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UNCOUPLED GPS ROAD CONSTRAINED POSITIONING BASED ON CONSTRAINED KALMAN FILTERING

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ABSTRACT

Car navigation systems take advantage of the synergies between the Global Positioning System (GPS) and digital road maps. For this kind of applications the digital road maps can provide a priori information to improve the positioning accuracy.

This paper presents a method for the estimation of the user's position, based on GPS positioning estimates, constrained to a road map. A low-cost GPS receiver was assumed as being the source of the positioning information. The techniques used in the proposed estimator were developed taking in consideration that the platforms where typically they would be implemented are characterized by having reduced computational capabilities.

The algorithm's positioning accuracy was characterized based on real data from a low-cost GPS receiver installed in a car. Different scenarios were used in the field trials in order to evaluate the impact of the satellite constellation visibility and geometry in the algorithm's performance.

I. INTRODUCTION

NOWADAYS, most car navigation systems rely mainly on a GPS receiver for positioning. For this kind of application, the required positioning accuracy can be achieved by most of the low-cost GPS receivers, available today. However, when the GPS positioning estimate is presented over a digital map, the user gets an additional perception of the positioning errors that should not be ignored.

Displaying the user's car position on the wrong lane, or even off-road, when the user knows he is on the right lane, would lead to a lack of confidence on the system. This a priori knowledge about the car possible positions, given by a road map, can be used to improve positioning accuracy. This approach is usually referred as road-constrained positioning.

Several strategies have been used to address the road-constrained positioning problem. Those strategies have lead to solutions based on multiple model (MM) approaches, [1-2], and/or particle filtering, [3-4]. Those strategies are characterized by requiring high computational loads, which may impair their use in small platforms with reduced computational capabilities.

In terms of positioning sensor measurements, two different possible strategies can be used to integrate the data from a GPS receiver. One strategy (coupled) is to use the receiver to satellite pseudorange measurements directly in the constrained positioning estimation. Another strategy (uncoupled) is to use the unconstrained positions, estimated by the GPS receiver, as observations for the constrained positioning estimation.

The former strategy requires the use of a GPS receiver with raw data output (satellite ephemeris and receiver to satellite pseudoranges), whereas the latter strategy can be used with low-cost GPS receivers, with standard NMEA data output (latitude, longitude, altitude...).

Taking in consideration that, the automotive navigation market is shifting from embedded systems, installed in cars, to personal navigation handheld devices, that user's can carry with them, the development of competitive solutions should be based on the integration of platforms with reduced computational capabilities and low-cost GPS receivers.

With this in mind, the approach presented in this paper explores the use of a constrained Kalman filter, [5], which application was already shown promising in theory [6]. The proposed algorithm aims to constrain the user's position to a digital road map, based on real unconstrained positioning estimates of a low-cost GPS receiver. The digital map here used is able to describe roads with a relatively complex shape.

In order to test the developed algorithm, several field trials were conducted with a low-cost GPS receiver installed in a car. Since the accuracy of the GPS (unconstrained) positioning is extremely dependent on the geometry of the visible satellite constellation, field trials were carried out to evaluate the algorithm's performance for different satellite configurations. This is particularly important in urban navigation, more prone to the canyon effect.

This paper is organized as follows. In section II the theoretical framework behind the constrained positioning estimation is presented. Section III describes the main blocks of the constrained positioning algorithm. In section IV the results of the experimental field trials are presented and analyzed. The paper ends with conclusions in section V.

II. THEORETICAL FORMULATION

The following sub-sections present the main tools required to constrain a set of GPS positioning estimates into a known track, given by a digital road map.

A. Coordinates Frames

Typically, GPS receivers express their position estimates in Latitude, Longitude and Altitude (LLA). However, since all the processing required in the road constrained positioning algorithm was developed in terms of cartesian coordinates, the GPS LLA positioning estimates are converted to a local East, North, Up cartesian frame (ENU). The algorithms to convert from LLA frame to the ENU frame and vice-versa can be found in [7] and [8].

B. GPS Estimation Errors

Besides the interference of factors like multipath and ionospheric delays in the propagation of GPS signals, its estimation error also depends strongly on the relative position between the receiver and the visible satellite constellation when the measurements are made. This dependence can be quantified by dilution of precision parameters, such as Position Dilution of Precision (PDOP), which shall be as low as possible in order to provide more accurate estimates, [9]. This can be achieved in places with an unobstructed view of the sky, allowing line-of-sight communication with more satellites and also larger dispersion among them.

C. Unconstrained Kalman Filtering

The Kalman filter [5] is a useful algorithm to estimate the state vector $\vec{\hat{x}}$ of a system based on a group of noisy observations $\vec{y}$, thus providing a smooth evolution of its behaviour. On the other hand, it is also capable of quantifying the error covariance matrix P inherent to that estimation. This linear filter is an iterative discrete process consisting in the following steps per iteration:

$$\vec{\bar{x}}_{k+1} = A\vec{\hat{x}}_k \tag{1}$$

$$\bar{P}_{k+1} = AP_kA^T + Q \tag{2}$$

$$K_{k+1} = \bar{P}_{k+1}C^T(CP_kC^T + R)^{-1} \tag{3}$$

$$\vec{\hat{x}}_{k+1} = \vec{\bar{x}}_{k+1} + K_{k+1}(\vec{y}_{k+1} - C\vec{\bar{x}}_{k+1}) \tag{4}$$

$$P_{k+1} = (I_{4\times 4} - K_{k+1}C)\bar{P}_{k+1} \tag{5}$$

The matrices A and Q are related with the kinematics of the receiver. Assuming its movement is not subjected to high accelerations, the value of the position $\hat{x}$ and velocity $\hat{\dot{x}}$ in each spatial coordinate can be iteratively determined from [10]:

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{\dot{x}}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{T} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{\dot{x}}_k \end{bmatrix} + \begin{bmatrix} 0 \\ u_{a,k} \end{bmatrix}, \tag{6}$$

where T is the sampling interval (1 second between two consecutive GPS estimates) and u_a the acceleration noise with zero mean and variance q_a. This value must be carefully adjusted, taking into account the movement characteristics, to assure an optimal performance of the algorithm. It corresponds approximately to

$$q_a = \Delta V^2/T\ , \tag{7}$$

where ΔV is the expectable speed variation per sampling interval.

In order to estimate the position of a receiver in a bi-dimensional map, the adopted state vector is $\vec{\hat{x}} = [\hat{x} \quad \hat{\dot{x}} \quad \hat{y} \quad \hat{\dot{y}}]^T$, yielding:

$$A = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$Q = q_a\mathrm{T} \begin{bmatrix} T^2/3 & \mathrm{T}/2 & 0 & 0 \\ \mathrm{T}/2 & 1 & 0 & 0 \\ 0 & 0 & T^2/3 & \mathrm{T}/2 \\ 0 & 0 & \mathrm{T}/2 & 1 \end{bmatrix}. \tag{9}$$

The initial state vector $\vec{\hat{x}}_0$ and its uncertainty P_0 shall also be defined to start the filtering process.

The matrices C and R are established by the observations model. Since the GPS receiver express its position estimates in LLA, after converting them to ENU cartesian coordinates, the relation between these measurements and the state vector is given by:

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \hat{x}_k \\ \hat{\dot{x}}_k \\ \hat{y}_k \\ \hat{\dot{y}}_k \end{bmatrix} + v_k \tag{10}$$

v_k represents the observation noise in each iteration and, assuming that both spatial coordinates are affected with a typical standard deviation of 7 m,

$$R = \begin{bmatrix} 7^2 & 0 \\ 0 & 7^2 \end{bmatrix}. \tag{11}$$

D. Constrained Kalman Filtering

The objective of this subsection is to determine the most probable receiver position on a road. Such goal can be achieved through the minimization of a cost function $f(\vec{x})$, proportional to the deviation between a specific state vector $\vec{x}$ and the one coming from Kalman filtering $\vec{\hat{x}}$. Furthermore, it must be given more tolerance, i.e. less cost, to those variables estimated with more uncertainty. Based on the weighted least squares method, $f(\vec{x})$ can be defined as

$$f(\vec{x}) = \left(\vec{x} - \vec{\hat{x}}\right)^T W\left(\vec{x} - \vec{\hat{x}}\right), \tag{12}$$

where W is a symmetric positive defined weighting matrix. Accordingly to [6], when $W = P^{-1}$, the constrained state estimate reaches the minimum error covariance for this type of approach.

The domain of the cost function is, however, to be confined to the road path. Thus, with the purpose of modeling curved roads, it can be defined as a second-order constraint function:

$$g(\vec{x}) = \vec{x}^T M\vec{x} + m^T\vec{x} + \vec{x}^T m + m_0 = 0\ . \tag{13}$$

Usually, it is easy to determine a univariate quadratic regression with the following structure:

$$y = C_1x^2 + C_2x + C_3 \Leftrightarrow C_1x^2 + C_2x + C_3 - y = 0\ . \tag{14}$$

It suffices to most cases, although it is not possible to represent paths with several ordinates for the same abscissa. Moreover, problems are expected whilst calculating high-slope functions due to the finite range of its coefficients.

Combining (13) and (14) and assuming M is symmetric leads to:

$$M = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

$$m = \tfrac{1}{2}[C_2 \quad 0 \quad -1 \quad 0]^T \tag{16}$$

$$m_0 = C_3 \tag{17}$$

The Lagrangian multiplier method was then used in order to find the optimal constrained estimate. It is based on the minimization of the Lagrangian function, given by:

$$J(\vec{x},\lambda) = f(\vec{x}) + \lambda g(\vec{x})\ , \qquad (18)$$

being λ the Lagrangian multiplier. For positions subjected to the constraint (13), $J(\vec{x},\lambda) = f(\vec{x})$ and its minimum corresponds to the state vector that nulls the first derivative of the Lagrangian function. This method yields the following solution, depending on an unknown λ though:

$$\vec{\bar{\bar{x}}} = (W + \lambda M)^{-1}\left(W\vec{\hat{x}} - \lambda m\right)\ . \qquad (19)$$

For such, it was assumed that $(W + \lambda M)$ is invertible. Replacing (19) in (13) it is possible to obtain a non-linear expression to calculate λ:

$$g(\lambda) = \left(\vec{\hat{x}}^T W - \lambda m^T\right)(W + \lambda M)^{-1} \cdot \qquad (20)$$
$$\cdot \left[M(W + \lambda M)^{-1}\left(W\vec{\hat{x}} - \lambda m\right) + 2m\right] + m_0 = 0\ .$$

To find its roots it was chosen the Newton-Raphson method, which is known for its good commitment between complexity and quick convergence, whenever initialized near the desired root. Knowing the first derivative of the constraint function:

$$\dot{g}(\lambda) = -2\left[m^T + \left(\vec{\hat{x}}^T W - \lambda m^T\right)(W + \lambda M)^{-1} M\right] \cdot \qquad (21)$$
$$\cdot (W + \lambda M)^{-1}\left[M(W + \lambda M)^{-1}\left(W\vec{\hat{x}} - \lambda m\right) + m\right]\ ,$$

the iterative process is based on:

$$\lambda_{k+1} = \lambda_k - \frac{g(\lambda_k)}{\dot{g}(\lambda_k)} \qquad (22)$$

Once $|\lambda_{k+1} - \lambda_k| < \tau$, the procedure stops and λ_{k+1} is replaced in (19). In the following tests, $\tau = 10^{-6}$ and the initial estimate λ_0 was set to zero, since the Lagrangian multiplier may assume either positive or negative values.

III. Algorithm Description

Combining the mathematical tools presented in the previous sections, an application in *MATLAB* was developed, to process positioning coordinates obtained with a low-cost GPS receiver installed on a car. The operational diagram of this application is shown on Fig. 1.

In the block A, a set of LLA coordinates is converted in ENU positions. These points are registered in a Comma-Separated Values (CSV) file and correspond to spots along the roads to be analyzed.

The goal of block B is to create two objects with the information of all mathematical tracks to process. The first one keeps the validity domain of each track (defined by the user) and the coefficients of the regression concerning the points within each domain (the user can choose between linear and quadratic regression). In the other object is registered the identification of every track contiguous with each one.

The block C allows to load a CSV file containing the GPS measurements and also to convert them to an ENU frame, with the same origin used in block A.

An iterative process along all the observations is now started, with the Kalman filtering performed in block D. For that, it is necessary to use the parameters defined in section II.C and to know the first point of the map, and therefore the initial estimation.

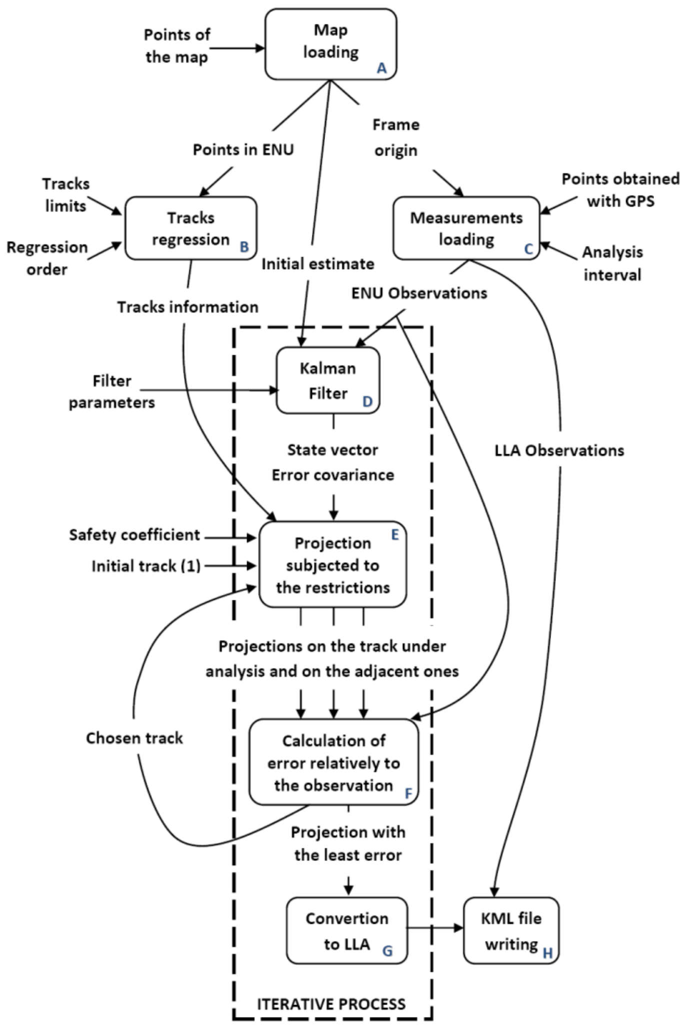

Figure 1: Algorithm overview.

The constraints are then applied, in block E, to each track and everyone contiguous with it, accordingly to section II.D. If one projection does not belong into the domain of validity of the track under analysis, it is suppressed. The amplitude of this domain is here raised by a safety coefficient, assuring mathematical continuity between every track. This situation will be addressed in detail along the next section.

In block F, the quadratic error between the observation and the projection, in each track, is calculated. As it was done with the cost function, the latter error is weighted with P^{-1}. The algorithm chooses the projection that minimizes the error and records the correspondent track for the next iteration.

The projections are then transformed into LLA coordinates by block G.

Block H writes a Keyhole Markup Language (KML) file, readable by *Google Earth*, [11]. This allows the user to visualize, in the *Google Earth* application, both the trajectory obtained with the GPS receiver (unconstrained) and the result of the road constrained positioning algorithm, proposed here.

IV. Experimental Results

After the algorithm development, several trials with real GPS data were made in order to evaluate its performance in different scenarios. In this section, an analysis of the results is presented. Using a Qstarz BT-Q1300 low-cost GPS receiver, the linear and quadratic strands of the algorithm were tested under low and high PDOP conditions in Lisbon.

A. *Low PDOP*

In this situation, the tests were made walking through a wide area outside the city. In Fig. 2 it is possible to conclude that the main objective of constraining the GPS data to the correct road is achieved. It is also possible to verify that the Kalman filter is an important tool, smoothing the observations and contributing to an easier constraint.

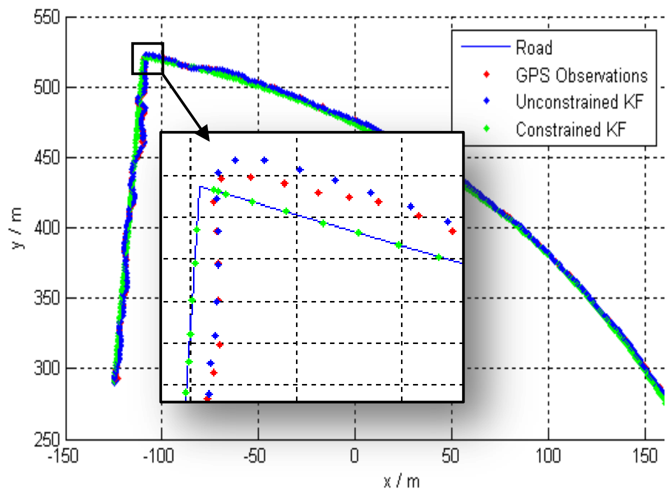

Figure 2: Comparison between the GPS estimates and the unconstrained and constrained Kalman filter estimates, for the linear regression.

Firstly, a linear regression was used and so a large number of segments were created and analyzed, as follows in Fig. 3. Due to an adoption of a linear strand of the algorithm and since each of these regressions is made with 2 points, the continuity between segments is practically assured. Therefore, a small safety coefficient was chosen. One last point should be enhanced at this stage and it is related with the occurrence near the hundredth iteration, which corresponds to a crossroad point. The algorithm appears to be in doubt between two segments, being this one of the major problems in the constraining subject. Anyhow, it is possible to observe that this small hesitation is negligible and the following result is completely reliable.

After this test, a quadratic regression was used and no significant differences were observed when comparing to the linear case. However, it was possible to conclude that much less segments were assumed due to the fact that more points of the map were used in each regression and, as a result, the computational load was reduced. Apart from this, a new problem arises in the transition of two quadratic segments, because significant discontinuities often appear, requiring a wise choice of the safety coefficient.

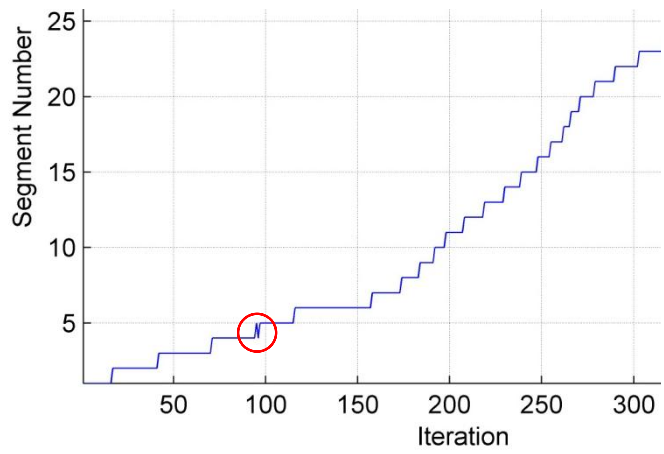

Figure 3: Road segment analysis in each iteration.

B. *High PDOP*

In order to test the algorithm in an urban scenario, with reduced view of the satellite constellation, a car circuit was selected in the town centre. The linear strand result is presented in Fig. 4. Again, the constraint is successfully obtained, even though the GPS estimates are sometimes quite off the road, due to multipath effects in the surrounding buildings.

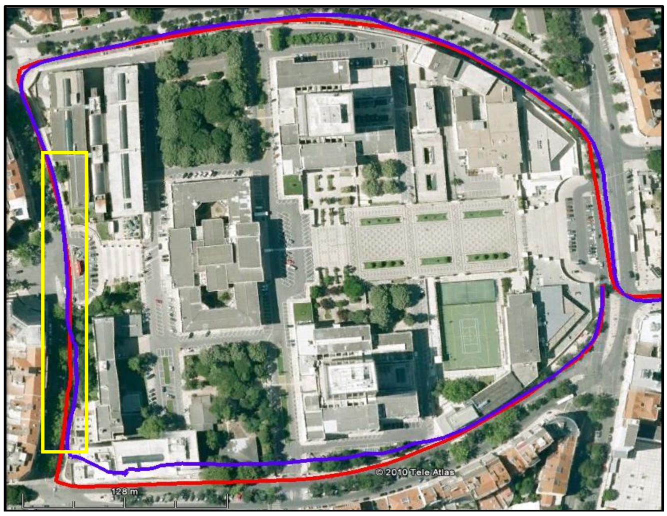

Figure 4: *Google Earth* view of the results with linear regression. In blue the GPS estimates and in red the constrained Kalman filter solution.

In Fig. 5 and 6 are presented the results obtained within the yellow rectangle of Fig. 4 using linear and quadratic constraints, respectively. For quasi-vertical roads it is impossible to choose an infinite slope for a univariate function and, as for an infinitesimal xx axis fluctuation a tremendous yy variation is verified, the function that represents the road should be carefully designed. Comparing the linear with the quadratic case, it is easy to verify that between different road segments it is more difficult to the non linear case to guarantee continuity and so different safety coefficients should be used. In the linear strand, 80 cm is enough to obtain optimal results. However, the discontinuous sections are bigger in the quadratic form and so, 2 m were adopted to ensure that the constrained solution does not fit in a discontinuity.

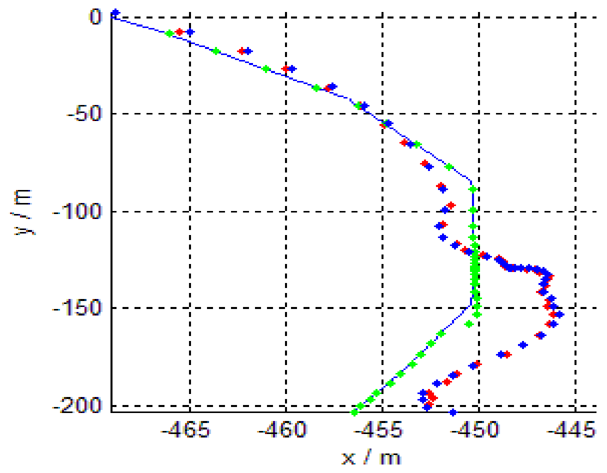

Figure 5: Enlargement of a critical vertical path, with linear regression.

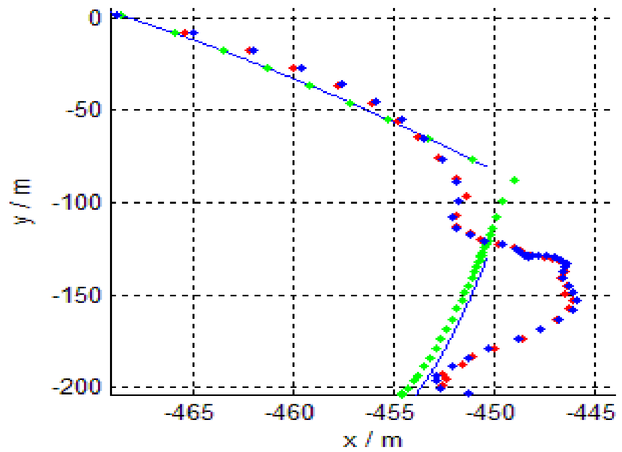

Figure 6: Enlargement of a critical vertical path, with quadratic regression.

Analyzing Fig. 7, there is one point to enhance for both linear and quadratic variants and it lies in the deviation at the crossroad. As presented, as the observations get closer to the cross, the algorithm has a small doubt and instantly assumes the upright road as the correct one, because the values are closer to this road instead of the real one. Anyhow, as the movement proceeds, it realizes that a wrong assumption was made and a correct constrain takes place again.

Figure 7: Google Earth view of the results with linear regression in linear crossroads. In blue the GPS estimates and in red the constrained Kalman filter solution.

V. Conclusions

The success of the results obtained in both low and high PDOP trials supports the effectiveness of the proposed algorithm. The Lagrangian multiplier method was revealed capable of solving the problem of constraining a set of observations to a road path. Results not presented in this paper also showed that, the performance of the Newton-Raphson method, in this context, was very efficient.

Thus, the proposed algorithm provides a fast and simple algorithm, applicable to car navigation systems, with reduced computational capabilities, and based on low-cost GPS receivers.

When using a linear regression to model the roads, a higher number of tracks has to be defined (increasing the complexity of the problem), but a closed-form (and, therefore, faster) solution exists, [6]. Despite assuring good results in all studied scenarios, this kind of regression is particularly useful in rectilinear paths.

On the other hand, the quadratic regression allows a more accurate description of the road with a fewer number of tracks required, but in order to guarantee mathematical continuity between them, adjustments have to be carefully done. Considering these adjustments, in the safety coefficient and in the tracks limits, this type of regression contributes to an optimization of the algorithm, especially when dealing with curved roads.

Acknowledgements

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