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MIMO Channel Dimension Estimation in Interference Channels with Antenna Disparity

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***Abstract*—While obtaining the channel state information (CSI) required to perform Interference Alignment (IA) in a centralised MIMO network is a challenge, this is compounded in unplanned wireless networks that may be decentralised, not to mention composed of devices with varying degrees of complexity and capability. This is of particular interest to heterogeneous networks and the Internet of Things (IoT). The disparity in antennas caused by this variation in capability can be taken advantage of, increasing the sum rate of the network providing the users are aware of the capabilities of their neighbours. This paper presents a method for estimating with a high degree of certainty the number of antennas each user possesses, endowing the receiver with the vital CSI required before channel equalisation or estimation can take place. The method is correlation based and so is not restricted to full rank channels where the receiving device has an equal or greater number of antennas than the transmitter. This permits the use of the method in situations where the rank of the channel is restricted, either by design or by nature (e.g. 'keyhole' channels).**

I. Introduction

Multiple-Input Multiple-Output (MIMO) systems offer the opportunity to improve the sum rate of networks in many ways; one such way is to exploit the spatial signatures of the channels between unintended users (i.e. undesired communications paths giving rise to interference) and precoding transmissions such that they avoid said signatures, thus limiting the interference caused and improving Signal to Interference plus Noise (SINR) levels throughout the network. A significant proportion of the literature on interference management and alignment lies in the context of centralised systems such as cellular networks, but this does not preclude the possibility that such techniques could be applied to decentralised networks as well. In fact, it could be suggested that dense unplanned networks such as those seen in rapidly deployable networks (or 'scatternets') and the emerging IoT have more to gain from alignment techniques; the potential to limit interference in networks that can easily become congested (becoming interference-limited, if not rendered totally unusable) is a significant application advantage.

Considering the two-user MIMO Interference Channel (IC), the network is composed of two user pairs; pair 1 consisting of a transmitter and its intended receiver; pair 2 consisting of its transmitter and intended receiver. Each user's transmitters and receivers are separate devices within the network. In order to permit both transmitters to use the channel simultaneously, each receiver will suffer a degree of interference from the other user pair's transmitter with a high degree of probability. Given the interference received at each receiver is strong enough, successive interference cancellation can be used to decode and cancel out the interference, leaving the desired signal. The simultaneous rate achievable by all users in the symmetric Interference Channel using this approach can be characterised by the 'Generalised Degrees of Freedom' (GDoF), which is itself a measure normalised to the single-user AWGN channel capacity for a given SNR [1]. [1] also introduces the 'GDoF W-curve', which plots the symmetric GDoF achievable in the two-user MIMO IC against α, the level of interference as a function of SNR. The level of interference tolerable at each receiver varies not only with the interference power, but with the transmitted rates of both users; this means any modulation and coding scheme (MCS) capable of sustaining simultaneous transmission must have this side information in order to prevent excessive interference.

Many transmission schemes assume that all receivers and transmitters within the network have the same number of antennas, or at least that the number of transmitters M and receivers N is the same for all users during the transmission period. However, this is unlikely to be the case in heterogeneous networks or networks comprised of many devices created by different manufacturers and fulfilling different roles (for instance in the IoT) - while all the devices are connected to the same network, there is the possibility that additional Degrees of Freedom accessible to better-equipped devices can offer an improvement to the simultaneous rate of the network. With no channel knowledge a transmitter cannot be aware of these additional DoF available to it, so the number of independent streams that the transmitter is able to send is limited to the minimum of the receiving antennas, i.e. $d_{sym} = \min(M_i, N_i)\ \forall i \in \mathcal{K}$ [2], aligned along a suitable precoding vector.

One such MCS that takes advantage of this antenna disparity is the scheme proposed by Karmakar and Varanasi [3]. This scheme uses the 'null spaces' created when one receiver has fewer antennas than the transmitter to hide additional streams of symbols in a subspace that causes no interference to receivers that are not equipped to receive it. This additional stream may be used simply for additional payload data between users, or for side information that may facilitate network operation (e.g. as in physical network coding). By using this hidden stream for payload data, this approach alters the sum GDoF achievable within the network; with only one additional receive antenna introduced to the network the GDoF 'W-curve' begins to appear more 'V' shaped (see Figure 1). This simplifies the optimum transmission strategy for the channel, yielding an improvement in symmetric GDoF at marginal interference levels. [3] presents the achievable rate region in GDoF achievable by such an MCS for an arbitrary number of transmit and receive antennas at each user. In [3] the achievable GDoF region is defined as a function of six variables; the number of antennas at both receivers and antennas, and the SNR/INR between the desired/undesired users respectively. This GDoF region is easily determined using Linear Goal Programming techniques, but in order to do so these parameters must be estimated in the first instance.

The collection of CSI for IA is usually by means of a training sequence; simple equalisation of a known training sequence in the absence of interference is easily performed by zero forcing, but is only possible when the channel matrix is full-rank and well-conditioned. Crucial to the zero-forcing approach is knowing the training sequence used; while the length of a sequence might be inferred from inspection of the time correlation over a number of sequences, the rank of the signal (i.e. the number of individual streams) is a more daunting task. In order for zero-forcing to be an option, therefore, it is necessary to restrict the number of transmitted streams to a known quantity. In a heterogeneous network or network with wide variety of node capabilities this will be limited to the smallest number of antennas a user is equipped with, in order to permit rank estimation on this user. The ability to determine the signal rank is also of use in channels where the 'keyhole' effect causes rank deficiency (i.e. $N < M$) [4].

In [5] the authors present an optimum training sequence for correlated channels, but requires prior knowledge of the channel covariance. [6] addresses the problem of rank deficiency with a chaos-based estimation method for rank-deficient channels, employing joint detection and sphere decoding.

In this paper a correlation-based approach is taken, which exploits the properties imbued on the signal both by its structure and the correlation between the antennas that make up the arrays at either end of the channel.

II. System Model

Consider again the two-user Interference Channel comprised of two pairs of devices (hereafter referred to as 'users'). Each device takes a turn to transmit on the channel without interference from any other device. As well as the transmitting device, either or both receiving devices may possess more than one antenna.

Channel coherence and multipath conditions are assumed to be such that for the duration of training the channel is frequency flat and invariant, allowing the full matrix between the transmitter and receiver to be described in a complex-valued $N \times M$ channel matrix, where N is the number of receive antennas and M the number of transmit antennas.

As part of training the transmitter transmits independent sequences of length L on each of its M antennas, represented by the matrix of column vectors $\mathbf{S}$. The received baseband sequence at the receiver can therefore be described by the expression

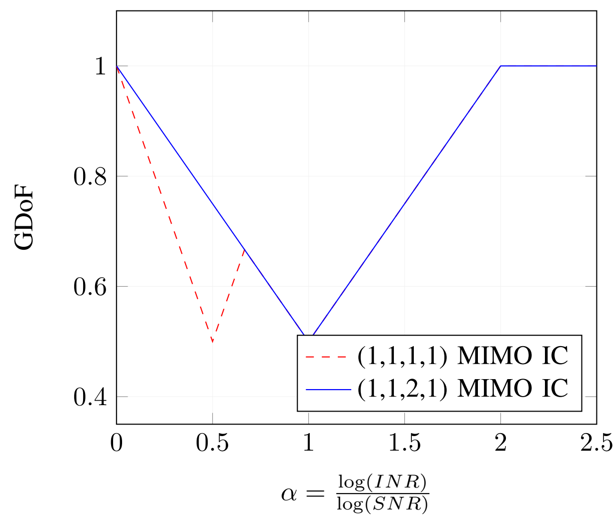

Fig. 1. Additional transmit antenna permits increase in symmetric GDoF (from [3])

$$\mathbf{Y} = \sqrt{SNR}\mathbf{H}\mathbf{S}^T + \mathbf{W} \quad (1)$$

where T denotes the transpose of a matrix and $\mathbf{W}$ represents the additive white Gaussian noise apparent at the receiver, which has zero mean and unity power.

Several stochastic models for the channel entries in $\mathbf{H}$ exist; in this paper the *Kronecker Model* [7] is adopted, which models the channel matrix as the product of the two 'one-sided' antenna correlation matrices at the receiver ($\mathbf{R}_r$) and transmitter ($\mathbf{R}_t$)

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_{iid}\mathbf{R}_t^{1/2} \quad (2)$$

where $\mathbf{H}_{iid}$ is a matrix whose elements are i.i.d. zero-mean circular symmetric Gaussian random values with unity variance.

Using the vector operator $\mathbf{a} = \text{vec}(\mathbf{A}) = [a_{11} \ \dots \ a_{m1} \ a_{12} \ \dots \ a_{mn}]^T$, the columns of matrix $\mathbf{A}$ are stacked on top of each other. Applying the identity $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B})$ to the channel matrix $\mathbf{h} = \text{vec}(\mathbf{H}) = (\mathbf{R}_r^{1/2} \otimes \mathbf{R}_t^{1/2})\mathbf{h}_{iid}$ where $\mathbf{h}_{iid}$ is a $LN \times 1$ complex Gaussian random variable with unity variance. From this the channel covariance matrix can be computed, i.e. $\mathbf{R_H} = \mathbb{E}[\mathbf{hh}^H]$, where $\cdot^H$ denotes the Hermitian transpose. The channel covariance matrix therefore has the structure

$$\mathbf{R_H} = \mathbf{R}_r \otimes \mathbf{R}_t \quad (3)$$

here $\otimes$ denotes the Kronecker product, hence the name of the channel model. The defining feature of this model is that correlations at either 'end' of the channel are taken into account, but it is assumed that the correlations are separable, and the channel exhibits no coupling between the scatterers surrounding either side of the link.

III. Estimation of Signal Rank

The covariance matrix of the training signal sent through the channel can be found in a similar manner.

$$\begin{aligned}\mathbb{E}[\mathbf{y}\mathbf{y}^H] &= \mathbb{E}[\text{vec}(\mathbf{Y})\text{vec}(\mathbf{Y})^H] \\ &= \mathbb{E}[\text{vec}(\sqrt{SNR}\mathbf{H}\mathbf{S}^T + \mathbf{w})\text{vec}(\sqrt{SNR}\mathbf{H}\mathbf{S}^T + \mathbf{w})^H] \\ &= \mathbb{E}[SNR(\mathbf{S}\mathbf{R}_t^{1/2} \otimes \mathbf{R}_r^{1/2})\mathbf{h}_{iid}\mathbf{h}_{iid}^H(\mathbf{R}_t^{1/2}\mathbf{S}^H \otimes \mathbf{R}_r^{1/2}) + \mathbf{I}] \\ &= \mathbb{E}[SNR(\mathbf{S}\mathbf{R}_t^{1/2}\mathbf{R}_r^{1/2}\mathbf{S}^H) \otimes (\mathbf{R}_r^{1/2}\mathbf{R}_r^{1/2}) + \mathbf{I}] \\ &= \mathbb{E}[SNR(\mathbf{S}\mathbf{R}_t\mathbf{S}^H \otimes \mathbf{R}_r) + \mathbf{I}] \\ &= SNR(\mathbf{Q} \otimes \mathbf{R}_r) + \mathbf{I}\end{aligned}$$

where in the last equation the matrix $\mathbf{Q}$ represents the combined temporal and spatial covariance matrix of the transmitted signal once passed through the transmit antenna array.

Since the receive antenna array configuration is assumed to be constant throughout the period of transmission (and likely the device's lifetime) it can be assumed that $\mathbf{R}_r$ is already known or can be modelled depending on the MIMO array structure employed.

Since $\mathbf{R}_r$ is known at the receiver, the Nearest Kronecker Product (NKP) can be found by solving the least squares problem shown in (4) [8].

$$\tilde{\mathbf{Q}} = \min_Q \left(\| \left(\frac{\mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right] - \mathbf{I}}{SNR} \right) - (\mathbf{Q} \otimes \mathbf{R}_r) \|_F^2 \right) \quad (4)$$

which is minimised by:

$$\tilde{q}_{ij} = \frac{\text{Tr}\left(\left(\frac{\mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right] - \mathbf{I}}{SNR} \right)^{[ij]T} \mathbf{R}_r \right)}{\text{Tr}(\mathbf{R}_r^T\mathbf{R}_r)} \quad (5)$$

where $\cdot^{[ij]}$ denotes the $ij^{th} N \times M$ submatrix of the larger matrix and $\| \cdot \|_F$ denotes the Frobenius norm.

The solution in (5) is applicable for real matrices, but since the Frobenius norm operates in the same manner for both real and complex matrices no extension to the decomposition is required. This introduces a degree of error into the NKP result, which is represented by ϵ for the rest of this paper.

The covariance matrix $\mathbf{Q}$ has the following structure [5]

$$\mathbf{Q} = \begin{bmatrix} \sum_{i=j}^{M}\sum_{j=1}^{N}\mathbf{R}_{ij}(0)\rho_{ij} & \dots & \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbf{R}_{ij}(L-1)\rho_{ij} \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbf{R}_{ij}(L-1)\rho_{ij} & \dots & \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbf{R}_{ij}(0)\rho_{ij} \end{bmatrix}$$

where ρ_{ij} is the i,j^{th} element of the one-sided transmit antenna correlation matrix $\mathbf{R}_t$, i.e. the correlation coefficient between antennas i and j of the array.

By selecting a suitable training sequence, the structure of $\mathbf{Q}$ (and therefore its approximation $\tilde{\mathbf{Q}}$) can be manipulated. Using an orthogonal sequence that exhibits low autocorrelation and cross-correlation the value of the main diagonal of the covariance matrix can be expressed as

$$\sum_{i=1}^{M}\mathbf{R}_{ii}(0)\rho_{ii} + \sum_{\substack{i=1 \\ i\neq j}}^{M}\sum_{j=1}^{N}\mathbf{R}_{ij}(0)\rho_{ij} + \epsilon \quad (6)$$

$$= \sum_{i=1}^{M} 1 + \sum_{\substack{i=1 \\ i\neq j}}^{M}\sum_{j=1}^{N}\mathbf{R}_{ij}(0)\rho_{ij} + \epsilon \quad (7)$$

$$= M + \sum_{\substack{i=1 \\ i\neq j}}^{M}\sum_{j=1}^{N}\mathbf{R}_{ij}(0)\rho_{ij} + \epsilon \quad (8)$$

where ϵ is the error term introduced by the complex nature of the original covariance matrix.

The main diagonal of the covariance matrix can therefore be used to estimate the number of streams that were transmitted through the channel; in the case of Gold sequences the values of $\mathbf{R}_{ii}(\tau)$ $\tau \neq 0$ and $\mathbf{R}_{ij}(\tau)$ $\forall$ τ have low (and deterministic) values.

Figure 2 shows a histogram of the signal covariance matrix and its approximation using (4) over 1000 instances of a 5×5 MIMO IC at SNR = 30 dB. The diagonal of the approximation $\tilde{\mathbf{Q}}$ can be observed to possess a similar mean value (equal to the number of transmit antennas $M = 5$) but larger variance than that of the original covariance matrix. From this it can be inferred that ϵ is a random variable with normal distribution and a small (< 0.05) mean value. The most appropriate estimation of the signal rank is therefore the mean of the covariance matrix's diagonal, i.e.

$$\hat{M} = \mathbb{E}[\text{diag}(\tilde{\mathbf{Q}})]. \quad (9)$$

This estimate is independent of the number of receive antennas (since R_r has already been removed from the equation through the NKP process) and in the next section will be demonstrated to work for various lengths of training sequence, blocks lengths, signal powers, and MIMO channel dimensions.

IV. Simulation Results

In this section the performance of the estimator is assessed. Estimation of the signal rank is made over a training sequence composed of B blocks of Gold sequences L symbols in length. The signal covariance matrix is formed by taking the expectation of the received signal over the B blocks, yielding an approximation of $\tilde{\mathbf{Q}}$ of dimension $L \times L$. For the time being we define the MSE as

$$\text{MSE}(\hat{M}) = \mathbb{E}[(\hat{M} - M)^2]. \quad (10)$$

This allows us to predict the likelihood of an erroneous decision, depending on the receiver's approach to mapping

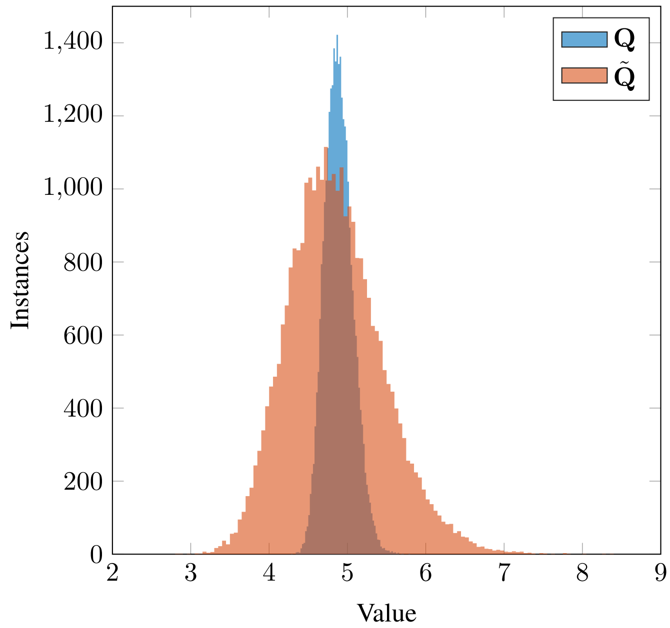

Fig. 2. Values of covariance matrices' diagonals

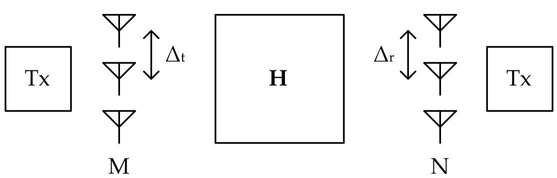

Fig. 3. MIMO Channel during Training

the result to an integer. The channel model follows from the analysis in Section II, where two known antenna correlation matrices are used with an AWGN matrix to generate a time invariant channel with Rayleigh fading that remains constant for the duration of the training process. The antennas are arranged in uniform linear arrays at both ends of the link (as shown in Figure 3, and scatterers are assumed to be sufficiently far away and distributed such that the correlation matrices may be approximated using the Bessel function [9]

$$\rho_{ij} = J_0(2\pi\Delta_{ij}) = \frac{1}{2\pi}\int_0^{2\pi} e^{j2\pi\Delta_{ij}\cos(\phi)}d\phi \qquad (11)$$

where Δ_{ij} is the distance between antennas i and j in wavelengths and $\Gamma()$ is the Gamma function. The training sequences remained constant throughout the training process, selected randomly from the $2^L - 2$ sequences that are not the generating sequences.

Figure 4 shows the MSE achieved in a 5×5 MIMO channel with $\Delta_t = \Delta_r = 0.3\lambda$. The results show that the MSE drops to acceptable levels at an SNR of around 7 dB, but improvements past 10 dB yield no improvement in performance. The estimator is invariant to the number of blocks B or the the Gold sequence length L, providing that L remains longer than the number of transmit antennas [10].

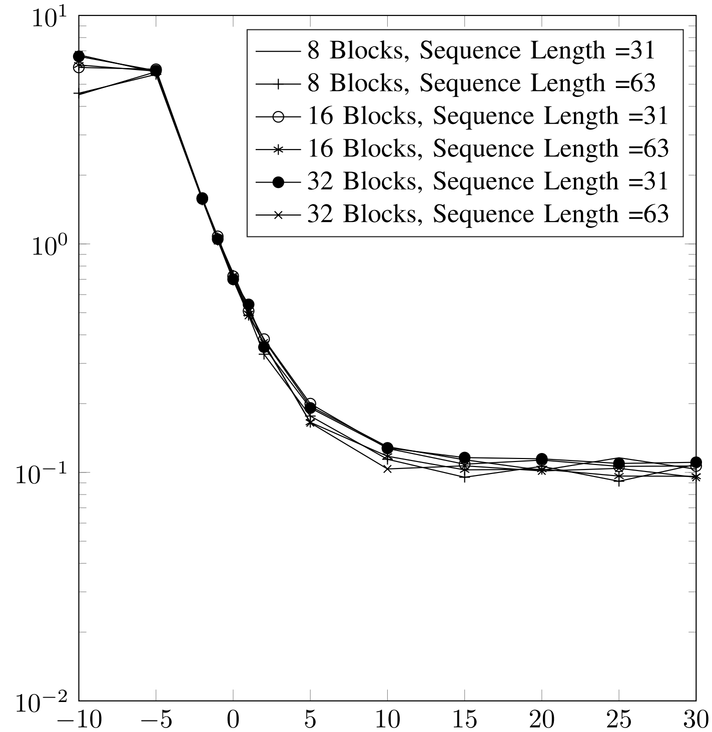

Fig. 4. Comparison of MSEs obtained for different block sizes and training sequence lengths

It can therefore be safely assumed that training time can be restricted (leaving more time for payload) by using the shortest possible sequences without affecting estimator performance.

Figure 5 shows the effect of antenna correlation on the estimate; since the value of transmit antenna cross-correlation ρ_{ij} determines the contribution the signal cross-correlation makes to the rank estimate, it follows that the MSE of the estimate should improve as the antennas become less correlated (i.e. the antennas move further away from the critical point of $\lambda/2$). This is in line with common wisdom that antennas should not be placed at critical intervals, causing greater correlation within the array. As expected, the receive antenna correlation makes no difference to the estimation error.

Figure 6 shows the MSE of the estimator against the MIMO channel dimension over 1000 channel instances at SNR = 30 dB. This clearly demonstrates the ability of the estimator to accurately estimate the signal rank for various numbers of both transmit and receive antennas. Crucially there is no penalty when the channel is rank-deficient. This estimation method is therefore suitable for all types of MIMO channel.

V. Conclusion

An estimator for the rank of a signal transmitted through a MIMO channel has been presented that is capable of reasonable accuracy regardless of the number of antennas at the receiver. It has been demonstrated that not only is this possible but that the estimator is robust against low levels

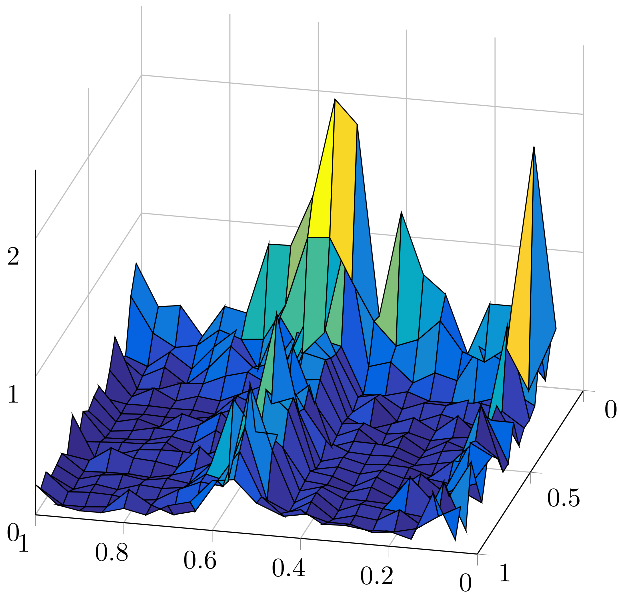

Fig. 5. MSEs obtained compared against transmitter and receiver separation (1000 iterations, 5×5 channel, $SNR = 30$dB, $B = 32$, $L = 31$)

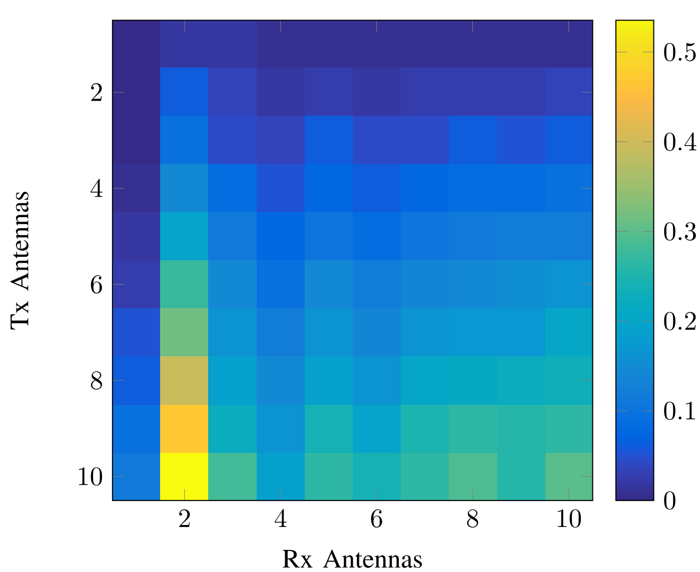

Fig. 6. Heatmap of MSE for MIMO channels of dimension $M \times N$ (SNR = 30 dB, B = 32, L = 31, $\Delta_r = \Delta_t = 0.3\lambda$)

of noise (SNR$>$ 10 dB) and can operate on relatively short training signal lengths.

It is noted however that the channel model assumed in this paper has its limitations; the Kronecker channel has been demonstrated to underestimate the channel capacity of real channels [11]. This is due to the assumption that the scattering environment in which the channel resides is such that there is no correlation between the paths taken from one group of transmit antennas to any group of receive antennas. In real-life scenarios this is not often the case, and physical objects in the channel environment create ‘clusters’ which induce correlation between the transmit and receive antennas.

This concept is introduced and modelled stochastically in Weichselberger, et al. [11], from which the channel gets its name, the *Weichselberger Model*. Clustered correlation is also discussed and deterministic models for antenna correlations presented in [12]. This additional correlation between antennas will impact the estimator’s ability to estimate the signal rank, as the transmit and receive antennas are no longer separable. It remains an open question if this additional correlation induced by clustering can be modelled and removed from the covariance matrix to recover the signal rank.

Acknowledgements

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