately, the time- dependent constitutive relationships of all of the component materials should be considered within the analytical model.

Fatigue performance of concrete

Some experimental results showed that the compressive stress-strain relationship of concrete changed continuously with the repetitions of a fatigue load due to the internal damage accumulation of the concrete (Holmen 1982). The typical concrete compressive stress-strain curve begins with an approximately linear shape, and then gradually enters into a characteristic convex shape as the peak strain is reached. Since the external load induced concrete strain is relatively low under service conditions and the shape of this curve is generally unsusceptible with the increase of load cycles, it is reasonable to assume an approximately linear stress-strain relationship for concrete in fatigue calculations, as shown in Figure 5.

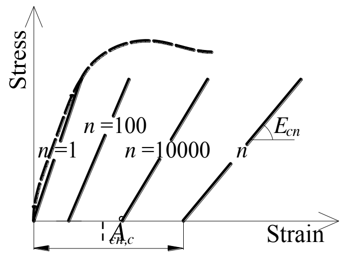

Figure 5 Stress-stain relationship for concrete

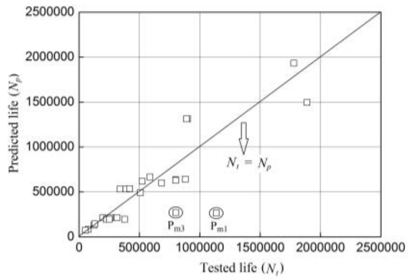

Figure 6 Predicted life versus tested life

The effective elastic modulus of concrete under a certain number of load cycles n can be written as (Sherif 2001):

$$E_{cn} = (1 - 0.33\frac{n}{N_f})E_c \tag{7}$$

where E_{cn} is the effective elastic modulus of concrete; n is the number of fatigue load cycles; E_c is the initial elastic modulus of concrete; N_f is the number of load cycles to failure for concrete, which can be calculated using the following equation (Holmen 1982):

$$\log N_f = 1.978 S_{\max}^{-3.033}(-\log K)^{0.0596} \tag{8}$$

where $S_{\max}$ is the maximum stress level and $S_{\max}=\sigma_{c,\max}/f_c$; f_c is compressive strength of concrete prism; K is defined by K=1-p; p is the probability of failure.

On the other hand, the total concrete strain (ε_{cn}) during the fatigue load is consist of two parts: e.g. elastic strain ($\varepsilon_{cn,e}$) and inelastic strain ($\varepsilon_{cn,c}$):

$$\varepsilon_{cn} = \varepsilon_{cn,e} + \varepsilon_{cn,c} \tag{9}$$

where $\varepsilon_{cn,e}$ is the elastic strain of concrete; $\varepsilon_{cn,c}$ is the inelastic strain and considered to be equal to the creep strain of concrete. Holmen (1982) proposed the following expressions to calculate the total concrete strain during fatigue loading:

$$
\varepsilon_{cn}=\begin{cases}\dfrac{1\times10^{-3}}{tg\alpha}\left|S_{\max}+3.180(1.183-S_{\max})(\dfrac{n}{N_f})^{0.5}\right|+0.413\times10^{-3}S_c^{1.184}\ln(t+1) & for\ 0<\dfrac{n}{N_f}\le 0.1\\ \dfrac{1.11\times10^{-3}}{tg\alpha}\left|1+0.677(\dfrac{n}{N_f})\right|+0.413\times10^{-3}S_c^{1.184}\ln(t+1) & for\ 0.1<\dfrac{n}{N_f}\le 0.8\end{cases} \tag{10}
$$

where $tg\alpha$ is secant modulus of the concrete ($tg\alpha=S_{\max}/\varepsilon_0$); ε_0 is the concrete strain caused by the upper limit of the fatigue load at the first cycle; S_c is the characteristic stress level and given as $S_c=S_m+RMS$; t is the duration of the fatigue load (unit in hours); S_m is the mean stress level, $S_m=(S_{\max}+S_{\min})/2$; $S_{\min}$ is the minimum stress level, $S_{\min}=\sigma_{c,\min}/f_c$; RMS is the root mean square value and for sinusoidal loading and $RMS=(S_{\max}+S_{\min})/2\sqrt{2}$.

Fatigue performance of steel and FRP

Although the repeated loading on steel reinforcement causes the accumulation of fatigue damage, Barsom (1987) and Joachim Rösler (2007) both demonstrated that the elastic modulus of steel reinforcement remains unchanged until just before failure, and no significant plastic deformation was observed by the action of high cycle fatigue loading. Besides, test results in Hull's (1981) research suggested that the mechanical behavior of FRP was virtually unaffected by fatigue loading. Hence, the constitutive relationships of steel and FRP materials are considered to be similar to the initial relationship in each load cycle.

Procedure to Estimate Fatigue Life

The detailed procedure for predicting the fatigue life is as follows: 1) Use Eqs. 4 and 5 to calculate the maximum and minimum stresses of the concrete layers with the applied maximum and minimum fatigue loads at the beginning. At the beginning, the elastic modulus of concrete is E_c and the creep strain of each concrete layer is zero; 2) Substitute these stresses into Eq. 7 to Eq. 10 to build the constitutive model for each layer of concrete. These constitutive models are assumed to represent the fatigue behavior during the whole process of the fatigue loading; 3) With the constitutive models for each concrete layer, the sectional analysis at the cracked section is conducted to calculate the maximum and minimum stresses and the stress amplitude of the tensile steel reinforcement in the each loading block using Eq. 4 to Eq. 6; 4) Substitute the value of the stress amplitude of the tensile steel reinforcement into Eqs. 1 and 2 to calculate the fatigue damage of the tensile steel reinforcement for each loading block and further obtain the total accumulated fatigue damage; 5) Adjust the constitutive model for each layer of concrete at the end of last loading block, then the corresponding stress amplitude and fatigue damage of steel reinforcement in the next loading block using the same method (i.e.sectional analysis); 6) Repeat from step 3 to step 5 until the total fatigue resistance is consumed and then the fatigue life can be obtained after summing the numbers of each loading block using Eq. 3. The above described procedure was implemented in a computer program based on MATLAB langue.

MODEL VERIFICATION

To validate the proposed model, an experimental database consisting of 28 prestressed/non-prestressed FRP sheets strengthened RC beams (Barnes *et al*. 1999; Papakonstantinou *et al*. 2001; Heffernan *et al*. 2004; Quattlebaum *et al*. 2005; Toutanji *et al*. 2006; Yu *et al*. 2011; Xie *et al*. 2012) was established. All beams were reported to have failed with the rupture of tensile steel reinforcement. Those specimens that failed with other modes or without essential parameters were not included in this database. Table 2 summarizes the geometric and material data for all 28 beams. In the table, the notations $F_{\max}$ and $F_{\min}$ denote the corresponding maximum and minimum fatigue load, respectively. All selected test beams had a rectangular section and were simply supported on the two rollers. The four-point or three-point fatigue loading was applied on the top face of the strengthened beams.

A comparison between measured fatigue lives (N_t) and those (N_p) predicted by the proposed model is presented in Table 2 and Figure 6. It is clearly shown that the predicted values for all FRP strengthened RC beams are distributed around the line of N_t/N_p=1.0, except for the test beams P_{m1} and P_{m3}. The main reasons for this big difference are believed to be the discreteness of material behavior, measuring error and model simplification. Therefore, the analytical model can be used to predict the fatigue life of FRP strengthened RC beams effectively.

Table 2 Comparisons between tested life and predicted life

Reference	Beam ID	E_c (GPa)	A_S (mm^2)	E_s (GPa)	A_f (mm^2)	E_f (GPa)	F_{max} (kN)	F_{min} (kN)	N_t (cycles)	N_p (cycles)
Papakonstantinou (2001)	S-2	34.5	253.4	200	191	72.4	46.7	2.2	880,000	642,879
	S-5	34.5	253.4	200	191	72.4	48.9	4	800,000	635,325
	S-6	34.5	253.4	200	191	72.4	64.5	4.4	126,000	132,492
	S-9	34.5	253.4	200	191	72.4	57.8	3.3	235,000	195,931
	S-10	34.5	253.4	200	191	72.4	44.5	3.3	685,000	599,712
Heffernan (2004)	M-CFa	34.5	628.3	210	89.4	233	98	28.2	900,000	1,312,025
	M-CFb	34.5	628.3	210	89.4	233	98	28.2	890,000	1,312,025
	H-CFa	34.5	628.3	210	89.4	233	112	28.2	340,000	531,520
	H-CFb	34.5	628.3	210	89.4	233	112	28.2	390,000	531,520
Quattlebaum (2005)	C-L(b)	31.5	398.2	200	71.4	216	29	7.9	587,000	666,240
	C-H	31.5	398.2	200	71.4	216	28.9	7.5	523,000	618,026
	N-H	31.5	398.2	200	71.4	216	28.9	7.9	800,000	629,553
Toutanji (2006)	3FI-9	36	141.8	210	55.74	228	34.7	6.23	259,432	213,064
	3FI-10	36	141.8	210	55.74	228	34.7	6.23	314,728	213,064
	3FI-11	36	141.8	210	55.74	228	34.7	6.23	197,954	213,064
	3FI-12	36	141.8	210	55.74	228	43.2	6.23	74,383	81,968
	3FI-13	36	141.8	210	55.74	228	43.2	6.23	74,579	81,968
Barnes (1999)	3	34.5	339.3	200	108	135	49	5	508,500	491,025
	4	34.5	339.3	200	108	135	40	4	1,889,200	1,495,732
Xie (2012)	P_{m1}	35.2	157.1	226	46	240	30	3	1,137,002	263,894
	P_{m3}	35.2	157.1	226	46	240	30	3	800,016	263,894
	P_{h1}	35.2	157.1	226	46	240	32.5	3.25	227,030	196,040
	P_{h2}	35.2	157.1	226	46	240	32.5	3.25	250,071	196,040
	P_{h3}	35.2	157.1	226	46	240	32.5	3.25	377,688	196,040
Yu (2011)	LJP-2	25.5	226.2	210	50	30.2	27.5	5	1,780,000	1,932,372
	LJP-3	25.5	226.2	210	50	30.2	36	5	420,789	536,258
	LJP-4	25.5	226.2	210	50	30.2	44	5	130,000	144,073
	LJP-5	25.5	226.2	210	50	30.2	53	5	54,000	73,294

CONCLUSIONS

An analytical model has been developed for predicting the fatigue life of FRP strengthened RC beam proposed in this paper. The model takes into account the degradation of the component material performance as well as the creep of concrete. After determining the failure criterion of steel reinforcement fracture, the load cycles are divided into several same loading blocks. The stress amplitude of the steel reinforcement is considered as a constant value in each loading block. To obtain the stress amplitude of the steel reinforcement, the traditional sectional analysis method and the Palmgren-Miner rule are applied. Comparisons between the model predictions and experimental ones reported by the relative researchers show a good correlation, which demonstrate the effectiveness of proposed model.

ACKNOWLEDGMENTS

The authors are grateful to the National Nature Science Foundation of China (Project codes: 51078079 and 51578135) and Program of China Scholarship Council for the financial support.

REFERENCES

American Concrete Institute (ACI). (1997). "Considerations for design of concrete structures subjected to fatigue loading". *ACI Committee 215*, Detroit, USA.

Barsom, J.M., and Rolfe, S.T. (1987). "Fracture and fatigue control in structures". New Jersey, USA.

Barnes, R.A., Mays, G.C. (1999). "Fatigue performance of concrete beams strengthened with CFRP plates". *Journal of Composites for Construction,* 3(2), 63-72.

BSI-BS5400. (1978). "Steel, concrete and composite bridges-part 10: code of practice for fatigue". London, British Standards Institution.

EIsayed, W., Ebead, U.A., and Neale, K.W. (2007). "Interfacial behavior and debonding failures in FRP-strengthened concrete slabs". *Journal of Composites for Constructio*n. 11(6), 619–628.

El-Tawil, S., Ogunc, C., Okeil, A., and Shahawy, M. (2001). "Static and fatigue analyses of RC beams strengthened with CFRP laminates". *Journal of Composites for Construction*. 5(4), 258–267.

Ferrier, E., Bigaud, D., Clement, J.C., and Hamelin,P. (2011). "Fatigue-loading effect on RC beams strengthened with externally bonded FRP". *Construction and Building Materials*, 25(2), 539-546.

Heffernan, P.J., Erki, M.A., and DuQuesnay, D. (2004). "Stress redistributions in a cyclically loaded reinforced concrete beam". *ACI Structural Joural*, 101(2), 261-268.

Holmen, J.O. (1982). "Fatigue of concrete by constant and variable amplitude loading". *ACI Special Publication*, 71-110.

Hull, D. (1981). "An introduction to composite materials". London, England.

Iwashita, K., Wu, Z.S., Ishikawa, T., Hamaguchi, Y and Suzuki, T. (2007). "Bonding and debonding behavior of FRP sheets under fatigue loading". *Advanced Composite Materials*, 1(16), 31-44.

Loo, K.Y.M., Foster, S.J., and Smith, S.T. (2012). "FE modeling of CFRP-repaired RC beams subjected to fatigue loading". *Journal of Composites for Construction*, *16*(5), 572-580.

Miner, M.A. (1945). "Cumulative damage in fatigue". *Journal of Applied Mechanics*, (12), 159-164.

Papakonstantinou, C.G., Petrou, M.F., and Harries, K.A. (2001). "Fatigue behavior of RC beams strengthened with GFRP sheets". *Journal of Composites for Construction,* 5(4), 246-253.

Quattlebaum, J.B., Harries, K.A., and Petrou, M.F. (2005). "Comparison of three flexural retrofit systems under monotonic and fatigue loads". *Journal of Bridge Engineering,* 10(6), 731-740.

Rösler, J., Bäker, M., and Harders, H. (2007). "Mechanical behaviour of engineering materials". Berlin, Germany.

Rougier, V.C., and Luccioni, B.M. (2007). "Numerical assessment of FRP retrofitting systems for reinforced concrete elements". *Engineering structures*, 29(8), 1664-1675.

Schijve, J. (2009). "Fatigue of structures and material". Delft, The Netherlands.

Shahawy, M., and Beitelman, T.E. (1999). "Static and fatigue performance of RC beams strengthened with CFRP laminates". *Journal of Structural Engineering*, 125(6), 613-621.

Sherif, E.T., Cahit, O., Ayman, O., and Mohsen, S. (2001). "Static and Fatigue Analyses of RC Beams Strengthened with CFRP Laminates". *Journal of Composites for Construction*, 4(5), 258-267.

Teng, J.G., Chen, J.F., Smith, S.T. and Lam, L. (2002). "FRP strengthened RC structures". Jon Wiley & Sons Ltd, England

Toutanji, H., Zhao, L., Deng, Y., Zhang, Y., and Balaguru, P. (2006). "Cyclic behavior of RC beams strengthened with carbon fiber sheets bonded by inorganic matrix". *Journal of Materials in Civil Engineering,* 18(1), 28-35.

Xie, J.H., Huang, P.Y., and Guo, Y.C. (2012). "Fatigue behavior of reinforced concrete beams strengthened with prestressed fiber reinforced polymer". *Construction and Building Materials,* 27(1), 149-157.

Yu, T.L., Li, C.Y., Lei, J.Q., and Zhang, H.X. (2011). "Fatigue of concrete beams strengthened with glass-fiber composite under flexure". *Journal of Composites for Construction,* 4(15), 557-564.

Zhang, R., and Shi, Z. (2008). "Numerical simulation of rebar/concrete interface debonding of FRP strengthened RC beams under fatigue load". *Materials and structures*, *41*(10), 1613-1621.