Proceedings of the International Association for Shell and Spatial Structures (IASS)
Symposium 2015, Amsterdam
Future Visions
17 - 20 August 2015, Amsterdam, The Netherlands

Engineered bamboo for shell structures

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Abstract

Engineered bamboo combines the benefits of a natural material with the advantages of a laminated composite, resulting in an efficient, light material well-suited to gridshell structures. Bamboo is a rapidly renewable material that can be harvested every 4-5 years. The round culm can either be used as is or it can be processed into a variety of laminated products. Engineered bamboo is currently promoted as a structural alternative to timber and glue-laminated timber, but also has potential in shell applications. In contrast to short fibre composites, engineered bamboo maintains fibre length and continuity within the raw material for exceptional mechanical properties. The composite section results in a high strength material in compression and tension, with bending properties comparable to timber products. The inherent flexibility of the material has advantages in comparison to timber, allowing for complex designs to be achieved. The present work explores the substitution of engineered bamboo in existing gridshells with a comparison to the original timber structure. As dowelled connections are key components at the boundary of gridshells, their potential for use with bamboo is also explored.

Keywords: gridshell, timber, engineered bamboo, structural analysis.

1. Gridshell structures

A shell is a structure which exhibits structural stiffness and strength due to its inherent double curved shape (Kelly et al. [1], Harris [2], Cabrinha [3], Harris et al. [4]). In particular, a gridshell is defined as a shell with regular large openings where the removed material is concentrated on the remaining surface, which consequently has a grid-like form (Kelly et al. [1]), Harris et al. [4]). Over the last century the construction of a limited number of innovative gridshells has revealed numerous advantages of these structures including: minimal material, lightweight, cost- and time-effective solutions for enclosing large volumes, coverage of any plan shape, structural efficiency, and environmental sustainability (Harris et al. [4], Harris [2], Cabrinha [3]). Until recently, challenges in construction and lack of established design precedents led to the restriction of the potential widespread use of these structures as efficient architectural and engineering solutions (Harris et al. [4], Harris [2], Cabrinha [3]). The majority of realised gridshells are constructed from timber due to its advantages in respect to other conventional structural materials.

1.2. Timber

While timber has been used in construction for centuries, it is not used as a conventional material for large-scale construction. However, the natural material is experiencing a renaissance, increasingly explored for structural applications due to cost and environmental benefits. In particular, timber is an ideal material for the formation of gridshells due to low torsional stiffness which enables the on-site forming of roof lattices from an initial flat mat of a uniform mesh of identical components, also known as laths (Happold and Liddell [5], Harris [2], Harris et al. [4]). Pioneering work of architects and engineers such as Frei Otto and Ted Happold throughout the 20th century, and the recent advances in computer aided design and computational power on early 21st century enable engineers of today to extract the potential of timber gridshell structures (Harris [2]). With increased design and analysis capabilities, additional alternative materials can be explored for gridshells.

1.3. Engineered bamboo

Engineered bamboo has potential for structural applications due to the inherent strength and rapid renewability of the material. The product uses raw bamboo in a laminated composite, creating a standard section that is comparable to timber and glue-laminated timber (Sharma et al. [6]). A high strength and lightweight material, engineered bamboo has advantages over conventional materials such as concrete and steel. The inherent flexibility of the material is potentially advantageous in gridshell structures, allowing additional complexity in design and form finding. A comparison of bamboo and timber experimental mechanical properties is presented in Table 1. The compressive and tensile strengths of laminated bamboo exceed that of the timber-based products, while the flexural properties are comparable.

	Density (kg/m^3)	*Compressive Stress Parallel to Grain* (MPa)	*Tensile Stress Parallel to Grain* (MPa)	*Shear Stress Parallel to Grain* (MPa)	*Flexural Modulus of Rupture* (MPa)	*Flexural Modulus of Elasticity* (GPa)
Laminated Bamboo [7]	686	77	90	16	77-83	10.8-12.9
Sitka Spruce [8,9]	383	36	59	9	67	8
Douglas-fir LVL [10,11]	520	57	49	11	68	13

Table 1: Mechanical properties for engineered bamboo and timber.

1.4. Dowel Connections

Connections are essential in gridshells, providing both stability and flexibility in the erection of the structure. At the nodal points over the surface of the gridshell, the laths must be allowed to move relative to one another as the structure is formed into its curved shape. As a result, the connections, shown in Figures 1-3, can be installed loose and tightened once the gridshell is in its final form. Friction holds the laths in place relative to one another. In contrast, the connections at the edge of each structure rely on bolts through the laths to transfer load to the supports. This form of connection is commonly used in timber and is known as a dowel-type connection. Since engineered bamboo is not widely used in construction, the behaviour of these connections in bamboo has yet to be fully characterised.

1.5 Objectives

Here we explore three prominent examples of timber gridshell structures: the timber lattice roof for the Mannheim Bundesgartenschau, the Downland Gridshell, and the Savill Garden gridshell. The case studies will form the background for the investigation of representative structures in engineered bamboo. We will also present a brief test series to compare the response of dowel connections in timber and engineered bamboo, which can be used as boundary connections in gridshell structures.

2. Methodology

The following section presents the case studies and analyses of the three selected structures. Additional experimental research is presented on the performance of dowel connections in both timber and engineered bamboo.

2.1. Case Studies

2.1.1. Timber lattice roof for the Mannheim Bundesgartenschau

The timber lattice roof for the Mannheim Bundesgartenschau (Figure 1) is one of the most significant structures of 20th century. Constructed between 1971 and 1975 in Mannheim, Germany, the structure at the time was the only example of double-layered gridshell worldwide (Kelly et al. [1]). Multiple challenges were faced throughout its design and construction since there was no previous experience available to the architects and engineers, and the only existing precedents were considerably less complex. The local architectural practice (Mutschler & Partners) won the competition and facing the challenges of such an innovative project sought for the specialised experience of Professor Frei Otto and Ove Arup & Partners (Happold and Liddell [5]).

The architectural purpose of the structure was to enclose, in an attractive way, two of the main facilities (the restaurant and Multihalle) at the central zone of the park, which hosted the biennial federal garden show of West Germany. Consequently, the architects envisioned a free-form, large-span, lightweight, curved structure which would blend harmonically with the landscape. In particular, the architectural form was comprised of two main domes interconnected with tunnels expressed through the gridshell geometry (Happold and Liddell [5], Happold and Liddell [12]).

Figure 1: Mannheim gridshell, interior view (left) [13] and joinery detail (right) [14].

The timber gridshell lattice was constructed of long (30–40m) Western Hemlock laths of square cross-section (50 x 50mm) that formed irregularly sized shells. The long laths were formed using finger jointing and the timber species was selected because of its straight-grain and long-length availability (Happold and Liddell [12], Happold and Liddell [5]). The overall final form of the structure was a result of form-finding techniques based on hanging chain physical models (Happold and Liddell [5]), since at the time computer aided design was in its infancy. Frei Otto's hanging chain net models initiated the conceptualisation of the structure, which were then abstracted through stereo-photographs into digital co-ordinates. The points were optimised, with respect to the net's equilibrium of forces, using custom-made computer programs (Professor Linkwitz from Stuttgart University) (Happold and Liddell [5]).

The construction process of the lattice roof started with a flat mat of laths, which was erected into shape from below using a proprietary system of scaffolding. This system of scaffolding towers was distributed in strategic locations and enabled lateral movement. When the shape was achieved, the gridshell boundary conditions were fixed and the nodal connections tightened. The curvature distribution of the lattice and the subsequent bending stresses required an increased lath size, which was impossible at that stage. The engineers addressed this by instead doubling the number of lath layers resulting in the first double-layer timber gridshell ever made (Happold and Liddell [5]).

The increased stiffness of this innovative lattice was exploited from the friction-based nodal joinery, shown in Figure 1. It consisted of a bolt and spring system that enabled relative slip between the layers of lath during formation. Moreover, steel wires were used to increase the structural diagonal stiffness of the lattice shells in response to the analysis of the expected main loads that the structure should withstand: self-weight, non-uniform wind and snow loading, and imposed loads for construction or maintenance purposes (Happold and Liddell [5]).

Overall, Mannheim's timber gridshell, which was initially programmed to function for two years but is still operational, is one of the landmark structures of 20th century. Due to its novelty at the time, there were a number of construction challenges (breakage of laths and joints, erection process) (Kelly et al. [1]). These challenges have been further refined and addressed in subsequent double-layer timber gridshell structures constructed in the UK in early 21st century.

2.1.2. Downland gridshell

The double-layered timber gridshell located at the Weald and Downland Open air museum in West Sussex (Figure 2) was the first of its kind in the UK and only the fifth such structure worldwide at the time of erection in 2002 (Kelly et al. [1]). The computational modelling advances since the construction of Mannheim gridshell in 1975 shifted the locus of form-finding from the hanging chain nets to modern three-dimensional computer models altering the nature both of the concept and construction phase (Harris et al. [4]). Edward Cullinan Architects joined Buro Happold Engineers, Dr Chris Williams, and Green Oak Carpentry Company to create an exemplar open space workshop reflecting the museum's commitment to craftsmanship and timber tradition (Kelly et al. [1]). The clear-span lattice, erected on top of a masonry structure, is described as a morphing triple-bulb, hourglass-shaped gridshell (Harris et al. [4]).

Figure 2: Downland gridshell, interior view (left) [15] and joinery detail (right) [16].

The dimensions of the initial flat, regular, rectangle lattice were 47 x 25m, the final curved structure would be 50m long with a height fluctuating between 12.5m at the valleys to 16m at the crowns and a corresponding width between 7.35m and 9.5m. The long (36m) laths were made out of locally sourced high-quality straight grain French Oak (50mm wide x 35mm deep) utilising both state-of the-art finger- and traditional scarf-joints (Kelly et al. [1], Harris et al. [4]). The resulting continuous length timber is advantageous both structurally and aesthetically, with axial forces carried more efficiently through long laths to the resistance points and there is minimal visual disruption due to joinery discontinuities. Special emphasis was also given for all the materials used in construction to be locally sourced and biodegradable or recyclable (Kelly et al. [1]). Finally, a number of innovations in gridshell construction were introduced during the realisation of this project.

Drawing experience and knowledge from previous projects, the initial laying platform for the flat lattice was erected to the height of the final shape's valleys; harnessing the gravity effects in that way rather than working against them. In particular, the lattice would be lowered into the desired position in contrast to the opposite procedure in the Mannheim gridshell example. Moreover, due to out-of-plane force considerations, it was decided that the formation of the valleys and crowns would happen simultaneously. This process was made possible by utilising a modular scaffolding system enabling lateral movement and built-in gage monitoring. In particular, the first central dome would be erected and fixed first and the outer two would follow morphing the desired final geometry (Kelly et al. [1]). This system enabled the accurate positioning and monitoring of the lattice during formation. The centre points of the three domes were initially placed in their final longitudinal and transverse location allowing only for vertical movement, which was obtained from the height of the multiprops. As a result, the actual shape could be easily compared at all stages with the theoretical one, which was output from the digital modelling (Harris et al. [4]). The form-finding process was largely based on a custom-made program developed by Dr. Chris Williams derived from dynamic relaxation analysis.

Further innovations included the invention of an alternative nodal joinery system (Figure 2), which would enable relative scissoring and sliding of lath layers, based on clamping plates rather that bolts and slotted laths as in the Mannheim lattice. This novel system had significant comparative advantages as it would be time- and cost- efficient without weakening the laths by reducing the surface area with holes (Harris et al. [4]). It should also be noted that during the formation, nodal connections needed to be loose to enable movement and rotation. Moreover, timber instead of steel tension cables (Mannheim gridshell) were utilised to enhance diagonal structural stiffness of the lattice shells (Kelly et al. [1]). The Downland timber gridshell was a relatively small but innovative structure, which provoked and renewed the international interest in such structures. The combined experience and technical knowledge gained from the project led to a second UK based double-layered timber roof lattice, the Savill Garden (Harris et al. [13]).

2.1.2. ***The Savill Garden gridshell***

When Crown Estates announced the winner of the architectural competition for a new building at Windsor Great Park, which should be roofed 'with an undulating gridshell of park-grown larch' (TRADA [18]), Glenn Howells Architects visited Downland gridshell with the client who was convinced both by the suitable form and feasibility of construction. As a result, the architectural practice joined Buro Happold as roof designers, Dr. Chris Williams, and Green Oak Carpentry Company to design a new timber lattice roof (Figure 3), four times the size of Downland gridshell (TRADA [18], Harris et al. [13]). The architectural brief called for a dramatic form that was environmentally sensitive and a structure that would melt into the surrounding landscape.

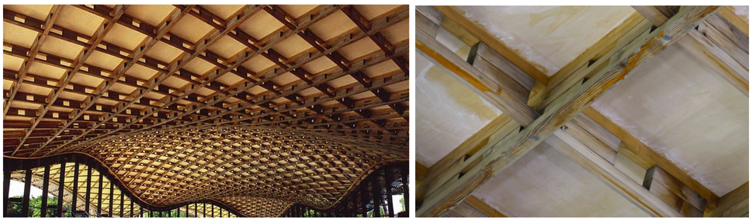

Figure 3: Savill gridshell, interior view (left) [19] and joinery detail (right) [20].

The proposed timber lattice was developed from an organic-like, doubly-curved, floating roof which could successfully mirror the adjacent forest skyline (Harris et al. [13]). The desired final shape was the output of a parametric definition developed by Dr. Chris Williams through synthesis of two intersecting circles, a sinusoidal centreline of varying amplitude and parabolic cross-sections (Harris et al. [13]). However, the form selected for the over-hanging roof introduced the need for steel elements due to the long openings towards the park. In particular, a steel tube perimeter ring was designed so as to support the timber structure and carry the imposed loads (Harris et al. [13], TRADA [18]). The steel structure supporting the timber roof played a significant role in the building as a whole since the loads would concentrate along the perimeter of the roof and particularly towards the steel legs. As a result the structural and finite element analysis was performed iteratively in a bidirectional way so as to achieve acceptable performance of both the roof and steel structure (Harris et al. [13], TRADA [18]).

The initial flat lattice was 90m long and 25m wide and was made out of regular larch laths of rectangular cross-section (80 x 50mm). Single laths were interconnected utilising both finger and scarf joints, which was interestingly the first instance of finger jointing in larch (TRADA [18]). Moreover, the timber species used was locally sourced and was divided into two quality categories (using criteria of limits of knots and slope grain) and consequently used in parts of the structure with corresponding structural and material behaviour needs. As a result, material wastage was minimised and structural efficiency increased (Harris et al. [13]).

The construction method of Savill Building gridshell differed from the previously discussed case-studies. In contrast to the previous example of the four layers being bent into shape together, only the first two bottom layers were formed initially. Subsequently, timber shear blocks were bolted on the laths and finally the upper two layers were fixed in place on top of them as shown in Figure 3 (Harris et al. [13]). It was argued that this technique increased not only the out-of-plane strength and stiffness, but also the layer spacing. Furthermore, diagonal stiffness and strength of the lattice was added through plywood covering, which was the first time plywood had been used for this purpose.

2.2. Analysis and Modelling

Based on the case-studies, analysis and modelling of representative structures was conducted. The structures were investigated utilising Timoshenko's Elastica theory (Timoshenko and Gere [21]) to approximate the initial lift force (P_{lift}) required for different types of materials (D'Amico et al. [22]). Using the geometry and mechanical properties, the following equation expresses the force required to bend a section:

$$P_{lift} = \left(\frac{K(m)}{l} \right)^2 EI \tag{11}$$

where *K(m)* is the elliptic integral (determined from the angle of the constructed gridshell), *l* is the span length, and *EI* is the flexural stiffness of the section. The results of the analysis are presented in Table 2. Due to the high bending modulus of elasticity, the engineered bamboo would perform comparably to the original timber used in each project in terms of the calculated forces required to lift the material in place and to anchor it to the boundary. To further analyse the three case studies, approximate models were created using Rhinoceros (McNeel and Associates [23]) with Grasshopper and Kangaroo (ModeLab [24], Kangaroo3d [25]). The input parameters for the models are listed in Table 3. For comparison, the initial lift force was also modelled (P_{model}), with the results summarised in Table 2 and shown in Figure 4. The required force (P_{model}) to displace the section had an error within 2% in comparison to the calculated values. Table 2 indicates that the laminated bamboo requires approximately 20-30% more lift force in comparison to the original timber, with the exception of the Downland gridshell due to the comparable bending modulus of French oak. In the Mannheim and Savill gridshells, reduction of the section size while maintaining P_{lift} could result in a stable but more radically curved gridshells. The particle spring method was utilised for form-finding (Kilian and Ochsendorf [26]). The Multihalle at the Mannheim Bundesgartenschau was modelled as a hanging chain to achieve the realised constructed form (Figure 4a).

Material	Western Hemlock[27]		French Oak[28]		Larch[27]		Engineered Bamboo[7]	
E (GPa)	10.4		13.1		9.9		12.9	
	P_{lift}	P_{model}	P_{lift}	P_{model}	P_{lift}	P_{model}	P_{lift}	P_{model}
Mannheim (N)	62.3	61.1					77.3	75.7
Downland (N)			84.7	85.2			83.4	83.9
Savill (N)					583.3	574.1	760.1	748.1

Table 2: Calculated and modelled initial lift forces for engineered bamboo and timber.

Gridshell	**Span (m)**	**Max Height (m)**	**Layers**	**Lath Size (mm)**	**Material**
Mannheim	60 x 80	20	4	50 x 50	Hemlock
Downland	9.5 x 50	16	4	50 x 35	French Oak
Savill Garden	25 x 90	8.5	4	50 x 80	Larch

Table 3: Design details and materials for case studies.

In comparison, the Downland (Figure 4b) and Savill (Figure 4c) gridshells were modelled as a double curvature shell structures lifted into place. The form finding models performed comparably based on the bending stiffness of engineered bamboo and the original timber structure. Additional structural analysis will determine the stability of the overall gridshell and connections.

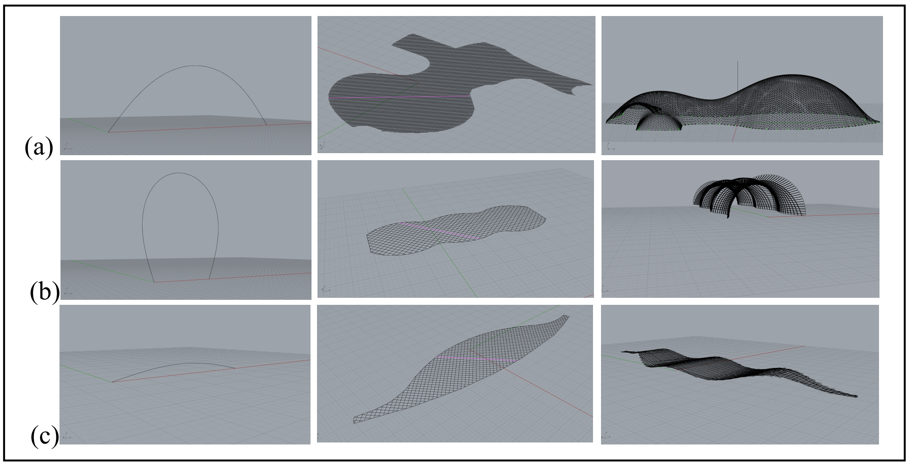

Figure 4: Elastica curve (left), plan (centre) and deformed shape (right) for (a) Mannheim Multihalle gridshell, (b) Downland Weald gridshell, and (c) Savill Garden gridshell.

2.4. Dowel Connections

To explore the performance of doweled connections in bamboo, experimental tests were conducted on both engineered bamboo and timber. The caramelised laminated bamboo board is made of

Phyllostachys pubescens, or Moso, bamboo with a soy-based resin [7]. The specimens are built up from a commercial laminated bamboo sheet (2440 x 1220 x19mm), which is cut and further laminated into a section with the desired dimensions using polyurethane adhesive (Purbond HB S309). The specimen dimensions are 72 (w) x 38 (t) x 508mm (l) in the grain direction. A 13mm hole was drilled 84mm from the edge of each specimen, corresponding to the minimum edge distance recommended in timber design (BSI [29]). The same geometry and method was used for solid timber specimens made of Scots pine to enable comparison to existing research results in timber (Dorn et al. [30]). The materials were loaded by a 12mm diameter steel dowel passing through a steel plate in a central slot in each specimen. A tensile load was applied to the steel plate, and resisted by embedment of the dowel into the timber or bamboo surrounding it. The displacement of the steel plate was measured relative to the edge of the timber or bamboo.

Figure 5 shows force-displacement diagrams for the two materials. It is shown that the timber specimen exhibited substantial ductility before fracture – in fact it continued to resist a substantial load until fracture at just over 6mm, and this corresponds to other tests on timber in the literature (Dorn et al. [30]). Fracture in the bamboo specimens occurred at much lower displacements, with only one of the specimens exhibiting significant ductility, and that one fracturing at just under 2mm. Design guidance based on plasticity theory by Johansen [31] allows prediction of their load-carrying capacity. The failure load in timber was close to the 12.2kN predicted by the Eurocode design standard, based on its density. The bamboo, on the other hand, significantly exceeded the 16.4kN predicted for timber of the same density.

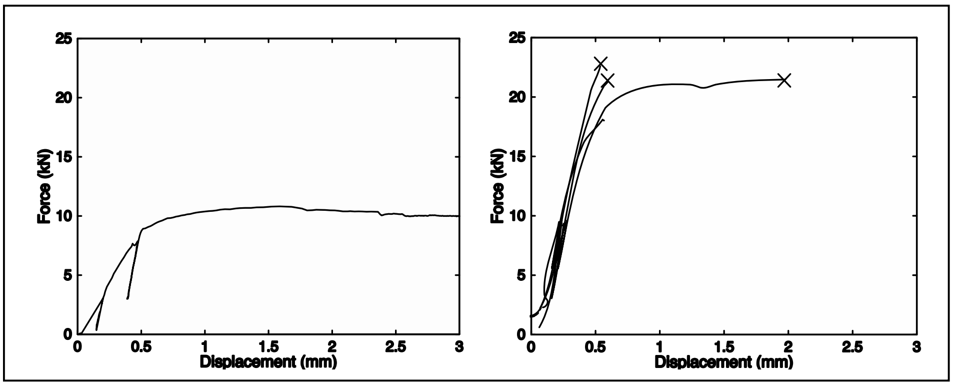

Figure 5: Force-displacement plots for timber (left) and bamboo (right) specimens – crosses indicate fracture in the bamboo.

The fracture in the timber specimen was by a single crack propagating from near the centre of the dowel hole in the direction of loading, as shown in the left hand image in Figure 6. This was also seen as the dominant type of failure in a large number of tests on single-fastener connections in timber by Jorissen [32]. The failure in bamboo connections, on the other hand, was generally by a pair of cracks propagating from nearer the sides of the hole, as shown in the right hand image in Figure 6.

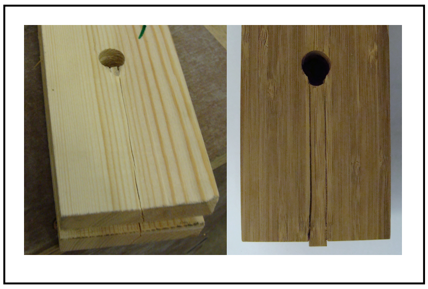

Figure 6: Fracture in timber (left) and engineered bamboo (right).

A numerical model of the connection was created in Matlab in which stress-field plots were generated from a stress function for an orthotropic material (Reynolds et al. [33]). The model utilised the material properties for timber given in EN 338 (BSI [34]) and the results for the shear and direct stress are shown in Figure 7. The strain fields show that the crack in timber propagates from the point with highest tensile stress perpendicular to the loaded direction, while the cracks in bamboo propagate from the points of highest shear stress.

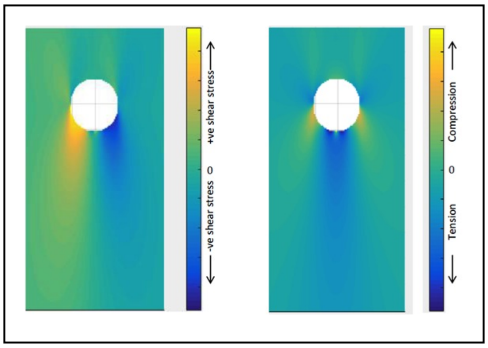

Figure 7: Stress fields for shear stress (left) and direct stress in the horizontal direction (right) in an orthotropic material with properties approximately appropriate for timber and bamboo.

The experimental study highlights the difference in fracture in dowel connections in timber and engineered bamboo, with the latter demonstrating a brittle failure mode. Additional work is needed to characterise the influence of the longtiduinal shear strength in bamboo connections. Furthermore, the use of digital image correlation to measure the strain field around holes in timber and bamboo plates under loading will establish the accuracy of elastic numerical models, which could be used for more general analysis.

3. Summary

The review of three case studies provided the background for the design and construction of timber gridshells. Due to the similarities in flexural properties, engineered bamboo has potential as a construction material in these types of structures. Through form-finding and analysis, the engineered bamboo was shown to be a comparable material to the original timber structure, with the potential for designs with greater curvature. Additional experimental work investigated boundary conditions in gridshells and specifically explored dowel connections in tension. The brief study highlighted the difference in failure modes in embedment in bamboo compared with timber, and indicates more research is required before dowel-type connections can be reliably used in engineered bamboo structures. Further structural analysis would allow for optimization of sections and determine the practicality of engineered bamboo in gridshell applications.

Acknowledgements

The presented work is supported by a Leverhulme Trust Programme Grant and EPSRC Grant EP/K023403/1. The research forms part of a collaboration between the University of Cambridge, Massachusetts Institute of Technology (MIT) and University of British Columbia (UBC).

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