Na, J., Yang, C., & Yang, R. (2017). Adaptive RBFNN control of robot manipulators with finite-time convergence. In IECON 2016 - 42nd Annual Conference of the IEEE Industrial Electronics Society : Proceedings of a meeting held 24-27 October 2016, Florence, Italy. (pp. 42-47). Institute of Electrical and Electronics Engineers (IEEE). DOI: 10.1109/IECON.2016.7792980

Adaptive RBFNN Control of Robot Manipulators with Finite-time Convergence

Chenguang Yang, Runxian Yang, Jing Na and Fei Chen

Abstract—**In this paper, a radial basis function neural network (RBFNN) based adaptive control is designed for nonlinear robot manipulators. Barrier Lyapunov function (BLF) technique and terminal sliding mode (TSM) technique are seamlessly integrated to achieve finite time convergence of both tracking performance and NN learning performance. BLF is employed to ensure position tracking error converge to a specified small bound in a finite time, and TSM is used to guarantee finite-time convergence of neural learning error to a small bound as well. Extensive simulation studies are performed to illustrate the effectiveness and efficiency of the proposed control method.**

Index Terms—**Robot manipulator; Adaptive control; Finite-time convergence; BLF, Neural Networks**

I. Introduction

With the increasing needs of robot by our modern society and industry, the research of robot control technologies have attracted enormous attention [1], [2], [3], [4]. In the recent decades, many robotic researchers focus on study of control design in the presence of various constraints since the violation of these constrains may cause collisions and threaten the safety of surrounding environment and the robot itself. In [5], an adaptive controller was developed for robot manipulators to constrained the operation in an circular area to guarantee the safety. A robust adaptive position/force control scheme was proposed to deal with the holonomic constraints of the mobile robots in [6]. Recently, BLFs have been developed in nonlinear control design to deal with the state and output constrains [7], [8], [9], [10], [11], [12]. By adding constraints to the behavior of the state variables or system's outputs, tracking errors are indirectly constrained with the BLF constraint control method. A BLF-based controller was developed to control a robot manipulator with uncertain dynamics and joint space constraints [7].

The integral BLFs were synthesize in controller to prevent the movement of joint to violate the predefined constraints. In [9], BLFs were incorporated in the adaptive neural network control for a class of nonlinear systems in the presence of unknown functions. In [10], by applying a error transformation, a convenient BLF was constructed in a robust position controller to achieve prescribed performance constraints for a strict feedback nonlinear multiple-input-multiple-output (MIMO) dynamic system. A BLF is employed to deal with the tracking control with full-state constraints for a n-link robot with uncertain dynamics [11]. While in [12], an asymmetric time-varying BLF was presented to ensure the control of strict feedback nonlinear systems to satisfy prescribed constraints. In practice, the transient performance is very important for robot systems. This is because the transient characteristics (e.g. overshoot and convergence rate of tracking errors, amplitudes and frequency of control signals) could greatly influence the system performance.

Inspired by the work in [12], this paper proposed a controller for robot manipulators by utilizing the asymmetric time-varying BLFs to ensure the tracking transient satisfying a prescribed performance as well the joint constraints not violated. Due to the complex configuration or mechanism of the robot, little knowledge about robot dynamics parameters are available in practical applications. Thus, the model free controller design approaches have been widely studied and NN based intelligent control has be regarded as powerful tool to deal with these unknown dynamics [13], [14], [15]. In [14], NN is used to approximate the hypersonic flight vehicle dynamics in the tracking control of strict-feedback systems. In [15], the RBFNN is used to compensate the complicated nonlinear terms in the closed-loop dynamics of the robotic system.

It is known that finite-time stabilization of dynamical systems may give rise to a high-precision performance besides finite-time convergence to the equilibrium. This can be achieved by some continuous nonsmooth feedback controllers in [16], the approach has been applied to control robot manipulators in [17]. Applying RBFNN control method in infinite-time for robotic system needs learning or renewing the weight terms, which can be considered as the unknown parameter for robotic system. The adaptive parameter estimation schemes are proposed in [18], [19], which exponential and finite-time error convergence are proved without using the derivative of the system states. In [20], [21] neural networks were incorporated into the TSM control design to relax the requirement of system model knowledge and achieve FT error convergence. However, it is noted that the parameter estimation was not addressed in the aforementioned schemes.

Motivated by the above mentioned work, in this paper, we combine BLF and TSM techniques together to design RBFNN based adaptive control for robot manipulators with unknown dynamics. A novel controller is developed with guaranteed tracking performance in both transient and steady state stages, and with finite-time convergence of neural learning performance.

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II. Problem Formulation and Model Dynamics

A. Problem Formulation

The control objective of this paper is to design a robot controller such that the end-effector position q could track a desired trajectory q_d specified in the joint space, while guarantee (i) the tracking errors could achieve predefined transient performances. (ii) all the signals in the robot system remain bounded.

B. Manipulator Dynamics

The dynamic equation of an n-link robot manipulator can be described as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where $M(q) \in \mathbb{R}^{n\times n}$, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ and $G(q) \in \mathbb{R}^n$ are the inertial matrix, Coriolis and centrifugal matrix and gravitational force vector, respectively, n is the number of robotic joints; $q \in \mathbb{R}^n$, $\dot{q} \in \mathbb{R}^n$ and $\ddot{q} \in \mathbb{R}^n$ are the vectors of the robot arm's joint position, joint velocity and joint acceleration, respectively and $\tau \in \mathbb{R}^n$ is torque applied on the joints. The following properties will be used in the control design and performance analysis. [15]

Property 1: The inertia matrix $M(q)$ is symmetric and positive definite.

Property 2: The $\dot{M}(q) + 2C(q,\dot{q})$ is a skew symmetric matrix, i.e.,

$$v^T\Big(\dot{M}(q) + 2C(q,\dot{q})\Big)v = 0 \quad \forall v \in \mathbb{R}^n \tag{2}$$

C. Preliminaries

In this paper, we use the RBFNN to approximate continues function $F(z) : \mathbb{R}^m \to \mathbb{R}$ as follows,

$$F_{nn}(z) = \sum_{i=1}^{N} w_i s_i(z) = W^T S(z) \tag{3}$$

where $Z \in \Omega_Z \subset \mathbb{R}^m$ is the input vector, $W^T \in \mathbb{R}^{N_s}$ is the weight vector, N_s is the number of RBFNN nodes, and $S(z) = [s_1, s_2, \cdots, s_N]^T$ is the regressor vector with $s_i(\cdot)$ being a radial basis function. The most commonly used Gaussian radial basis functions is used as follows:

$$s_i(\|Z - c_i\|) = \exp\left[\frac{-(Z-c_i)^T(Z-c_i)}{b_i^2}\right] \tag{4}$$

and b_i are distinct points in state space, and $b_i = [b_{i1}, b_{i2}, \cdots, b_{iq}]^T$ is the center of the receptive field and c_i is the width of the Gaussian function, $i = 1, \cdots, N_s$. It has been proven that, with sufficiently large node number, RBFNN (3) can approximate any continuous function $F(z)$ over a compact set Ω_Z to arbitrary accuracy as

$$f(Z) = W^{*T} S(Z) + \epsilon(Z), \quad \forall Z \in \Omega_Z \tag{5}$$

where W^* is the ideal constant weight vector, $\epsilon(Z)$ is the approximation error such that $|\epsilon(Z)| < \epsilon^*$ with constant $\epsilon^* > 0$ for all $Z \in \Omega_Z$.

Definition 1: [22] A vector S is persistently excited (PE) if there exist $T > 0$, $\iota > 0$ such that $\int_t^{t+T} S^T S \geq \iota$.

Lemma 1: [23] If a function $V(t) \geq 0$ with initial value $V(0) > 0$ satisfies the following condition

$$\dot{V} \leq -\kappa V^p, \quad 0 < p < 1.$$

Then, $V(t) \equiv 0$, $\forall t \geq t_c$, for a certain t_c that satisfies

$$t_c \leq \frac{V^{1-p}(0)}{\kappa(1-p)}$$

III. Control design

Let us defined the tracking error signals of the robot manipulator as

$$\begin{aligned} \zeta_e &= q - q_d \\ \zeta_v &= \dot{q} - \alpha \end{aligned} \tag{6}$$

where α is a virtual controller will be designed latter.
Then, the error equation can be derived from the robot dynamics 1 and (6) as

$$M\dot{\zeta}_v + C\zeta_v = \tau + F_1(z) \tag{7}$$

where $F_1(z) = -(M\dot{\alpha} + C\alpha + G)$ with $z = [q^T, \dot{q}^T, \alpha^T, \dot{\alpha}^T]^T$, G and C are the abbreviation of $G(q), C(q,\dot{q})$, respectively. It should be noted that $F_1(z) \in \mathbb{R}^n$ is an unknown function vector as the matrices M, C and the vectors G are unavailable. Therefore, the function $F_1(z)$ can not be directly applied in the controller design.
The following assumption is given

Assumption 1: The desired trajectory q_d is chosen so that the $S(z)$ is PE.
Define the symbols $i = 1, 2, \cdots, n$ and $j = 1, 2, 3$ in all following contents.
To formulate the system (7), let us define three alternative vectors as

$$\begin{cases} F_1(z) = -(M\dot{\alpha} + C\alpha + G) \\ F_2(z) = M\zeta_v \\ F_3(z) = -\dot{M}\zeta_v + C\zeta_v \end{cases} \tag{8}$$

where $F_j(z) \in \mathbb{R}^n$.
Thus, the system (7) can be rewritten by

$$\dot{F}_2(z) + F_3(z) - F_1(z) = \tau \tag{9}$$

It is well known that RBFNN (3) is applied to approximate the unknown dynamics function, an adaptive parameter estimation method are designed [24] .
Let us define alternative vectors as

$$\begin{cases} F_1(z) = W_{F_1}^{*T} S_1(z) + \varepsilon_1 \\ F_2(z) = W_{F_2}^{*T} S_2(z) + \varepsilon_2 \\ F_3(z) = W_{F_3}^{*T} S_3(z) + \varepsilon_3 \end{cases} \tag{10}$$

where $W_{F_j}^* \in \mathbb{R}^{N_j \times n}$ is optimal weigh matrix; $S_j(z) = [s_1^j, s_2^j, \cdots, s_{N_j}^j]^T \in \mathbb{R}^{N_j}$ are the corresponding regression vectors in (4); $\varepsilon_j = [\varepsilon_{j1}, \varepsilon_{j2}, \cdots, \varepsilon_{jn}]^T \in \mathbb{R}^n$ are the approximation error vectors, and $||\varepsilon_j|| \leq \varepsilon^*$ with a positive constant ε^*; N_j are the numbers of neural node of the RBFNN $F_j(z)$.
We can define three new RBFNN functions $\bar{S}_1(z) = [S_1^T(z), \mathbf{0}_{N_2}^T, \mathbf{0}_{N_3}^T]^T, \bar{S}_2(z) = [\mathbf{0}_{N_1}^T, S_2^T(z), \mathbf{0}_{N_3}^T]^T, \bar{S}_3(z) =$

$[\mathbf{0}_{N_1}^T, \mathbf{0}_{N_2}^T, S_3^T(z)]^T \in \mathbb{R}^{N_a}$, $N_a = N_1 + N_2 + N_3$; and define a new RBFNN weight matrix $W^*(z) \in \mathbb{R}^{N_a \times n}$ as

$$\begin{aligned} W^* &= \begin{bmatrix} W_1^{*T} & W_2^{*T} & \cdots & W_n^{*T} \end{bmatrix}^T = \begin{bmatrix} W_{F_1}^* \\ W_{F_2}^* \\ W_{F_3}^* \end{bmatrix} \\ &= \begin{bmatrix} W_{F_{1,1}}^* & W_{F_{1,2}}^* & \cdots & W_{F_{1,n}}^* \\ W_{F_{2,1}}^* & W_{F_{2,2}}^* & \cdots & W_{F_{2,n}}^* \\ W_{F_{3,1}}^* & W_{F_{3,2}}^* & \cdots & W_{F_{3,n}}^* \end{bmatrix} \end{aligned} \tag{11}$$

where $W_i^* = [W_{F_{1,i}}^{*T}, W_{F_{2,i}}^{*T}, W_{F_{3,i}}^{*T}]^T \in \mathbb{R}^{Na}$ with $W_{F_{j,i}}^* \in \mathbb{R}^{N_j}$ is the ith column of the jth RBFNN optimal weight matrix $W_{F_j}^*$.

Then, the equation (10) can be further formulated as

$$\begin{cases} F_1(z) = {W^*}^T \bar{S}_1(z) + \varepsilon_1 \\ F_2(z) = {W^*}^T \bar{S}_2(z) + \varepsilon_2 \\ F_3(z) = {W^*}^T \bar{S}_3(z) + \varepsilon_3 \end{cases} \tag{12}$$

Consequently, substituting (12) into (9), we have as

$${W^*}^T \bar{S}(z) = \tau + \bar{\varepsilon} \tag{13}$$

where $\bar{\varepsilon} = \varepsilon_1 - \varepsilon_3 - \dot{\varepsilon}_2 \in \mathbb{R}^n$ is RBFNN construction error vector with $||\bar{\varepsilon}|| < \bar{\varepsilon}^*$, $\bar{\varepsilon}^*$ is a positive constant, $\bar{S}(z) = \dot{\bar{S}}_2(z) + \bar{S}_3(z) - \bar{S}_1(z) = [-S_1^T(z), \dot{S}_2^T(z), S_3^T(z)]^T \in \mathbb{R}^{N_a}$ is a new RBFNN basic function vector.

Consequently, using RBFNN method, the system (13) can be divided into n subsystems as

$${W_i^*}^T \bar{S}(z) = \tau_i + \bar{\varepsilon}_i \tag{14}$$

where τ_i and $\bar{\varepsilon}_i$ are control input and RBFNN approximation error of the ith subsystem, respectively.

$$\tau_i = -k_{2i}\zeta_{vi} - \varrho_i \zeta_{ei} - k_{3i}\frac{\zeta_{vi}}{|\zeta_{vi}|} - \hat{W}_i^T \bar{S}_1(z) \tag{15}$$

where k_{2i} and k_{3i} are designed positive constant, ϱ_i is designed in (38), $\zeta_e = [\zeta_{e1}, \zeta_{e2}, \cdots, \zeta_{en}]^T$ and $\zeta_v = [\zeta_{v1}, \zeta_{v2}, \cdots, \zeta_{vn}]^T$ are defined in (6), $\hat{W}_i$ is the estimate of W^*.

To design the optimal adaptive estimation law of weight vectors W_i, an novel adaptive parameter estimation in are introduced [24].

We first design the following filters

$$\begin{cases} k\dot{\bar{S}}_{1_f} + \bar{S}_{1_f} = \bar{S}_1, & S_{1_f}|_{t=0} = \mathbf{0}_{[N_a]} \\ k\dot{\bar{S}}_{2_f} + \bar{S}_{2_f} = \bar{S}_2, & S_{2_f}|_{t=0} = \mathbf{0}_{[N_a]} \\ k\dot{\bar{S}}_{3_f} + \bar{S}_{3_f} = \bar{S}_3, & S_{3_f}|_{t=0} = \mathbf{0}_{[N_a]} \\ k\dot{\tau}_{fi} + \tau_{fi} = \tau_i, & \tau_{if}|_{t=0} = 0 \end{cases} \tag{16}$$

where, $k > 0$ is a filter parameter, $\bar{S}_{j_f} = \bar{S}_{jf}(z) \in \mathbb{R}^{N_a}$ and $\tau_{fi} \in \mathbb{R}$ are the filtered variables, respectively.

The filter operations are applied to the equation (14), such that a corresponding equation can be obtained as follows

$$W_i^{*T}(\frac{\bar{S}_2 - \bar{S}_{2_f}}{k} + \bar{S}_{3_f} - \bar{S}_{1_f}) = W^{*T}\bar{S}_f = \tau_i + \bar{\varepsilon}_{fi} \tag{17}$$

where $\bar{S}_f = \frac{\bar{S}_2 - \bar{S}_{2_f}}{k} + \bar{S}_{3_f} - \bar{S}_{1_f} \in \mathbb{R}_a^N$ is a new RBFNN function vector, $\bar{\varepsilon}_{fi}$ can only be used for analysis from $k\dot{\bar{\varepsilon}}_{fi} + \bar{\varepsilon}_{fi} = \bar{\varepsilon}_i$ with $\bar{\varepsilon}_{fi}(0) = 0$. It is clear that the W_i^* can be considered as unknown parameters in (17), which needs be estimated as $\hat{W}_i$ during control designation.

To accommodate parameter estimation, the matrix $P \in \mathbb{R}^{N_a \times N_a}$ and vector $Q_i \in \mathbb{R}^{1 \times N_a}$ are defined as follows

$$\begin{cases} \dot{P} = -\kappa_i P + \bar{S}_f \bar{S}_f^T, & P(0) = \mathbf{0}_{N_a \times N_a} \\ \dot{Q}_i = -\kappa_i Q_i + \tau_{fi}\bar{S}_f, & Q_i(0) = \mathbf{0}_{N_a} \\ R_i = P^T \hat{W}_i - Q_i \end{cases} \tag{18}$$

Considering (17), it is clear that $Q_i = P^T W_i^* - \mu_{\varepsilon i}$ with $\mu_{\varepsilon i} = \int_0^t e^{-\kappa_i(t-r)} \bar{\varepsilon}_{fi} \bar{S}_f(r) dr \in \mathbb{R}^{N_a}$ in (18). such that the parameter

$$\begin{aligned} R_i = P^T \hat{W}_i - Q_i &= P^T \hat{W}_i - P^T W_i^* + \mu_{\varepsilon i} \\ &= P^T \tilde{W}_i + \mu_{\varepsilon i} \end{aligned} \tag{19}$$

where $\mu_{\varepsilon i}$ is bounded, definition (1) implies $\bar{S}_i$ bounded, and $varepsilon_i$ is bounded according to (13), then, we have $||\mu_{\varepsilon i}|| \le \xi_\varepsilon^*$ for a constant $\xi_\varepsilon^* > 0$.

Lemma 2: [18] The matrix P is positive definite satisfying $\lambda_{min}(P(t)) > \delta_p$ for $t > T$ and $\sigma > 0, T > 0$, provided the NN function $S(z)$ is PE in Definition (1).

The RBFNN weight estimation $\hat{W}_i$ in (18) can be obtained by designing the following adaptive law

$$\dot{\hat{W}}_i = \Gamma_i \left(\zeta_{vi} \bar{S}_1 - \gamma_i \frac{P^T R_i}{||R_i||} \right) \tag{20}$$

where $\Gamma_i \in \mathbb{R}^{N_a \times N_a}$ is a positive definitive matrix, and γ_i is a positive constant.

A. Predefined Tracking Performance

As mention above, for tracking errors ζ_e in (2), our controller design objective is to make $q(t)$ track a predefined trajectory $q_d(t)$ while guarantee $\zeta_e(t)$ satisfying the predefined transient performance. At first, let us define a smooth decreasing performance function which could describe the transient performance of tracking errors as $\phi(t) = [\phi_1(t), \phi_2(t), \cdots, \phi_n(t)]^T$

$$\phi(t) = \begin{bmatrix} (\rho_{01} - \rho_{\infty 1})e^{-a_1 t} + \rho_{\infty 1} \\ (\rho_{02} - \rho_{\infty 2})e^{-a_2 t} + \rho_{\infty 2} \\ \vdots \\ (\rho_{0n} - \rho_{\infty n})e^{-a_n t} + \rho_{\infty n} \end{bmatrix} \tag{21}$$

where $(\rho_{01}, \rho_{02} \cdots, \rho_{0n})$, $(\rho_{\infty 1}, \rho_{\infty 2}, \cdots, \rho_{\infty n})$ and $(a_1, a_2, \cdots, a_n)$ are properly chosen positive constants.

The performance functions ϕ_1, ϕ_2, $\cdots$, ϕ_n are smooth, bounded and positive functions with $\lim_{t\to\infty} \phi_i(t) = \rho_{\infty i}$. To ensure the tracking error satisfying the prescribed tracking transient, we depicted the bounds of the tracking errors as $-\beta_1\phi(t) < \zeta_e < \beta_2\phi(t)$, where β_1 and β_2 are positive design constants. The functions $\beta_1\phi_i(t)$ and $-\beta_2\phi_i(t)$ describe the tracking transient performance with a_i regulates the lower bounded of the required convergence rate of tracking errors, while $\beta_1\rho_{0i}$ and $-\beta_2\rho_{0i}$ define the maximum overshoot and undershoot of the tracking errors. Thus we can regulate the transient performance and the steady-state stages by properly select the function ϕ_i and the designed parameters β_1, β_2.

B. Stability Analysis

Theorem 1: Consider the robot manipulator (1) with the tracking error (6), employ the global NN controller design (15) with the NN weight adaptive law (20) and the prescribed transient performance (21), then, we have all the tracking signals are UUB and the tracking error coverage to a small neighborhood of zero; the predefined transient and tracking performance is guaranteed.

Considering the following Lyapunov function

$$V = V_1 + V_2 + V_3 \tag{22}$$

V_1 is designed according to the work in [12], we will proceed the controller design using backstepping technique. Let us designed an asymmetric time-varying barrier function as

$$V_1 = \sum_{i=1}^{n}\left(\frac{h_i}{2}\ln\frac{1}{1-\xi_{bi}^2} + \frac{1-h_i}{2}\ln\frac{1}{1-\xi_{ai}^2}\right) \tag{23}$$

where ξ_{ai} and ξ_{bi} are designed by applying coordinate transformations on the tracking error ζ_e, we have

$$\begin{aligned} \xi_a &= \left[\frac{\zeta_{e1}}{\varphi_{11}}, \frac{\zeta_{e2}}{\varphi_{12}}, \cdots, \frac{\zeta_{en}}{\varphi_{1n}}\right] \\ \xi_b &= \left[\frac{\zeta_{e1}}{\varphi_{21}}, \frac{\zeta_{e2}}{\varphi_{22}}, \cdots, \frac{\zeta_{en}}{\varphi_{2n}}\right] \\ \xi &= h_i(\zeta_{ei})\xi_{bi} + (1-h_i(\zeta_{ei}))\xi_{ai} \end{aligned} \tag{24}$$

where $\varphi_{1i}(t) = -\beta_1\phi_i(t)$, $\varphi_{2i}(t) = \beta_2\phi_i(t)$, $i = 1, 2, \cdots, n$, ξ_{ai}, ξ_{bi} are ith element of the vectors ξ_a, ξ_b, respectively, and $h_i(\zeta_e)$ is defined as

$$h_i = \begin{cases} 1 & \zeta_{ei} \geq 0 \\ 0 & \zeta_{ei} < 0 \end{cases} \tag{25}$$

V_2 and V_3 are considered as follows:

$$\begin{aligned} V_2 &= \frac{1}{2}\zeta_v^T M \zeta_v \\ V_3 &= \frac{1}{2}R_i^T P^{-T}\Gamma_i^{-1}P^{-T}R_i \end{aligned} \tag{26}$$

where Γ_i is defined in (20).

The time differentiation of (23), we can obtain that

$$\dot{V}_1 = \sum_{i=1}^{n}\left(h_i\frac{1}{1-\xi_{bi}^2}\xi_{bi}\dot{\xi}_{bi}\right) \tag{27}$$

$$+ \sum_{i=1}^{n}\left((1-h_i)\frac{1}{1-\xi_{ai}^2}\xi_{ai}\dot{\xi}_{ai}\right) \tag{28}$$

According to definition of ξ_{ai}, ξ_{bi}, we have

$$\dot{V}_1 = \sum_{i=1}^{n}\left(\frac{h_i}{(1-\xi_{bi}^2)\varphi_{2i}}\xi_{bi}(\dot{\zeta}_{ei} - \zeta_{ei}\frac{\dot{\varphi}_{2i}}{\varphi_{2i}})\right) \tag{29}$$

$$+\sum_{i=1}^{n}\left(\frac{1-h_i}{(1-\xi_{ai}^2)\varphi_{1i}}\xi_{ai}(\dot{\zeta}_{ei} - \zeta_{ei}\frac{\dot{\varphi}_{1i}}{\varphi_{1i}})\right) \tag{30}$$

Substituting (6) into (27), we have

$$\begin{aligned} \dot{V}_1 = &\sum_{i=1}^{n}\left(\frac{h_i\xi_{bi}}{(1-\xi_{bi}^2)\varphi_{2i}}(\zeta_{vi} + \alpha_i - \dot{q}_{di} - \zeta_{ei}\frac{\dot{\varphi}_{2i}}{\varphi_{2i}})\right) \\ &+\sum_{i=1}^{n}\left(\frac{(1-h_i)\xi_{ai}}{(1-\xi_{ai}^2)\varphi_{1i}}(\zeta_{vi} + \alpha_i - \dot{q}_{di} - \zeta_{ei}\frac{\dot{\varphi}_{1i}}{\varphi_{1i}})\right) \end{aligned} \tag{31}$$

Then, we design a virtual controller as

$$\alpha_i = \dot{q}_{di} - k_{1i}\zeta_{ei} + \sigma_i(t)\zeta_{ei} \tag{32}$$

where

$$\sigma_i(t) = \sqrt{(\frac{\dot{\varphi}_{1i}}{\varphi_{1i}})^2 + (\frac{\dot{\varphi}_{2i}}{\varphi_{2i}})^2 + k_{ai}} \tag{33}$$

k_{ai} and k_{1i} are designed positive constants. Notice that the following inequality holds

$$\sigma_i(t) + h_i\frac{\dot{\varphi}_{1i}}{\varphi_{1i}} + (1-h_i)\frac{\dot{\varphi}_{2i}}{\varphi_{2i}} \geq 0 \tag{34}$$

Substituting (32) into (31) yields

$$\begin{aligned} \dot{V}_1 = &\sum_{i=1}^{n}\left((\frac{h_i\xi_{bi}}{(1-\xi_{bi}^2)\varphi_{2i}} + \frac{(1-h_i)\xi_{ai}}{(1-\xi_{ai}^2)\varphi_{1i}})\zeta_{vi}\right) \\ &-\sum_{i=1}^{n}k_{1i}\left(\frac{h_i\xi_{bi}}{1-\xi_{bi}^2}\frac{\zeta_{ei}}{\varphi_{2i}} + \frac{(1-h_i)\xi_{ai}}{1-\xi_{ai}^2}\frac{\zeta_{ei}}{\varphi_{1i}}\right) \\ &+\sum_{i=1}^{n}\left(\frac{h_i\xi_{bi}}{(1-\xi_{bi}^2)\varphi_{2i}}(\sigma_i - \zeta_{ei}\frac{\dot{\varphi}_{2i}}{\varphi_{2i}})\right) \\ &+\sum_{i=1}^{n}\left(\frac{(1-h_i)\xi_{ai}}{(1-\xi_{ai}^2)\varphi_{1i}}(\sigma_i - \zeta_{ei}\frac{\dot{\varphi}_{1i}}{\varphi_{1i}})\right) \end{aligned} \tag{35}$$

And in terms of (34), we have

$$\begin{aligned} &\dot{V}_1 \leq \\ &\sum_{i=1}^{n}\left((\frac{h_i}{(\varphi_{2i}^2 - \zeta_{ei}^2)} + \frac{(1-h_i)}{(\varphi_{1i}^2 - \zeta_{ei}^2)})\zeta_{ei}\zeta_{vi} - \frac{k_{1i}\xi_i^2}{(1-\xi_i^2)}\right) \end{aligned} \tag{36}$$

Noting that the following inequality exists,

$$\frac{\xi_i^2}{(1-\xi_i^2)} \geq \text{In}\frac{1}{(1-\xi_i^2)} \qquad \forall|\xi_i| < 1. \tag{37}$$

Substituting (37) into (36), and using the ϱ_i to represent $h_i/(\varphi_{2i}^2 - \zeta_{ei}^2) + (1-h_i)/(\varphi_{1i}^2 - \zeta_{ei}^2))$, we can obtain that

$$\dot{V}_1 \leq \sum_{i=1}^{n}\left(-\frac{k_{1i}}{(1-\xi_i^2)} + \varrho_i\zeta_{ei}\zeta_{vi}\right) \tag{38}$$

Let us take the derivative of V_2 in (26), substitute (1) and (6) into (39), we can obtain that

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2}\zeta_v^T\dot{M}\zeta_v + \zeta_v^T M\dot{\zeta}_v \\ &= \frac{1}{2}\zeta_v^T\dot{M}\zeta_v + \zeta_v^T(\tau - C\dot{q} - G - M\dot{\alpha}) \\ &= \zeta_v^T(\tau + F_1(z)) \end{aligned} \tag{39}$$

where $F_1(z)$ have be described in (7).

Substituting the control input (15) into (39) as

$$\dot{V}_2 = \sum_{i=1}^{n}\left(\eta_i + \zeta_{vi}(\hat{W}_i^T\bar{S}_1(z) - W_i^{*T}S(z) + \varepsilon_i)\right) \tag{40}$$

where $\eta_i = -k_{2i}\zeta_{vi}^2 - k_{3i}\frac{\zeta_{vi}^2}{|\zeta_{vi}|} - \varrho_i\zeta_{ei}\zeta_{vi}$.

According to the Young's inequality, the following relation can be easily obtained

$$\zeta_{vi}\varepsilon_{1i} \leq \frac{1}{2}\zeta_{vi}^2 + \frac{1}{2}\varepsilon_{1i}^2 \tag{41}$$

Substituting (41) into (40), and considering $||\varepsilon_j|| < \varepsilon^*$, we have

$$\dot{V}_2 \leq \sum_{i=1}^{n}\left(\eta_i - \zeta_{vi}\tilde{W}_i^T\bar{S}_1 + \frac{1}{2}\zeta_{v2}^2 + \frac{1}{2}\varepsilon^{*2}\right) \quad (42)$$

Differentiating the second equation of (26) with respect to time, we can obtain $\dot{V}_3$ as

$$\dot{V}_3 = \sum_{i=1}^{n}\left(R_i^T P^{-T}\Gamma_i^{-1}\frac{\partial(P^{-T}R_i)}{\partial t}\right) \quad (43)$$

According to (18) and (19), we have

$$\begin{aligned}\frac{\partial(P^{-T}R_i)}{\partial t} &= \frac{\partial(\tilde{W}_i + P^{-T}\mu_{\varepsilon i})}{\partial t}\\ &= \dot{\tilde{W}}_i - P^{-T}\dot{P}^T P^{-T}\mu_{\varepsilon i} + P^{-T}\dot{\mu}_{\varepsilon i}\\ &= \dot{\tilde{W}}_i + \bar{\mu}_{\varepsilon i} = \dot{\hat{W}}_i + \bar{\mu}_{\varepsilon i}\end{aligned} \quad (44)$$

where $\bar{\mu}_{\varepsilon i} = P^{-T}\dot{\mu}_{\varepsilon i} - P^{-T}\dot{P}^T P^{-T}\mu_{\varepsilon i} \in \mathbb{R}^{N_a}$.
Substituting (44) into (43), and considering the equation (19) differentiation of V_3 can be written as

$$\begin{aligned}\dot{V}_3 &= \sum_{i=1}^{n}\left(R_i^T P^{-T}\Gamma_i^{-1}(\dot{\hat{W}}_i + \bar{\mu}_{\varepsilon i})\right)\\ &= \sum_{i=1}^{n}\left(R_i^T P^{-T}\Gamma_i^{-1}(\Gamma_i(\zeta_{vi}\bar{S}_1 - \gamma_i\frac{PR_i}{||R_i||}) + \bar{\mu}_{\varepsilon i}))\right)\\ &= \sum_{i=1}^{n}\left(R_i^T P^{-T}\zeta_{vi}\bar{S}_1 - \gamma_i\frac{R_i^T P^{-T}PR_i}{||R_i||}\right)\\ &+ \sum_{i=1}^{n}R_i^T P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}\\ &\leq \sum_{i=1}^{n}\left(\zeta_{vi}\tilde{W}_i^T\bar{S}_1 + |\zeta_{vi}|||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||\right)\\ &- \sum_{i=1}^{n}\left((\gamma_i - ||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||)||R_i||\right)\end{aligned} \quad (45)$$

Let us combine (38), (42) with (45),

$$\begin{aligned}\dot{V}_1 + \dot{V}_2 + \dot{V}_3 &\leq \sum_{i=1}^{n}\left(-\frac{k_{1i}}{1-\xi_i^2} - k_{2i}\zeta_{vi}^2 + \frac{1}{2}\zeta_{vi}^2 + \frac{1}{2}\varepsilon_i^2\right)\\ &+ \sum_{i=1}^{n}\left(-k_{3i}\frac{\zeta_{vi}^2}{|\zeta_{vi}|} + |\zeta_{vi}|||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||\right)\\ &- \sum_{i=1}^{n}\left((\gamma_i - ||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||)||R_i||\right)\\ &\leq -\sum_{i=1}^{n}\left((k_{2i} - \frac{1}{2})\zeta_{vi}^2\right)\\ &- \sum_{i=1}^{n}\left(\frac{k_{1i}}{1-\xi_i^2} - \frac{1}{2}\varepsilon^{*2}\right)\\ &- \sum_{i=1}^{n}\left(\gamma_i - ||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||)||R_i||\right)\\ &- \sum_{i=1}^{n}\left((k_{3i} - ||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||)|\zeta_{vi}|\right)\end{aligned} \quad (46)$$

According assumption (1), and noting that ε_j and $\bar{S}_j$ are bounded with $||\varepsilon_j|| \leq \varepsilon^*$ and $\int_t^{t+T}\bar{S}_j\bar{S}_j^T \geq \iota$, $T > 0, \iota > 0$, then, $\mu_{\varepsilon i}$ and $\dot{\mu}_{\varepsilon i}$ are bounded in finite-time interval. Considering the lemma (2), P is bounded in magnitude, thus $||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||$ and $||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||$ exist and are bounded as long as all closed-loop system parameters are suitably chosen. Then, the equation (22) is semiglobal stability for enough large k_{1i}, γ_i, and k_{3i} and $k_{2i} \geq \frac{1}{2}$, we have

$$\dot{V} \leq \sum_{i=1}^{n}\left((k_{2i} - \frac{1}{2})\zeta_{vi}^2\right) \leq 0 \quad (47)$$

The inequation (47) further implies $\lim_{t\longrightarrow\infty}\zeta_v \equiv 0$. Thus, the control error ζ_v converges to zero and all other signals in the closed-loop are bounded.
To further prove finite-time convergence, we substitute (26) into $V_{23} = V_2 + V_3$, thus

$$V_{23} = \frac{1}{2}\zeta_v^T M\zeta_v + \frac{1}{2}R_i^T P^{-T}\Gamma_i^{-1}P^{-T}R_i \quad (48)$$

Then, the time differentiation of (48) is written as

$$\begin{aligned}\dot{V}_{23} &\leq \sum_{i=1}^{n}\left(-k_{2i}\zeta_{vi}^2 - k_{3i}\frac{\zeta_{vi}^2}{|\zeta_{vi}|} - \varrho_i\zeta_{ei}\zeta_{vi} + \zeta_{vi}\varepsilon_{1i}\right)\\ &+ \sum_{i=1}^{n}\left(|\zeta_{vi}|||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1|| - (\gamma_i - ||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||)||R_i||\right)\\ &\leq \sum_{i=1}^{n} -k_{2i}\zeta_{vi}^2 - \sum_{i=1}^{n}\left((\gamma_i - ||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||)||R_i||\right)\\ &- \sum_{i=1}^{n}\left((k_{3i} - \varrho_i|\zeta_{ei}| - \varepsilon^* - ||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||)|\zeta_{vi}|\right)\end{aligned} \quad (49)$$

The bounded analysing for $||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||$ and $||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||$ represented above as long as all closed-loop system parameters are suitably chosen. Then, V_{23} is semiglobal stability for enough large γ_i and k_{3i}, the semiglobal stability of (49) follows such that

$$\dot{V}_{23} \leq -\sum_{i=1}^{n}k_{2i}\zeta_{vi}^2 = -\zeta_v^T k_2\zeta_v \leq 0 \quad (50)$$

with $k_2 = [k_{21}, k_{22}, \cdots, k_{2n}]$.
The inequation (49) can be represented as

$$\begin{aligned}\dot{V}_{23} &\leq -\sum_{i=1}^{n}\left((k_{3i} - \varrho_i|\zeta_{ei}| - \varepsilon^* - ||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||)|\zeta_{vi}|\right)\\ &- \sum_{i=1}^{n}\left((\gamma_i - ||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||)||R_i||\right)\end{aligned} \quad (51)$$

According Lemma (2) and the Lyapunov function V_{23} in (48), the inequation (51) can be rewritten as

$$\dot{V}_{23} \leq \mathcal{K}V_{23}^l \quad (52)$$

with $l = 1/2$ and

$$\begin{aligned}\mathcal{K} = \sum_{i=1}^{n}(&min[(k_{3i} - \varrho_i|\zeta_{ei}| - \varepsilon^* - ||\mu_{\varepsilon i}^T P^{-T}\bar{S}_1||)\\ &\times\sqrt{2/\lambda_{max}(M)},\\ &(\gamma_i - ||P^{-T}\Gamma_i^{-1}\bar{\mu}_{\varepsilon i}||)\delta_p\sqrt{2/\lambda_i(\Gamma_i^{-1})}\ ])\end{aligned}$$

Noting the inequation (52) and applying Lemma 1, we have $V_{23} \equiv 0, \forall t \geq t_c$ with the finite-time

$$t_c \leq 2\mathcal{K} V_{23}^{1/2}(0) \tag{53}$$

Consequently, combination V (22), $\dot{V}$ (47), V_{23} (48) and $\dot{V}_{23}$ (52), finite-time convergence of the tracking error ζ_e, ζ_v and R to zero is guaranteed, which implies $\lim_{t\to\infty} \tilde{W}^T P = \mu_\varepsilon$. This complete the proof.

IV. Simulation Studies

In this section, simulation studies are carried out to illustrate the effectiveness of the proposed adaptive RBFNN control algorithm (15). In the simulation, we employ a 2-link manipulator model whose dynamics is given by [11]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{54}$$

where $q = [q_1, q_2]$ is a vector of joint variables, and

$$G(q) = \begin{bmatrix} G_{11} \\ G_{21} \end{bmatrix} \tag{55}$$

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ m_{21} & M_{22} \end{bmatrix} \tag{56}$$

$$C(q,\dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{57}$$

The plant parameters are chosen as follows:

$$\begin{aligned}
G_{11} &= (m_1 l_{c2} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2) \\
G_{21} &= m_2 l_{c2} g \cos(q_1 + q_2) \\
M_{11} &= m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos q_2) + I_1 + I_2 \\
M_{12} &= m_2(l_{c_2}^2 + l_1 l_{c2} \cos q_2) + I_2 \\
M_{22} &= m_2 l_{c2}^2 + I_2 \\
C_{11} &= -m_2 l_1 l_{c2}(\dot{q}_1 + \dot{q}_2) \sin q_2 \\
C_{12} &= -m_2 l_1 l_{c2}(\dot{q}_1 + \dot{q}_2) \sin q_2 \\
C_{21} &= m_2 l_1 l_{c1} \dot{q}_1 \sin q_2 \\
C_{22} &= 0
\end{aligned}$$

with $m_1 = 2kg$, $m_2 = 0.85kg$, $l_1 = 0.35m$, $l_2 = 0.31m$, $I_1 = 0.061kgm^2$ and $I_2 = 0.020kgm_2$, and m_i and l_i are the mass and length of link i, l_{ci} is the distance between $i-1$th joint and the ith link's mass center, $i = 1, 2$. And I_i is the inertia of link i.

The reference trajectory q_d is chosen as $q_d = [sin(0.5t), 2cos(0.5t)]^T$, where $t \in [0, t_f]$ and $t_f = 15s$. The initial state are set as $q = [-1, 3]$. While to guarantee the transient performance, the prescribed performance functions are designed as $\phi_1(t) = (1 - 0.05)e^{-t} + 1$, $\phi_2(t) = (1 - 0.03)e^{-t} + 1$, i.e. the tracking error are bounded by

$$-\beta_1 \phi_i(t) < z_{11}(t) < \beta_2 \phi_i(t) \quad i = 1, 2 \tag{58}$$

with $\beta_1 = 1$, $\beta_2 = 1$. The control gains are selected as $k_1 = [20, 20]^T$, $k_2 = [10, 10]^T$,$k_3 = [30; 30]$, the gains of NN adaptive laws are chosen as $\Gamma_1 = 0.01\mathbf{I}_{N_a \times N_a}$, $\Gamma_2 = 0.01\mathbf{I}_{N_a \times N_a}$, and the parameter $\gamma = [1; 1]$. The simulation results are shown in Figs.1-4. As shown in Fig.1 and Fig.3, we see clearly that the q_1, q_2, $\dot{q}_1$ and $\dot{q}_2$ could effectively follow the reference trajectories, which means that the proposed controller can achieve a good tracking in the presence of unknown manipulator dynamics. While Fig.2 shows the tracking errors ζ_{e1}, ζ_{e2} coverage to a small value close to zero quickly. The simulation results in Fig.2 also illustrate that our proposed adaptive RBFNN with time-vary-BLF controller has guaranteed the tracking errors always remain in the predefined region and the prescribed transient performances are never violated. While the corresponding control input is depicted as shown in Fig.4.

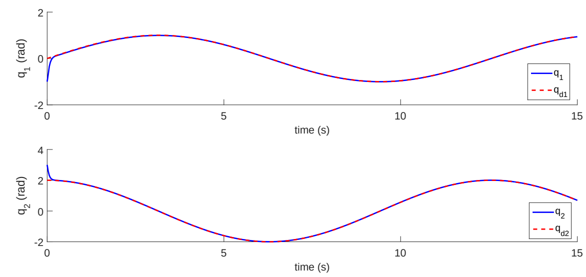

Fig. 1. Position tracking q_1 and q_2

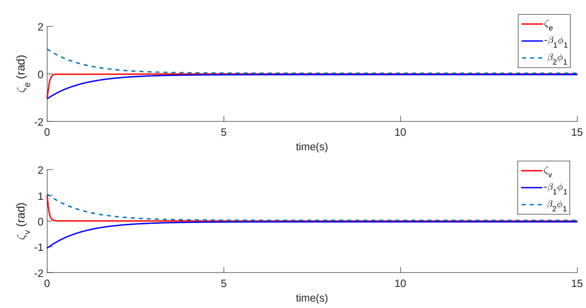

Fig. 2. Tracking performance of the position errors ζ_e and ζ_v

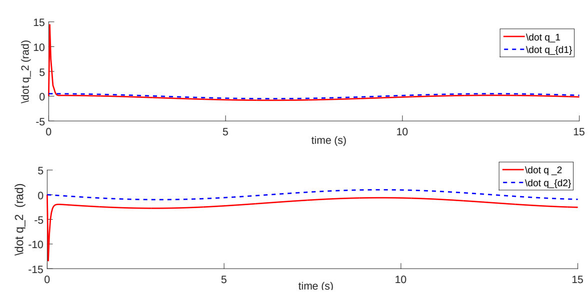

Fig. 3. Veloctiy tracking $\dot{q}_1$ and $\dot{q}_2$

V. Conclusion

In this paper, we have investigated the adaptive neural network control for robot manipulators with unknown dynamics. Controller designed using adaptive RBFNN and time-varying BLF techniques achieves predefined transient performance and guarantee finite time convergence of RBFNN learning. Leakage terms, functions of the estimation error, are incorporated into the adaptation laws to avoid windup of the adaptation

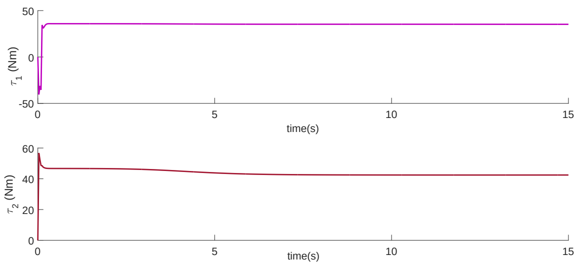

Fig. 4. Control input τ_1 and τ_2

algorithms. Simulation results have demonstrated the effectiveness and efficiency of the proposed control scheme.

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