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Quantum speedup of the Travelling Salesman Problem for bounded-degree graphs

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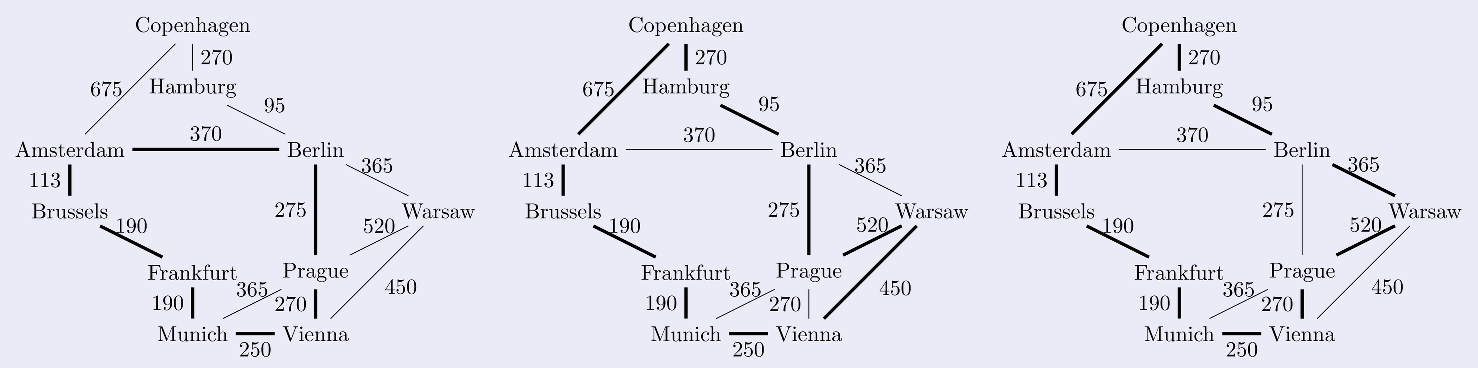
Main Results

- We show near-quadratic speedups for the Travelling Salesman Problem (TSP) when the degree of any vertex is at most 4.
- This is through applying a quantum speedup for backtracking [Mon15] to two TSP algorithms [XN16a,XN16b].
- We then demonstrate polynomial speedups up to degree-6.
- See Physical Review A 95(3), 032323 (2017) [arXiv:1612.06203] for further details.

1. The Travelling Salesman Problem

- Let G be a graph with n vertices and m edges.
- A cycle H on G is *Hamiltonian* if it visits every vertex in G.
- The TSP is to find the shortest Hamiltonian cycle.



(a) ✗ Not a Hamiltonian cycle. (b) ✗ Not the shortest Hamiltonian cycle. (Length: 3028 Minutes) (c) ✓ Solution to the TSP. (Length: 2938 Minutes)

- The best general classical algorithms take exponential time in n.

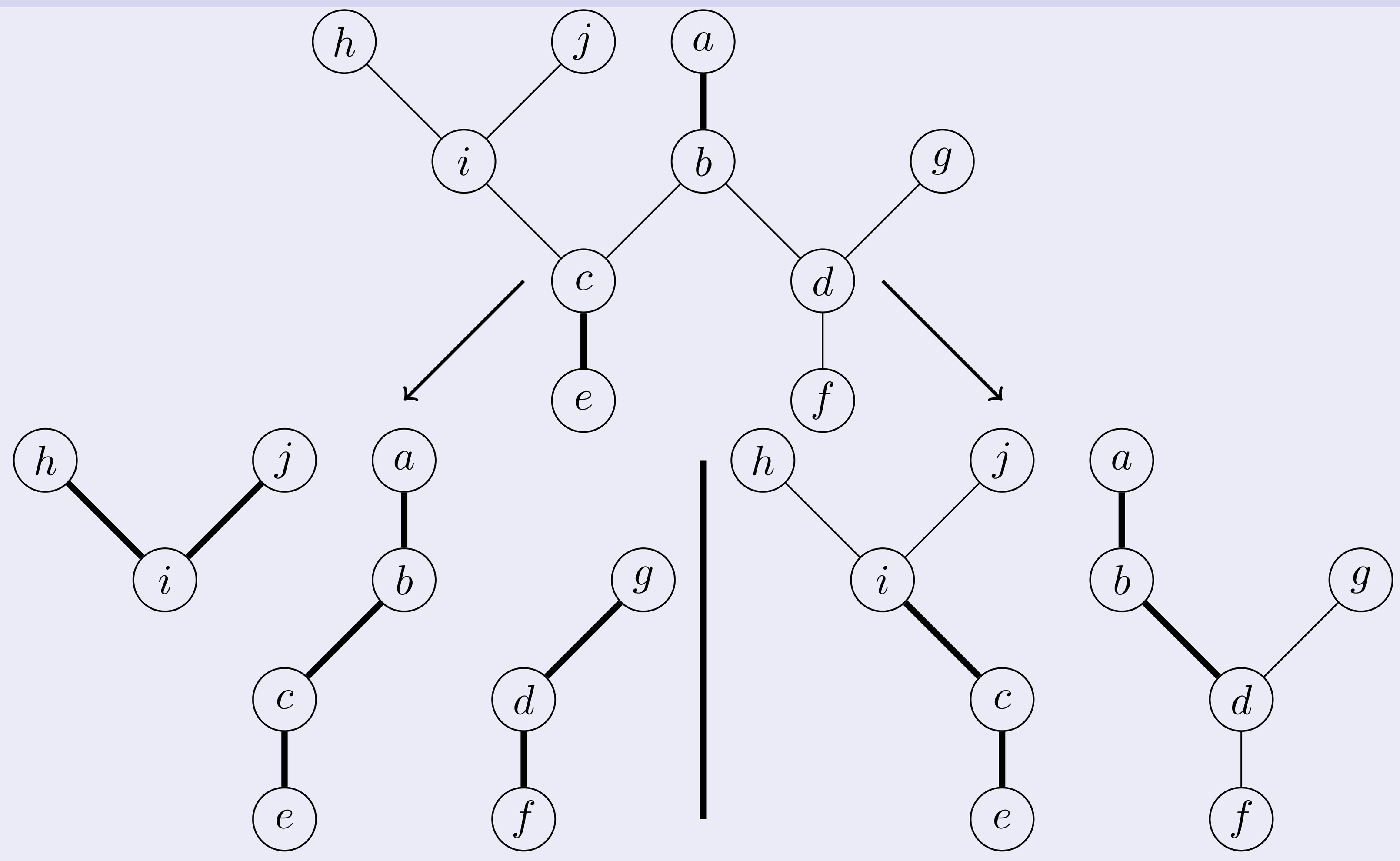
2. Backtracking algorithms

- Backtracking algorithms are a way of solving constraint satisfaction problems.
- They have two parts:
  1. A predicate, which checks if the constraints are satisfiable;
  2. and a heuristic, which chooses the next variable to assign.
- When called with a partial assignment, the predicate checks if the constraints are satisfiable. If not, we return.
- Otherwise, the heuristic picks a variable which we assign a value to and recursively call ourselves with this new partial assignment.
- Montanaro [Mon15] developed a quantum backtracking algorithm which has a quadratic speedup for finding a solution.

3. Quantum speedup for degree-3 graphs

- Backtracking algorithms can solve the TSP with "forced" edges which me must travel down and "removed" edges which we must avoid travelling on.
- The predicate checks if a Hamiltonian cycle is possible, and the heuristic selects another edge to force or remove.
- The best backtracking algorithm on degree-3 graphs runs in $O^*(2^{3n/10})$ time and polynomial space [XN16a].
- We apply [Mon15] to this algorithm to find a Hamiltonian cycle, failing to find one when one exists with probability δ, in $O^*(2^{3n/20}\log(1/\delta))$ time, where O^* hides polynomial factors.
- We find the shortest Hamiltonian cycle with bounded error (finding a sub-optimal cycle or no cycle) via binary search with $O(\log L \log\log L)$ overhead, where L is the longest edge length.

4. Example backtracking step



- Forcing bc, as shown on the left, means that b and c are incident to two forced edges, so ci and bd are removed. Now d and i are of degree 2, so edges df, dg, hi and ij are forced.
- Removing bc, as shown on the right, means that b and c are of degree 2, so edges bd and ci are now forced.

5. Expanding to higher-degree graphs

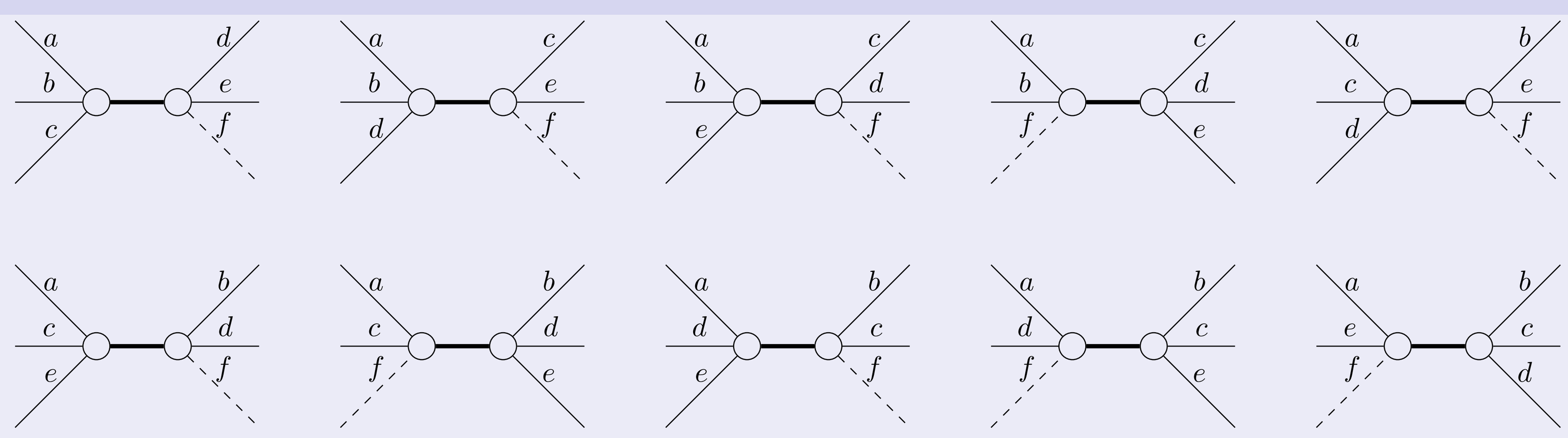


Figure: Ways of splitting a vertex of degree 5 or 6 into two lower-degree vertices.

- For degree-4 graphs, we apply the same technique to the algorithm of [XN16b] in $O^*(1.301^n \log L \log\log L)$ time.
- Other speedups can be found by breaking higher-degree vertices into degree 4 vertices connected by forced edges.
- We find the shortest way of splitting each vertex via [DH99].
- For degree-5/6 graphs, there are 10 ways of splitting each vertex, of which 6 will preserve the shortest Hamiltonian cycle. Thus we get an additional $O((10/6)^{n/2})$ overhead.
- For degree-7 graphs, this method is slower than classical algorithms for the general TSP [HK62, Bjö14].

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