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SHRINKAGE AND EARLY-AGE TEMPERATURE INDUCED CRACKING AND CRACK CONTROL IN CONCRETE STRUCTURES

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ABSTRACT

Cracks occur in reinforced concrete structures wherever and whenever the tensile stress in the concrete reaches the tensile strength of the concrete. After concrete sets and hardens, tensile stress at any location may be caused by factors such as early-age heat of hydration, applied loads, restrained shrinkage, temperature changes, settlement of the supports and so on. This paper deals with the control of cracking caused by restraint to early-age cooling and shrinkage of concrete. Such cracking is inevitable in many situations and a significant amount of reinforcement crossing each crack is required for crack control. Rational procedures are proposed for determining the effects of internal restraint and external restraint, including restraint provided by embedded reinforcement and both end-restraint and side-restraint that may exist at the supports of beams, slabs and walls. Guidance is also provided for estimating the maximum width and spacing of cracks in a variety of situations.

KEYWORDS

Cracking, crack control, heat of hydration, shrinkage, reinforced concrete, restraint.

INTRODUCTION

Cracking in reinforced concrete structures is common and normal. In some situations, it is inevitable. Cracks occur wherever and whenever the tensile stress in the concrete reaches the tensile strength of the concrete. After the concrete sets and hardens, tensile stress at any location may be caused by many different factors, including early-age heat of hydration, applied loads, restrained shrinkage, temperature changes, settlement of supports and so on. Cracks caused predominantly by the internal actions resulting from the applied loads are often termed *structural cracks*, while cracks caused by restraint to load-independent deformation, including deformations due early-age cooling, shrinkage or ambient temperature changes, are termed *intrinsic cracks*. Often cracks are initiated by a combination of causes. For example, the bending moment at which cracking occurs in a beam or slab may be significantly reduced if tensile stresses caused by restraint to early-age temperature contractions and shrinkage have developed in the member before loading. Shrinkage induced deformation may also cause significant increases in crack widths with time.

Many variables influence the width and spacing of cracks, including the magnitude and duration of loading, the quantity, orientation and distribution of the reinforcement crossing the crack, the cover to the reinforcement, the bond characteristics of the reinforcement, the deformational properties of the concrete (including its creep and shrinkage characteristics) and the size of the member. Considerable variations exist in the crack width from crack to crack and in the spacing between adjacent cracks, because of random variations in the properties of the in-situ concrete.

Control of cracking in concrete structures is often achieved by limiting the stress in the bonded reinforcement at the cracked section to some appropriately low value and ensuring that the bonded reinforcement is suitably distributed within the tensile zone. Building codes usually specify the maximum bar spacing for bonded reinforcement and the maximum concrete cover. Some codes specify deterministic procedures for calculating crack widths, with the intention to control cracking by limiting the calculated crack width to some appropriately low value. However, the influence of shrinkage on crack widths is not properly considered in the major building codes and is therefore often not adequately considered in structural design. As a consequence, excessively wide cracks are a relatively common problem for many reinforced concrete structures throughout the world.

This paper deals with the control of intrinsic cracking caused by restraint to early-age cooling and shrinkage of concrete. Rational procedures are proposed for determining the effect of restraint and the development of tensile stresses in the concrete. Guidance is also provided for estimating the maximum width and spacing of cracks in a variety of situations

EARLY-AGE THERMAL STRESSES AND STRAINS

Heat of hydration in a concrete element in the first day or so after casting rises to a peak value and then dissipates. The peak temperature T_{peak} depends on the cement content, the thickness of the concrete element and the placement temperature. As the concrete element cools, restraint to the early-age contraction may cause cracking in the immature concrete. In many situations, early-age thermal cracking cannot be avoided, but it can be controlled by avoiding excessive heat of hydration, reducing restraint where possible and using an adequate quantity and distribution of reinforcement crossing the cracks.

In concrete elements, calculation of the tensile stresses that initiate cracking is complicated by the changing elastic modulus of the young concrete and the relaxation of stress resulting from tensile creep of the concrete. The change in temperature with time due to heat of hydration in a restrained concrete element is $\Delta T = \pm(T - T_a)$, where T is the temperature at any time and T_a is the mean ambient temperature. ΔT is taken to be positive for a rise in temperature and negative for a drop in temperature. The corresponding free temperature strain is $\varepsilon_T = \alpha_c \Delta T$, where α_c is the coefficient of thermal expansion for concrete. The actual strain ε_{actual} measured at a point in the member is significantly different from ε_T and the difference $\varepsilon_r = (\varepsilon_{actual} - \varepsilon_T)$ is the stress related strain resulting from restraint and consists of elastic and creep strains. After the temperature has dropped from T_{peak} to T_a at time t_a (i.e. $\Delta T_{max} = T_a - T_{peak}$), the tensile restrained strain due to temperature is given by:

$$\varepsilon_{r.ta} = -\alpha_c \, \Delta T_{max} \, R \tag{1}$$

where R is a restraint factor that depends on the thickness of the concrete element, the shape of the temperature differential across the member and the restraint provided by adjacent members and supports (i.e. the external boundary conditions). Determination of the restraint factor R is discussed subsequently.

In the design for early-age crack control, along with the temperature induced strain, it is prudent to include the autogenous shrinkage strain ε_{cse} that will have developed at time t_a, so that the total restrained strain at time t_a is given by:

$$\varepsilon_r = -\left(\alpha_c \Delta T_{max} + \varepsilon_{cse}\right) R \tag{2}$$

remembering that ε_{cse} is contraction (negative) and so too is $\alpha_c \Delta T_{max}$. Bamforth (2007) suggested that for the assessment of early-age cracking, the autogenous shrinkage at age 3 days should be considered.

The stress induced by the restraint to early thermal strains is initially compressive during heating, but due to the low elastic modulus and the high creep strains in the first few hours after initial set when the temperature is rising, the compressive stresses are relatively small. The stress becomes tensile as the concrete cools. Creep also relieves the tensile stress caused by cooling. Provided cracking has not occurred, the restrained strain in Eq. 2 can be expressed in terms of the tensile stress (σ_r) at time t_a as

$$\varepsilon_r = \varepsilon_{elastic} + \varepsilon_{creep} = (\sigma_r / E_c) + \chi\varphi(\sigma_r / E_c) \tag{3}$$

where φ is the creep coefficient associated with the heat of hydration time period and χ is an aging coefficient to account for the fact that σ_r is gradually applied to the concrete. Before cracking, the stress caused by restraint σ_r may therefore be determined from the restrained strain (given by Eq. 2) using:

$$\sigma_r = \varepsilon_r \bar{E}_e \tag{4}$$

where $\bar{E}_e$ is the age-adjusted effective modulus of the concrete given by:

$$\bar{E}_e = E_c / (1 + \chi\varphi_{cc}) \tag{5}$$

If cracking occurs, part of the average restrained strain (measured over a gauge length greater than the crack spacing) is relieved by the crack formation. This portion of the restrained strain is termed the *crack-induced strain* $\varepsilon_{r.cr}$ and is important for the calculation of crack widths. The restrained stress at the crack is zero and the average tensile stress between the cracks is now

$$\sigma_r = (\varepsilon_r - \varepsilon_{r.cr}) \bar{E}_e \tag{6}$$

Bamforth (1982) suggested that over the early age thermal cycle, creep reduces the stress by about 35%, while others have suggested that the reduction of stress due to creep is up to 50%. Taking $\chi\varphi$ =0.65 in Eq. 5 corresponds to reduction in stress of about 40% due to creep and is recommended here.

To avoid early-age thermal and shrinkage cracking, the tensile stress induced by restraint σ_r (Eq. 4) must be less than the direct tensile strength of the concrete at time t_a, $f_{ct}(t_a)$, which may be taken from:

For t_a = 3 days: $f_{ct}(t_a) = 0.24\sqrt{f_{cm}(28)}$ in MPa (7a)

For $t_a \geq 28$ days: $f_{ct}(t_a) = 0.40\sqrt{f_{cm}(28)}$ in MPa (7b)

and $f_{cm}(28)$ is the mean compressive strength of the concrete at age 28 days in MPa.
Even if early-age thermal cracking does not occur, the stress σ_r should not be ignored. Additional stresses caused by restraint to subsequent shrinkage strains (both autogenous and drying shrinkage) may increase the tensile restraining stress to the tensile strength of the concrete and cause cracking. The design process for the control of cracking is the same, whether the tensile stress that causes cracking is caused by restraint to early-age temperature change or due to subsequent shrinkage, and the control of such cracking requires an adequate quantity and distribution of steel reinforcement.

RESTRAINT FACTORS

Internal restraint to temperature differentials

Consider a concrete element subjected a temperature differential across its thickness, as shown in Figure 1, with no external restraint and no embedded reinforcement. During hydration soon after setting, the temperature rises to its peak value at the centre of the member, dropping to its minimum value at the surfaces of the member, as shown in Figure 1a. The change in temperature between the concrete surface and the interior of the member is ΔT, as shown. As the temperature rises and the interior expands to a greater extent than at the surface, the surface is subjected to tensile stresses (due to the restrained tensile strain) and the interior of the member is in compression. These restrained stresses, known as *eigenstresses*, are self-equilibrating. If the temperature differential is high enough, cracking may occur at the concrete surfaces and the restrained stresses drop substantially. Even if cracking does not occur, at these elevated temperatures and at this very early age, the elastic modulus of concrete is low, creep is high and the stresses that develop during heating are rapidly relieved.

The restrained strain is calculated ignoring autogenous shrinkage which is uniform through the member thickness and does not contribute to the strain differential. As cooling takes place, after the peak temperature has been reached, the interior of the member cools and contracts more than the surface and tension develops at the interior of the member, as shown in Figure 1b. The concrete is now a little more mature, with a higher modulus, and the concrete stresses are often sufficiently high to cause interior cracks to develop, as shown. The maximum tensile stress that develops at the interior of the member before cracking is

$$\sigma_r = -\alpha_c \Delta T \, R \, \overline{E}_e \tag{8}$$

In Eq. 8, ΔT is negative, as it represents a drop in temperature. After cracking, the restrained stress drops, the crack-induced strain $\varepsilon_{r.cr}$ increases and the cracks will open with time due to restrained drying and autogenous shrinkage. Shrinkage may cause the interior cracks and the surface cracks that have developed during the heat of hydration cycle to join, resulting in full depth cracking.

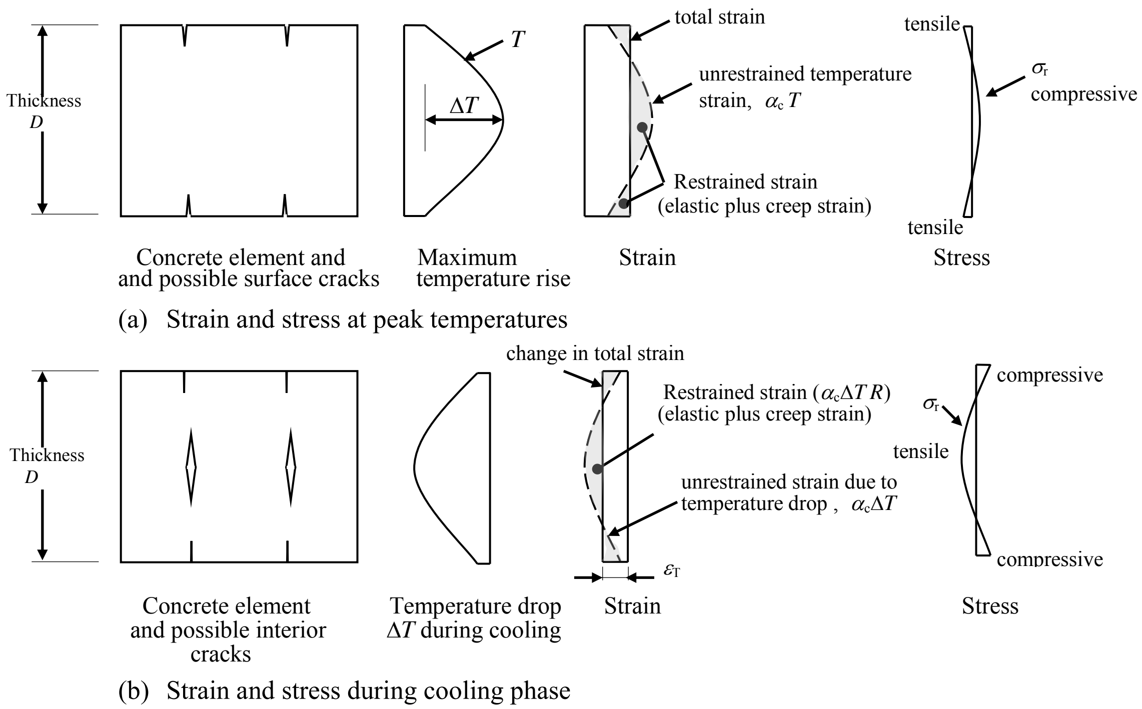


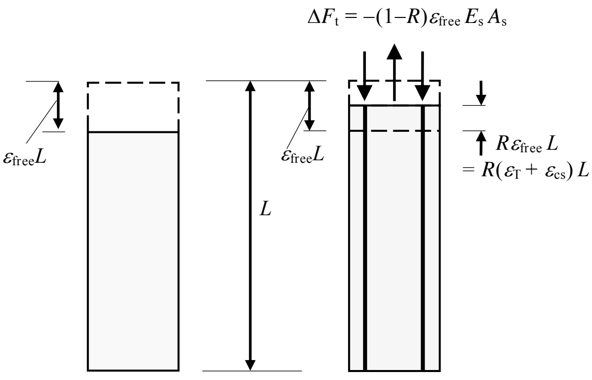
Figure 1 Development of strain, stress and possible cracking in a thick element due to early-age temperature differentials.

At the interior of the member, where both the temperature drop and the restrained tensile stress is greatest, the restraint factor R is readily determined by simple mechanics and depends on the temperature profile, which in turn depends on the mix characteristics, the member thickness and the environmental conditions. For a parabolic temperature profile, R = 0.33. For a triangular temperature gradient, R = 0.5. Bamforth (2007) recommended that R is taken as 0.42. A value of 0.4 is recommended here.

Internal restraint provided by embedded reinforcement

Symmetrically reinforced sections:

Consider the unreinforced and unrestrained concrete member of length L shown in Figure 2a and the symmetrically reinforced concrete member shown in Figure 2b. Except for the inclusion of longitudinal steel reinforcement of area A_s symmetrically placed about the centroid of the cross-section in the second member, the two members are identical. A gradual compressive strain in the concrete ε_{free} caused by early age cooling ε_T (shown in Figure 1b) and concrete shrinkage ε_{cs} would cause the unreinforced member to shorten by an amount $\varepsilon_{free} L = (\varepsilon_T + \varepsilon_{cs})L$, as shown in Figure 2a. If early-age cracking is being considered immediately after the heat of hydration cycle, ε_{cs} is equal to the autogenous shrinkage at 3 days. If restraint cracking is being investigated at a later time t, ε_{cs} is the sum of the autogenous and drying shrinkage strain components at that time (i.e. $\varepsilon_{cs} = \varepsilon_{cse} + \varepsilon_{csd}$).



(a) Unreinforced member (b) Symmetrically reinforced member

Figure 2 Internal restraint in a symmetrically reinforced concrete member.

The shortening deformation ε_{free} of the concrete in the reinforced member (Figure 2b), causes a gradual build-up of compression in the bonded reinforcement and this is opposed by an equal and opposite tensile force ΔF_t applied to the concrete. The gradually increasing tensile force results in tensile elastic strain and tensile creep strain, and the overall shortening of the member is reduced to $(1-R)\varepsilon_{free} L$ (as shown in Figure 2b), where R is the restraint factor ($0 \leq R \leq 1.0$), that depends on the amount of reinforcement. The stress in the steel σ_s and the compressive force in the steel F_s are:

$$\sigma_s = (1-R)\varepsilon_{free} E_s \tag{9a}$$

and

$$F_s = (1-R)\varepsilon_{free} E_s A_s \tag{9b}$$

where E_s is the elastic modulus of the steel. The reinforced concrete member is shortening, but it is subjected to tensile force that could possibly result in, or contribute to, cracking.

Using the age-adjusted effective modulus method for the time-dependent analysis of the member in Figure 2b, it can be readily shown (Gilbert and Ranzi, 2011) that the compressive concrete strain ε_c at time t caused by a free shortening strain of ε_{free} (= $\varepsilon_T + \varepsilon_{cs}$) and the restraint factor R are given by:

$$\varepsilon_c = (1-R)\varepsilon_{free} = \varepsilon_{free} / (1 + \bar{n}_e p) \tag{10}$$

$$R = \bar{n}_e p / (1 + \bar{n}_e p) \tag{11}$$

and the concrete tensile stress σ_{cs} at time t induced by the restraint to ε_{free} is given by

$$\sigma_{cs} = -\varepsilon_{free} E_s p / (1 + \bar{n}_e p) \tag{12}$$

where $\bar{n}_e$ is the age adjusted modular ratio ($= E_s / \bar{E}_e$), p is the reinforcement ratio (= A_s/A_c), and $\bar{E}_e$ is the age-adjusted effective modulus of the concrete given in Eq. 5.

The creep coefficient for use in Eq. 5 (φ) is the tensile creep coefficient at time t due to a stress first applied when contraction commenced. As already mentioned, when considering restraint immediately after the heat of hydration cycle, $\chi\varphi$ may be taken as 0.65. When considering the possibility of cracking after say 1 month, $\chi\varphi$ should not be taken greater than 1.8.

Taking E_s = 200000 MPa, E_c = 20000 MPa, $\chi\varphi$ = 1.625 (typical concrete properties for 30 MPa concrete when considering the possibility of restrained cracking at age 1 month), the restraint factor R is determined using Eq. 11 and is given in Table 1 for a wide range of reinforcement ratios. Also shown in Table 1 is the concrete tensile stress σ_{cs} induced by a free strain of $\varepsilon_{free} = \varepsilon_T + \varepsilon_{cs} = -600\times10^{-6}$. Even when these concrete stresses may not initiate cracking in the absence of other actions, such as when p is less than about 2.0%, they will substantially reduce the applied load required to cause cracking. Of course, this analysis assumes that the concrete is uncracked and that the tensile stress σ_{cs} can develop in the concrete. If any early-age cracking occurs at the end of the initial cooling period, the concrete in the vicinity of that crack cannot carry tension and the crack will open due to the subsequent shrinkage.

Table 1 Internal restraint factor for symmetrically reinforced sections.

$p = A_s/A_c$	0.002	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
R	0.050	0.116	0.208	0.283	0.344	0.396	0.441	0.479	0.512
σ_{cs} (MPa)	0.23	0.53	0.95	1.29	1.57	1.81	2.01	2.19	2.34

Unsymmetrically reinforced sections:

If the reinforcement is not symmetrically placed on a section, restraint to early-age thermal and shrinkage contraction will induced a curvature on the cross-section and a concrete tensile stress that may initiate cracking. Consider the singly-reinforced member shown in Figure 3a and the small segment of length, Δz. The shrinkage/temperature-induced stresses and strains on an uncracked and on a previously cracked cross-section are shown in Figures 3b and 3c, respectively. As the concrete shrinks, with a free strain of $\varepsilon_{free} = \varepsilon_T + \varepsilon_{cs}$, the steel reinforcement is compressed and, in turn, the steel imposes an equal and opposite tensile force δF_t on the concrete at the level of the steel. This gradually increasing tensile force, acting at some eccentricity to the centroid of the concrete cross-section produces elastic plus creep strains and a resulting curvature on the section. The shrinkage-induced curvature often leads to significant load independent deflection of the member. The magnitude of δF_t (and hence the shrinkage induced curvature) depends on the quantity and position of the reinforcement and on whether or not the cross-section has previously cracked.

The curvature caused by ΔF_t obviously depends on the size of the (uncracked) concrete part of the cross-section, and hence on the extent of cracking, and this in turn depends on the magnitude of the applied moment and the quantity of reinforcement. For an uncracked section, the restraint factor depends on the reinforcement ratio. Taking E_s = 200000 MPa, E_c = 20000 MPa, $\chi\varphi$ = 1.625 and d/D = 0.9, the restraint factor R for the uncracked rectangular singly-reinforced cross section shown in Figure 3b has been determined using the age-adjusted effective modulus method and is given in Table 2 for a wide range of reinforcement ratios p (=A_s/bD). The restraint factor R, the restraining force ΔF_t and the extreme fiber concrete tensile stress σ_{cs}, caused by a uniform free strain of magnitude $\varepsilon_{free} = \varepsilon_T + \varepsilon_{cs}$ may be approximated by:

$$R = \frac{\bar{n}_e p(1+\lambda_1)}{1+\bar{n}_e p(1+\lambda_1)} \; ; \; \Delta F_t = \frac{-\varepsilon_{free} E_s A_s}{1+\bar{n}_e p(1+\lambda_1)} \; \text{and} \; \sigma_{cs} = \frac{-\varepsilon_{free} E_s p(1+\lambda_1\lambda_2)}{1+\bar{n}_e p(1+\lambda_1)} \qquad (13a,b,c)$$

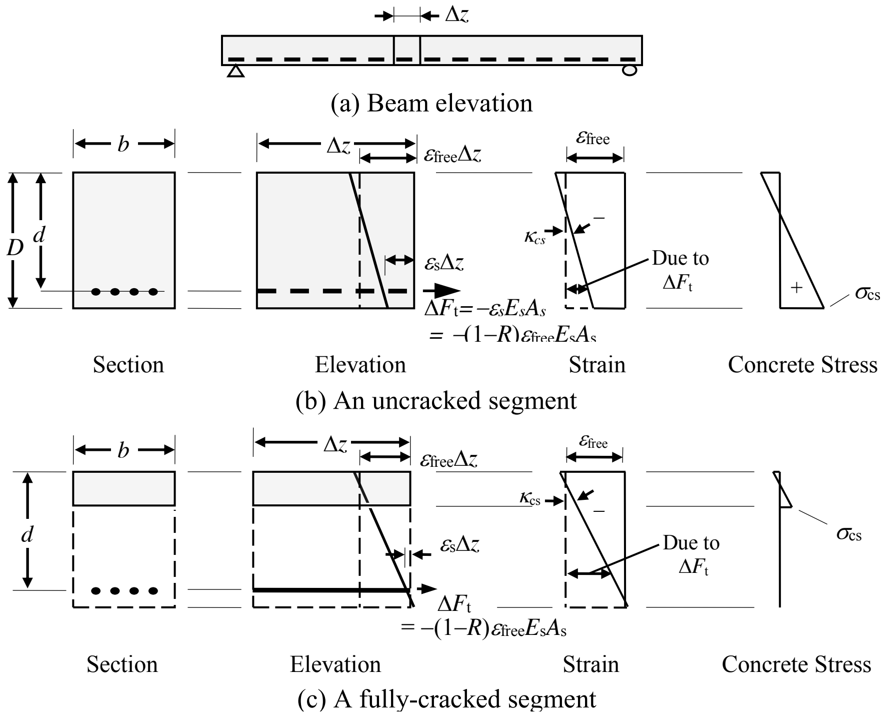


Figure 3 Shrinkage-induced deformation and stresses in a singly-reinforced beam.

where λ_1 and λ_2 depend on the geometry of the cross-section and are given by:

$$\lambda_1 = 12\,[(d/D) - 0.5]^2 \qquad \text{and} \qquad \lambda_2 = 0.5D/(d - 0.5D) \tag{14a, b}$$

Also shown in Table 2 is the extreme fiber concrete tensile stress σ_{cs} induced by a uniform free strain of $\varepsilon_{free} = \varepsilon_T + \varepsilon_{cs} = -600\times10^{-6}$.

For a cracked section, the restraint factor is close to 1.0. Taking E_s = 200000 MPa, E_c = 20000 MPa, $\chi\varphi$ = 1.625, the restraint factor R for the cracked singly-reinforced cross-section shown in Figure 3c has been determined using the age-adjusted effective modulus method and is given in Table 3 for a wide range of reinforcement ratios p (=A_{st}/bD). Also shown in Table 3 is the extreme fiber concrete tensile stress σ_{cs} at the bottom of the uncracked part of the concrete induced by a uniform free strain of magnitude $\varepsilon_{free} = \varepsilon_T + \varepsilon_{cs} = -600\times10^{-6}$. Although shrinkage strain is independent of stress, it appears that shrinkage curvature is not independent of the external load. The shrinkage induced curvature on a previously cracked cross-section is considerably greater than on an uncracked cross-section, as can be seen in Figure 3.

Table 2 Internal restraint factor R for an uncracked singly-reinforced section (d/D = 0.9).

$p = A_s/bD$	0.002	0.004	0.006	0.008	0.01	0.012	0.014	0.016
R	0.134	0.237	0.319	0.386	0.441	0.488	0.528	0.563
σ_{cs} (MPa)	0.711	1.260	1.697	2.053	2.348	2.598	2.811	2.995

Table 3 Internal restraint factor R for an already cracked singly-reinforced cross-section.

$p = A_s/bD$	0.002	0.004	0.006	0.008	0.01	0.012	0.014	0.016
R	0.987	0.966	0.942	0.916	0.891	0.865	0.841	0.817
σ_{cs} (MPa)	0.485	0.676	0.808	0.908	0.986	1.047	1.096	1.135

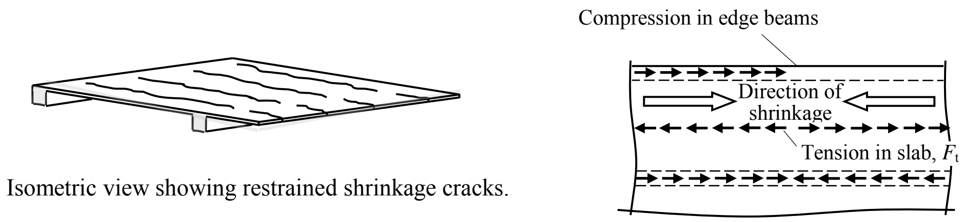
Edge restraint in a slab or wall

Frequently, walls and slabs are subjected to *edge (or side) restraint*, where shortening due to early-age contraction and shrinkage is restrained on one or more sides of the element. Examples of side restraint are shown again in Figure 4. In Figure 4a, contraction in the secondary direction of the one-way slab is restrained by the more massive supporting beams (that will be contracting at a slower rate than the slab). The restraining forces applied to the slab along each supported edge cause a direct tension in the secondary direction of the slab that may cause the cracking shown in the isometric view. The spacing and width of these cracks depend on the amount and distribution of reinforcement in the secondary direction of the slab. Figure 4b shows a wall where contraction ($\varepsilon_{free} = \varepsilon_T + \varepsilon_{cs}$) is restrained on one edge by the footing. The restraint to contraction may result in cracking, initiated at the base of the wall, where the resultant of the tensile restraint force is acting, and then extending the full height of the wall, as shown in the isometric view. A wall restrained on two adjacent edges is shown in Figure 4c, together with crack pattern initiated by restrained shrinkage in the two orthogonal directions. The cracks resulting from restrained shrinkage are direct tension cracks caused by the tensile restraining force(s), F_t. They generally extend completely through the restrained slab and, if uncontrolled, can become unserviceable and lead to waterproofing and corrosion problems. They may even compromise the integrity of the member.

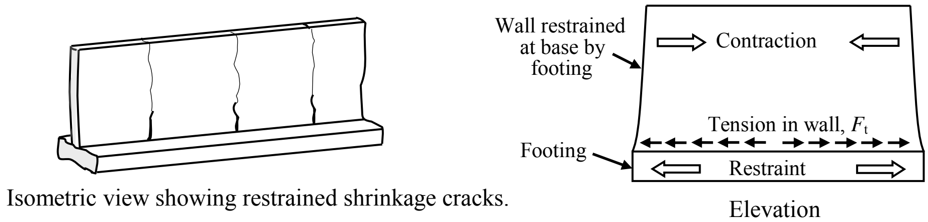
Consider the wall and footing shown in cross-section in Figure 5a. Using the notation specified in the figure, the area, section modulus and second moment of area about the centroid of the wall are $A_1 = D_1h_1$, $Z_1 = D_1h_1^2/6$ and $I_1 = D_1h_1^3/12$ and, for the footing, $A_2 = D_2h_2$, $Z_2 = D_2h_2^2/6$ and $I_2 = D_2h_2^3/12$. If the wall is cast at some time after the base, it will be cooling and shrinking at a faster rate than the base. At some time t after the wall is cast, the free contraction strain in the wall is $\varepsilon_{free.1} = \varepsilon_{T.1} + \varepsilon_{cs.1}$, while the contraction of the base is $\varepsilon_{free.2} = \varepsilon_{cs.2}$ (where $\varepsilon_{free.2} < \varepsilon_{free.1}$). The elastic modulus, creep coefficient, aging coefficient for the wall and base are $E_{c.1}$, φ_1, χ_1 and $E_{c.2}$, φ_2, χ_2, respectively. The corresponding age-adjusted effective moduli $\bar{E}_{e.1}$ and $\bar{E}_{e.2}$ are determined using Eq. 5. The self-equilibrating restraining forces that develop with time, F_t (tensile) in the wall and $-F_t$ (compressive) in the base, act at a distance $\bar{y}$ below the interface between the wall and the base, as shown in the elevation in Figure 5b. The longitudinal strains and stresses are shown in Figures 5c and 5d. It can be readily shown that distance $\bar{y}$ depends on the age-adjusted flexural rigidities of the wall and the base, $\bar{R}_{I.1} = \bar{E}_{e.1}I_1$ and $\bar{R}_{I.2} = \bar{E}_{e.2}I_2$, and is given by:

$$\bar{y} = 0.5(h_2\bar{R}_{I.1} - h_1\bar{R}_{I.2})/(\bar{R}_{I.1} + \bar{R}_{I.2}) \tag{15}$$

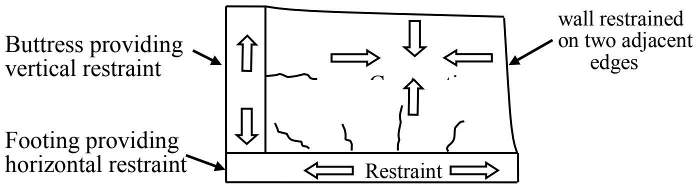
The restraining force F_t depends on the age-adjusted axial and first moment rigidities and the difference in free contraction of the wall and base, $\Delta\varepsilon_{free} = \varepsilon_{free.1} - \varepsilon_{free.2}$ and may be obtained by enforcing strain compatibility at the wall-footing interface giving:



(a) Contraction in the secondary direction of a one-way slab.

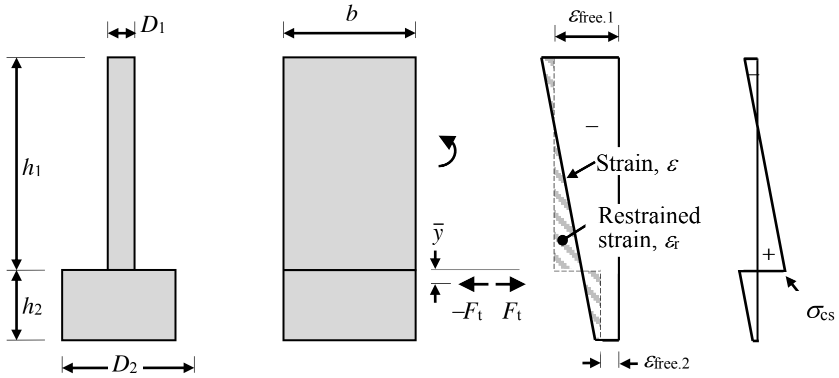


(b) Contraction in a wall restrained on one edge.



(c) Contraction of wall with two adjacent edges restrained and potential

Figure 4 Typical edge restraint to contraction in walls and slabs.



(a) Section (b) Elevation and restraining actions (c) Strain (d) Stress

Figure 5 Restraint actions, strains and stresses in an edge-restrained wall.

$$F_t = -\Delta\varepsilon_{free} / (\bar{a}_1 + \bar{a}_2 + \bar{b}_1 + \bar{b}_1) \tag{16}$$

where $\bar{a}_{.1} = 1/(\bar{E}_{e.1}A_1)$, $\bar{a}_2 = 1/(\bar{E}_{e.2}A_2)$, $\bar{b}_1 = e_1/(\bar{E}_{e.1}Z_1)$ and $\bar{b}_2 = e_2/(\bar{E}_{e.2}Z_2)$. The terms e_1 and e_2 are the distances from the line of action of F_t to the centroids of the wall and the base, respectively, i.e. $e_1 = 0.5h_1 + \bar{y}$ and $e_2 = 0.5h_2 - \bar{y}$. The tensile stress at the bottom of the wall caused by the restraining force F_t is

$$\sigma_{cs} = (F_t / A_1) + F_t(\bar{y} + 0.5h_1)/Z_1 \tag{17}$$

and the restrained strain ε_r at this point and the corresponding restraint factor R is

$$\varepsilon_r = \sigma_{cs} / \bar{E}_{e.1} \quad \text{and} \quad R = \varepsilon_r / \Delta\varepsilon_{free} = \sigma_{cs} / \bar{E}_{e.1}\Delta\varepsilon_{free} \tag{18a, b}$$

CRACKING CAUSED BY EDGE RESTRAINT

Before cracking, the deformation resulting from a restraining force F_t over a gauge length L_o is $\varepsilon_r L_o = (\sigma_{cs} / \bar{E}_e) L_o$. When the concrete stress σ_{cs} exceeds the tensile strength f_{ct}, cracking occurs. The crack opens and the concrete stress at the crack drops to zero. Either side of the crack, the concrete stress gradually increases due to the steel-concrete bond until at some distance s_o from the crack the concrete stress is unaffected by the crack. Cracks form at a spacing of between s_o and $2s_o$. If the gauge length L_o is long enough to contain *m* cracks, the deformation caused by restraint $\varepsilon_r L_o$ may now be expressed as

$$\varepsilon_r L_o = (\varepsilon_{r.cr} + \varepsilon_{r1}) L_o \tag{19}$$

where the *residual restrained strain* ε_{r1} is the sum of the elastic and creep strains caused by the average tensile concrete stress between the cracks and $\varepsilon_{r.cr}$ is the *crack-induced strain* (introduced in Eq. 6). The length $\varepsilon_{r.cr} L_o$ is the sum of the widths of the *m* cracks within the length L_o, i.e.

$$\varepsilon_{r.cr} = \left(\sum_{i=1}^{m} w_i \right) / L_o \qquad \text{and} \qquad \varepsilon_{r1} = \varepsilon_r - \varepsilon_{r.cr} = \frac{\sigma_{av}}{\overline{E}_e} \tag{20a, b}$$

In a member subjected to edge restraint (or one with restraint provided by eccentric reinforcement), the average spacing between cracks depends on the concrete cover, the bond characteristics between the reinforcement and the concrete, the bar diameter, the ratio of reinforcement area to the effective area of the tensile concrete. The maximum crack spacing $s_{r.max}$ recommended by Eurocode 2 (2004) is:

$$s_{r.max} = 3.4c + 0.425 k_1 d_b / p_{p.eff} \tag{21}$$

where *c* is the concrete cover to the reinforcement; k_1 depends on the bond characteristics of the reinforcement and may be taken as 0.8 for high bond bars (Eurocode 2, 2004), but when good bond cannot be guaranteed, such as when early-age cracking at t_a = 3 days is being considered, k_1 should be increased to 1.14; d_b is the diameter of the reinforcing bars; $p_{p.eff}$ is the ratio of the tensile reinforcement area to the effective area of the tensile concrete ($=A_{st}/A_{c.eff}$). For a wall or slab subjected to edge restraint, $A_{c.eff}$ may be taken as the gross area of the cross-section. For a member in bending, $A_{c.eff}$ is the product of the member width at the tensile steel level and $h_{c.ef}$, where $h_{c.ef}$ is the smaller of (0.5 × member thickness) and $2.5(c + d_b/2)$.

The maximum crack width w_{max} is determine from

$$w_{max} = s_{r.max} \varepsilon_{r.cr} = s_{r.max}(\varepsilon_r - \varepsilon_{r1}) \tag{22}$$

Bamforth (2007) conservatively approximated the residual strain ε_{r1} (given by Eq. 20b) by f_{ct}/E_c and therefore the crack-induced strain $\varepsilon_{r.cr}$ may be taken as:

$$\varepsilon_{r.cr} = \varepsilon_r - \varepsilon_{r1} = \varepsilon_r - (f_{ct} / E_c) \tag{23}$$

WORKED EXAMPLE

Determine the stress and strain distribution due to the restraint to early-age contraction and shrinkage of the wall shown in Figure 6a. Also determine the crack spacing and crack width at the base of the wall, if each face of the wall is reinforced with 12mm deformed bars running horizontally at 250 mm centres. The concrete cover to the steel bars is 30 mm and the bond conditions are assumed to be good. Assume that cracking occurs when f_{ct} = 2.0 MPa.

For the wall: $E_{c.1}$ = 20000 MPa, φ_1 = 2.5, χ_1 =0.65, $\varepsilon_{free.1}$ = –0.0006; and, from Eq. 5, $\overline{E}_{e.1}$ = 7619 MPa.

For the footing: $E_{c.2}$ = 35000 MPa, φ_2 = 1.5, χ_2 =0.65; $\varepsilon_{free.2}$ = –0.0002, and, from Eq. 5, $\overline{E}_{e.1}$ = 17722 MPa.

For the wall, D_1= 200 mm and h_1 = 4000 mm, and for the base, D_2 = 1000 mm and h_2 = 600 mm. The age-adjusted rigidities are: $\overline{R}_{A.1} = \overline{E}_{e.1}A_1$ = 6.095×10^9 N; $\overline{R}_{B.1} = \overline{E}_{e.1}Z_1$ = 4.063×10^{12} Nmm;
$\overline{R}_{I.1} = \overline{E}_{e.1}I_1$ = 8.127×10^{15} Nmm^2; $\overline{R}_{A.2} = \overline{E}_{e.2}A_2$ = 1.063×10^{10} N;
$\overline{R}_{B.2} = \overline{E}_{e.2}Z_2$ = 1.063×10^{12} Nmm; and $\overline{R}_{I.2} = \overline{E}_{e.2}I_2$ = 3.190×10^{14} Nmm^2.

From Eq. 15, the distance $\overline{y}$ below the interface between the wall and the base at which the resultant restraining force F_t acts is:

$$\overline{y} = 0.5(h_2 \overline{R}_{I.1} - h_1 \overline{R}_{I.2}) / (\overline{R}_{I.1} + \overline{R}_{I.2}) = 213.1 \text{ mm}$$

and, with $\Delta\varepsilon_{free}$ = –0.0004, $e_1 = 0.5h_1 + \overline{y}$ = 2213.1 mm and $e_2 = 0.5h_2 - \overline{y}$ = 86.9 mm, Eq. 16 gives:

$$F_t = -\Delta\varepsilon_{free} / (\overline{a}_1 + \overline{a}_2 + \overline{b}_1 + \overline{b}_1) = 452.3 \text{ kN}$$

From Eqs. 17 and 18:

$$\sigma_{cs} = (F_t / A_1) + F_t(\overline{y} + 0.5h_1)/Z_1 = 2.44 \text{ MPa};$$

$$\varepsilon_r = \sigma_{cs} / \overline{E}_{e.1} = +321\times10^{-6} \text{; and } R = \varepsilon_r / \Delta\varepsilon_{free} = 0.801$$

The longitudinal stress and strain distributions due to restrained contraction are shown in Figures 6b and 6c.

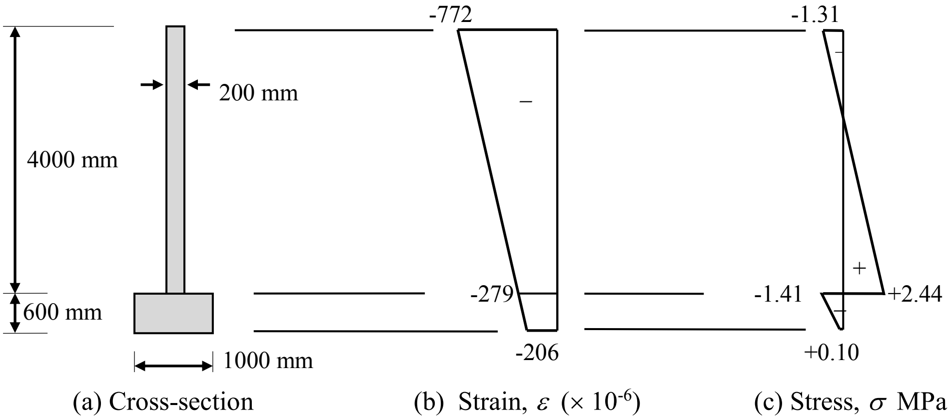


Figure 6 Cross-section of wall and footing and strain and stress distributions due to restrained deformation.

For this example, with A_{st} = 452 mm^2/m on each face of the wall, the reinforcement ratio is $p_{p.eff}$ = (2×452)/(200×1000) = 0.00452 and the maximum crack spacing is determined using Eq. 21:

$$s_{r.max} = 3.4 \times 30 + \frac{0.425 \times 0.8 \times 12}{0.00452} = 1005 \text{ mm}$$

Due to the restrained contraction at the base of the wall, ε_r = 321 × 10^{-6} and, after cracking, Eq. 23 gives:

$$\varepsilon_{r.cr} = 321 \times 10^{-6} - \frac{2.0}{20000} = 221 \times 10^{-6}$$

and the maximum crack width is calculated using Eq. 22:

$$w_{max} = 1005 \times 221 \times 10^{-6} = 0.222 \text{ mm}$$

CONCLUSIONS

Rational procedures have been proposed for determining the degree of restraint and the control of cracking caused by early-age cooling of concrete and by shrinkage of concrete. Restraining forces and concrete tensile stresses caused by temperature and shrinkage strains in a variety of situations have been considered, including concrete members subjected to temperature and/or shrinkage differentials, reinforced concrete members containing embedded reinforcement, and reinforced concrete slabs and walls subjected to edge-restraint. Calculation of the width and spacing of cracks caused by any one, or any combination, of these restraining forces is outlined and illustrated by a worked example.

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