SUSTAINABLE DESIGN USING THE ADVANCED PLASTIC ANALYSIS FOR IMPERFECT STRUCTURAL SYSTEMS

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ABSTRACT

The old linear and approximate second order analysis (known as P-Δ-only Analysis, which was first appeared in American LRFD 2010 and Hong Kong Steel Code 2005 and 2011) methods of design is considered non-sustainable as they have many limitations for future development such as those demonstrated by the leaning column paradox. Direct second order plastic P-Δ-δ analysis has been applied to the design of a number of structures in Hong Kong and Macau during the past decade since the introduction of the Hong Kong Steel Code in 2005 and 2011. The method could be extended to study the collapsing behavior of a building by an incremental-iterative procedure to determine the collapse load factor. This paper discusses the progress of the method in dealing with the design of buildings under various extreme event scenarios at ultimate limit state. The key issue related to this paper is that the study could now be conducted by a practicing structural engineer conveniently by the use of advance structural analysis computer method with robust element and nonlinear solution schemes. The method allows engineers to dictate more on the design of buildings and structures rather than relying on arguable assessment on structural performance under extreme events. Benchmark example on verifying structural analysis software is proposed in this paper. Finally, application of the method by a single element per member in the economical and safe design of a constructed steel structure without any use of effective length is demonstrated with saving in time and cost. The achieved reliability of the design by the new method will be discussed, which is believed to be pioneering in design technology in structural engineering.

KEYWORDS

Sustainable design, second-order direct analysis, second-order indirect analysis.

INTRODUCTION

Bare steel frames are widely used in the construction industry as they can span across large distances while being easy to construct. Failure modes of this structural form are not simply due to elastic buckling, material yielding or individual member buckling, instead it can be due to more complicate failure modes, which is the elasto-plastic buckling of members. The first-order linear elastic analysis, which has been widely used in the design of steel structures, is not able to predict the failure load accurately and reliably since most of the non-linear effects due to interactive buckling modes cannot be simulated. The use of second-order "direct" analysis can trace the behaviors and predict the failure load of the structures accurately up to the first plastic hinge for conventional "first-plastic hinge design" or the plastic collapse load for the "advanced analysis" in AS4100(1998). The inclusion of both P-Δ and P-δ moments and their associated imperfections are important in the analysis and would result in safer design since they are unavoidable in practice. Moreover, the second-order indirect analysis or the P-Δ-only analysis could only give the sway moment that can be found alternatively by using sway amplification factor multiplied to the bending moment from a linear analysis and it does not consider member buckling. Its limitation in application should be aware of. Using the rigorous second order direct plastic analysis, a steel building frame or other structures could be easily checked for stability and ultimate load capacity in a user-friendly and convenient manner. Fig.1 shows the load vs. deflection plot and thus the collapse load of the Vogel frame with lowest mid column removed or damaged completely, where it can be used to investigate the mechanism and load resistance under progressive collapse check resulted by a failed structural member. This checking can be done in a simple manner using a PC or desktop computer at minimal cost and maximum convenience using software such as NIDA.

Studies of bare steel structural system or the portal frame system covers specifically in this paper have been comprehensive because of its extensive uses (see, for example, Lim et. al 2005). In many countries such as the Japan and UK, a high proportion of construction steel material is used for the construction of portal frames for industrial buildings. Lim el al (2005) proposed a method of analysis and design mainly based on the Merchant-Rankine formula and the moment amplification approaches. Telue and Mahendran (2004) use a sophisticated finite element to study the response of portal frames and it appears that it could hardly be used for daily portal frame design with many load cases. Furthermore, these works mainly concentrate on single bay portals, while a more practical and economical design method allowing for plastic yielding and large deflection for general framed structures is not yet reported. This paper is aimed to resolve this problem.

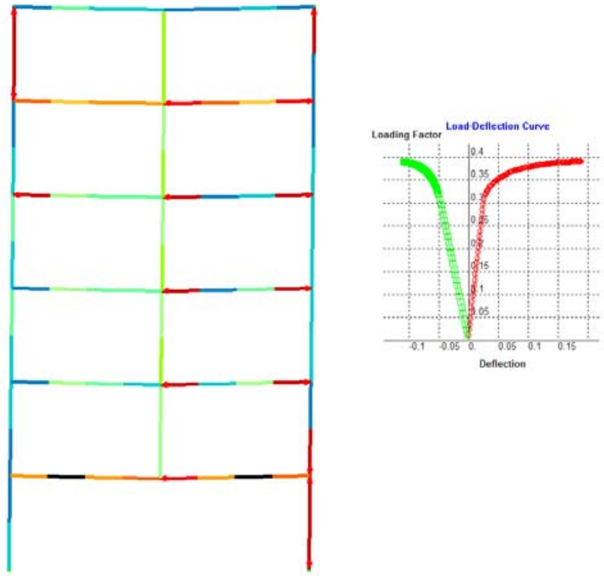

Figure 1 Plastic load resistance of the 6-storey Vogel's frame upon removal of the center lowest column

Different elements for nonlinear analysis have been proposed by many researchers. The stability function is one of the most popular methods used. The stability function allows the use of only one single element to model a member and gives an accurate result. However, the formulation of the stability function is too complicated and different forms of stability functions would be obtained under tensile and compressive axial forces which lead to loss of generality. Cubic element is also commonly used in nonlinear analysis to model the one-dimensional beam-column member used by researchers such as Meek and Tan (1984), Liew and Tang (2000), Albermani and Kitipornchai (2003) and so on. However, the element over-estimates or under-estimates when only one single element is used to model a member as demonstrated by Chan and Zhou (1994). The use of several elements to model a member leads to complexity in modeling and increase the computational time. Besides, the direction of member initial imperfection is not easy to assess. The use of pointwise equilibrium polynomial (PEP) function proposed by Chan and Zhou (1994) allows the model of one element per member model in most practical cases with a provision of accurate result. The P-Δ and P-δ moments, initial and geometry imperfection and material nonlinearity can be included in their method. Separating the compressive and the tensile load cases with the matrix being valid for positive, negative and zero axial force is not needed in their approach. Moreover, Chan and Zhou (1995) included the equivalent initial imperfection, which simulates the effect of geometric imperfection and residual stress, into the PEP element formulations to complete the modeling for buckling and sectional action computation. In this paper, the pointwise equilibrium polynomial (PEP) function is used. Simple columns with different boundary conditions and a two storey frames are analyzed and a practical transmission line tower is designed, which show the advantages and convenience of using the proposed second-order direct analysis method of design.

Second-order Direct Analysis Method of Design

The use of one element to model each member will not only saves the computational time which is especially important in large structures conveniently, it also includes member initial imperfection which is unavoidable in all members and structures. Ignoring imperfections may resulted to an unsafe design. The PEP element is capable of modelling one member by using only one element in second-order analysis. The detailed element formation, tangent and secant stiffness matrix and the numerical method for analysis is summarized below.

Element Formation

The initial imperfection as shown in Fig.2 is assumed in the element as follows.

$$v_0 = v_{m0}\left(1-(\frac{2x}{L})^2\right), \quad -L/2 \le x \le L/2 \tag{1}$$

v_{mo} is amplitude of initial imperfection equal to the imperfection at mid-span and L is the length of the member.

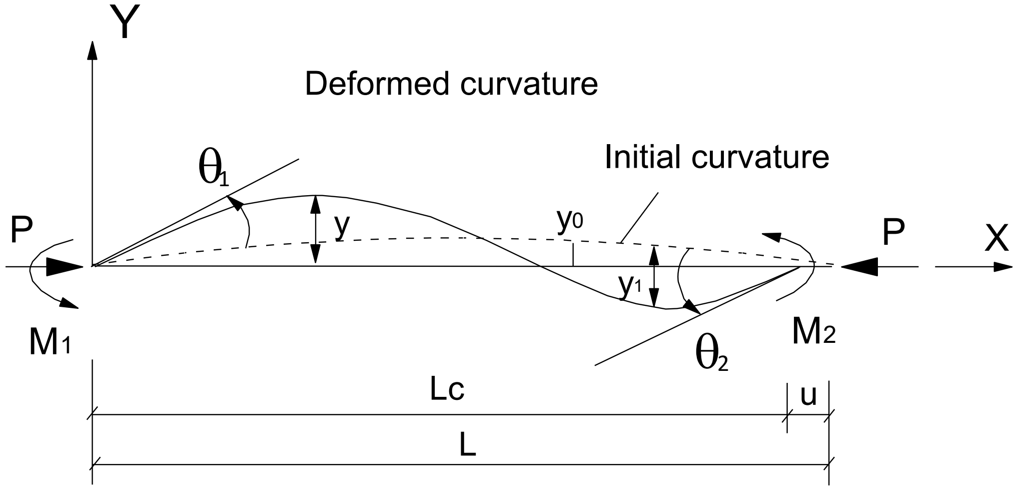

Figure 2 The beam-column element

The fifth order polynomial function given below with six coefficients can be solved by using six constrained equations,

$$v = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \tag{2}$$

in which x is non-dimensional distance along the element, v_o is lateral initial imperfection,

Six constrained equations include the four equations in compatibility condition and two equations in equilibrium condition are used to solve the six coefficients in the polynomial function.

In equilibrium condition,

$$EIv'' = P\left(v+v_0\right)+M_1\left(\frac{x}{L}-\frac{1}{2}\right)+M_2\left(\frac{x}{L}+\frac{1}{2}\right) \tag{3}$$

$$EIv''' = Pv' + \frac{M_1 + M_2}{L} \tag{4}$$

In compatibility condition,

$$\text{At } x = -\frac{L}{2} \qquad v = 0 \text{ and } v' = \theta_1 \tag{5}$$

$$\text{At } x = \frac{L}{2} \qquad v = 0 \text{ and } v' = \theta_2 \tag{6}$$

in which E is Young's modulus of elasticity; I is second moment of area; v is lateral displacement due to applied loads; P is axial force; M_1 and M_2 are nodal end moments. Please note that the shape function derived by PEP element is valid for positive, negative and zero value of axial force.

The final displacement function about the axis passing through the two ended nodes can be expressed as,

$$v = N_1(L\theta_1) + N_2(L\theta_2) + N_0 v_{mo} \tag{7}$$

in which N_1 and N_2 are the shape functions for perfectly straight element and

$$N_o = -q\frac{(1-t^2)^2}{H_2}, \qquad H_2 = 48 + q\,, \qquad q = \frac{PL^2}{EI}$$

Secant and Tangent Stiffness Matrix

The secant stiffness matrix is used to check the equilibrium convergence, to compute the equilibrium error due to change in the geometry and to check the cross sectional strength. The secant stiffness matrix can be obtained by applying the stationary energy principle as follows:

The total potential energy function, Π, is given by:

$$\Pi = U - V \tag{8}$$

in which U is the strain energy and V is the external work done as:

$$V = \sum F_i u_i \tag{9}$$

in which F_i is the applied forces and u_i is the displacements.

The strain energy, U, can be expressed as:

$$U = \frac{1}{2}\int_L EAu'^2 dx + \frac{1}{2}\int_L EIv'' dx + \frac{1}{2}\int_L P\left(v'^2 + 2v'v_0'\right)dx \tag{10}$$

The tangent stiffness matrix used for prediction of the displacement increment can be found by second variation of the total potential energy function.

$$\delta^2\Pi = \frac{\partial^2\Pi}{\partial_{r_i}\partial_{r_j}} = \frac{\partial_{s_i}}{\partial_{r_j}} + \frac{\partial s_i}{\partial_q}\frac{\partial_q}{\partial_{r_j}} \qquad i,j = 1,2,3 \tag{11}$$

in which s_i and r_i are respectively the force and displacement vectors given by,

$$s = [M_1 M_2 P]^T \tag{12}$$

$$r = [\theta_1 \theta_2 e]^T \tag{13}$$

Section Capacity Check for Second-order Direct Analysis

The use of second-order analysis can included the P-Δ and P-δ effects automatically in the section capacity check equation, as a result, the individual member check is replaced by the section capacity check as:

$$\frac{P}{A_g p_y} + \frac{M_y + P(\delta_y + \Delta_y)}{M_{cy}} + \frac{M_z + P(\delta_z + \Delta_z)}{M_{cz}} = \phi \le 1 \tag{14}$$

In which, P is the axial force in the member, p_y is the design strength of the member, A_g is the gross cross-sectional area , M_y and M_z are the applied first-order moments about the y- and z- axes, M_{cy} and M_{cz} are the moment capacities about the y- and z- axes. $P(\delta_y)$ and $P(\delta_z)$ are the second order P-δ moments about the y- and z- axes of which the consideration allows us to include automatically the bending effect due to axial force and second-order deflections, and $P(\Delta_y)$ and $P(\Delta_z)$ are the second order P-Δ moments about the two axes of which the consideration allows us to include automatically the bending effect due to axial force and second-order deflections.

Therefore, the estimation of effective length is not required as the P-Δ and P-δ effects have been included in the Eq.14 for section capacity check. Moreover, the initial imperfection is also included in analysis that the Perry-Robertson formula for imperfect columns can be directly applied in the integrated analysis and design procedure.

In the analysis procedure, a small load increment of, say 5 to 10% of the expected design load, is applied to the structure by an incremental-interactive procedure and the design load is attained when the section capacity factor φ in the Eq. 14 is equal to unity.

Numerical Method

Load control Newton-Raphson method combined the minimum residual displacement method (Chan 1988) and arch-length load control (Crisfield, 1980) is used and it is capable of tracing the path up to and beyond the limit point without numerical divergence. Minimum value for the equilibrium error for residual displacement in each interaction is guaranteed by using this method. This numerical method not only traces the pre-buckling behaviors of the dome, but it also charts the post-buckling load versus deflection curve.

Examples

Two examples are selected for application of the proposed method. The first example is a gable portal frame by Ziemian (1992). The second example is a project for transmission towers done by the authors utilizing the purposed method.

Ziemian's two-story frame

The two storey frame shown in Fig.3 is proposed by Ziemian (1992) as a calibration frame. He adopted both plastic hinge and plastic zone methods in his analysis and the geometrical dimensions are shown in Fig.3 and its detailed material properties are available in the original text (Ziemian, 1992).

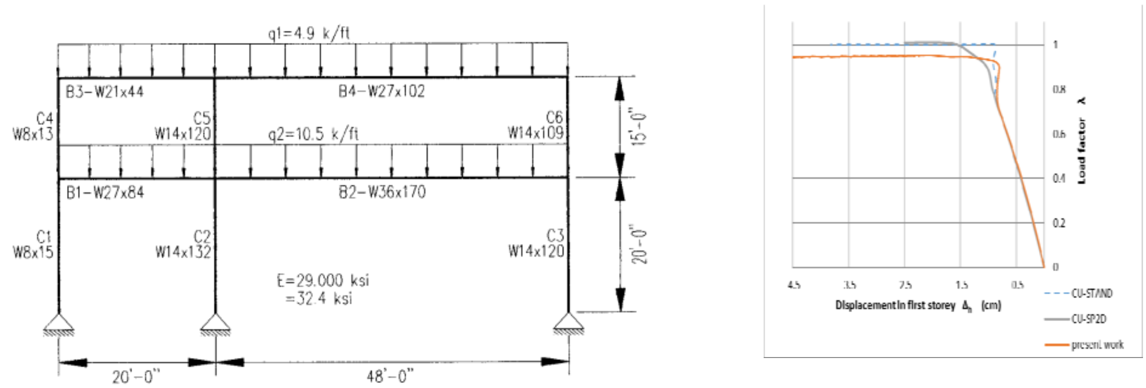

Figure 3 Comparison of the first story load-deflection response by Ziemian (1992) and NIDA(2015)

It can be seen in the comparison the present results are slighly conservative and it seems to be acceptable and preferrable in practical design. This conservatism is due to use of the linear moment-axial force interaction equation which underestimates the plastic capacity of a section. A more refined yield or plastic function could be adopted to more economize the design. On the other hand, the proposed method is simpler to use than the plastic zone method. Also, the plastic hinge method is based on the section capacity check which is easily found in design codes for the yield function of the section so engineers could have a better physical grasp of the yield condition for the yield section.

Another observation in this example shows that the present method converges very well in both elastic and plastic ranges. The problem of reaching divergence for nonlinear analysis in the past seems to have been overcome by the proposed method using the self-equilibrium element and the iterative scheme, which unfortunately unavailable in most frame analysis and design software in market.

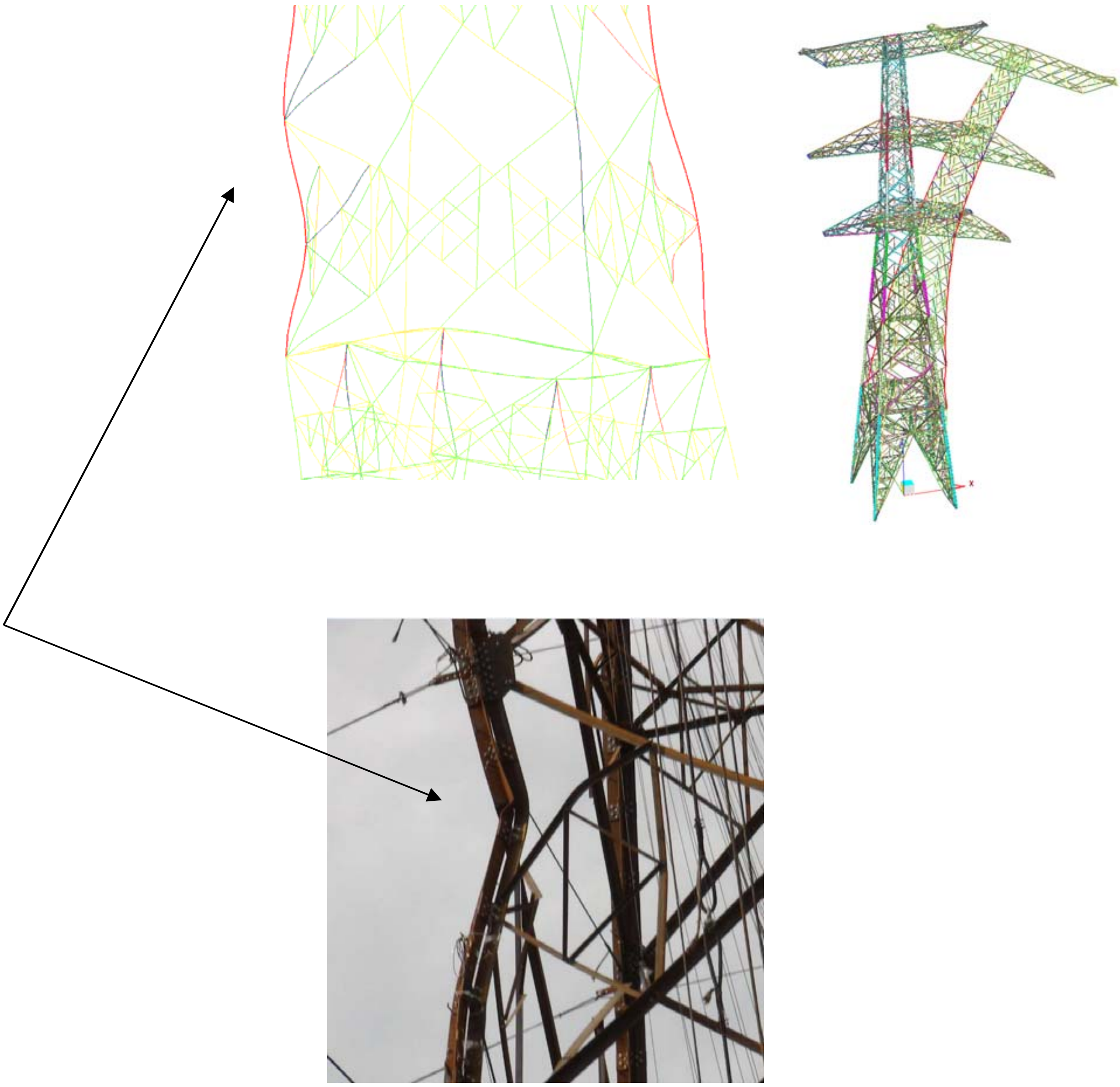

Figure 4 The tested and simulated tower in a completed project by authors

Simulation test of transmission line towers

Transmission line towers collapse quite frequently in different parts of the world, likely because of the wide spread use of inadequate second order analysis not allowing for member P-δ effect. When collapse, engineers in government departments in these countries will impose full-scale tests for the towers to ensure they would not collapse under the design loads so there have been a practice of testing towers in many countries.

On the other hand, some places like Hong Kong do not generally have problem in tower collapse therefore they do not require such physical tests. These places use nonlinear and full direct analysis for simulation of tower collapse to ensure they would not fail before reaching the design loads. To this, the authors were involved in a number of simulation tests for countries in countries in Southeast Asian. Using the plastic hinge approach, the method and the associated program NIDA is applied to practical simulation of transmission line towers in Hong Kong, China and Myanmar. Fig.4 below shows the analysis and test results using the actual material and geometrical properties in the test. The effects from material yielding, imperfections eccentric connections and slips are considered in the model. Interestingly, the numerical and test results give failure loads with 5%

discrepancy for several tested load cases and two types of tested towers. Fig.4 shows the failed tower in test and in our simulation with NIDA respectively.

CONCLUSIONS

The design method of Second-order Direct Analysis has been developed and applied to benchmark and practical examples. Unlike the old method where assumption of effective length is uncertain, change of stiffness due to the initial stress for members is ignored, the method can handle not only daily design under static loads, it could be extended rationally to other scenarios such as studies for progressive collapse, seismic design, fire engineering design using material properties under elevated temperature and so on. The concept results in a more sustainable solution and the uncertainties from the effective length method can be eliminated when dealing with design under truly ultimate or collapse limit state. The popularity of this method is showcased by the numerous projects in Hong Kong and Macau undertook by the authors.

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REFERENCES

Albermani, F.G.A. and Kitipornchai, S., "Numerical simulation of structural behaviour of transmission towers", Engineering Structures, vol 43, 2003, pp.167-177.

AS4100 (1998), Australian Standard for Steel Structures, Standards Australia.

Chan, S. L. (1988). "Geometric and Material Non-Linear Analysis of Beam-Columns and Frames Using the Minimum Residual Displacement Method." *International Journal for Numerical Methods in Engineering*, 26(12), 2657-2669.

Chan, S. L., and Zhou, Z. H. (1994). "Pointwise Equilibrating Polynomial Element for Nonlinear-Analysis of Frames." *Journal of Structural Engineering-ASCE*, 120(6), 1703-1717.

Chan, S. L., and Zhou, Z. H. (1995). "Second-Order Elastic Analysis of Frames Using Single Imperfect Element Per Member." *Journal of Structural Engineering-ASCE*, 121(6), 939-945.

Crisfield, M. A. (1980): Incremental / iterative solution procedures for nonlinear structural analysis. In: Numerical methods for nonlinear problems. Vol. 1. C. Taylor & E. Hinton & D. R. J. Owen editors. Pineridge Press, Swansea, UK.

Gu, J. X., and Chan, S. L. (2005). "Second-order analysis and design of steel structures allowing for member and frame imperfections." *International Journal for Numerical Methods in Engineering*, 62(5), 601-615.

Liew J.Y.R. and Tang L.K. "Advanced plastic hinge analysis for design of tubular space frames", Engineering Structures, 22, 2000, pp. 769-783.

Lim, J.B.P., King, C.M., Rathnone , A.J., Davies, J.M. and Edmonson, V., "Eurocode 3 and the in-plane stability of portal frames", November 2005, The Structural Engineer, pp.43-49.

Meek, J. L., and Tan, H. S. (1984). "Geometrically Nonlinear-Analysis of Space Frames by an Incremental Iterative Technique." *Computer Methods in Applied Mechanics and Engineering*, 47(3), 261-282.

NIDA. (2015). "Non-linear Integrated Design and Analysis user's manual, NAF-NIDA series, Version 9. Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong. (http://www.nidacse.com)."

Telue, Y, Mahendran, M, (2004) Behaviour and Design of Cold-formed Steel Wall Frames Lined With Plasterboard on Both Sides, *Engineering Structures,* p567-579

Vogel , U. (1985), "Calibration frames, Stahlbau, 54, October, pp.295-311.

Ziemian, R.D. (1992). *Advanced Methods of Inelastic Analysis for in the Limit States Design of Steel Structures*, Ph.D. Thesis, Cornell University, Ithaca, N.Y.