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Non-coherent MIMO Scheme Based on OFDM-MFSK

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Abstract—We propose a non-coherent transmission system based on orthogonal frequency division multiplexing (OFDM) for low-power applications. The proposed system uses M-ary frequency shift keying (MFSK) and employs two transmitting and two receiving antennas, with encoding across the space, frequency and time domains. For the resulting scheme, we develop a novel optimal non-coherent detector that produces soft bit information for a state-of-the-art error correction decoder. The proposed solution eliminates the need for channel knowledge and enables a simple receiver structure. Simulation results demonstrate an up to 8 dB coding gain of the proposed scheme over single-antenna OFDM-MFSK. When compared with coherent systems based on OFDM and binary phase shift keying (BPSK), the designed scheme offers a receiver sensitivity gain of 9 dB and beyond.

Index Terms—OFDM, MFSK, MIMO, non-coherent detection.

I. INTRODUCTION

Orthogonal modulation such as M-ary frequency shift keying (MFSK) combined with non-coherent detection is an appealing choice for low-rate low-power machine-to-machine (M2M) applications and has received significant attention in the literature [?], [?]. The authors of [?] were the first to propose MFSK to be used with orthogonal frequency division multiplexing (OFDM) as an alternative to bandwidth-efficient modulation schemes traditionally used with OFDM. OFDM-MFSK improves receiver sensitivity at the expense of data rate and enables non-coherent reception, eliminating the need for channel knowledge.

To further improve receiver sensitivity, multiple antennas with space-time (ST) coding can be employed. Traditional ST codes, such as the Alamouti code [?], assume perfect channel knowledge. Several studies into ST codes, which do not require channel knowledge, have been made over the years. The initial attempt [?] considered unitary ST codes, but only for the frequency-flat channel. The problem of non-coherent multiple-input multiple-output (MIMO) OFDM communication in the frequency-selective channel was treated in [?], where the authors introduced the concept of space-frequency codes. However, the emphasis was put on bandwidth-efficient communication, while in the power-efficient mode relevant to MFSK the information rate of the suggested codes was much lower than in the single-antenna case.

In this work, we adopt the MIMO ST encoding scheme proposed previously [?] for M-ary pulse position modulation (MPPM) and integrate it in the OFDM-MFSK transmitter. Our main contribution, however, is a simple, non-coherent decoder, as opposed to the more complex sphere decoder used in [?] that requires channel knowledge. In addition, the proposed decoder generates soft inputs (in contrast with
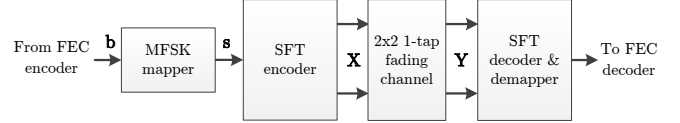
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Fig. 1. Block diagram of the proposed 2×2 MIMO OFDM-MFSK system. The 2×2 single-tap fading channel represents the combination of the OFDM modem and multiple-tap channel.

the decoder of [?]) and therefore can be combined with a state-of-the-art error correction decoder. The proposed scheme can be seamlessly integrated into existing OFDM modems to complement bandwidth-efficient modulations for low-power applications.

II. SYSTEM MODEL

A single-input single-output (SISO) OFDM-based MFSK system is described in [?]. For modulation purposes, each set of $\log_2 M$ bits, where M is the alphabet size, is mapped to M subcarriers, with only one subcarrier being non-zero. In this way, information is encoded in the index of the non-zero subcarrier. The total number of subcarriers N is chosen to be a multiple of M, hence the system can be viewed as a set of N/M parallel independent subsystems, each operating in its own subband.

The block diagram of the proposed 2×2 MIMO OFDM-MFSK system is depicted in Fig. 1. Here, the combination of the OFDM modem and multiple-tap fading channel is represented by an equivalent 2×2 single-tap channel for which each subcarrier experiences narrowband fading. The input of the system are bits encoded by a forward error correction (FEC) encoder. The output is the soft information about each bit for the FEC decoder. We adopt the rate-1 2×2 ST code proposed in [?] by deploying it in the frequency domain and renaming it to a space-frequency-time (SFT) code.

Let $\mathbf{b}$ denote a vector of $2\log_2 M$ input bits and $\mathbf{s} = \left[\mathbf{s}_1^T, \mathbf{s}_2^T\right]^T$ denote a vector of two consecutive M-tuples at the output of the MFSK mapper. Based on [?], the corresponding SFT codeword $\mathbf{X}$ can be written as follows:

$$\mathbf{X} = \left[\begin{array}{cc} \mathbf{s}_1 & \Omega\mathbf{s}_2 \\ \mathbf{s}_2 & \mathbf{s}_1 \end{array}\right] \triangleq [\mathbf{X}_1\,\mathbf{X}_2], \qquad (1)$$

where the subscript denotes the time index and Ω is an $M \times M$ cyclic permutation matrix defined as

$$\Omega = \left[\begin{array}{cc} \mathbf{0}_{1 \times M-1} & 1 \\ \mathbf{I}_{M-1} & \mathbf{0}_{M-1 \times 1} \end{array}\right]. \qquad (2)$$

In effect, the permutation matrix cyclically shifts the components of the vector it applies to by one position down.

For each codeword (1), the corresponding matrix of received samples in the frequency domain can be written as

$$\mathbf{Y} \triangleq [\mathbf{Y}_1 \, \mathbf{Y}_2]. \tag{3}$$

The input-output relationship for each time slot can be expressed as follows:

$$\mathbf{Y}_t = \begin{bmatrix} \mathbf{y}_t^{(1)} \\ \mathbf{y}_t^{(2)} \end{bmatrix} = \sqrt{E_s}\mathbf{H}_t\mathbf{X}_t + \mathbf{W}_t, \, t = 1, 2. \tag{4}$$

Here, the superscript denotes the antenna index, E_s is the average symbol energy per receiving antenna, $\mathbf{W}_t \sim C\mathcal{N}(\mathbf{0}_{2M\times 1}, N_0\mathbf{I}_{2M})$ is a vector of AWGN samples at time t and $\mathbf{H}_t$ is a MIMO channel matrix:

$$\mathbf{H}_t \triangleq \begin{bmatrix} \mathbf{H}_t^{(11)} & \mathbf{H}_t^{(12)} \\ \mathbf{H}_t^{(21)} & \mathbf{H}_t^{(22)} \end{bmatrix}, \tag{5}$$

where $\mathbf{H}_t^{(ij)} = \mathrm{diag}(\mathbf{h}_t^{(ij)})$ and $\mathbf{h}_t^{(ij)} \sim C\mathcal{N}(\mathbf{0}_{M\times 1}, \mathbf{I}_M)$, $t = 1, 2$ is a vector of the channel frequency response samples at the subcarriers in question between the j-th transmitting and i-th receiving antennas. It is assumed that the channel remains constant over a single OFDM symbol period but changes in an i.i.d. fashion from one OFDM symbol to another.

III. PROPOSED NON-COHERENT RECEIVER

The authors of [?] proposed an ML decoding algorithm assuming perfect channel knowledge. In this section we derive a non-coherent way of joint SFT decoding and MFSK demapping assuming that no channel knowledge is available.

Let (m, n), $m, n = 0, ..., M - 1$, denote a pair of indices of non-zero subcarriers in $\mathbf{s}_1$ and $\mathbf{s}_2$ obtained from the vector of source bits $\mathbf{b}$. This set of indices uniquely determines the SFT codeword $\mathbf{X}$, as per (1). Given equiprobable $\mathbf{b}$, the optimum maximum likelihood (ML) detector produces the following decision:

$$(\hat{m}, \hat{n}) = \arg\max_{(m,n)} p(\mathbf{Y}|\mathbf{X}), \tag{6}$$

Given the knowledge of the transmitted codeword $\mathbf{X}$, the received samples are statistically independent across the two received antennas and two time slots. As a result, the likelihood in (6) can be factored as follows:

$$p(\mathbf{Y}|\mathbf{X}) = \prod_{t=1,2} \prod_{k=1,2} p(\mathbf{y}_t^{(k)}|\mathbf{X}_t), \tag{7}$$

where $\mathbf{y}_t^{(k)}$ is a vector of M samples received at antenna k and time slot t. By defining $q = (n+1) \bmod M$, the following cases can now be considered:

1) $m \neq n, q$: In this case, assuming no knowledge of the channel, each of the four constituent vectors of $\mathbf{Y}$ has two components distributed as $C\mathcal{N}(0, E_s + N_0)$, while the other $M - 2$ components are distributed as $C\mathcal{N}(0, N_0)$. The likelihood for $t = k = 1$ can be expressed as follows:

$$p(\mathbf{y}_1^{(1)}|\mathbf{X}_1) = \frac{1}{(\pi N_0)^M} \exp\left(-\frac{\|\mathbf{y}_1^{(1)}\|^2}{N_0}\right)\left(\frac{E_s}{N_0}+1\right)^{-2}$$
$$\cdot \exp\left[\frac{E_s}{N_0}\left(\frac{\left|y_{1,m}^{(1)}\right|^2 + \left|y_{1,n}^{(1)}\right|^2}{E_s + N_0}\right)\right], \tag{8}$$

where $\|\cdot\|$ denotes the Euclidean norm. Expressing the other three likelihoods in a similar manner, the total likelihood (7) can now be written as

$$p(\mathbf{Y}|\mathbf{X}) = C\exp\left(\mathcal{E} - \delta\right), \tag{9}$$

where $C = (\pi N_0)^{-4M}\exp\left(-\|\mathbf{Y}\|^2/N_0\right)$,

$$\mathcal{E} = \frac{E_s}{N_0}\sum_{k=1}^{2}\frac{\left|y_{1,m}^{(k)}\right|^2 + \left|y_{1,n}^{(k)}\right|^2 + \left|y_{2,q}^{(k)}\right|^2 + \left|y_{2,m}^{(k)}\right|^2}{E_s + N_0}, \tag{10}$$

$$\delta = \ln\left(\frac{E_s}{N_0}+1\right)^8. \tag{11}$$

2) $m = n$: In this case, vectors $\mathbf{y}_1^{(k)}$, $k = 1, 2$, have only a single component distributed as $C\mathcal{N}(0, 2E_s + N_0)$, with the other $M - 1$ components distributed as $C\mathcal{N}(0, N_0)$. At the same time, vectors $\mathbf{y}_2^{(1)}$ and $\mathbf{y}_2^{(2)}$ have components distributed identically to the previous case. The total likelihood can now be expressed as (9), but with $\mathcal{E}$ and δ this time calculated as follows:

$$\mathcal{E} = \frac{E_s}{N_0}\sum_{k=1}^{2}\left(\frac{2\left|y_{1,m}^{(k)}\right|^2}{2E_s + N_0} + \frac{\left|y_{2,q}^{(k)}\right|^2 + \left|y_{2,m}^{(k)}\right|^2}{E_s + N_0}\right), \tag{12}$$

$$\delta = \ln\left(\frac{2E_s}{N_0}+1\right)^2\left(\frac{E_s}{N_0}+1\right)^4. \tag{13}$$

We note that (12) is distinct from (10) due to the different denominator $(2E_s + N_0)$ of the first fraction under the summation.

3) $m = q$: This time the constituent vectors corresponding to the second time slot have only a single signal component, while in the first time slot both vectors have two signal components. By analogy to the previous case, $\mathcal{E}$ in the total likelihood can be expressed as follows:

$$\mathcal{E} = \frac{E_s}{N_0}\sum_{k=1}^{2}\left(\frac{\left|y_{1,m}^{(k)}\right|^2 + \left|y_{1,n}^{(k)}\right|^2}{E_s + N_0} + \frac{2\left|y_{2,m}^{(k)}\right|^2}{2E_s + N_0}\right). \tag{14}$$

The value of δ is calculated using (13).

Having obtained the total likelihood expression for all cases and noting that C in (9) does not depend on the transmitted bits, the ML rule (6) can be rewritten as

$$(\hat{m}, \hat{n}) = \arg\max_{(m,n)} (\mathcal{E} - \delta), \tag{15}$$

where $\mathcal{E}$ and δ, depending on the combination of m and n, are calculated using the equations above. The expressions for δ can be further simplified by observing common terms in (11) and (13) not affecting the decision rule. Removing these terms, δ can be redefined as follows:

$$\delta = \begin{cases} 4\ln\left(\frac{E_s}{N_0}+1\right), & m \neq n, q; \\ 2\ln\left(\frac{2E_s}{N_0}+1\right), & \text{otherwise.} \end{cases} \tag{16}$$

The ML rule (15) can be generalized to an arbitrary number of received antennas N_r by replacing the upper limit of the

summation index k in (10), (12) and (14) with N_r, and the coefficient in front of the logarithm in (16) with $2N_r$ if $m \neq n, q$ and N_r otherwise.

It can be observed that if no shifting was applied during encoding based on (1) (i.e., if $\Omega = \mathbf{I}_M$), then tuples (m, n) and (n, m) would have the same value of the decision metric $\mathcal{E} - \delta$, resulting in ambiguity and a high error rate. The shift eliminates such ambiguity and creates a unique decision metric for each possible bit sequence.

The decision rule (15) produces optimum indices based on which the original bits can be identified. However, the channel decoder that follows the SFT detector requires soft inputs. Soft information for each individual bit of $\mathbf{b}$ in the form of the log-likelihood ratio can be obtained as follows:

$$\Lambda(b_j) = \ln \left[\frac{\sum\limits_{m,n:\, b_j=1} p(\mathbf{Y}|\mathbf{X})}{\sum\limits_{m,n:\, b_j=0} p(\mathbf{Y}|\mathbf{X})} \right], \, j = 0, ..., 2\log_2 M - 1. \tag{17}$$

Using the general expression for the likelihood (9) and utilizing the approximation $\ln\left[\sum_i \exp(a_i)\right] \approx \max_i(a_i)$, (17) can be expressed as

$$\Lambda(b_j) = \max_{m,n:\, b_j=1} (\mathcal{E} - \delta) - \max_{m,n:\, b_j=0} (\mathcal{E} - \delta). \tag{18}$$

The expression can be interpreted as follows. For each of the two possible bit values, two sets of possible tuples (m, n) are identified, with $M^2/2$ tuples in each set. For each set and each tuple within the subset, the value of $\mathcal{E} - \delta$ is calculated, where, depending on the relationship between m and n, $\mathcal{E}$ and δ are defined by (10), (12), (14) and (16). Finally, the largest value of $\mathcal{E} - \delta$ is determined for each set, and the LLR is calculated as the difference between the two largest values.

It can be observed that the ML detector scales and sums up subcarrier energies across the two receiving antennas and time slots, with an additional offset applied to account for the constellation asymmetry. It should be noted that although the channel knowledge is not required, the knowledge of E_s and N_0 is assumed. The former can be estimated using a synchronization preamble normally represented by a constant-amplitude sequence, while the latter can be measured in the absence of transmission.

We emphasize that the derived optimum non-coherent decoder does not require channel knowledge and produces soft outputs, as opposed to the original decoder presented in [?].

Remark 1. The authors of [?] also proposed generalized codes for more than two transmitting antennas. While it would still be possible to derive a non-coherent ML decoder for such codes following the same logic as above, the computational complexity of the receiver can become prohibitive. For instance, for N_t transmitting antennas, M^{N_t} combinations of N_t unique indices would need to be analyzed for each group of $N_t \log_2 M$ bits. The number of possible relationships between the indices would grow exponentially with N_t too.

A. Theoretical Error Performance Analysis

In this section, we provide brief error performance analysis of the proposed scheme. Detailed mathematical analysis is the subject of future work.

A detection error occurs if

$$\mathcal{E} < \tilde{\mathcal{E}} \tag{19}$$

for some $\tilde{\mathcal{E}}$ corresponding to $(\tilde{m}, \tilde{n}) \neq (m, n)$. Let $\tilde{q} = (\tilde{n} + 1) \bmod M$. The diversity provided by the scheme depends on how many signal subcarriers versus those carrying noise only are used when comparing the energy metrics. In the SISO arrangement, the energy of one signal subcarrier is compared with that of one noise subcarrier, and no diversity gain is provided. It is clear that some pairs of tuples $(\tilde{m}, \tilde{n}, \tilde{q})$ and (m, n, q) have overlapping indices. In such cases, the number of independent signal subcarriers contributing the left side of (19) is reduced, and so is the diversity gain. By inspecting possible pairs of tuples, it can be observed that $\mathcal{E}$ contains at least two signal terms, with not less than one index from each receiving antenna. At the same time, some pairs of tuples do not overlap at all, thus maximizing the diversity gain. It can be concluded, therefore, that on average the code provides a diversity gain by a factor of two.

IV. Performance Analysis

The performance of the proposed system was evaluated via Monte Carlo simulation. A 6-tap multipath fading channel, where all taps have the same variance and experience Rayleigh fading, was employed. The channels between different pairs of antennas were assumed to be uncorrelated. The total number of subcarriers was set to 64, with a cyclic prefix length of 16 symbols. A rate-$1/2$ $(408, 204)$ low-density parity-check (LDPC) code was employed as a FEC code [?]. For this code, a log likelihood decoder based on the sum-product algorithm [?] was implemented. For each signal-to-noise ratio (SNR) point, 10^5 random 204-bit packet realizations were simulated. The SNR used in the analysis below was defined as a ratio of the average symbol energy at the receiver input to the noise spectral density. Perfect time synchronization was assumed, as was the knowledge of E_s and N_0 at the receiver.

Fig. 2 illustrates the packet error rate (PER) performance of the proposed SFT-based MIMO system in comparison with a SISO equivalent as a function of SNR per receive antenna, for various modulation alphabet sizes. It can be observed that the MIMO system has a significant SNR gain compared with the SISO counterpart, which grows with M: from 4 dB for 2FSK to 8 dB for 64FSK at $\text{PER} = 10^{-3}$. The nature of the gain is twofold. First, there is a power gain due to multiple receivers. The power gain suffers when non-zero subcarriers transmitted from the two antennas have the same index: in this case, fewer terms are added in the decision rule. As a result, the power gain increases with M, since the relative number of coinciding non-zero indices becomes smaller for a larger alphabet size. The second contribution of the SNR improvement is a diversity gain that arises from four independent signal paths. Based on the steepness of the curves, it can be observed from Fig. 2 that the SFT code introduces additional diversity by a factor of two.

For benchmarking purposes, the performance of coherent SISO and 2×2 MIMO OFDM systems based on binary phase
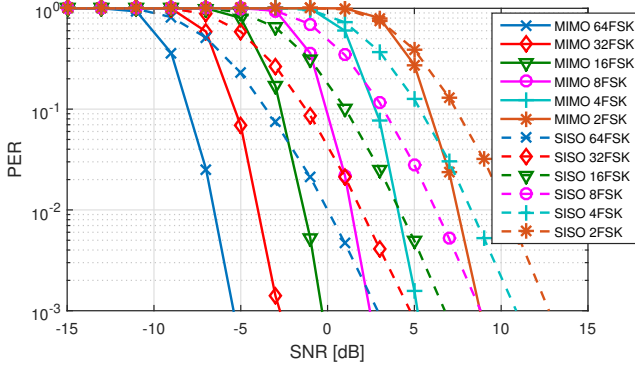
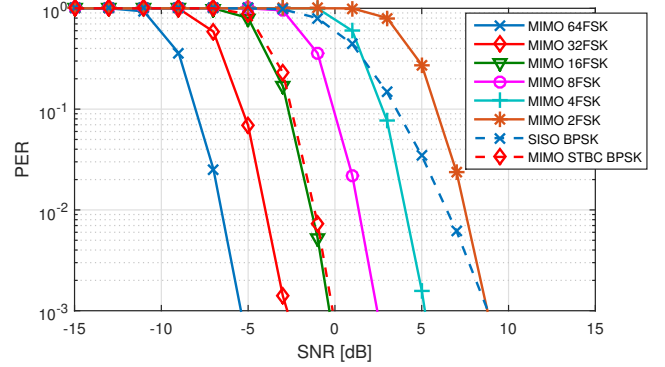Fig. 2. PER performance of MIMO and SISO OFDM-MFSK as a function of SNR per receive antenna.

Fig. 3. PER performance of non-coherent MIMO OFDM-MFSK and coherent SISO and MIMO STBC OFDM-BPSK as a function of SNR per receive antenna.

shift keying (BPSK) was also simulated assuming perfect channel knowledge. In the MIMO case, the Alamouti Space Time Block Code (STBC) code [**?**] was implemented, with a coherent ML detector at the receiver [**?**] generating soft bit information for the LDPC decoder. Fig. 3 compares the PER performance of the proposed non-coherent MIMO MFSK system and coherent SISO and MIMO STBC BPSK counterparts. It can be observed that SISO BPSK system outperforms MIMO MFSK for $M = 2$, since the MFSK-based modes suffer from a penalty due to non-coherent detection. However, as M increases, the combination of MFSK and the SFT code outweighs the penalty, providing up to 9 dB gain in SNR at $\mathrm{PER} = 10^{-3}$. When comparing with the MIMO STBC BPSK system, it is clear that the proposed scheme provides the same diversity gain as the STBC. Although only high-order MFSK modes have an SNR benefit over the coherent case, it should be noted that the latter assumes perfect channel knowledge. As a result, in practice the performance of a real coherent MIMO STBC system will be penalized. By increasing the alphabet size beyond 64 (if a larger number of subcarriers is available), the SNR gain of the MIMO MFSK system can be further increased.

It should be noted that the SNR gain of MFSK comes at the cost of poorer bandwidth utilization, which for a given M is equal to $(\log_2 M)/M$. Nevertheless, the proposed scheme offers a solution for applications such as long-range communication, where better receiver sensitivity has a priority over throughput. In addition, since the proposed solution is based on the standard OFDM modem, it can be combined with bandwidth-efficient modulation schemes. This would result in an adaptive system able to operate in both bandwidth- and power-efficient modes over a wide range of SNR values.

V. CONCLUSION

In this correspondence, a novel communication system was proposed that significantly improves the receiver sensitivity of standard OFDM-based systems. The solution is based on OFDM-MFSK modulation combined with an SFT code to enable multiple transmitting and receiving antennas. A simple energy-based non-coherent detector is derived for a 2×2 MIMO case. The designed detector eliminates the need for

channel estimation, which is especially beneficial in the low SNR region where standard pilot-based estimation techniques usually fail. In addition, the proposed detector produces soft bit information, which allows state-of-the-art channel codes to be used. The performance of the proposed system was simulated and compared with its SISO counterpart, and it was demonstrated that the SFT code introduces additional power and diversity gains. When compared with coherent BPSK-based SISO and MIMO STBC counterparts, the proposed system exhibited an SNR gain of up to 9 and 5 dB, respectively. This gain can be attractive in applications where improved receiver sensitivity is more important than data rate. The proposed solution can be seamlessly integrated into existing OFDM-based modems and can coexist with traditional, bandwidth-efficient modulation schemes and coherent receivers.

REFERENCES

[1] M. Valenti, "Iterative demodulation and decoding of turbo-coded M-ary noncoherent orthogonal modulation," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 9, pp. 1739–1747, Sep. 2005.

[2] P. N. Alevizos, Y. Fountzoulas, G. N. Karystinos, and A. Bletsas, "Log-Linear-Complexity GLRT-Optimal Noncoherent Sequence Detection for Orthogonal and RFID-Oriented Modulations," *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1600–1612, Apr. 2016.

[3] M. Wetz, I. Periša, W. G. Teich, and J. Lindner, "Robust Transmission Over Fast Fading Channels on the Basis of OFDM-MFSK," *Wirel. Pers. Commun.*, vol. 47, pp. 113–123, Oct. 2008.

[4] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[5] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communications in Rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 543–564, Mar. 2000.

[6] M. Borgmann and H. Bolcskei, "Noncoherent space-frequency coded MIMO-OFDM," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 9, pp. 1799–1810, Sep. 2005.

[7] C. Abou-Rjeily and W. Fawaz, "Space-Time Codes for MIMO Ultra-Wideband Communications and MIMO Free-Space Optical Communications with PPM," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 6, pp. 938–947, Aug. 2008.

[8] D. J. C. MacKay, "Encyclopedia of sparse random graph codes." [Online]. Available: http://www.inference.eng.cam.ac.uk/mackay/codes/data.html

[9] ——, "Good Error-Correcting Codes Based on Very Sparse Matrices," *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 399–431, Mar. 1999.

[10] A. Naguib, N. Seshadri, and A. Calderbank, "Increasing data rate over wireless channels," *IEEE Signal Process. Mag.*, vol. 17, no. 3, pp. 76–92, May 2000.