COMPARSION OF UNCERTAINTY IN MODAL IDENTIFICATION UNDER KNOWN AND UNKNOWN INPUT EXCITATIONS

Ching-Tai Ng[1,*] and Siu-Kui Au[2]
[1] School of Civil, Environmental & Mining Engineering, The University of Adelaide, Adelaide, SA 5005, Australia. *Email: alex.ng@adelaide.edu.au
[2] Centre for Engineering Dynamics and Institute for Risk and Uncertainty, University of Liverpool, L69 3GH Liverpool, United Kingdom

ABSTRACT

Modal identification is a technique that can assess modal properties of structures based on vibration data. This technique can be categorized into known and unknown input modal identification. Known input modal identification, e.g. forced vibration tests, is more economically demanding because of the need of special devices to generate artificial loading but the data obtained has higher signal-to-noise ratio. Unknown input modal identification, e.g. ambient vibration, could be performed economically with structures under working conditions. This study employs a fast Bayesian FFT method to not only identify the modal parameters, such as natural frequencies and damping ratios, but also quantify the uncertainties associated with the modal identification results. This provides a tool to investigate the uncertainties in the modal identification. In this study two numerical examples are used to generate synthetic data for investigating and comparing the uncertainties in the known and unknown input modal identification.

KEYWORDS

Known input, unknown input, modal identification, uncertainty, Bayesian, FFT.

INTRODUCTION

Modal identification is a process to assess the actual dynamic properties, such as the natural frequencies, damping ratios and mode shapes, of structures using vibration data. It is one of the important components in vibration control and structural health monitoring. Modal identification can be classified into known and unknown input excitation condition. The former usually refers to force vibration test (Edwin 2000; Au and Ni 2013) using artificial input loading, in which both loading and response of structures are simultaneously measured during the test. The latter (Brownjohn 2003; Au 2011; Au 2012; Au et al. 2013; Au and Zhang 2016) usually refers to a case where only response is measured during the test, e.g. unknown loading from wind acting on buildings.

The known input excitation modal identification requires special devices, such as shaker or impulse hammer, to generate the artificial input loading, and hence, it is more economically demanding. However, this approach is usually able to achieve a good signal-to-noise (s/n) ratio for the measured vibration data. The unknown input excitation modal identification can be performed economically. It assumes the loading is statistically random, and hence, it does not require specific knowledge of loading, special device to generate the artificial input loading, and can be carried out with structure under working condition. Since the loading is not measured, the process of modal identification is more sophisticated.

In the literature different methods have been developed to identify the modal parameters from vibration data for known and unknown input excitation modal identification. For the known input excitation condition, half-power bandwidth method (Zembaty and Kowalski 2000) and least-square fitting of frequency response function (Maia et al. 2003) have been used to determine the modal parameters from vibration data consisting of the input loading and responses of structures. For the unknown input excitation condition, natural excitation technique (James et al. 1995) and stochastic subspace identification method (Reynders et al. 2007) have been developed for ambient modal identification in the literature.

There are a number of challenges in determining the modal parameters in practical situation through vibration data, such as the limited number of sensors, limited frequency bandwidth in response, modelling error and measurement noise. There are always uncertainties associated with the identified modal parameters. Recently a Bayesian system identification approach has been proposed to address these challenges (Beck and Katafygiotis 1998; Papadimitriou et al. 2001; Beck 2010). Time-domain Bayesian formulation for system identification with known input excitation data (Yuen and Katafygiotis 2002) and ambient data have been developed in the literature (Yuen and Katafygiotis 2001). However, a frequency-domain formulation can provide a more robust approach in stochastic modelling of the prediction error that account for the discrepancy between the model and measured data. In addition it also allows the modal identification carried out using only the spectral data in a selected frequency band dominated by the contributing modes of interest, and hence, it can improve the accuracy in the modal identification. In frequency-domain, a Bayesian approach (Yuen and Katafygiotis 2003) using fast Fourier transform of vibration data has been developed for unknown input excitation modal identification previously. Recently efficient algorithm that allows practical implementation in the modal identification has been proposed for known and unknown input excitation condition. The fast Bayesian modal identification approach for modal identification of structures has been developed for known single-input force vibration (Au and Ni 2013) and unknown input excitation condition (Au 2012). Although methods have been developed to quantify the uncertainties associated with the identified modal parameters, the relationship between known and unknown input excitation conditions and insights into the uncertainties associated with the identified modal parameters are still unclear.

The objective of this study is to investigate the uncertainties associated with identified modal parameters of structures under known and unknown input excitation condition. In this study the fast Bayesian approaches for known and unknown input excitation condition have been employed to not only identify the modal parameters, but also the associated uncertainties. The next section first presents the Bayesian formulations for known and unknown input excitation condition, which includes the Bayesian fast Fourier transform (FFT) formulation for known and unknown input condition, calculation of posterior uncertainties and definition of modal s/n ratio. After that a numerical case study is carried out identify the modal parameters and quantify the associated uncertainties. Finally, conclusions are then drawn.

BAYESIAN FFT MODAL IDENTIFICATION THEORY

Bayesian FFT Formulation

In this study the fast Bayesian FFT modal identification approach (Yuen et al. 2002; Au 2012; Au and Ni 2013) is employed to not only identify the modal parameters but also their associated uncertainties. The following sections briefly explain methods for known and unknown input condition in the modal identification. In the context of modal identification, the measured acceleration $\{\hat{\ddot{\mathbf{x}}}_j \in R^n : j = 1,...,N\}$, where N is the number of samples per measurement channel, is modeled as

$$\hat{\ddot{\mathbf{x}}}_j = \ddot{\mathbf{x}}_j(\theta) + \varepsilon_j \qquad (1)$$

where $\ddot{\mathbf{x}}_j(\theta)$ is the model acceleration response given by a set of modal parameters . $\{ {}_j \quad R^n\}$ is the prediction error that is the discrepancy between the measured response and the model response. Taking FFT on Eq. (1), the prediction error model in the frequency domain is

$$\hat{\mathcal{F}}_k = \mathcal{F}_k(\theta) + \varepsilon_k \qquad (2)$$

where $\mathcal{F}_k(\theta)$ is the FFT of the model acceleration response corresponding to frequency $f_k = (k \quad 1)/N \quad t$. ${}_k \quad C^n$ is FFT of the prediction error at frequency f_k and assumed has a flat PSD in the frequency band of interest. The FFT of the measured acceleration response $\hat{\mathcal{F}}_k$ is defined as

$$\hat{\mathcal{F}}_k = \sqrt{\frac{2\Delta t}{N}} \sum_{j=1}^{N} \ddot{\mathbf{x}}_j \exp\left[-2\pi\mathbf{i}\frac{(k-1)(j-1)}{N}\right] \quad \text{for} \quad k = 1,...,N \qquad (3)$$

where t is the sampling interval and $\mathbf{i}^2 = \quad 1$. For $k = 2,3,...,N_q$, the FFT corresponds to frequency $f_k = (k \quad 1)/N \quad t$, where $N_q = \text{int}[N/2]+1$ is the frequency index at the Nyquist frequency and $\text{int}[\cdot]$ denotes the integer part of its argument. The scaling factor $\sqrt{2 \quad t/N}$ is defined such that the spectral density is one-sided with respect to the physical frequency in Hz rather than circular frequency in rad/s. In the modal identification the values for $k=1$, N_q+1, $N_q+2,...,N$ are ignored because the former simply gives a scaled sample average of the signal due to the voltage offset of the measurement channel; the latter are the conjugate mirror image of those values at $k = 2,3,...,N_q$, hence, providing no addition information. Thus only $(N_q \quad 1)$ FFT values are used in the modal identification. In practice only FFT data within the selected resonant frequency band covering the modes of interest are used for modal identification as remaining band contains only negligible or irrelevant information, which tend to increase the error in the modal identification. Hence, the FFT data used in the modal identification only over N_f frequencies in the selected resonant frequency bands.

Known Input Modal Identification

For modal identification with measured acceleration response and known input force, the modal parameters to be identified consist of the natural frequencies $\{f_i : i = 1,...,m\}$, damping ratios $\{\zeta_i : i = 1,...,m\}$ mode shapes $\{\Phi(i) \in R^n : i = 1,...,m\}$, modal mass $\{M_i : i = 1,...,m\}$ and the power spectral density (PSD) of prediction error S_e. Let $\mathbf{Z}_k = [\mathrm{Re}\,\mathcal{F}_k(\theta), \mathrm{Im}\,\mathcal{F}_k(\theta)]^T \in R^{2n}$ and $\hat{\mathbf{Z}}_k = [\mathrm{Re}\,\hat{\mathcal{F}}_k, \mathrm{Im}\,\hat{\mathcal{F}}_k]^T \in R^{2n}$ are an augmented vector comprising the real and imaginary part of $\mathcal{F}_k(\theta)$ and $\hat{\mathcal{F}}_k$, respectively. Using the Bayes' theorem and assuming a non-informative prior distribution, the posterior probability density function (PDF) of θ given the FFT of the measured data is given by

$$p\left(\theta \middle| \left\{\hat{\mathbf{Z}}_k\right\}\right) \propto p(\theta)\, p\left(\left\{\hat{\mathbf{Z}}_k\right\} \middle| \theta\right) \tag{4}$$

where $p(\theta)$ is the prior PDF that reflects the plausibility of θ in the absence of data. A non-informative prior is common condition because the variation in the posterior PDF is often dominated by the likelihood function with large amount of data, hence, the posterior PDF $p(\theta | \{\hat{\mathbf{Z}}_k\})$ is directly proportional to the likelihood function $p(\{\hat{\mathbf{Z}}_k\} | \theta)$. The likelihood function in individual frequency is given by (Au and Ni 2013)

$$p\left(\left\{\hat{\mathbf{Z}}_k\right\} \middle| \theta\right) = \prod_k \frac{1}{(2\pi)^n \det\left(\mathbf{C}_{ik}\right)^{1/2}} \exp\left[-\frac{1}{2}\left(\hat{\mathbf{Z}}_k - \mathbf{Z}_k\right)^T \mathbf{C}_k^{-1}\left(\hat{\mathbf{Z}}_k - \mathbf{Z}_k\right)\right] \tag{5}$$

where $\mathbf{C}_k$ is the covariance matrix of $\hat{\mathbf{Z}}_k$. Since it is assumed that the sample size N is large, the $\mathrm{Re}\,\hat{\mathcal{F}}_k$ and $\mathrm{Im}\,\hat{\mathcal{F}}_k$ of $\hat{\mathbf{Z}}_k$ are asymptotically independent and have a variance of $S_e/2$. The likelihood function can be simplified and its compact form is

$$p\left(\left\{\hat{\mathbf{Z}}_k\right\} \middle| \theta\right) = (2\pi)^{-nN_f}\left(\frac{S_e}{2}\right)^{-nN_f} \exp\left[-S_e^{-1}\sum_k \left[\hat{\mathcal{F}}_k - \mathcal{F}_k(\theta)\right]^* \left[\hat{\mathcal{F}}_k - \mathcal{F}_k(\theta)\right]\right] \tag{6}$$

where

$$\mathcal{F}_k(\theta) = S_k \sum_{i=1}^{m} h_{ik} \Phi_r(i) \tag{7}$$

and $h_{ik} = [(\beta_{ik}^2 - 1) + \mathbf{i}(2\zeta_i\beta_{ik})]^{-1}$ is the (complex) transfer function of the ith mode evaluated at frequency f_k. $\beta_{ik} = f_i / f_k$ is a frequency ratio. S_k is FFT of the measured acceleration of the moving mass of the shaker at frequency f_k. S_e is the spectral density of the prediction error and $\Phi_r(i) = r_i\,\Phi(i)$. In this study the mode shape is normalized such that it is equal to unity at the input degree-of-freedom (dof) I, and hence, $\Phi_r(I,i) = r_i$.

It is more convenient to work with the negative log-likelihood function (NLLF), so that

$$p\left(\theta \middle| \left\{\hat{\mathbf{Z}}_k\right\}\right) \propto \exp\left[-L(\theta)\right] \tag{8}$$

where

$$L(\theta) = nN_f \ln \pi + nN_f \ln S_e + S_e^{-1}\sum_k \left[\hat{\mathcal{F}}_k - \mathcal{F}_k(\theta)\right]^* \left[\hat{\mathcal{F}}_k - \mathcal{F}_k(\theta)\right] \tag{9}$$

For globally identifiable case under the condition of high sampling rate and long duration of data in the modal identification, the posterior PDF can be well approximated by a Gaussian PDF, which is equivalent to second order approximation of $L(\theta)$. Let $\hat{\theta}$ be the most probable value (MPV), which maximises the posterior PDF, and hence, minimises the NLLF. Consider the second-order Taylor series about $\hat{\theta}$

$$L(\theta) \approx L\left(\hat{\theta}\right) + \frac{1}{2}\left(\theta - \hat{\theta}\right)^T \mathbf{H}_L\left(\hat{\theta}\right)\left(\theta - \hat{\theta}\right) \tag{10}$$

where $\mathbf{H}_L(\theta)$ is the Hessian of Eq. (9) at the MPV. Substituting Eq. (10) into Eq. (8), the posterior PDF becomes a Gaussian PDF

$$p\left(\theta \middle| \hat{\mathbf{Z}}_k\right) \propto \exp\left[-\frac{1}{2}\left(\theta - \hat{\theta}\right)^T \hat{\mathbf{C}}^{-1}\left(\theta - \hat{\theta}\right)\right] \tag{11}$$

where $\hat{\mathbf{C}}$ is the posterior covariance matrix and is given by

$$\hat{\mathbf{C}} = \mathbf{H}_L\left(\hat{\theta}\right)^{-1} \tag{12}$$

It can be seen that the Gaussian PDF is completely characterised by the MPV and the covariance matrix, hence, a fast computation of these quantities is important for practical implementation of the Bayesian method

Unknown Input Modal Identification

In unknown input modal identification, the prediction error and modal forces can be modelled as independent and identically distributed (i.i.d.) Gaussian white noise and stationary process with a constant spectral density matrix, i.e. independent of *k*, respectively. The modal parameter to be identified consists of natural frequencies $\{f_i : i = 1,...,m\}$, damping ratios $\{\ _i : i = 1,...,m\}$, mode shapes $\{\ (i)\quad R^n : i = 1,...,m\}$, Hermitian PSD matrix of modal forces $\mathbf{S}\quad R^{m\ m}$, and power spectral density of prediction error S_e. For large *N* and small time step in the data acquisition, $\hat{\mathbf{Z}}_k$ are asymptotically independent at different frequencies and their real and imaginary part follow a Gaussian distribution (Yuen and Katafygiotis 2003). The likelihood function $p(\hat{\mathbf{Z}}_k|\ )$ is then given by

$$p\left(\left\{\hat{\mathbf{Z}}_k\right\}|\ \right) = (2\ )^{\ nN_f}\left[\prod_k \det \mathbf{C}_k(\ )\right]^{\frac{1}{2}} \exp\left[\ \frac{1}{2}\sum_k \hat{\mathbf{Z}}_k^T \mathbf{C}_k(\ )^{\ 1}\hat{\mathbf{Z}}_k\right] \tag{13}$$

where $\det(\times)$ denotes the determinant. The sum and product are over the N_f frequencies in the selected frequency band. $\mathbf{C}_k$ depends on , and is given by

$$\mathbf{C}_k = \frac{1}{2}\begin{bmatrix} \ \left(\mathrm{Re}\,\mathbf{H}_k\right)\ ^T & \ \left(\mathrm{Im}\,\mathbf{H}_k\right)\ ^T \\ \ \left(\mathrm{Im}\,\mathbf{H}_k\right)\ ^T & \ \left(\mathrm{Re}\,\mathbf{H}_k\right)\ ^T \end{bmatrix} + \frac{S_e}{2}\mathbf{I}_{2n} \tag{14}$$

where the first and second term account for the contribution from the model response and prediction error, respectively. $\ = [\ (1),...,\ (m)]\quad R^{n\ m}$ is the mode shape matrix confined to the measured dofs. $\mathbf{I}_{2n}$ is the $2n\quad 2n$ identity matrix. $\mathbf{H}_k$ is the theoretical spectral density matrix of the model response with (*i*,*j*) entry given by

$$\mathbf{H}_k(i,j) = S_{ij} h_{ik} h_{jk}^* \tag{15}$$

where S_{ij} is the cross spectral density between the *i*th and *j*th modal force. The posterior PDF $p(\ |\hat{\mathbf{Z}}_k)$ in terms of the NLLF $L(\ )$ as

$$p\left(\ \left|\hat{\mathbf{Z}}_k\right.\right) \propto \exp\left[\ L(\ )\right] \tag{16}$$

where the log-likelihood function $L(\ )$ is defined as

$$L(\ ) = \frac{1}{2}\sum_k \left[\ln \det \mathbf{C}_k(\ ) + \hat{\mathbf{Z}}_k^T \mathbf{C}_k(\ )^{\ 1}\hat{\mathbf{Z}}_k\right] \tag{17}$$

For a sufficiently large amount of data, the posterior PDF can be well-approximated by a Gaussian PDF centred at the most probable value. Considering the second order Taylor series about $\hat{\ }$ with the first term vanishes due to optimality of $\hat{\ }$, it can be shown that the posterior covariance matrix is equal to the inverse of the Hessian of $L(\ )$ evaluated at the most probable value.

Posterior Uncertainties

In known and unknown input forced modal identification, the MPV of the modal parameters are determined by maximizing $p(\ |\hat{\mathbf{Z}}_k)$, which is equivalent to minimizing NLLF. The posterior covariance matrix is then approximated by the inverse of the Hessian of the NLLF. In this study, coefficient of variation (c.o.v.) is used as an indicator of the uncertainties associated with the identified modal parameters. The c.o.v. is calculated as the ratio of the square root of variance, which is the corresponding diagonal element of the posterior covariance matrix, to the MPV value.

Modal Signal-to-noise Ratio

This section defines the modal s/n ratio, and hence, to provide an indication of the quality of the data for modal identification and assist interpreting the identification results. A modal s/n ratio is defined as the PSD of acceleration response to the PSD of the prediction error as

$$\ = \frac{S}{4S_e\ ^2} \tag{18}$$

This is organically defined in the case of unknown input forced modal identification. In the case of known input force modal identification, $S = S_F r^2 \|\ \|^2\ \ _I^2$ where S_F is the PSD of the shaker acceleration within the selected frequency band and $\ _I$ is the mode shape value at the shaker dof.

NUMERICAL CASE STUDIES

Effect of Modal Signal-to-noise Ratio

A two-story shear building was considered to study the effect of different levels of measurement noise on uncertainties associated with the identified modal parameters. The height of the first and second story of the shear building is 4 m and 3 m, respectively. The shear building has uniform mass 5600 kg, and the inter-story stiffness of the first and second story is 1.769×10^7 N/m and 1.244×10^7 N/m. The natural frequencies are 3.261 Hz and 11.211 Hz. The damping ratio is assumed to be 1% in all modes. It is assumed that an accelerometer was installed at each story to measure the acceleration responses in horizontal direction. The measured responses are assumed with sampling rate at 100 Hz. It is assumed that the measured acceleration was contaminated by i.i.d. Gaussian white noise with a root PSD of 1×10^{-6} $g/\sqrt{Hz}$. Pseudo-random excitation with different magnitudes and having a flat root PSD from 0.1 Hz to 20 Hz frequency band is applied to the second floor of the shear building. The excitation signal was generated by band-passing a Gaussian white noise signal through a second order Butterworth filter. 185 s acceleration responses were measured, which covers 10 s before the shaker is turned on, 140 sec pseudo-random excitation, and 35 s free vibration after the shaker is turned off.

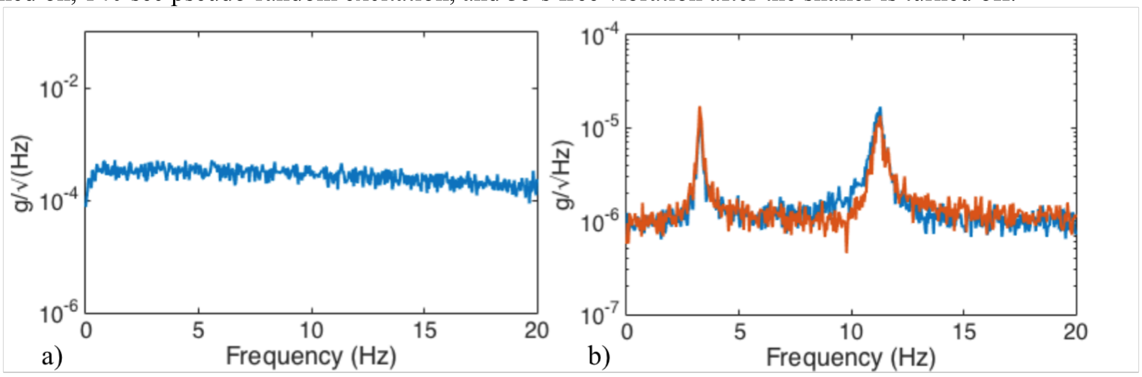

Figure 1 a) Root PSD spectrum of shaker mass applied on the two-storey shear building and b) corresponding root PSD spectrum of acceleration data

Figure 1a shows one of the considered Pseudo-random excitations applied on the second floor of the two-story shear building. The root PSD spectrum of the measured acceleration responses at the first and second story of the two-story shear building are shown in Figure 1b. Figure 1b shows that two modes are adequately excited with their resonance peaks apparent. The known and unknown input fast Bayesian FFT modal identification are used to identify the modal parameters, i.e. natural frequencies and damping ratios, and quantify the associated uncertainties. In the case of the unknown input modal identification, the two-story shear building is subjected to the same excitation as the known input case. However, the measured input loading is not used, and hence, it is output-only modal identification. Two modes are identified separately with a single mode (m=1) assumed within each band. In this study the FFT data within frequency bands (3.065 – 3.457 Hz) and (10.539 – 11.884 Hz) are used to identify the 1st and 2nd mode, respectively.

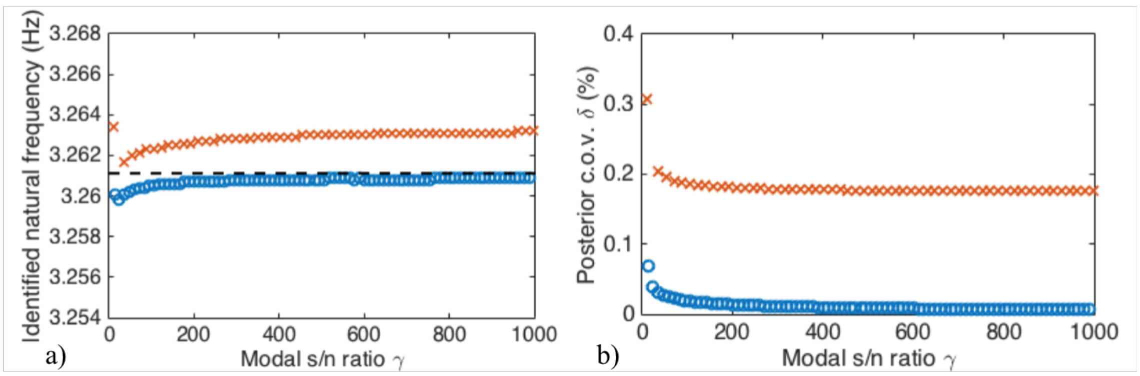

Figure 2 a) Identified 1st mode of natural frequency and b) corresponding posterior c.o.v. versus modal s/n ratios (circles: known input; crosses: unknown input; dashed line: true value)

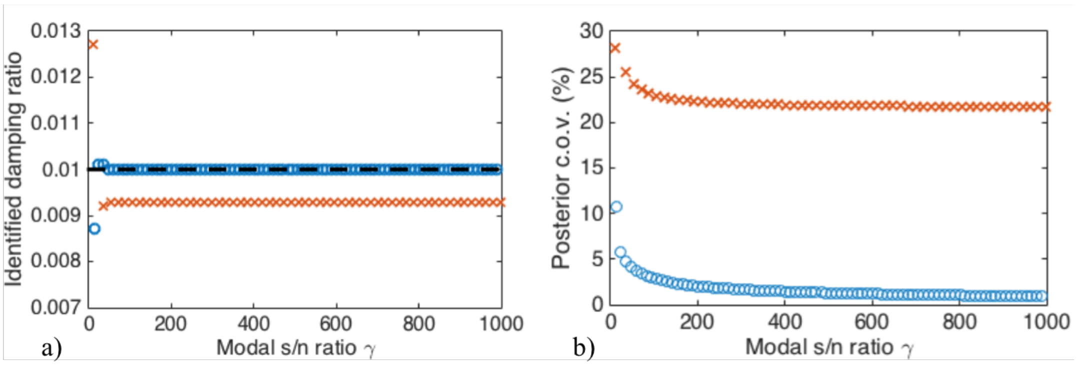

Figure 3 a) Identified 1st mode of damping ratio and b) corresponding posterior c.o.v. versus modal s/n ratios (circles: known input; crosses: unknown input; dashed line: true value)

Figures 2a and 3a show the identified 1st mode natural frequencies and damping ratios using known and unknown input modal identification. The modal s/n ratio rather than the excitation magnitude is shown, as it is dimensionless and more informative. The results show that the natural frequencies and damping ratios identified by known and unknown input modal identification are close to the true value. However, the natural frequencies and damping ratios identified by the known input modal identification have better agreement with the true value than the unknown input modal identification. Figures 2b and 3b shows the posterior c.o.v. of the identified natural frequencies and damping ratios, respectively. It can be seen that the value of the posterior c.o.v. decreases with the increase in the modal s/n ratio. For the case of unknown input modal identification, the posterior c.o.v. of natural frequency and damping ratio tend to be a constant at 0.174% and 21.635%, respectively, which are much higher than the posterior c.o.v. in the known input excitation modal identification (0.003% and 0.491% for natural frequencies and damping ratios). The reason is that the unknown input modal identification does not have the loading information, and hence, it always has larger uncertainty associated with the identified natural frequencies and damping ratios. The identified natural frequencies and damping ratios and the associated posterior c.o.v. for the 2nd mode versus different modal s/n ratios are also shown in Figures 4a and 5a, and Figures 4b and 5b, respectively. A similar phenomenon is observed in the results for 2nd mode.

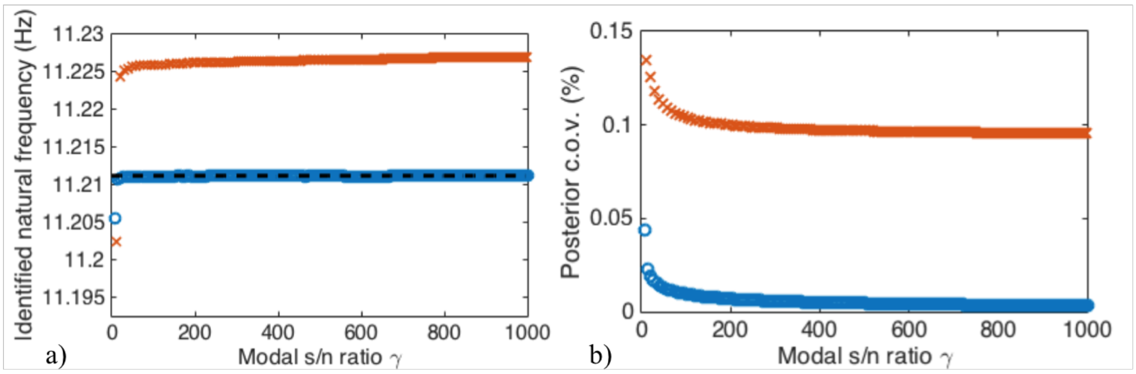

Figure 4 a) Identified 2nd mode of natural frequency and b) corresponding posterior c.o.v. versus modal s/n ratios (circles: known input; crosses: unknown input; dashed line: true value)

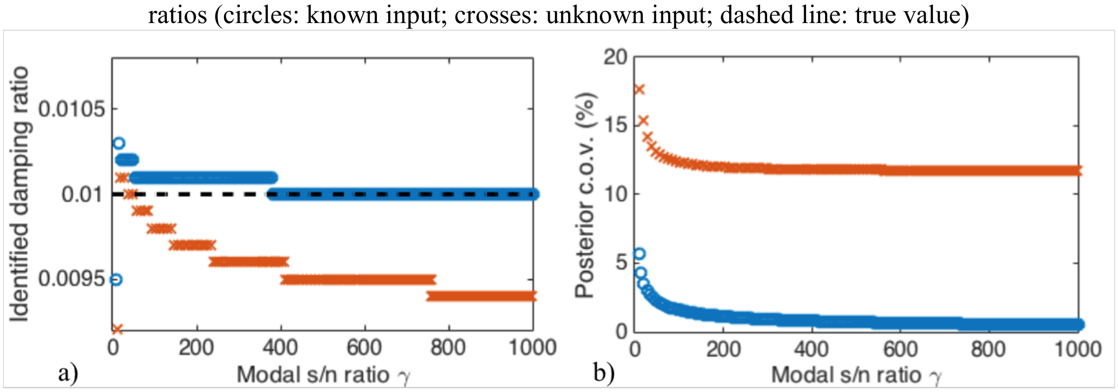

Figure 5 a) Identified 2nd mode of damping ratio and b) corresponding posterior c.o.v. versus modal s/n ratios (circles: known input; crosses: unknown input; dashed line: true value)

Effect of Measured Number of Degrees-of-freedom

The second example is a ten-story shear building. It was used to study the effect of measured number of dofs. The ten-storey shear building has uniform floor mass of 100 ton, inter-story stiffness of 177 kN/mm and damping ratio of 1% in all modes. The natural frequency of the first two modes are 1 Hz and 2.98 Hz. It is assumed that a shaker is installed on the top of the building to generate horizontal excitation. The excitation signal is a pseudo-random excitation with a flat root PSD from 0.1 Hz to 20 Hz. This example considers increasing number of accelerometers installed from the top to the bottom of the ten-story shear building. The sampling rate of each accelerometer is 100 Hz in the simulation and the data is contaminated by i.i.d. channel noise with a root PSD of $1\times10^{-6}\ g/\sqrt{Hz}$. The data covers 50 s before the shaker is turned on, 500 s forced vibration during the shaker is turned on, and 50 s free vibration after the shaker is turned off. Table 1 summarized the number of measured dof and the locations for the ten-story shear building.

Number of measured dof *n*	Floors with accelerometer installed
2	10/F, 9/F
3	10/F, 9/F, 8/F
4	10/F, 9/F, 8/F, 7/F
5	10/F, 9/F, 8/F, 7/F, 6/F
6	10/F, 9/F, 8/F, 7/F, 6/F, 5/F
7	10/F, 9/F, 8/F, 7/F, 6/F, 5/F, 4/F
8	10/F, 9/F, 8/F, 7/F, 6/F, 5/F, 4/F, 3/F
9	10/F, 9/F, 8/F, 7/F, 6/F, 5/F, 4/F, 3/F, 2/F
10	10/F, 9/F, 8/F, 7/F, 6/F, 5/F, 4/F, 3/F, 3/F, 2/F, 1/F

Table 1 Summary of the number of measured dof *n* and locations for the ten-story shear building (shaker located at 10/F)

Figure 6 shows the mode shapes for 1st and 2nd mode of the ten-story shear building. The study focuses on identifying the natural frequency and damping ratio of the 2nd mode. The reason is that the value of the mode shape component at the 7th floor is almost close to zero, which means the accelerometer installed at 7th floor does not contain much information of the 2nd mode. This is used to study the effect of the modal information measured by each accelerometer on the uncertainties associated with identified natural frequencies and damping ratios.

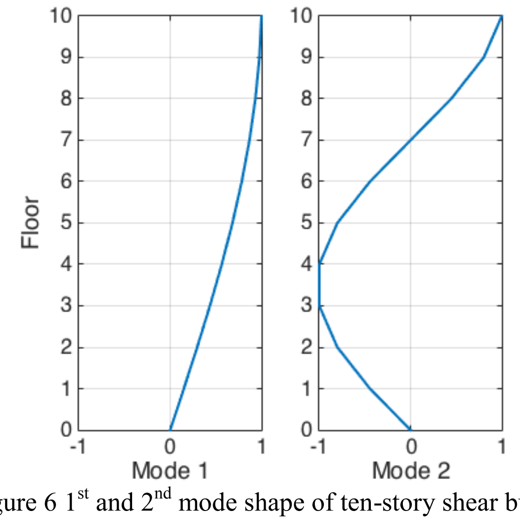

Figure 6 1st and 2nd mode shape of ten-story shear building

Figure 7 shows the modal s/n ratio versus different number of measured dofs for 1st and 2nd mode. For the 1st mode, the modal s/n ratio increases with diminishing rate as *n* increases. This is because the increasing number of accelerometers are placed from the top to the bottom on the ten-story shear building and the information measured for 1st mode at each dof increases with diminishing rate as evidenced by the mode shape of the 1st mode in Figure 6. Different to 1st mode, the modal s/n ratio of the 2nd mode does not increase much even an accelerometer is added on 7th floor. The reason is that the mode shape of the 2nd mode having an almost zero value component at this floor, which means the accelerometer installed at this floor does not provide much information of the 2nd mode.

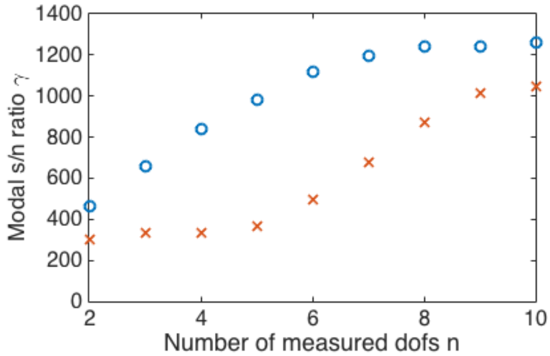

Figure 7 Modal s/n ratio versus the measured number of dofs for ten-story shear building (circles: 1st mode; crosses 2nd mode)

Figures 8 and 9 show the posterior c.o.v. of the identified natural frequencies and damping ratio for the 1st mode under known and unknown input excitation condition. As expected, the posterior c.o.v. of the identified damping ratio is much larger than the identified natural frequency. Comparing the posterior c.o.v. in known and unknown input modal identification, the uncertainty associated with the identified modal parameters is always larger than that in known input modal identification. This is because the input excitation information is not provided, and hence, there is less information available in the modal identification process. Figures 8a and 9a show that the posterior c.o.v. of the identified natural frequencies and damping ratios in known input modal identification decrease with diminishing rate as the n increases, which is consistent with modal s/n ratio as shown in Figure 7. However, the posterior c.o.v. in the unknown input modal identification is insensitivity to n as shown in Figures 8b and 9b.

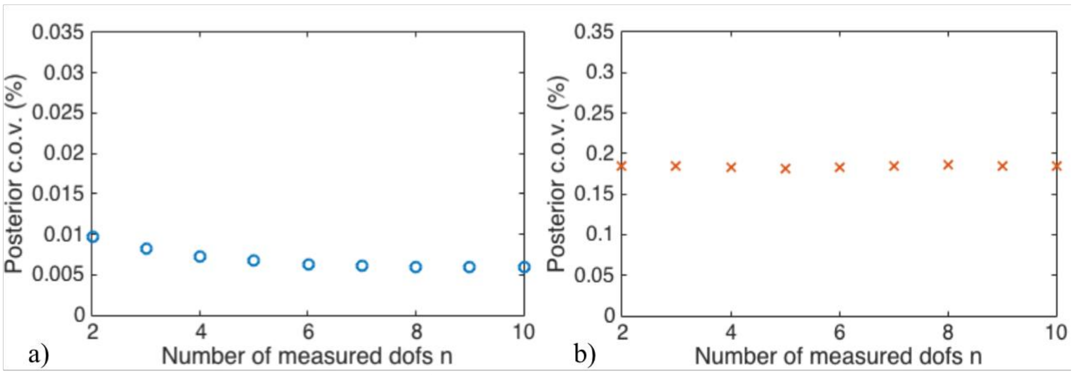

Figure 8 Posterior c.o.v. of the identified 1st mode natural frequency using a) known and b) unknown input modal identification for ten-story shear building

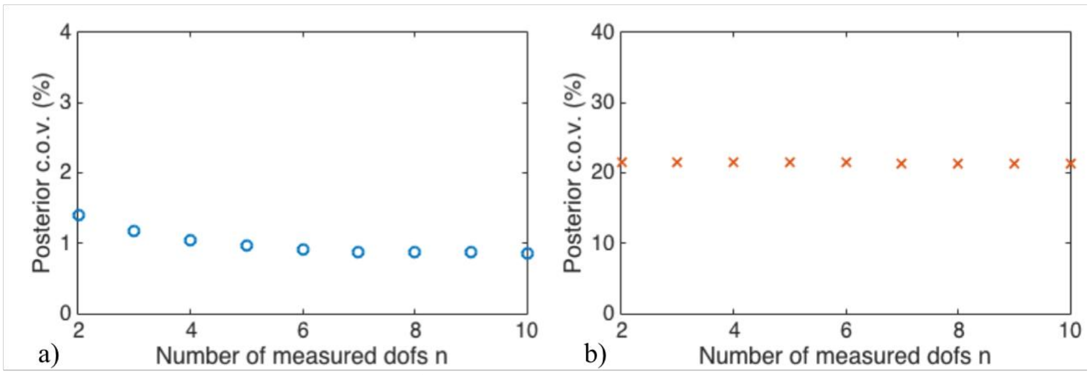

Figure 9 Posterior c.o.v. of the identified 1st mode damping ratio using a) known and b) unknown input modal identification for ten-story shear building

Figures 10 and 11 shows the identified 2nd mode natural frequency, damping ratio and the corresponding posterior c.o.v. using known and unknown input modal identification. As shown in Figure 7, the modal s/n ratio of the 2nd mode does not increase much as the 4th accelerometer (n = 4) is added to the 7th floor, where the value of the mode shape component is almost close to zero, and hence, the accelerometer at the 7th floor does not provide much information about the 2nd mode. This is consistent with the posterior c.o.v. of the identified 2nd mode natural frequency and damping ratio under known input modal identification as shown in Figures 10a and 11a. Both figures show that the values of the posterior c.o.v. of the identified natural frequency and damping ratio do not reduce after the 4th accelerometer was added at the 7th floor. For unknown input modal identification, the values of the posterior c.o.v. are not as sensitive as the results of known input modal identification to *n*.

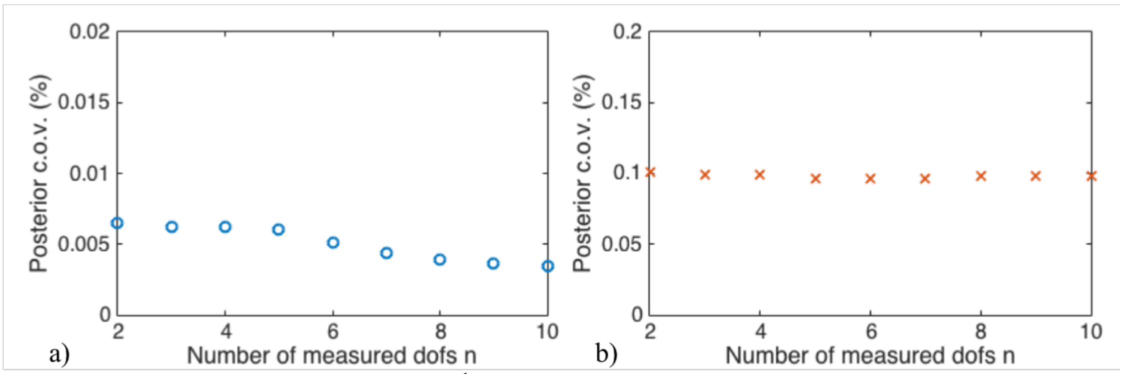

Figure 10 Posterior c.o.v. of the identified 2nd mode natural frequency using a) known and b) unknown input modal identification for ten-story shear building

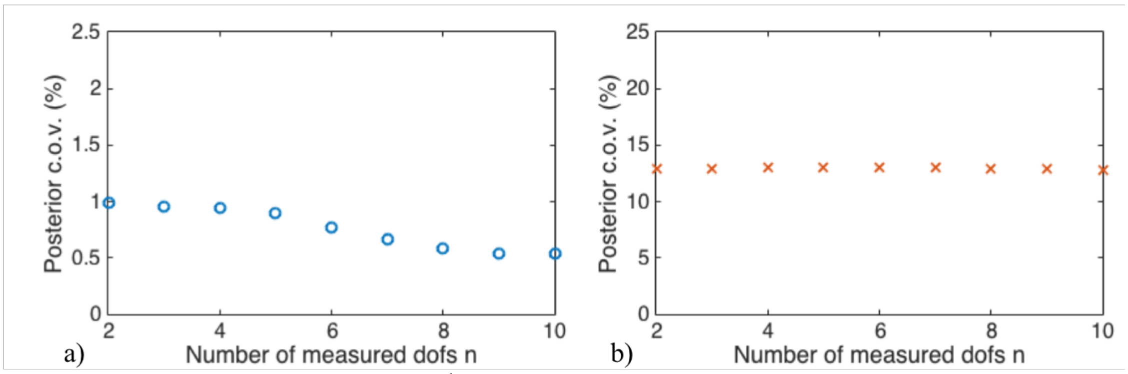

Figure 11 Posterior c.o.v. of the identified 2nd mode damping ratio using a) known and b) unknown input modal identification for ten-story shear building

CONCLUSIONS

A comparison of the uncertainties associated with the modal parameters in modal identification has been presented in this paper. The study employs a frequency-domain fast Bayesian FFT method to identify the modal parameters, such as natural frequencies and damping ratios, from vibration data. In addition to identifying modal parameters, the fast Bayesian FFT method also quantifies the associated uncertainties in modal identification. This study has analysed and compared the uncertainties in known and unknown input condition. In general the uncertainties associated with the modal parameters in unknown input modal identification is much larger than and as not as sensitive as known input modal identification to modal s/n ratio. This study has compared and provided insights on the uncertainties of known and unknown input modal identification.

REFERENCES

Au, S.K. (2011). “Fast Bayesian FFT method for ambient modal identification with separated modes”. Journal of Engineering Mechanics, 137, 214-226.

Au, S.K. (2012). “Fast Bayesian ambient modal identification in the frequency domain, Part I: posterior most probable value”. Mechanical Systems and Signal Processing, 26, 60-75.

Au, S.K. and Ni Y.C. (2013). “Fast Bayesian modal identification of structures using known single-input forced vibration data”. Structural Control and Health Monitoring, 21(3), 381-402.

Au, S.K., Zhang, F.L. and Ni, Y.C. (2013). “Bayesian operational modal analysis: theory, computation, practice”. Computers and Structures, 126, 3-4.

Au, S.K. and Zhang, F.L. (2016). “Fundamental two-stage formulation for Bayesian system identification, Part I: general theory”. Mechanical Systems and Signal Processing, 66, 31-42.
Beck, J.L. (2010). “Bayesian system identification based on probability logic”. Structural control and Health Monitoring, 17(7), 825-847.
Beck, J.L. and Katafygiotis, L.S. (1998). “Updating models and their uncertainties I: Bayesian statistical framework”. Journal of Engineering Mechanics, 124(4), 455-461.
Brownjohn, J.M.W. (2003). “Ambient vibration studies for system identification of tall buildings”. Earthquake Engineering and Structural Dynamics, 32, 71-95.
Ewins, D.J. (2000). *Modal testing: theory, practice, and application*. Research Studies Press, Hertfordshire.
James, G.H., Carne, T.G., and Lauffer, J.P. (1995). “The Natural excitation technique (NExT) for modal parameter extraction from operating structures”. Journal of Analytical and Experimental Modal Analysis, 10(2), 260-277.
Maia, N.M.M., Silva, J.M.M., Almas, E.A.M. and Sampaio, R.P.C. (2003). “Damage detection in structures: from mode shape to frequency response function method”. Mechanical Systems and Signal Processing, 17(3), 489-498.
Papadimitrious, C., Beck, J.L. and Katafygiotis, L.S. (2001). “Updating robust reliability using structural test data”. Probabilistic Engineering Mechanics, 16(2), 103-113.
Reynders, E. Pintelon, R. and De Roeck, G. (2007). “Uncertainty bounds on modal parameters obtained from stochastic subspace identification”. Mechanical Systems and Signal Processing, 22, 945-969.
Yuen, K.V. and Katafygiotis, L.S. (2001). “Bayesian time-domain approach for modal updating using ambient data”. Probabilistic Engineering Mechanics, 16(3), 219-231.
Yuen, K.V. and Katafygiotis, L.S. (2002). “Bayesian modal updating using complete input and incomplete response noisy measurement. Journal of Engineering Mechanics”, 128(3), 340-350.
Yuen, K.V. and Katafygiotis, L.S. (2003). “Bayesian fast Fourier transform approach for modal updating using ambient data”. Advance in Structural Engineering, 6(2), 81-95.
Yuen, K.V., Katafygiotis, L.S. and Beck, J.L. (2002). “Spectral density estimation of stochastic vector processes’. Probabilistic Engineering Mechanics, 17(3), 265-272.
Zembaty, Z. and Kowalski, M. (2000). “Dynamic identification of a model of brick masonry building”. Archives of Civil Engineering, 46(1), 106-136.