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# Model Updating based on Nonlinear Normal Modes Identified Under Broadband Excitation

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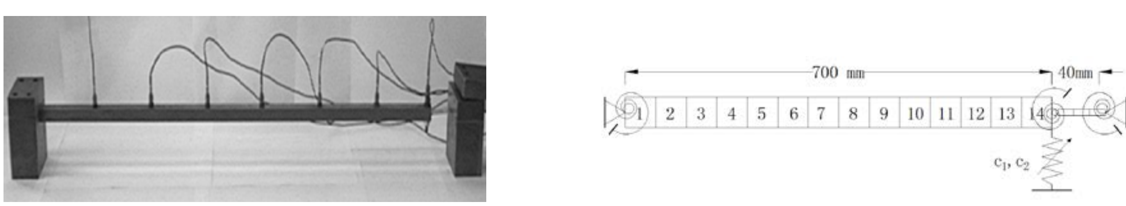
**ABSTRACT**

The present paper investigates the updating of numerical models based on nonlinear normal modes (NNMs) extracted using phase separation. The proposed model updating procedure comprises three steps. First, a broadband excitation signal is applied to the structure of interest and input-output data are collected. Second, NNMs are identified by integrating a nonlinear subspace identification method and numerical continuation in a phase separation approach. Third, model parameters are updated by minimizing the difference between numerically-predicted and experimentally-estimated NNMs calculated at multiple energy levels. A numerical cantilever beam with geometrical nonlinearity is exploited herein for demonstration purposes. Synthetic vibration data are generated under a white-noise excitation. The performance of the model updating procedure is verified versus NNM identification issues, like noise perturbations.

**Benchmark Structure and Data Simulation**

The benchmark structure was developed during the European COST Action F3 [1] and consists of a clamped beam with a local nonlinearity due to a thin beam bolted at one end, as shown in Figure 1(a). Based on the dynamic charateristics of the nonlinear beam, the structure is modeled as follows: the main beam is discretized into 14 beam elements; the left end of the main beam is perfectly fixed in translation but not in rotation, a rotational spring (114,700 N/rad) is considered as the constraint; the seven accelerometers are modeled as lumped masses (2.1 g each); the bolt connection of the two beams is modeled as a lumped mass (11.15 g) and a rotational spring (42.2 N/rad); the thin beam is discretized into 3 beam elements; the right end of the thin beam is perfectly fixed in translation, but the rotational constraint is modeled as a rotational spring (40 N/rad); the local nonlinearity is modeled as a combination of cubic spring and a quadratic spring with nonlinear coefficients $c_1$ and $c_2$ respectively. Figure 1(b) shows the finite element model of the nonlinear beam. Table 1 shows the mechanical properties of the nonlinear beam.




(a) (b)

Figure 1. (a) The nonlinear beam (ULg set up) [1], (b) FE model of the nonlinear beam.

Table 1. Mechanical properties of the nonlinear beam [2].

| Young's modulus (N/m$^2$) | Density (kg/m$^3$) | $c_1$(N/m$^3$) | $c_2$(N/m$^2$) |
|---|---|---|---|
| $2.05\times10^{11}$ | 7800 | $8\times10^9$ | $-1.05\times10^7$ |

The FE model is used to simulate the response of the beam. A broadband excitation signal is applied to the nonlinear beam and acceleration response at the location of seven accelerometers shown in Figure 1(a) are simulated. The measurements are polluted with Gaussian white noise signals to represent realistic noisy data.

**Model Updating**

The NNMs are identified from noisy input-output data using the phase separation method developed in [2]. A separate model with similar model assumptions but unknown parameters (Young's modulus, $c_1$ and $c_2$) is also created for the model updating study. The created model is considered to be in the same model class as the one used for numerical simulation, so no modeling error is considered in this study. The three updating parameters are estimated by minimizing the difference between model-predicted and identified NNMs at multiple energy levels. The model predicted NNMs can be produced using a numerical continuation algorithm proposed by [3]. Figure 2 shows the algorithm of the proposed nonlinear model updating method.

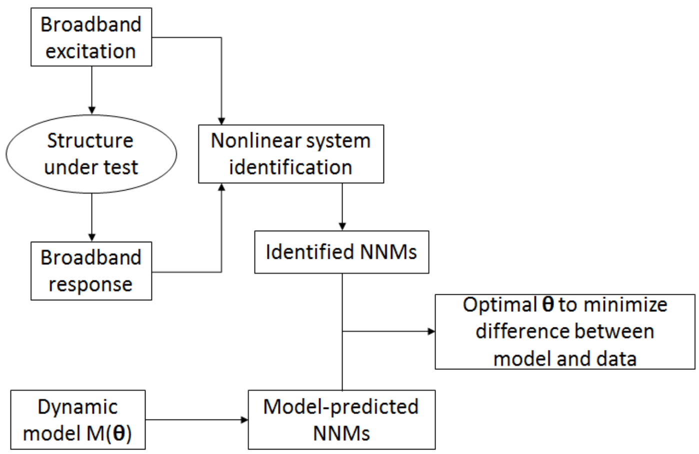



Figure 2. Block diagram of the nonlinear model updating method

To study the effects of measurement noise on model updating resuls, three levels of noise are added to the simulated data: 1%, 2% and 5% in root-mean-square. Figure 3 shows a sample identified first NNM with 1%, 2% and 5% level of noise in the output signal.

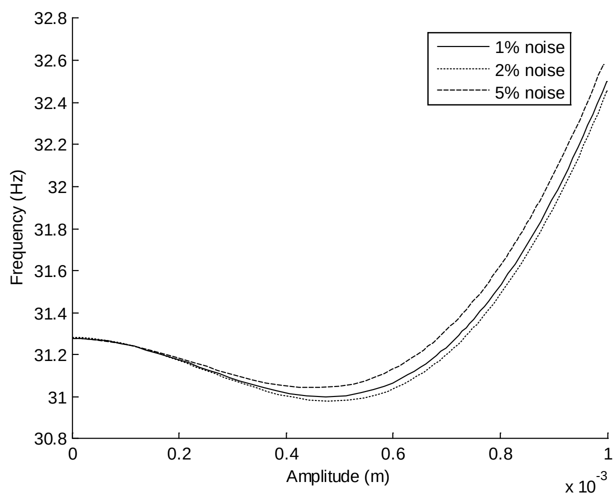



Figure 3. First NNM of the nonlinear beam (identified with 1%, 2% and 5% of noise in output signal).

The optimal model parameters $\boldsymbol{\theta}$ are achieved by minimizing the objective function which is defined as the square sum of the difference of periods and mode shapes between model-predicted NNMs and system identified ones. The objective function $f(\boldsymbol{\theta})$ is defined as:

$$f(\boldsymbol{\theta}) = \sum_{m=1}^{3} \sum_{i=1}^{n} \left[ r_m^{Period}(i) \quad \mathbf{r}_m^{NNM}(i)^T \right] \begin{bmatrix} r_m^{Period}(i) \\ \mathbf{r}_m^{NNM}(i) \end{bmatrix} \tag{1}$$

where

$$r_m^{Period}(i) = \frac{T_m^{\text{model}}(i) - T_m^{\text{measured}}(i)}{T_m^{\text{measured}}(i)}, \qquad \mathbf{r}_m^{NNM}(i) = \frac{\mathbf{z}_m^{\text{model}}(i) - \mathbf{z}_m^{\text{measured}}(i)}{\left\| \mathbf{z}_m^{\text{measured}}(i) \right\|} \tag{2}$$

In this equation, $T_m(i)$ and $\mathbf{z}_m(i)$ are period and mode shape of the $m^{\text{th}}$ mode at energy level (i). The superscript "model" and "measured" refer to model-computed and identified modes. $r_m^{Period}$ and $\mathbf{r}_m^{NNM}$ are recidual period and mode shape of mode $m$. The total number of points on the NNM branch at different energy levels which are included in objective function is defined by $n$.

Figure 4 illustrates the sensitivity of the first NNM with respect to the updating parameters. The plot shows the first NNM corresponding to three values of the parameter vector $\boldsymbol{\theta}$ shown in Table 2. It can be seen that slight variation in each of the updating parameters cause observable changes in the NNMs.

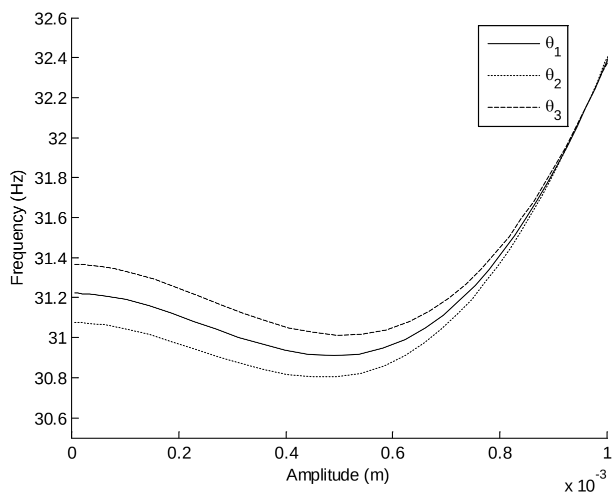



Figure 4. First NNMs predicted by FEA model with different parameters.

Table 2 Considered parameters in Figure 4 (the values of $\boldsymbol{\theta}$ are ratios to the exact values shown in Table 1)

| | Young's modulus | $c_1$ | $c_2$ |
|---|---|---|---|
| $\boldsymbol{\theta}(1)$ (exact) | 1 | 1 | 1 |
| $\boldsymbol{\theta}(2)$ | 0.99 | 1.01 | 0.99 |
| $\boldsymbol{\theta}(3)$ | 1.01 | 0.99 | 1.01 |

To study the accuracy of updating results, 20 model updating cases will be performed at each level of considered measurement noise. The statistical properties of updated model parameters will be reported and compared to the exact values.

## REFERENCES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