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Incipient Fault Detection Based on Robust Threshold Generators: A Sliding Mode Interval Estimation Approach ★

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Abstract: This paper presents an incipient fault detection framework for systems with process disturbances and sensor disturbances based on a novel proposed threshold generator. Firstly, the definition of incipient faults is given using the $\mathcal{H}_-$ from the quantitative point of view. Then, from the generated residuals and RMS evaluation function, the threshold generator is proposed based on sliding mode interval estimation module to ensure that the RMS evaluation of residuals is less than the generated threshold. By using recent results of the bounded real lemma for internally positive systems, a set of sufficient conditions to detect incipient faults via linear matrix inequality (LMI) is presented. Case study on an electrical traction device is presented to verify the effectiveness of the proposed method.

Keywords: Incipient fault detection, threshold generator, sliding mode, interval estimator.

1. INTRODUCTION

Generally speaking, model-based fault diagnosis approaches consist of two important parts: residual generators and evaluation functions. In common ideologies of designing robust fault detection systems such as Frank (1990), the dynamics of the residual generators are firstly designed ensuring that they have a good trad-off between sensitivity to faults and robustness against disturbances. Then a threshold (possibly adaptive threshold) is selected to ensure that the evaluation of residuals is smaller than it in fault free scenario. The above ideology is the so-called active robustness (see e.g. de Oca S et al. (2012)), where the design freedom depends on the dynamics design of residual generators. Due to that incipient faults are usually submerged by disturbances, the detectability always can not satisfy the requirement for incipient fault detection (IFD). An alternative approach, known as passive robustness, is proposed by enhancing the threshold robustness through the dynamics of threshold generators (see e.g. Johansson et al. (2006), de Oca S et al. (2012), Puig et al. (2013) and Raïssi et al. (2010)). Johansson et al. (2006) propose an inequality for a linear system with uncertain parameters which is shown to be a valuable tool for developing dynamic threshold generators for fault detection. However, a general method for finding a tight realizable upper bound of the modulus of an impulse response is still an open problem. In de Oca S et al. (2012) and Puig et al. (2013), the domain shape zonotope is used to propagate uncertainties, but it needs fast computations. The appearance of interval observers proposed in Gouzé et al. (2000) motivates researchers to apply them to fault diagnosis (for instance Raïssi et al. (2010)) due to that it does not need fast computations compared with the approach in de Oca S et al. (2012) and Puig et al. (2013), which also motivates this paper.

In this paper, an IFD method is presented based on new proposed robust threshold generators by combining the interval estimation technique with the sliding mode technique. During the past decades, sliding mode observer based FDI has been extensively studied (see, e.g. Hermans and Zarrop (1996), Edwards et al. (2000), Yan and Edwards (2007) and Zhang et al. (2016a)). However, results based on both interval estimation and sliding mode techniques for IFD have not been available. It has been shown in Zhang et al. (2016a) that sliding mode observer can effectively improve the detectability of residual-based fault detection when compared with Luenberger observers due to its reduced order sliding motion and robustness to 'observer matched' disturbances. In this paper, an $\mathcal{H}_-$ index induced from the 2-norm of faults and disturbances is introduced to define a worst-case scale variable. For certain practical systems, incipient faults and serious abrupt faults are distinguished using this defined scale variable. Then based on the designed residual generator, the interval sliding mode estimators are proposed to ensure that the residuals stay in an envelop composed by the interval sliding motion. Furthermore, the dynamics characterizing the threshold generators are proposed based on the designed interval sliding mode observers. The further most important step is to optimize the parameters of the designed dynamics of threshold generators such that the considered incipient faults can be detected, which is formulated

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as an $\mathcal{H}_\infty$ optimal problem for internally positive systems, and solved by linear matrix inequality (LMI) technique. The contribution of this paper is summarized as follows.

(i) The definition of incipient faults is given from the quantitative point of view;

(ii) The dynamics characterizing the robust threshold generators are proposed based on novel designed interval sliding mode estimators;

(iii) The optimal parameters for IFD are obtained based on LMI technique.

Notation: The notation $\oplus$ represents the *Minkowski* sum, $|\cdot|$ denotes the element-wise absolute value, $\mathbf{B}^r$ is a r-dimensional unitary box. If there is no special note, $\|\cdot\|$ represents the 2−norm of a matrix or a vector. For a real matrix or a vector M, $M > 0$ ($M \geq 0$) means that its entries are positive (nonnegative). The symbol diag(v) denotes a diagonal matrix with the diagonal elements formed by the elements of the vector v. For two vectors $x_1,\ x_2 \in \mathcal{R}^n$ or matrices $A_1,\ A_2 \in \mathcal{R}^{n\times n}$, the relations $x_1 \leq x_2$ and $A_1 \leq A_2$ are defined in element wise, respectively. Given a matrix $A \in \mathcal{R}^{m\times n}$ or a vector $x \in \mathcal{R}^n$, defining $A^+ = \max\{0, A\}$, $A^- = A^+ - A$ and $x^+ = \max\{0, x\}$, $x^- = x^+ - x$, respectively, then $A^+,\ A^-,\ x^+,\ x^-$ are nonnegative.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Preliminaries

The $\mathcal{H}_\infty$ norm of a transfer function $G_{y\eta}(s)$ is denoted by $\left\|G_{y\eta}(s)\right\|_\infty = \sup_\omega \bar{\sigma}\left(G_{y\eta}(j\omega)\right)$, and, as in Liu et al. (2005), the $\mathcal{H}_-$ index of a transfer function $G_{y\eta}(s)$ is defined as $\left\|G_{y\eta}(s)\right\|_- = \inf_\omega \underline{\sigma}\left(G_{y\eta}(j\omega)\right)$, where $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ are the largest and smallest singular values of $G_{yn}(\cdot)$ respectively. Note that

$$\left\|G_{y\eta}(s)\right\|_\infty = \sup_{\eta\neq 0}\frac{\|y\|_2}{\|\eta\|_2} = \sup_{\eta\neq 0}\frac{\|y\|_{RMS}}{\|\eta\|_{RMS}}. \tag{1}$$

where the $\|\cdot\|_{RMS}$ is defined in Emami-Naeini et al. (1988).

A commonly used lemma in interval estimator design is shown as follows.

Lemma 1. (Efimov et al. (2012)) Let $x,\ \underline{x},\ \bar{x} \in \mathcal{R}^n$ satisfy that $\underline{x} \leq x \leq \bar{x}$. Then, for any matrix A with appropriate dimensions, $A^+\underline{x} - A^-\bar{x} \leq Ax \leq A^+\bar{x} - A^-\underline{x}$.

Consider LTI systems with process faults and actuator faults in a compact form described by

$$\dot{x} = Ax + Bu + D_p f_p + D_a f_a + \eta_p(x, u, \omega, t) \tag{2}$$
$$y = Cx + \eta_s(x, u, \omega, t) \tag{3}$$

where $x \in \mathcal{R}^n$ is state, $u \in \mathcal{R}^h$ is control, $y \in \mathcal{R}^p$ is measurement output. The unknown function $\eta_p(\cdot)$ represents lumped process uncertainties, including modeling errors, parameters perturbation, external and internal disturbances. The unknown function $\eta_s(\cdot)$ represents unknown lumped sensor disturbances. The signal $f_p \in \mathcal{R}^{q_1}$ and $f_a \in \mathcal{R}^{q_2}$ represent process faults and actuator faults respectively. All the matrices A, B, C, D_p and D_f are known with appropriate dimensions, D_p and D_f are of full column rank.

It is assumed throughout this paper that rank($C[D_p, D_f]$) = rank($[D_p, D_f]$) = $\tilde{q}$ which is a popular assumption (see, e.g., Edwards et al. (2000) and Yan and Edwards (2007)). From Yan and Edwards (2007), there exists a coordinate transformation T such that $T[D_p, D_f] = \begin{bmatrix} 0 & 0 \\ D_{p2} & D_{f2}\end{bmatrix}$ with $D_{p2} \in \mathcal{R}^{\tilde{q}\times q_1}$ and $D_{f2} \in \mathcal{R}^{\tilde{q}\times q_2}$ full column rank, and $CT^{-1} = [0, C_2]$ with C_2 non-singular. Then, with coordinate transformation T, system (2)-(3) is rewritten as

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + \eta_{p1}(\cdot) \tag{4}$$
$$\dot{x}_{21} = A_{211}x_1 + A_{22}^{11}x_{21} + A_{22}^{12}x_{22} + \eta_{p21}(\cdot), \tag{5}$$
$$\dot{x}_{22} = A_{212}x_1 + A_{22}^{21}x_{21} + A_{22}^{22}x_{22} + \eta_{p22}(\cdot) + D_{p2}f_p + D_{a2}f_a, \tag{6}$$
$$y = C_2x_2 + \eta_s(\cdot) \tag{7}$$

where $x_1 \in \mathcal{R}^{n-p}$, $x_2 := \text{col}(x_{21}, x_{22})$ with $x_{21} \in \mathcal{R}^{n-p-\tilde{q}}$ and $x_{22} \in \mathcal{R}^{\tilde{q}}$, and specially the matrix A_{11} is Hurwitz.

Assumption 1. For lumped uncertainties $\eta_{p1}(\cdot), \eta_{p21}(\cdot), \eta_{p22}(\cdot)$ and $\eta_s(\cdot)$ in system (4)-(7), there exists a function $\bar{\eta}_{p21}(y, u, t)$ such that $\|\eta_{p21}(\cdot)\| \leq \bar{\eta}_{p21}(y, u, t)$. Moreover, there exist zonotopes W_{p1}, W_{p22} and W_s such that $\eta_{p1}(\cdot) \in W_{p1}$, $\eta_{p22}(\cdot) \in W_{p22}$ and $\eta_s(\cdot) \in W_s$ where

$$W_{p1} = \eta_{p1}^c \oplus H_{\bar{\eta}_{p1}}\mathbf{B}^{n-p} \tag{8}$$
$$= \left\{\eta_{p1}(\cdot) \in R^{n-p} \,\middle|\, \left|\eta_{p1}(\cdot) - \eta_{p1}^c\right| \leq \bar{\eta}_{p1}, \eta_{p1}^c, \bar{\eta}_{p1} \in \mathcal{R}^{n-p}\right\}$$
$$W_{p22} = \eta_{p22}^c \oplus H_{\bar{\eta}_{p22}}\mathbf{B}^{\tilde{q}} \tag{9}$$
$$= \left\{\eta_{p22}(\cdot) \in R^{\tilde{q}} \,\middle|\, \left|\eta_{p22}(\cdot) - \eta_{p22}^c\right| \leq \bar{\eta}_{p22}, \eta_{p22}^c, \bar{\eta}_{p22} \in \mathcal{R}^{\tilde{q}}\right\},$$
$$W_s = \eta_s^c \oplus H_{\bar{\eta}_s}\mathbf{B}^p$$
$$= \left\{\eta_s(\cdot) \in R^p \,\middle|\, \left|\eta_s(\cdot) - \eta_s^c\right| \leq \bar{\eta}_s, \eta_s^c, \bar{\eta}_s \in \mathcal{R}^p\right\}. \tag{10}$$

Remark 1. The bound on $\eta_{p21}(\cdot)$ is used to ensure that the sliding motion takes place in finite time and maintains on the sliding surface thereafter (see Edwards et al. (2000) and Yan and Edwards (2007)). The definition of zonotope is given in de Oca S et al. (2012), and in (8)-(10), $\bar{\eta}_{p1}$, $\bar{\eta}_{p22}$, $\bar{\eta}_s$, η_{p1}^c η_{p22}^c and η_s^c are all vectors. Zonotopes are usually used to propagate system uncertainties, see, for instance, de Oca S et al. (2012) and Xu et al. (2013), which provide an interval bound. ∇

2.2 Problem formulation

Suppose that the designed residual generator for system (2)-(3) is represented by the transfer function as

$$r(s) = G_{rf}(s)f(s) + G_{r\eta}(s)\eta(s) \tag{11}$$

where $r(s)$, $f(s)$ and $\eta(s)$ are the Laplace transform of the residuals $r(t)$, lumped faults $f(t)$ and lumped uncertainties $\eta(t)$, respectively. The terms $G_{rf}(s)$ and $G_{r\eta}(s)$ are the transfer functions from $f(s)$ and $\eta(s)$ to $r(s)$ respectively.

Then based on the interval model of system (2)-(3), the upper bound $\bar{r}(s)$ and the lower bound $\underline{r}(s)$ of $r(s)$ are estimated in the fault-free operation, respectively by

$$\bar{r}(s) = \bar{G}_{r\eta}(s)\bar{G}(\underline{\eta}(s), \bar{\eta}(s)),\ \underline{r}(s) = \underline{G}_{r\eta}(s)\underline{G}(\underline{\eta}(s), \bar{\eta}(s)) \tag{12}$$

where $\bar{G}(\cdot)$ and $\underline{G}(\cdot)$ are functions of $\underline{\eta}(s)$ and $\bar{\eta}(s)$ with $\bar{\eta}(s) \leq \eta(s) \leq \underline{\eta}(s)$. Then, $\underline{r}(s) \leq r(s) \leq \bar{r}(s)$ when $f(s) = 0$.

Suppose that the RMS of $r(s)$ is chosen as the residual evaluation function as in Emami-Naeini et al. (1988). Then the residual evaluation $J = \|r\|_{RMS}$. Recall that the threshold is the tolerant limit for unknown inputs and model uncertainties during the fault-free operation. It requires that $J \leq J_{th}$ when $f = 0$. Accordingly, the threshold $J_{th}(t, T)$ is chosen based on the threshold generator (12) as

$$J_{th}(t, T) = \max\left(\|\bar{r}(t)\|_{RMS}, \left\|\underline{r}(t)\right\|_{RMS}\right). \tag{13}$$

Generally speaking, the incipient faults are typically small and are usually not easy to be detected. In fact, the definition of incipient faults are mostly from the qualitative point of view (see, e.g. Frank (1990) and Chen and Patton (1997)), and the corresponding IFD system is developed based on those qualitative definition. The residuals generated by incipient faults are difficult to exceed the thresholds based on bounds of disturbances, which is the main problem of the existing IFD system. In this paper, a worst-case scale variable between norm bound of faults and norm bound of disturbances is proposed to define the incipient faults. This scale variable is expressed in terms of

$$\Gamma = \|G_{\eta f}(s)\|_{-} = \inf_{\eta \neq 0} \frac{\|f\|}{\|\eta\|}, \tag{14}$$

where $G_{\eta f}(s)$ represents the transfer function matrix from f to η. Note that $\|G_{\eta f}(s)\|_{-}$ is the $\mathcal{H}_{-}$ index of $G_{\eta f}(s)$.

It is well known that without disturbances and uncertainties, the incipient faults are easily detected. Thus, it is supposed throughout this paper that $\|\eta\| \geq \epsilon$ (ϵ is a small positive constant).

For some practical systems, there exist constants $\underline{\Gamma}$ and $\bar{\Gamma}$ such that

(a) the faults satisfying that $0 \leq \Gamma < \underline{\Gamma}$ are unnecessary to detect;
(b) the faults satisfying that $\underline{\Gamma} < \Gamma < \bar{\Gamma}$ are considered as incipient faults;
(c) The faults satisfying that $\underline{\Gamma} < \Gamma < +\infty$ are considered as serious faults.

The objective of this paper is to detect the incipient faults satisfying that $\underline{\Gamma} < \Gamma < \bar{\Gamma}$ before they develop to serious faults. To achieve this objective, the parameters in residual generator $r(s)$ and threshold generator $J_{th}(t, T)$ characterized by $\bar{r}(s)$ and $\underline{r}(s)$ should be optimized. The **optimization principles** are shown as follows.

(i) The generated residual r should be as sensitive as possible to fault f and as robust as possible to disturbance η.
(ii) The residual r under incipient faults scenario should exceed the threshold $J_{th}(t, T)$ in finite time, i.e. there is time t_d with $t_0 < t_d < t_0 + T$ (t_0 is the fault occurrence time) such that $\|J_f\| \geq J_{th}(t_d, T)$ (J_f is the residual evaluation in fault scenario).

3. FAULT DETECTION SCHEMES

3.1 Residual generation

It is well known that key part in fault detection system is the residual generator, which is often constructed by state estimation algorithm. The residual r in this paper is generated based on the variable x_{22} in (6). Consider following system

$$\dot{\hat{x}}_{22} = A_{212}\hat{x}_1 + A_{22}^{21}\hat{x}_{21} + A_{22}^{22}\hat{x}_{22} \tag{15}$$

where $\hat{x}_1$ and $\hat{x}_{21}$ are the estimations of x_1 and x_{21} respectively. Denote $r = x_{22} - \hat{x}_{22}$. Then by comparing (15) with (6), the residual generator is obtained by

$$\dot{r} = A_{212}e_1 + A_{22}^{21}e_{21} + A_{22}^{22}r + \eta_{p22}(\cdot) + D_{p2}f_p + D_{a2}f_a \tag{16}$$

where $e_1 = x_1 - \hat{x}_1$ and $e_{21} = x_{21} - \hat{x}_{21}$ with $\hat{x}_1$ and $\hat{x}_{21}$ being determined later.

3.2 Residual evaluation

Evaluation of the generated residual is an important task for FDI due to the existence of model uncertainties and disturbances. One of the widely adopted approach is to choose a so-called threshold $J_{th} > 0$ and, then use the following logical relationship for fault detection

$$J = \|r\|_{RMS} > J_{th} \Rightarrow \text{a fault is detected, alarm triggered} \tag{17}$$

$$J = \|r\|_{RMS} \leq J_{th} \Rightarrow \text{fault free, no alarm} \tag{18}$$

where $\|r\|_{RMS}$ is the RMS of $r(t)$ in time interval $(t, t+T)$ with T being the finite time window.

3.3 Threshold generation

For subsystem (4), consider the following systems

$$\dot{\bar{x}}_1 = A_{11}\bar{x}_1 + \hat{A}_{12}y - \hat{A}_{12}^{+}\underline{\eta}_0 + \hat{A}_{12}^{-}\bar{\eta}_0 + \bar{\eta}_1 + F(\bar{x}_1 - \underline{x}_1), \tag{19}$$

$$\dot{\underline{x}}_1 = A_{11}\underline{x}_1 + \hat{A}_{12}y - \hat{A}_{12}^{+}\bar{\eta}_0 + \hat{A}_{12}^{-}\underline{\eta}_0 + \underline{\eta}_1 - F(\bar{x}_1 - \underline{x}_1) \tag{20}$$

where $\hat{A}_{12} = A_{12}C_2^{-1}$, the constant vectors $\bar{\eta}_1$ and $\underline{\eta}_1$ are obtained based on the zonotope W_{p1}, given by $\bar{\eta}_1 = \bar{\eta}_{p1} + \eta_{p1}^c$ and $\underline{\eta}_1 = -\bar{\eta}_{p1} + \eta_{p1}^c$, and $\bar{\eta}_0$ and $\underline{\eta}_0$ are obtained based on the zonotope W_s, given by $\bar{\eta}_0 = \bar{\eta}_s + \eta_s^c$ and $\underline{\eta}_0 = -\bar{\eta}_s + \eta_s^c$. It is assumed that $\underline{x}_1(0) \leq x_1(0) \leq \bar{x}_1(0)$. The gain matrix $F \in \mathcal{R}^{(n-p)\times(n-p)}$ is a nonnegative matrix to be determined in the sequel such that $\underline{x}_1 \leq x_1 \leq \bar{x}_1$ where x_1 is the states of subsystem (4).

Let $\hat{x}_1 \in [\bar{x}_1, \underline{x}_1]$. For subsystem (5), consider the following system

$$\dot{\hat{x}}_{21} = A_{211}\hat{x}_1 + A_{22}^{11}\hat{x}_{21} + \hat{A}_{22}^{12}y + (A_{22}^{11} - \hat{A}_{22}^{11})(C_{21}y - \hat{x}_{21}) + \nu \tag{21}$$

where $\hat{A}_{22}^{12} = A_{22}^{12}[0, I_{\tilde{q}}]C_2^{-1}$, $C_{21} = [I_{p-\tilde{q}}, 0]C_2^{-1}$, and $\hat{A}_{22}^{11}$ is chosen to be symmetric negative definite. The function ν is defined by

$$\nu = m(\cdot)\mathrm{sgn}([I_{p-\tilde{q}}, 0]C_2^{-1}y - \hat{x}_{21}) \tag{22}$$

where $m(\cdot)$ is a positive scalar function to be determined later to ensure that sliding motion is established.

By comparing (21) with (5), the error subsystem is obtained by

$$\dot{e}_{21} = A_{211}e_1 + \hat{A}_{22}^{11}e_{21} + \hat{A}_{22}^{12}\eta_s + \eta_{p21}(\cdot) - \nu. \tag{23}$$

The concept of threshold selector is firstly proposed in Emami-Naeini et al. (1988), which is actually an optimal threshold generator. Then the threshold generator is developed in Johansson et al. (2006) by solving an inequality involving a convolution operator. In this paper, a novel threshold generator will be proposed. Consider the following dynamics

$$\begin{aligned}\dot{\bar{x}}_{22} =& A_{212}^{+}\bar{x}_1 - A_{212}^{-}\underline{x}_1 + \bar{\Phi} \\ &+ A_{22}^{21}\hat{x}_{21} + A_{22}^{22}\bar{x}_{22} + L_1(\bar{x}_{22} - \underline{x}_{22}) + \bar{\eta}_3,\end{aligned} \tag{24}$$

$$\begin{aligned}\dot{\underline{x}}_{22} =& A_{212}^{+}\underline{x}_1 - A_{212}^{-}\bar{x}_1 + \underline{\Phi} \\ &+ A_{22}^{21}\hat{x}_{21} + A_{22}^{22}\underline{x}_{22} - L_2(\bar{x}_{22} - \underline{x}_{22}) + \underline{\eta}_3\end{aligned} \tag{25}$$

where $\bar{\eta}_3$ and $\underline{\eta}_3$ are obtained based on the zonotope W_{p22} and given by $\bar{\eta}_3 = 2(\bar{\eta}_{p22} + \eta_{p22}^c)$ and $\underline{\eta}_3 = 2(-\bar{\eta}_{p22} + \eta_{p22}^c)$. The gain matrices L_1 and L_2, and function vectors $\bar{\Phi}$ and $\underline{\Phi}$ are determined later.

Denote $\bar{r} = \bar{x}_{22} - x_{22}$ and $\underline{r} = x_{22} - \underline{x}_{22}$. Then by comparing (24) and (25) with (6), the dynamics of $\bar{r}$ and $\underline{r}$ are obtained by

$$\begin{aligned}\dot{\bar{r}} =& A_{212}^{+}\bar{e}_1 + A_{212}^{-}\underline{e}_1 + \bar{\Phi} \\ &- A_{22}^{21}e_{21} + (A_{22}^{22} + L_1)\bar{r} + L_1\underline{r} + \bar{\eta}_3 - \eta_{p22},\end{aligned} \tag{26}$$

$$\begin{aligned}\dot{\underline{r}} =& A_{212}^{+}\underline{e}_1 + A_{212}^{-}\bar{e}_1 - \underline{\Phi} \\ &+ A_{22}^{21}e_{21} + (A_{22}^{22} + L_2)\underline{r} + L_2\bar{r} + \eta_{p22} - \underline{\eta}_3\end{aligned} \tag{27}$$

Suppose that the sliding motion has bee established. During sliding motion, $e_{21} = 0$. Under the initial condition that $\underline{x}_{22}(0) < x_{22}(0) < \bar{x}_{22}(0)$, which ensures that $\bar{r} > 0$ and $\underline{r} > 0$ for all $t > 0$, it requires that the matrix $\bar{A}_{22} = \begin{bmatrix} A_{22}^{22}+L_1 & L_1 \\ L_2 & A_{22}^{22}+L_2 \end{bmatrix}$ is Metzler, at the same time, $\mathrm{col}(A_{212}^{+}\bar{e}_1 + A_{212}^{-}\underline{e}_1 + \bar{\Phi}, A_{212}^{+}\underline{e}_1 + A_{212}^{-}\bar{e}_1 - \underline{\Phi}) > 0$ and $\mathrm{col}(\bar{\eta}_3 - \eta_{p22}, \eta_{p22} - \underline{\eta}_3) > 0$. Furthermore, during the sliding motion in fault-free operation, by comparing (26) and (27) with (16) and denoting $\bar{\Delta} = \bar{r} - r$ and $\underline{\Delta} = r + \underline{r}$, it follows that

$$\begin{aligned}\dot{\bar{\Delta}} =& A_{212}^{+}\bar{e}_1 + A_{212}^{-}\underline{e}_1 + \bar{\Phi} - A_{212}e_1 \\ &+ (A_{22}^{22} + L_1)\bar{\Delta} + L_1\underline{\Delta} + \bar{\eta}_3 - 2\eta_{p22}, \quad (28)\\ \dot{\underline{\Delta}} =& A_{212}^{+}\underline{e}_1 + A_{212}^{-}\bar{e}_1 - \underline{\Phi} + A_{212}e_1 \\ &+ (A_{22}^{22} + L_2)\underline{\Delta} + L_2\bar{\Delta} + 2\eta_{p22} - \underline{\eta}_3. \quad (29)\end{aligned}$$

Under the condition that $\underline{\Delta}(0) > 0$ and $\bar{\Delta}(0) > 0$, to ensure $\underline{\Delta}(t) > 0$ and $\bar{\Delta}(t) > 0$ for all $t > 0$, it requires that $\bar{A}_{22}^{22}$ is Metzler, and $\mathrm{col}(A_{212}^{+}\bar{e}_1 + A_{212}^{-}\underline{e}_1 + \bar{\Phi} - A_{212}e_1, A_{212}^{+}\underline{e}_1 + A_{212}^{-}\bar{e}_1 - \underline{\Phi} + A_{212}e_1) > 0$ and $\mathrm{col}(\bar{\eta}_3 - 2\eta_{p22}, 2\eta_{p22} - \underline{\eta}_3) > 0$.

Recalling the zonotope of W_{p22} and the values of $\bar{\eta}_3$ and $\underline{\eta}_3$, it is easy to see that $\mathrm{col}(\bar{\eta}_3 - 2\eta_{p22}, 2\eta_{p22} - \underline{\eta}_3) > 0$. Then $\mathrm{col}(\bar{\eta}_3 - \eta_{p22}, \eta_{p22} - \underline{\eta}_3) > 0$. An available solution of $\bar{\Phi}$ and $\underline{\Phi}$ satisfying the requirements that $\mathrm{col}(A_{212}^{+}\bar{e}_1 + A_{212}^{-}\underline{e}_1 + \bar{\Phi}, A_{212}^{+}\underline{e}_1 + A_{212}^{-}\bar{e}_1 - \underline{\Phi}) > 0$ and $\mathrm{col}(A_{212}^{+}\bar{e}_1 + A_{212}^{-}\underline{e}_1 + \bar{\Phi} - A_{212}e_1, A_{212}^{+}\underline{e}_1 + A_{212}^{-}\bar{e}_1 - \underline{\Phi} + A_{212}e_1) > 0$ is given by

$$\bar{\Phi} = \frac{1}{2}(A_{212}^{+} + A_{212}^{-})(\bar{x}_1 - \underline{x}_1), \quad (30)$$

$$\underline{\Phi} = \frac{1}{2}(A_{212}^{+} + A_{212}^{-})(\underline{x}_1 - \bar{x}_1). \quad (31)$$

Therefore, the threshold generator can be given by (13) characterized by the dynamics (26) and (27). In fault-free case, the inequality $J < J_{th}$ holds.

4. DESIGN SCHEMES

Firstly, the dynamics $\underline{x}_1$ and $\bar{x}_1$ in (19) and (20) should be designed such that $\underline{x}_1 < x_1 < \bar{x}_1$. Denote $\bar{e}_1 = \bar{x}_1 - x_1$ and $\underline{e}_1 = x_1 - \underline{x}_1$. By comparing (19) and (20) with (4), it follows that

$$\dot{\bar{e}}_1 = (A_{11} + F)\bar{e}_1 + F\underline{e}_1 + \bar{H}, \quad (32)$$

$$\dot{\underline{e}}_1 = F\bar{e}_1 + (A_{11} + F)\underline{e}_1 + \underline{H}, \quad (33)$$

where $\bar{H} = \hat{A}_{12}\eta_s - \hat{A}_{12}^{+}\underline{\eta}_0 + \hat{A}_{12}^{-}\bar{\eta}_0 + \bar{\eta}_1 - \eta_{p1}(\cdot)$ and $\underline{H} = \eta_{p1}(\cdot) - \underline{\eta}_1 - \hat{A}_{12}\eta_s + \hat{A}_{12}^{+}\bar{\eta}_0 - \hat{A}_{12}^{-}\underline{\eta}_0$. From Lemma 1, it is easy to see that $\bar{H} > 0$ and $\underline{H} > 0$. It is worth pointing out that if $\bar{x}_1(0)$ is chosen sufficiently small and $\underline{x}_1(0)$ sufficient large, the initial condition that $\bar{e}_1(0) = \bar{x}_1(0) - x_1(0) > 0$ and $\underline{e}_1(0) = x_1(0) - \underline{x}_1(0) > 0$ can be guaranteed. Thus, refereing to Gouzé et al. (2000), the requirement $\underline{x}_1 < x_1 < \bar{x}_1$ for all $t > 0$ can be realized by designing appropriate gain matrix F such that $\bar{A}_{11} = \begin{bmatrix} A_{11}+F & F \\ F & A_{11}+F \end{bmatrix}$ is Metzler using linear matrix inequality (LMI) technique.

In addition, from Bolajraf and Rami (2016), if $\bar{A}_{11}$ is Hurwitz, then $\mathrm{col}(\bar{e}_1, \underline{e}_1)$ converges towards the box

$$\mathcal{B}(0, v) := \{z \in \mathcal{R}^{2(n-p)} | 0 \le z \le v\} \quad (34)$$

where $v = \bar{A}_{11}^{-1}\mathrm{col}(\bar{H}, \underline{H})$. Then it follows that

$$\|\bar{e}_1\|_1 + \|\underline{e}_1\|_1 \le \sum_{i=1}^{2(n-p)} v_i \quad (35)$$

where v_i is the ith row of v in (34).

Note that from Lemma 1, it is obtained that $(A^{+}\bar{x} - A^{-}\underline{x}) - (A^{+}\underline{x} - A^{-}\bar{x}) \le (A^{+} + A^{-})(\bar{x} - \underline{x})$. Applying this property to $\bar{H}$ and $\underline{H}$, it follows that $\bar{H} \le (\hat{A}_{12}^{+} + \hat{A}_{12}^{-})(\bar{\eta}_0 - \underline{\eta}_0) + \bar{\eta}_1 - \underline{\eta}_1$ and $\underline{H} \le (\hat{A}_{12}^{+} + \hat{A}_{12}^{-})(\bar{\eta}_0 - \underline{\eta}_0) + \bar{\eta}_1 - \underline{\eta}_1$. For the zonotopes W_{p1} and W_s, $\bar{\eta}_0 - \underline{\eta}_0 \le 2\eta_{p1}^{c}$ and $\bar{\eta}_1 - \underline{\eta}_1 \le 2\eta_s^{c}$. Thus, $\bar{H} \le 2(\hat{A}_{12}^{+} + \hat{A}_{12}^{-})\eta_{p1}^{c} + 2\eta_s^{c}$ and $\underline{H} \le 2(\hat{A}_{12}^{+} + \hat{A}_{12}^{-})\eta_{p1}^{c} + 2\eta_s^{c}$. In light of that $\|\bar{e}_1\| + \|\underline{e}_1\| \le \sqrt{n-p}(\|\bar{e}_1\|_1 + \|\underline{e}_1\|_1)$, it follows from (35) that $\|\bar{e}_1\| + \|\underline{e}_1\| \le \sqrt{n-p}\sum_{i=1}^{2(n-p)} v_i$.

Consider the sliding surface

$$\mathscr{S} = \{\mathrm{col}(e_1, e_{21}) | e_{21} = 0\}. \quad (36)$$

The gain $m(\cdot)$ in rejection function (22) need to be designed such that the sliding motion takes place and maintains on sliding surface (36) thereafter. Suppose that $\underline{x}_1 \le x_1 \le \bar{x}_1$ has been guaranteed. Denote $\hat{x}$ as the midpoint of $[\underline{x}_1, \bar{x}_1]$ given by

$$\hat{x}_1 = \mathrm{mid}([\underline{x}_1, \bar{x}_1]) = (\underline{x}_1 + \bar{x}_1)/2. \quad (37)$$

It follows that $e_1 = x_1 - \hat{x}_1 = (\underline{e}_1 - \bar{e}_1)/2$, and then $\|A_{211}e_1\| \le 1/2\|A_{211}\|(\|\underline{e}_1\| + \|\bar{e}_1\|)$. In addition, it follows from the zonotope W_s that $\|\eta_s(\cdot)\|_2 \le \sqrt{p}\|\eta_s(\cdot)\|_\infty = \sqrt{p}\max_{1\le i\le p}((|\bar{\eta}_s + \eta_s^{c}|)_i, (|-\bar{\eta}_s + \eta_s^{c}|)_i)$ with $(\cdot)_i$ being the ith row. Thus, the gain $m(\cdot)$ is chosen to satisfy that

$$\begin{aligned}m(\cdot) \ge& \frac{1}{2}\|A_{211}\|\sqrt{n-p}\sum_{i=1}^{2(n-p)} v_i + \bar{\eta}_{p21}(\cdot) + \varpi \\ &+ \|\hat{A}_{22}^{12}\|\sqrt{p}\max_{1\le i\le p}((|\bar{\eta}_s + \eta_s^{c}|)_i, (|-\bar{\eta}_s + \eta_s^{c}|)_i) \quad (38)\end{aligned}$$

where ϖ is a positive scalar. With that $\hat{A}_{22}^{11}$ being symmetric negative definite, based on Edwards and Spurgeon (1998), the reachability condition is satisfied. Therefore, it is concluded that with regard to $\hat{x}_1 = \mathrm{mid}([\underline{x}_1, \bar{x}_1])$, system (23) is driven to the sliding surface $\mathscr{S}$ in (36) in finite time t_s and remains on it thereafter.

Remark 2. The inequality (38) can not be used directly since v_i in (35) is not known. Fortunately, the bounds of $\bar{H}$ and $\underline{H}$ are known, and v_i can be replaced by the corresponding bounds given by the components of $v = \bar{A}_{11}^{-1}\mathrm{col}(\bar{H}, \underline{H})$. ∇

To detect the incipient faults, as in Emami-Naeini et al. (1988), it requires that

$$\inf_{\eta\in W} J_f > \sup_{\eta\in W} J_{th} \quad (39)$$

where J_f represents the residual evaluation function of (16) in fault scenario. The symbol η represents the lump uncertainties and W represents the zonotope of η, and f represents the lumped faults.

Since the right hand side of (39) satisfies

$$J_{th} = \max(\|\bar{r}\|, \|\underline{r}\|) \le (\|\bar{r}\| + \|\underline{r}\|) \le \sqrt{2\left(\|\bar{r}\|^2 + \|\underline{r}\|^2\right)}, \quad (40)$$

a sufficient condition for (39) is obtained by

$$\sup_{\eta\in W}\left(\|\bar{r}\|^2 + \|\underline{r}\|^2\right) \le \frac{1}{2}\inf_{\eta\in W} J_f^2. \quad (41)$$

Then,

$$\sup_{\eta\ne 0}\frac{\|\bar{r}\|^2 + \|\underline{r}\|^2}{\|\eta\|^2} \le \inf_{\eta\in W}\frac{J_f^2}{\|\eta\|^2} = \left(\inf_{\eta\in W}\frac{\|r_f\|}{\|\eta\|}\right)^2 \quad (42)$$

where r_f is the residual in fault scenario.

In light of $\left(\inf\limits_{\eta\in W}\frac{\|r_f\|}{\|\eta\|}\right)^2 \geq \left(\inf\limits_{\eta\in W}\frac{\|G_{rf}(s)f(s)\|}{\|\eta\|} - \inf\limits_{\eta\in W}\frac{\|G_{r\eta}(s)\eta(s)\|}{\|\eta\|}\right)^2$, inequality (42) is replaced by

$$\sup_{\eta\neq 0}\frac{\|\bar{r}\|^2+\|\underline{r}\|^2}{\|\eta\|^2} \leq \left(\inf_{\eta\in W}\frac{\|G_{rf}(s)f(s)\|}{\|\eta\|} - \inf_{\eta\in W}\frac{\|G_{r\eta}(s)\eta(s)\|}{\|\eta\|}\right)^2. \tag{43}$$

To calculate the right hand side of (43), the transfer function from $f(s)$ and $\eta(s)$ to $r(s)$ in (11) should be presented. Recalling the expression of e_1 given after (37), it follows from (32) and (33) that

$$\dot{e}_1 = \frac{1}{2}A_{11}e_1 + \frac{1}{2}\left(\underline{H}-\bar{H}\right). \tag{44}$$

During sliding motion, $e_{21} = 0$, then $G_{rf}(s)$ and $G_{r\eta}$ are obtained based on (16) and (44), and represented by

$$G_{rf}(s) = \left(sI - A_{22}^{22}\right)^{-1}\begin{bmatrix} D_{p2} & D_{a2}\end{bmatrix}, \tag{45}$$

$$G_{r\eta}(s) = \begin{bmatrix} 0 & I_{p-\tilde{q}}\end{bmatrix}\left(sI - \begin{bmatrix}\frac{1}{2}A_{11} & 0\\ A_{212} & A_{22}^{22}\end{bmatrix}\right)^{-1} \tag{46}$$

with $f = \mathrm{col}(f_p, f_a)$ and $\eta = \mathrm{col}(\eta_{p22}, \frac{1}{2}(\underline{H}-\bar{H}))$.

It should be pointed out that $\inf\limits_{\eta\in W}\frac{\|G_{rf}(s)f(s)\|}{\|\eta\|}$ and $\inf\limits_{\eta\in W}\frac{\|G_{r\eta}(s)\eta(s)\|}{\|\eta\|}$ are not norm here. It has been defined in Hou and Patton (1996) and Liu et al. (2005) that $\|G_{r\eta}(s)\|_- = \inf\limits_{\eta\in W}\frac{\|G_{r\eta}(s)\eta(s)\|}{\|\eta(s)\|} = \min_\omega \underline{\sigma}(G_{r\eta}(j\omega))$, which is the so-called $\mathcal{H}_-$ index. Also, $\inf\limits_{\eta\in W}\frac{\|G_{rf}(s)f(s)\|}{\|\eta\|} = \inf\limits_{\eta\in W}\frac{\|G_{rf}(s)f(s)\|}{\|f(s)\|}\frac{\|f(s)\|}{\|\eta(s)\|} \geq \min_\omega \underline{\sigma}(G_{rf}(j\omega))\Gamma$ where Γ is the scale variable defined in (14). Then inequality (43) is replaced by

$$\sup_{\eta\neq 0}\frac{\|\bar{r}\|^2+\|\underline{r}\|^2}{\|\eta\|^2} \leq \left(\min_\omega \underline{\sigma}(G_{rf}(j\omega))\Gamma - \min_\omega \underline{\sigma}(G_{r\eta}(j\omega))\right)^2. \tag{47}$$

In light of $\left(\sup\limits_{\eta\in W}\frac{\|r_f\|}{\|\eta\|}\right)^2 \leq \left(\sup\limits_{\eta_0\in W_0}\frac{\|r_f\|}{\|\eta_0\|}\right)^2\left(\sup\limits_{\eta\in W,\eta_0\in W_0}\frac{\|\eta\|}{\|\eta_0\|}\right)^2$ with bounded η_0, inequality (47) is replaced by

$$\sup_{\eta_0\in W_0}\frac{\|\bar{r}\|^2+\|\underline{r}\|^2}{\|\eta_0\|^2} \leq \gamma^2(\Gamma) \tag{48}$$

where $\gamma^2(\Gamma) = \frac{\left(\min_\omega \underline{\sigma}(G_{rf}(j\omega))\Gamma - \min_\omega \underline{\sigma}(G_{r\eta}(j\omega))\right)^2}{\left(\sup\limits_{\eta\in W,\eta_0\in W_0}\frac{\|\eta\|}{\|\eta_0\|}\right)^2}$.

Note that during the sliding motion, $\hat{x}_{21} = x_{21}$. The augmented system formed by (26), (27), (32) and (33) with regulated output z is represented by a compact form as

$$\dot{\bar{x}} = \bar{A}\bar{x} + \eta_0,\ z = \bar{C}\bar{x} \tag{49}$$

where $\bar{x} = \mathrm{col}(\bar{e}_1, \underline{e}_1, \bar{r}, \underline{r})$, $\eta_0 = \mathrm{col}(\bar{H}, \underline{H}, \eta_3 - \eta_{p22}, \eta_{p22} - \underline{\eta}_3)$, $\bar{A} = \begin{bmatrix}\bar{A}_{11} & \\ \bar{A}_{21} & \bar{A}_{22}\end{bmatrix}$ with $\bar{A}_{21} = \begin{bmatrix}\frac{3}{2}A_{212}^+ + \frac{1}{2}A_{212}^{-1} & \frac{1}{2}A_{212}^+ + \frac{3}{2}A_{212}^-\\ \frac{1}{2}A_{212}^+ + \frac{3}{2}A_{212}^{-1} & \frac{3}{2}A_{212}^+ + \frac{1}{2}A_{212}^{-1}\end{bmatrix}$, and $\bar{C} = \begin{bmatrix}0 & 0 & I_{p-\tilde{q}} & 0\\ 0 & 0 & 0 & I_{p-\tilde{q}}\end{bmatrix}$.

Due to that the matrices $\bar{A}_{11}$ and $\bar{A}_{22}$ are Metzler and $\bar{A}_{21} > 0$, $\bar{A}$ is Metzler. Thus, from the fact that $\eta_0 > 0$, system (49) is a internally positive system.

According to the bounded real lemma for internally positive systems Tanaka and Langbort (2011), for system (49), the inequality (48) is satisfied if and only if there exists a diagonal matrix $\bar{P} \in \mathcal{R}^{2(n-\tilde{p}+p)\times 2(n-\tilde{p}+p)} > 0$ such that

$$\begin{bmatrix}\bar{A}^T\bar{P} + \bar{P}\bar{A} + \bar{C}^T\bar{C} & \bar{P}\\ * & -\gamma^2(\Gamma)\end{bmatrix} < 0. \tag{50}$$

Let $\bar{A} = \bar{A}_0 + \bar{L}$ where $\bar{A}_0 = \left[\begin{array}{c|c}\mathrm{diag}\{A_{11}, A_{11}\} & 0\\ \hline \bar{A}_{21} & \mathrm{diag}\{A_{22}, A_{22}\}\end{array}\right]$ and $\bar{L} = \mathrm{diag}\left\{\left[\begin{smallmatrix}F & F\\ F & F\end{smallmatrix}\right], \left[\begin{smallmatrix}L_1 & L_1\\ L_2 & L_2\end{smallmatrix}\right]\right\}$, and let $\bar{P} = \mathrm{diag}(\bar{P}_1, \bar{P}_1, \bar{P}_2, \bar{P}_3)$ where $\bar{P}_1 \in \mathcal{R}^{(n-\tilde{p})\times(n-\tilde{p})}$, $\bar{P}_2 \in \mathcal{R}^{p\times p}$ and $\bar{P}_3 \in \mathcal{R}^{p\times p}$. Denote $\bar{P}_1F = Y_F$, $\bar{P}_2L_1 = Y_{L_1}$ and $\bar{P}_3L_2 = Y_{L_2}$. Then $\bar{P}\bar{Y} = \mathrm{diag}\left\{\left[\begin{smallmatrix}Y_F & Y_F\\ Y_F & Y_F\end{smallmatrix}\right], \left[\begin{smallmatrix}Y_{L_1} & Y_{L_1}\\ Y_{L_2} & Y_{L_2}\end{smallmatrix}\right]\right\}$, and (50) becomes that

$$\begin{bmatrix}\bar{A}_0^T\bar{P} + \bar{P}\bar{A}_0 + Y + Y^T + \bar{C}^T\bar{C} & \bar{P}\\ * & -\gamma^2(\Gamma)\end{bmatrix} < 0. \tag{51}$$

Therefore, the inequality (39) holds for the incipient faults f with scale variable Γ if there exist matrices $\bar{P}$ and Y such that (51) holds for any Γ satisfying that $\underline{\Gamma} < \Gamma < \bar{\Gamma}$.

The following theorem is ready to be presented.

Theorem 1. For system (2)-(3) and IFD system with the residual generator (16), threshold generator (13) characterized by dynamics (26) and (27), and RMS evaluation function, the following results hold:

(i) The subsystems (19)-(20) and (21) are driven to the sliding surface (36) in finite time and remains on it thereafter if the gain $m(\cdot)$ in (22) is chosen to satisfy (38).

(ii) After sliding motion takes place, the incipient faults satisfying $\underline{\Gamma} < \Gamma < \bar{\Gamma}$ can be detected, if there exists a spd matrix $\bar{P} = \mathrm{diag}(\bar{P}_1, \bar{P}_1, \bar{P}_2, \bar{P}_3)$ and nonnegative gain matrices $F = Y_F\bar{P}_1^{-1}$, $L_1 = Y_{L_1}\bar{P}_2^{-1}$ and $L_2 = Y_{L_2}\bar{P}_3^{-1}$ such that $\bar{A}_{11}$ and $\bar{A}_{22}$ are Metzler, and (51) holds for $\Gamma_0 = \arg\min\limits_{\Gamma\in[\underline{\Gamma},\bar{\Gamma}]}\gamma^2(\Gamma)$.

Proof. Based on the synthesis after (38), the result **(i)** is obtained directly. As for result **(ii)**, it is obtained from dynamics (32)-(33) and (26)-(27) that the Metzler matrices $\bar{A}_{11}$ and $\bar{A}_{22}$ ensure that $\underline{x}_1 < x_1 < \bar{x}_1$ and $\underline{r} < r < \bar{r}$. Furthermore, if (51) holds for $\min\limits_{\Gamma\in[\underline{\Gamma},\bar{\Gamma}]}\gamma^2(\Gamma)$, then (51) holds for any Γ satisfying $\underline{\Gamma} < \Gamma < \bar{\Gamma}$. Hence, the results **(i)** and **(ii)** are obtained.

5. CASE STUDY

Consider state space expression for inverter devices in electric railway traction systems given in Zhang et al. (2016b). Based on Yan and Edwards (2007), there exists a coordinate transformation such that the system is expressed by (4)-(7) where the matrices are given by

$$A_{11} = \begin{bmatrix}-8851 & 4630\\ 17130 & -8851\end{bmatrix}, A_{12} = \begin{bmatrix}3676 & 8475\\ 8764 & 20915\end{bmatrix},$$
$$A_{21} = \begin{bmatrix}-11785 & 11785\\ -11785 & -11785\end{bmatrix}, A_{22} = \begin{bmatrix}0 & 8796\\ -8796 & -1667\end{bmatrix}$$

and $C = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$. In this case study, the zonotopes to propagate disturbances are set as $\eta_{p1}^c = 0$, $\bar{\eta}_{p1} = 10000[1.5, 1.5]$, $\eta_{p21} = 15000$, $\eta_{p22}^c = 0$, $\bar{\eta}_{p22} = 15000$, $\eta_s^c = 0$ and $\bar{\eta}_s = 150$. In addition, only process incipient faults are considered for the system. Then $D_p = 1$ and $D_a = 0$.

It should be pointed out that the inverter devices in electric railway traction systems usually work at frequency from 10db to 1000db. In this case study, the frequencies of the both considered disturbances and faults are also located in 10db to 1000db.

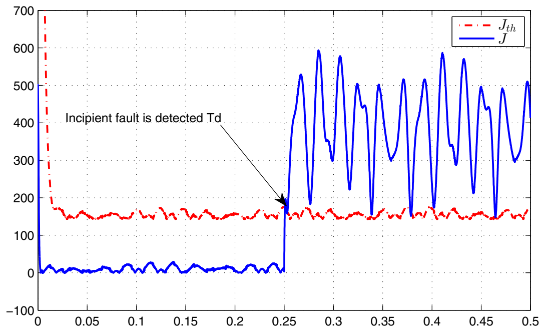

Fig. 1. The time responses of J and J_{th}.

Then from the Bode plot of $G_{rf}(s)$ and $G_{r\eta}(s)$, it follows that $\min_{\omega\in[10,10^3]} \underline{\sigma}(G_{rf}(j\omega)) = 0.005145$ and $\min_{\omega\in[10,10^3]} \underline{\sigma}(G_{r\eta}(j\omega)) = 0.0004318$. In addition, from the given zonotopes propagating disturbances, $\sup_{\eta\in W, \eta_0\in W_0} \frac{\|\eta\|}{\|\eta_0\|} = 0.58$. For inverter devices, two constant scale variables $\underline{\Gamma}$ and $\bar{\Gamma}$ used to distinguish incipient faults are given by $\underline{\Gamma} = 10$ and $\bar{\Gamma} = 20$. Therefore, $\min_{\Gamma\in[10,20]} \gamma^2(\Gamma) = 0.0077$. Based on Theorem 1, the gain matrices are calculated and given by

$$F = \begin{bmatrix} 660 & 237 \\ 17912 & 400 \end{bmatrix}, \; L_1 = L_2 = 113.8176. \qquad (52)$$

In the simulation, the disturbances belong to the given zonotopes given by $\eta_{p1} = 10000[1.5\sin(300t), 1.5\sin(300t)]$, $\eta_{p21} = 15000\sin(200t)$, $\eta_{p22} = 15000\sin(250t)$ and $\eta_s = 100[1.5\sin(350t), 1.5\sin(350t)]$. The simulated incipient faults satisfy that the scale variable Γ in (14) is larger than 10, and appears at time 0.25s. The simulation result is shown in Fig. 1.

It can be seen from Fig. 1 that the incipient fault is detected at time $T_d > 0.25$s, which demonstrates the effectiveness of the proposed IFD method.

6. CONCLUSION

This paper has presented an IFD framework by the novel proposed threshold generator for the systems with process disturbances and measurement disturbances. The definition of incipient faults has been given for the first time from the quantitative point of view. The interval sliding mode estimation module has been proposed to characterizing the threshold generators. A set of sufficient conditions to detect certain considered incipient faults via linear matrix inequality (LMI) has also been presented. Case study on an electrical traction device has been presented to demonstrate the practicability and the effectiveness of the proposed method.

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