



Wang, S., Ren, X., & Na, J. (2016). Adaptive dynamic surface control based on fuzzy disturbance observer for drive system with elastic coupling. *Journal of the Franklin Institute*, 353(8), 1899-1919. DOI: 10.1016/j.jfranklin.2016.03.006

Peer reviewed version

Link to published version (if available):
[10.1016/j.jfranklin.2016.03.006](https://doi.org/10.1016/j.jfranklin.2016.03.006)

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Adaptive Dynamic Surface Control Based on Fuzzy Disturbance Observer for Drive System with Elastic Coupling

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Abstract

This paper proposes an adaptive control strategy by employing dynamic surface control (DSC) technique and fuzzy disturbance observer (FDO) for two-inertia system with uncertainties and disturbance. Firstly, the unknown elements including uncertainties and external disturbance are estimated by using a fuzzy disturbance observer which does not need a priori information of these unknown dynamics. Next, the estimations of unknown disturbance are integrated into DSC design by using recursive feedbacks to damp torsional vibration. The 'explosion of complexity' ~~inherent~~ in conventional backstepping technique is avoided by introducing first-order filters. The stability analysis of the design scheme is verified based on the Lyapunov stability theory. All the signals in the closed-loop system are guaranteed to be uniformly ultimately bounded and the tracking error can be made arbitrarily small by adjusting the design parameters. Comparative simulations and experiments demonstrate the effectiveness and applicability of the proposed method.

Keywords: Dynamic surface control, fuzzy logic system, disturbance observer, elastic coupling

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1. Introduction

Two-inertia systems are widely used for the drive system in industry plants, which can be found in various electromechanical systems, such as robot-arm, wind turbine generators, crane system and automotive industry. The drive system is composed of a motor connected to a load machine through a stiffness shaft and flexible coupling. This configuration may excite torsional oscillations and lead to the failure of the entire drive system in some operation cases. To maintain a good control performance of drive system, it is necessary to eliminate the torsional vibrations.

Over the past decades, many control methods have been proposed to damp torsional vibrations for the two-inertia system. For instance, PI/PID control, sliding mode control, resonance-ratio control, digital filters, adaptive robust control and model predictive control [1, 2, 3, 4, 5]. These control approaches are based on additional feedbacks from different state variables of the control system (motor speed, torsional torque and load speed). Among these methods, the information of the state variables must be known. However, in the real drive systems, the state variables may not be directly measured in terms of hardware transducers due to the lack of assemble space and the increased cost. Therefore, the estimation and observer technique are adopted to estimate unknown variables. The artificial intelligent techniques, e.g. neural network, fuzzy logic system and genetic algorithm, were employed to estimate the unknown state variables [6, 7, 8, 9, 10, 11]. A torsional vibration control approach was presented in [6], which was based on the additional feedback from the torsional torque and the load-side speed estimated by a neural network estimator. To estimate the motor-side speed for suppressing the torsional vibration, the neuro-fuzzy system was employed [7]. In [8], an adaptive sliding-mode neuro-fuzzy speed controller based on the model reference adaptive structure was used to suppress torsional vibration. The modified fuzzy Luenberger observer was designed via the difference between the electromagnetic and estimated shaft torque [9]. Additionally, disturbance observer approaches have also been investigated for the vibration

suppression of the two-inertia system. The disturbance observer (DOB) with a low-pass filter, called Q-filter, was developed in [12]. In [13], a fraction-order DOB and PI fuzzy controller was proposed to improve the response and accuracy of the control system. Yun *et al.* [14] proposed a systematic design method of a robust DOB with Q-filter for suppressing vibrations in two-inertia system, where a robust DOB with Q-filter was utilized to compensate for the effect of the parameter variations. However, in many practical applications, the information of the parameter uncertainties or disturbance are generally unknown. The aforementioned control methods which do not consider the nonlinear elements. The nonlinearities including uncertainties, nonlinear friction and extra disturbance are difficult to measured. However, these nonlinearities can affect the performance of the control system, and even result in instability of the control system. Therefore, these nonlinearities must be considered in the control design for two-inertia systems. In this paper, the drive system with elastic coupling is considered, and system uncertainties and external disturbance are all considered in the nonlinear control design.

To solve the nonlinear control problem, the backstepping technique [15] has been utilized to design a stabilizing controller for a class of nonlinear systems due to its systematic and recursive procedure. Many applications of backstepping technique have been addressed in [16, 17, 18]. In [16], a robust tracking control approach of airbreathing hypersonic vehicles was designed by using backstepping method and non-linear disturbance observer technique, where disturbance estimations were integrated into the virtual controller design in each step to compensate for the unknown disturbance. The potential problem with backstepping technique is the so-called 'explosion of complexity' which was caused by the repeated differentiations of the virtual controller. To overcome the drawback of backstepping method, a method named dynamic surface control was investigated by introducing a first-order filter at each step of backstepping design [19, 20, 21]. An adaptive DSC combined with neural networks was proposed for a kind of pure-feedback nonlinear systems with unknown dead zone and disturbance [22]. However, the unknown nonlinear dynamics in the aforemen-

tioned methods are assumed to be linearly parameterized. For generic unknown nonlinear functions, approximator-based ~~adaptive control~~ methods have been proposed to approximate unknown nonlinearities via fuzzy logic system (FLS) owing to their nonlinear approximation and learning abilities [23, 24, 25]. A fuzzy adaptive dynamic surface control was proposed for a single-link flexible-joint robot in [26], and the fuzzy state observer was utilized to observe the unknown nonlinear function. In [23], Fei proposed a robust adaptive control using fuzzy compensator for MEMS triaxial gyroscope, where the fuzzy compensator was employed to compensate for the model uncertainties and external disturbances. Additionally, the FLS was used to observe the extra disturbance [27]. However, to our best knowledge, the FLS combined with dynamic surface control technique has not yet been applied for the nonlinear two-inertia system.

Motivated by the above observations, an adaptive tracking control approach, which is performed by incorporating dynamic surface control technique and fuzzy disturbance observer, is proposed for a drive system with elastic couplings of the two-inertia system. The fuzzy disturbance observer is employed to estimate the unknown disturbance, and the estimations are introducing into ~~virtual controller~~ to compensate for the unknown disturbance. The dynamic surface controller is designed by using recursive feedback from state variables to damp the torsional vibration. Compared with the existing results, the main advantages of the proposed controller can be summarized as follows:

1. The unavoidable nonlinear dynamics of two-inertia systems, e.g. nonlinear friction, uncertainties and external ~~disturbances are~~ all considered in the control design. These nonlinear elements are lumped as an augmented unknown nonlinearity, which can be estimated by using a fuzzy disturbance observer.
2. An adaptive DSC based on FDO is proposed for vibration suppression of the two-inertia system by using recursive feedbacks from all state variables. Comparing to PID and neural network control methods in [1, 6], the proposed controller can accurately track the reference input.

The rest of the paper is organized as follows. First, the problem formulation is introduced in Section 2. In Section 3, the FDO is designed to estimate the

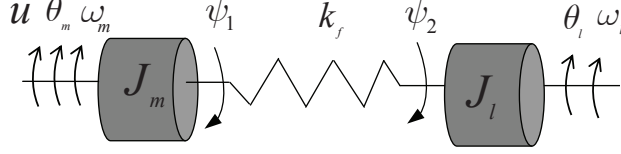


Figure 1: Two-inertia system model

unknown disturbance, and the stability of FDO is proved by using the Lyapunov stability theory. In Section 4, the DSC based on FDO is presented. The stability of the closed-loop system is analyzed in Section 5. Simulation and experiment results are included in Section 6, and conclusion is given in Section 7.

2. Problem Formulation

A typical two-inertia system is composed of a motor connected to a load through a shaft torque (Figure 1), which can be described by the following set of equations

$$\begin{cases} \dot{\theta}_l = \omega_l \\ \dot{\omega}_l = \alpha_l(t)(\theta_m - \theta_l) - \psi_1(\omega_l) \\ \dot{\theta}_m = \omega_m \\ \dot{\omega}_m = -\alpha_m(t)(\theta_m - \theta_l) + b_m(t)u - \psi_2(\omega_m) \end{cases} \quad (1)$$

where θ_l and θ_m are the positions of the load and motor, ω_l and ω_m are the velocities of the load and motor, respectively. The control input u is the torque applied to the motor side. In this paper, it is assumed that all the variables are measured by sensors. Here, the coefficients α_l , α_m and b_m are given by

$$\alpha_l(t) = \frac{k_f(t)}{J_l(t)} \quad \alpha_m(t) = \frac{k_f(t)}{J_m(t)} \quad b_m(t) = \frac{1}{J_m(t)} \quad (2)$$

where J_l and J_m represent the inertia of the load and motor, respectively. k_f is the spring coefficient. The term $\psi_1(\omega_l)$ contains friction, gravity and disturbance, and the term $\psi_2(\omega_m)$ contains the coulomb friction, damping and disturbance. In other words, all the modeled uncertainties and errors are lumped into the unknown signals $\psi_1(\omega_l)$ and $\psi_2(\omega_m)$.

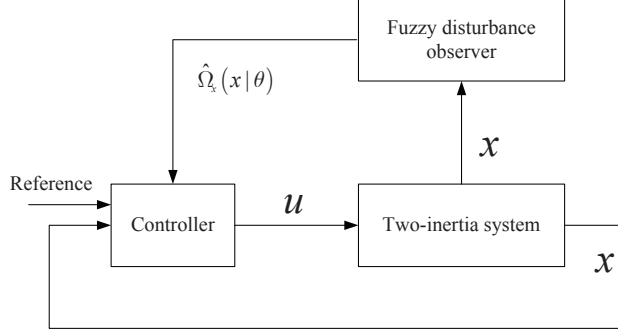


Figure 2: Configuration of two-inertia system with the fuzzy disturbance observer

Choose the state variable $x = [x_1 \ x_2 \ x_3 \ x_4]^T = [\theta_l \ \omega_l \ \theta_m \ \omega_m]^T$, equation (1) can be written in the form of

$$\dot{x} = \begin{bmatrix} x_2 \\ \alpha_l(t)(x_3 - x_1) \\ x_4 \\ -\alpha_m(t)(x_3 - x_1) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_m \end{bmatrix} u + \begin{bmatrix} 0 \\ \psi_1(x_2) \\ 0 \\ \psi_2(x_4) \end{bmatrix} \quad (3)$$

Then, (3) can be written as

$$\dot{x} = f(x) + g(x)u + \Omega_x(x) \quad (4)$$

In this paper, the FLS is used in a disturbance observer to estimate total disturbance because a fuzzy system can approximate the nonlinear function. Figure 2 shows the configuration of the two-inertia system with the fuzzy disturbance. In this configuration, the $\hat{\Omega}_x(x|\theta)$ represents the unknown disturbance $\Omega_x(x|\theta)$ of the two-inertia system.

3. Fuzzy Disturbance Observer

3.1. Fuzzy logic system

The basic configuration of a FLS consists of the fuzzifier, the fuzzy inference engine working on fuzzy rules and the defuzzifier (see Figure 3). The fuzzy

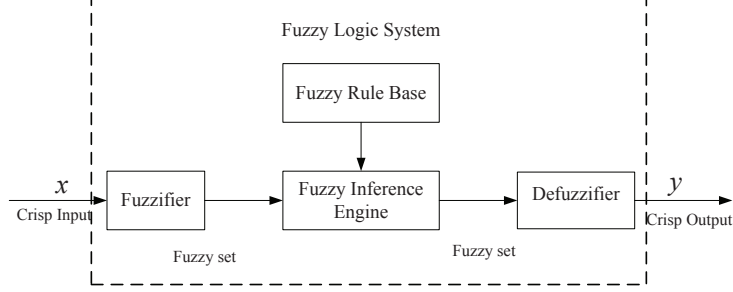


Figure 3: Structure of Fuzzy Logic system

inference engine employs fuzzy rules to perform a mapping from an input linguistic vector $x = [x_1, \dots, x_n]^T \in R^n$ to an output linguistic variable $y \in R$. The knowledge base is composed of a collection of fuzzy IF-THEN rules[28]. The i th fuzzy IF-THEN rule is given

$R^j : IF x_1 \text{ is } A_1^j \text{ and } x_2 \text{ is } A_2^j \dots \text{ and } x_n \text{ is } A_n^j. THEN y \text{ is } y^j (j = 1, 2, \dots, M).$

where $A_1^j, A_2^j, \dots, A_n^j$ are fuzzy sets, and y^j is the fuzzy singleton for the output in the j th fuzzy, M is the number of rules. Then the resulting FLS can be represented as

$$y(x) = \frac{\sum_{j=1}^M \bar{y}_j \left(\prod_{i=1}^n \mu_{A_i^j}(x_i) \right)}{\sum_{j=1}^M \left(\prod_{i=1}^n \mu_{A_i^j}(x_i) \right)}. \quad (5)$$

where $\mu_{A_i^j}(x_i)$ is the fuzzy member function. Then equation (5) can be written as

$$y(x) = \theta^T \varphi(x) \quad (6)$$

where $\theta = (\bar{y}_1, \dots, \bar{y}_M)^T$ is adjustable parameter vector, $\varphi(x) = (\varphi_1(x), \dots, \varphi_M(x))^T$ and $\varphi_j(x)$ are the fuzzy basis functions, which can be defined as

$$\varphi_j(x) = \frac{\prod_{i=1}^n \mu_{A_i^j}(x_i)}{\sum_{j=1}^M \left(\prod_{i=1}^n \mu_{A_i^j}(x_i) \right)} \quad (7)$$

Assumption 1 [29]: Let $f(x)$ be a continuous function defined on a compact set M_x . Then for any constant $\varepsilon > 0$, there exists a fuzzy logic system (5) such that

$$\sup_{x \in M_x} |f(x) - \theta^T \varphi(x)| \leq \varepsilon \quad (8)$$

3.2. Fuzzy Disturbance Observer

In this subsection, the fuzzy disturbance observer is presented to estimate the unknown lumped disturbance. Considering nonlinear system (4) with an unknown disturbance $\Omega_x(x)$:

$$\dot{x} = f(x) + g(x)u + \Omega_x(x) \quad (9)$$

where $\Omega_x(x)$ is the compound disturbance of control system. Consider the following system:

$$\dot{z} = -\sigma z + p(x, \hat{\theta}) \quad (10)$$

where $p(x, \hat{\theta}) = \sigma x f(x) + g(x)u + \hat{\Omega}_x(x, |\hat{\theta})$.

Define an auxiliary variable as $\varsigma = x - z$. Then subtracting (10) from (9), one can obtain as follows

$$\varsigma = \dot{x} - \dot{z} = -\sigma \varsigma + \Omega_x(x) - \hat{\Omega}_x(x, |\hat{\theta}) \quad (11)$$

Following the universal approximation capability of the fuzzy system, the unknown disturbance $\Omega_x(x)$ can be described by the output $\hat{\Omega}_x(x, |\hat{\theta})$ of the fuzzy system plus a reconstruction error ε :

$$\Omega_x(x) = \hat{\Omega}_x(x, |\theta^*) + \varepsilon \quad (12)$$

Now we proposed an adaptive law for $\hat{\theta}$ of the estimator $\hat{\Omega}_x(x, |\hat{\theta})$ to observe $\Omega_x(x)$ in the following theorem.

Theorem 1: Consider the two-inertia system (3). If the adjustable fuzzy disturbance observer parameter vector is tuned by

$$\dot{\hat{\theta}} = \ell_1(\varphi(x)\varsigma + \ell_2\hat{\theta}) \quad (13)$$

where ℓ_1 and ℓ_2 are design parameters, respectively. Then the disturbance observer error ς is uniformly ultimately bounded (UUB, $\varsigma \in L_\infty$) within a arbitrarily small region.

Proof: Choose the following Lyapunov function candidate

$$V_0 = \frac{1}{2}\varsigma^2 + \frac{1}{2\ell_1}\tilde{\theta}^T\tilde{\theta} \quad (14)$$

where $\tilde{\theta} = \theta^* - \hat{\theta}$ is the parameter error. The differential of V_0 is

$$\begin{aligned} \dot{V}_0 &= \varsigma\dot{\varsigma} + \frac{1}{\ell_1}\tilde{\theta}^T\dot{\tilde{\theta}} \\ &= -\sigma\varsigma^2 - \varsigma\tilde{\theta}^T\varphi(x) + \varsigma\varepsilon + \frac{1}{\ell_1}\tilde{\theta}^T\dot{\tilde{\theta}} \\ &= -\sigma\varsigma^2 + \tilde{\theta}^T\left\{\frac{1}{\ell_1}\dot{\tilde{\theta}} - \varsigma\varphi(x)\right\} + \varsigma\varepsilon \\ &= -\sigma\varsigma^2 + \varsigma\varepsilon + \tilde{\theta}^T\left\{\frac{1}{\ell_1}(\ell_1(\varphi(x)\varsigma + \ell_2\hat{\theta})) - \varsigma\varphi(x)\right\} \\ &= -\sigma\varsigma^2 + \varsigma\varepsilon + \ell_2\tilde{\theta}^T\hat{\theta} \end{aligned} \quad (15)$$

Applying the following inequalities

$$\begin{aligned} \varsigma\varepsilon &\leq \frac{1}{2}\sigma\varsigma^2 + \frac{1}{2\sigma}\varepsilon^2 \\ \ell_2\tilde{\theta}^T\hat{\theta} &\leq -\frac{\ell_2}{2}\tilde{\theta}^T\tilde{\theta} + \frac{\ell_2}{2}\theta^{*T}\theta^* \end{aligned} \quad (16)$$

The following inequality holds

$$\dot{V}_0 \leq -\frac{1}{2}\sigma\varsigma^2 - \frac{\ell_2}{2}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\sigma}\varepsilon^2 + \frac{\ell_2}{2}\theta^{*T}\theta^* \quad (17)$$

Then under the assumption that $\hat{\theta}$ is bounded, the disturbance observation error is UUB. This completes the proof.

~~*Remark 1:* Since $V_0(0)$ is bounded and $V_0(0)$ is non-increasing, if $\varepsilon \in L_2$, then $\varsigma \in L_2$. Also, $\dot{\varsigma} \in L_\infty$ and $\hat{\theta} \in L_\infty$. According to the Barbalat lemma [30], one have $\lim_{t \rightarrow \infty} \|\varsigma(t)\| = 0$.~~

4. Controller Design

In this section, we will design a DSC based on FDO for the system (1). The structure of adaptive DSC is shown in Figure 4. Similar to traditional backstepping design methods, the recursive design procedure is based on the following change of coordinates: $S_1 = x_1 - x_r$, $S_i = x_i - x_{id}$, $y_i = x_{id} - \bar{x}_i$ ($i = 2, 3, 4$), where S_i is the error surface, x_{id} is the desired state command, which

is produced by the first order low pass filter, y_i is the filter error, and $\bar{\chi}_i$ is the visual control which will be designed for the system (1). Finally, the actual controller u will be designed in the last step.

Step1: At this step, the first error surface is defined as

$$S_1 = x_1 - x_r \quad (18)$$

where x_r is the desired trajectory.

Then the derivative of (18) is

$$\dot{S}_1 = \dot{x}_1 - \dot{x}_r = x_2 - \dot{x}_r \quad (19)$$

Here, a Lyapunov candidate is considered in the quadratic form as follows:

$$V_1 = \frac{1}{2} S_1^2 \quad (20)$$

The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= S_1 \dot{S}_1 = S_1(S_2 + y_2 + \bar{\chi}_2 - \dot{x}_r) \\ &= S_1(S_2 + y_2 + \bar{\chi}_2 - \dot{x}_r) \end{aligned} \quad (21)$$

Define a virtual controller $\bar{\chi}_2$ as follows

$$\bar{\chi}_2 = -k_1 S_1 + \dot{x}_r \quad (22)$$

where k_1 is positive design parameter.

To avoid the problem of "explosion of complexity" in traditional backstepping design ~~method~~, we introduce a new variable x_{2d} and let $\bar{\chi}_2$ pass through a first-order filter with time constant τ_2 to obtain x_{2d} as

$$\bar{\chi}_2 = \tau_2 \dot{x}_{2d} + x_{2d}, \quad x_{2d}(0) = \bar{\chi}_2(0) \quad (23)$$

where x_{2d} is the output variable. Define a filter error as

$$y_2 = x_{2d} - \bar{\chi}_2 \quad (24)$$

Then

$$\dot{x}_{2d} = -\frac{y_2}{\tau_2} \quad (25)$$

and

$$\dot{y}_2 = \dot{x}_{2d} - \dot{\bar{\chi}}_2 = -\frac{y_2}{\tau_2} + B_1(\cdot) \quad (26)$$

where $B_1(\cdot)$ is a continuous function with the following expression:

$$B_1(\cdot) = -k_1 \dot{S}_1 + \ddot{x}_r \quad (27)$$

Step 2: The second error surface is defined as

$$S_2 = x_2 - x_{2d} \quad (28)$$

According to the Theorem 1, the $\psi_1(x_2)$ can be expressed as $\hat{\psi}_2(x_2|\hat{\theta}_1)$ by FDO. Equation (3) can be rewritten as

$$\dot{x}_2 = \alpha_l(x_3 - x_1) + \hat{\psi}_2(x_2|\hat{\theta}_1) + \varepsilon_1 \quad (29)$$

The derivative of S_2 is

$$\dot{S}_2 = \dot{x}_2 - \dot{x}_{2d} = \alpha_l(x_3 - x_1) + \hat{\psi}_2(x_2|\hat{\theta}_1) + \varepsilon_1 - \dot{x}_{2d} \quad (30)$$

Choose a Lyapunov function candidate as

$$V_2 = \frac{1}{2}S_2^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 \quad (31)$$

The time derivative of V_2 is

$$\begin{aligned} \dot{V}_2 &= S_2\dot{S}_2 + \frac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 \\ &= S_2[\alpha_l(x_3 - x_1) + \hat{\psi}_1(x_2|\hat{\theta}_1) + \varepsilon_1 - \dot{x}_{2d}] + \frac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 \\ &= S_2[\alpha_l(S_3 + y_3 + \bar{\chi}_3 - x_1) \hat{\psi}_2(x_2|\hat{\theta}_1) + \varepsilon_1 - \dot{x}_{2d}] + \frac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 \end{aligned} \quad (32)$$

Choose a virtual controller $\bar{\chi}_3$ as

$$\bar{\chi}_3 = x_1 - \frac{1}{\alpha_l}[\hat{\psi}_1(x_2|\hat{\theta}_1) + k_2 S_2 + \dot{x}_{2d}] \quad (33)$$

Introduce a new state variable x_{3d} and let $\bar{\chi}_3$ pass through a first-order with a time constant τ_3 to obtain x_{3d} as

$$\tau_3 \dot{x}_{3d} + x_{3d} = \bar{\chi}_3, \quad x_{3d}(0) = \bar{\chi}_3(0). \quad (34)$$

Define the output error of this filter as

$$y_3 = x_{3d} - \bar{\chi}_3. \quad (35)$$

It yields

$$\dot{x}_{3d} = -\frac{y_3}{\tau_3} \quad (36)$$

and

$$\dot{y}_3 = -\frac{y_3}{\tau_3} + B_2(\cdot) \quad (37)$$

where

$$B_2(\cdot) = -\dot{x}_1 + \frac{1}{a_l} [\dot{\psi}_1(x_2|\hat{\theta}_1) + \ddot{x}_{2d} + k_2 \dot{S}_2]. \quad (38)$$

Step 3: From (3), define the third error surface is

$$S_3 = x_3 - x_{3d}. \quad (39)$$

The time derivative of S_3 is

$$\dot{S}_3 = \dot{x}_3 - \dot{x}_{3d} = S_4 + y_4 + \bar{\chi}_4 - \dot{x}_{3d}. \quad (40)$$

Consider the Lyapunov function candidate as

$$V_3 = \frac{1}{2} S_3^2 \quad (41)$$

The derivative of V_3 is

$$\dot{V}_3 = S_3 [S_4 + y_4 + \bar{\chi}_4 - \dot{x}_{3d}] \quad (42)$$

Choose a virtual controller $\bar{\chi}_4$ as

$$\bar{\chi}_4 = -k_3 S_3 + \dot{x}_{3d}. \quad (43)$$

Introduce a new state variable x_{4d} and let $\bar{\chi}_4$ pass through a first-order filter with a time constant τ_4 to obtain

$$\tau_4 \dot{x}_{4d} + x_{4d} = \bar{\chi}_4 \quad x_{4d}(0) = \bar{\chi}_4(0). \quad (44)$$

Define the output error of this filter as

$$y_4 = x_{4d} - \bar{\chi}_4. \quad (45)$$

Then

$$\dot{x}_{4d} = -\frac{y_4}{\tau_4} \quad (46)$$

and

$$\dot{y}_4 = \dot{x}_{4d} - \dot{\bar{\chi}}_4 = -\frac{\bar{\chi}_4}{\tau_4} + B_3(\cdot) \quad (47)$$

$$B_3(\cdot) = k_3 \dot{S}_3 - \ddot{x}_{3d}. \quad (48)$$

Step 4: Define the last error surface is

$$S_4 = x_4 - x_{4d} \quad (49)$$

where we employ another fuzzy disturbance observer $\hat{\psi}_2(x_4|\hat{\theta}_2)$ to estimate the unknown term $\psi_2(x_4)$, the \dot{x}_4 can be written as

$$\dot{x}_4 = \alpha_m(t)(x_1 - x_3) + b_m(t)u + \hat{\psi}_2(x_4|\hat{\theta}_2) + \varepsilon_2 \quad (50)$$

Then the time derivative of S_4 is

$$\dot{S}_4 = \dot{x}_4 - \dot{x}_{4d} = \alpha_m(t)(x_1 - x_3) + b_m(t)u + \hat{\psi}_2(x_4|\hat{\theta}_2) + \varepsilon_2 - \dot{x}_{4d} \quad (51)$$

Choose a Lyapunov function candidate as

$$V_4 = \frac{1}{2}S_4^2 + \frac{1}{2\gamma_2}\tilde{\theta}_2^T\dot{\tilde{\theta}}_2 \quad (52)$$

where $\gamma_2 > 0$ is a design parameter.

The derivative of V_4 is

$$\dot{V}_4 = S_4[\alpha_m(t)(x_1 - x_3) + b_m(t)u + \hat{\psi}_2(x_4|\hat{\theta}_2) + \varepsilon_2 - \dot{x}_{4d}] + \frac{1}{\gamma_2}\tilde{\theta}_2^T\dot{\tilde{\theta}}_2 \quad (53)$$

From (53), one can obtain the actual controller u as follows

$$u = \frac{1}{b_m} [-S_3 - k_4 S_4 - \alpha_m(t)(x_1 - x_3) - \hat{\psi}_2(x_4|\hat{\theta}_2) - \dot{x}_{4d}] \quad (54)$$

5. The Stability Analysis

In this section, we will analysis the stability of the closed-loop system. Before illustrating ~~this~~, we have the following assumption.

Assumption 2 [19]: For the reference signal x_r which is a sufficiently smooth function of t , x_r , \dot{x}_r , and \ddot{x}_r are bounded, that is, there exists a known positive constant $B_0 > 0$, and the first and second order derivative of x_r satisfy $\Omega_r := \{(x_r, \dot{x}_r, \ddot{x}_r) : x_r + \dot{x}_r + \ddot{x}_r \leq B_0\} \subset R^3$ and denotes $\Omega_i := \{[S_i, y_i, \tilde{\theta}_i, \varsigma_i] : \sum_{i=0}^4 V_i + \sum_{i=1}^3 y_i^2 \leq 2p\} \subset R^{4i}$ as the compact set of the initial conditions with p a positive constant. Then for any $B_0 > 0$ and $p > 0$, the sets Ω_r and Ω_i are compact in R^3 and R^{4i} , respectively. Thus, there exists a positive constant M_j such that $|M_j| < B_j$ on $\Omega_r \times \Omega_i$.

Consider the closed-loop system composed of the DSC controller (54), the virtual controller (22), (33), (43), and parameter adaptive law (13), thus the closed-loop system is semi-global stability. Moreover, the tracking error and the observer error can be made arbitrarily small by choosing design parameters.

A Lyapunov function is chosen as

$$V = \frac{1}{2} \sum_{i=0}^4 V_i + \frac{1}{2} \sum_{i=2}^4 y_i^2 \quad (55)$$

The derivative of V is

$$\begin{aligned} \dot{V} = & S_1(S_2 + y_2 + \bar{x}_2 - \dot{x}_r) + S_2[\alpha_l(S_3 + y_3 + \bar{x}_3 - x_1) + \hat{\psi}_1(x_2|\hat{\theta}_1) \\ & + \varepsilon_1 - \dot{x}_{2d}] + S_3[S_4 + y_4 + \bar{x}_4 - \dot{x}_{3d}] + S_4[\alpha_M(t)(x_1 - x_3) \\ & + b_M(t)u + \hat{\psi}_2(x_4|\hat{\theta}_2) + \varepsilon_2 - \dot{x}_{4d}] + \sum_{i=2}^4 y_i \dot{y}_i + \frac{1}{2} \sum_{i=1}^2 \varsigma_i^2 + \frac{1}{2} \sum_{i=1}^2 \tilde{\theta}_i^T \tilde{\theta}_i \end{aligned} \quad (56)$$

Substituting (22), (33), (43) and (54) into (56), one have

$$\begin{aligned} \dot{V} = & -k_1 S_1^2 - k_2 S_2^2 - k_3 S_3^2 - k_4 S_4^2 + S_1(S_2 + y_2) + S_2(S_3 + y_3) + S_3(S_4 + y_4) \\ & + \sum_{i=1}^3 y_i \left(-\frac{\bar{x}_{i+1}}{\tau_{i+1}} + B_{i+1}(\cdot) \right) + S_2 \varepsilon_1 + S_4 \varepsilon_2 + \frac{1}{2} \sum_{i=1}^2 \varsigma_i^2 + \frac{1}{2\ell_1} \sum_{i=1}^2 \tilde{\theta}_i^T \tilde{\theta}_i \end{aligned} \quad (57)$$

According to Young's inequality, one have

$$\begin{aligned}
S_j y_{j+1} &\leq \frac{1}{2} S_j^2 + \frac{1}{2} y_{j+1}^2 \\
S_2 \varepsilon_1 &\leq \frac{1}{2} S_2^2 + \frac{1}{2} \varepsilon_1^2 \\
S_4 \varepsilon_2 &\leq \frac{1}{2} S_4^2 + \frac{1}{2} \varepsilon_2^2 \\
y_{j+1} B_j(\cdot) &\leq \frac{1}{2\delta} y_{j+1}^2 B_j^2(\cdot) + \frac{\delta}{2}
\end{aligned} \tag{58}$$

where δ is a design constant.

Substituting (58) and (16) into (57), one have

$$\begin{aligned}
\dot{V} &\leq -\left(k_1 - \frac{1}{2}\right) S_1^2 - (k_2 - 1) S_2^2 - \left(k_3 - \frac{1}{2}\right) S_3^2 - (k_4 - 1) S_4^2 \\
&\quad - \sum_{i=1}^3 \left(\frac{1}{\tau_{1+i}} - \frac{1}{2} - \frac{B_{i+1}^2(\cdot)}{2\delta}\right) y_{i+1}^2 - \sum_{i=1}^2 \left[\frac{1}{2} \sigma_i \varsigma_i^2 - \frac{\ell_2}{2} \tilde{\theta}_i^T \tilde{\theta}_i\right] + \phi
\end{aligned} \tag{59}$$

Define an auxiliary variable ϕ as follows

$$\phi = \sum_{i=1}^2 \left[\left(\frac{1}{2\sigma} + \frac{1}{2}\right) \varepsilon_i^2 + \frac{\ell_2}{2} \theta_i^{*T} \theta_i^*\right] + \frac{3}{2} \delta. \tag{60}$$

According to Assumption 2, equation (59) can be written as

$$\begin{aligned}
\dot{V} &\leq -\left(k_1 - \frac{1}{2}\right) S_1^2 - (k_2 - 1) S_2^2 - \left(k_3 - \frac{1}{2}\right) S_3^2 - (k_4 - 1) S_4^2 \\
&\quad - \sum_{i=1}^3 \left(\frac{1}{\tau_{1+i}} - \frac{1}{2} - \frac{M_{i+1}^2(\cdot)}{2\delta}\right) y_{i+1}^2 - \sum_{i=1}^2 \left[\frac{1}{2} \sigma_i \varsigma_i^2 - \frac{\ell_2}{2} \tilde{\theta}_i^T \tilde{\theta}_i\right] + \phi
\end{aligned} \tag{61}$$

Choose the design controller parameters such that $k_1 - \frac{1}{2} > 0$, $k_2 - 1 > 0$, $k_3 - \frac{1}{2} > 0$, $k_4 - 1 > 0$, $\frac{1}{\tau_j} - \frac{1}{2} - \frac{M_{i+1}^2(\cdot)}{2\delta} > 0$, $\frac{1}{2} \sigma > 0$, $\frac{1}{2} \ell_2 > 0$. Define $\pi = \left\{k_1 - \frac{1}{2} > 0, k_2 - 1 > 0, k_3 - \frac{1}{2} > 0, k_4 - 1 > 0, \frac{1}{\tau_j} - \frac{1}{2} - \frac{M_{i+1}^2(\cdot)}{2\delta} > 0, \frac{1}{2} \sigma > 0, \frac{1}{2} \ell_2 > 0\right\}$.

Then, one have

$$\dot{V}_4 \leq -\pi V_4 + \phi. \tag{62}$$

By by selecting sufficiently large $k_i (i = 1, 2, 3, 4)$, ℓ_1, ℓ_2 , we can make $\pi > \phi/p$, then $V_4 \leq 0$ on $V_4 = p$. Thus, $V_4 \leq p$ is an invariant set, i.e., if $V_4(0) \leq p$, then $V_4 < p$ for all $t \geq 0$. Therefore, $V_4(t)$ is bounded. This completes the proof.

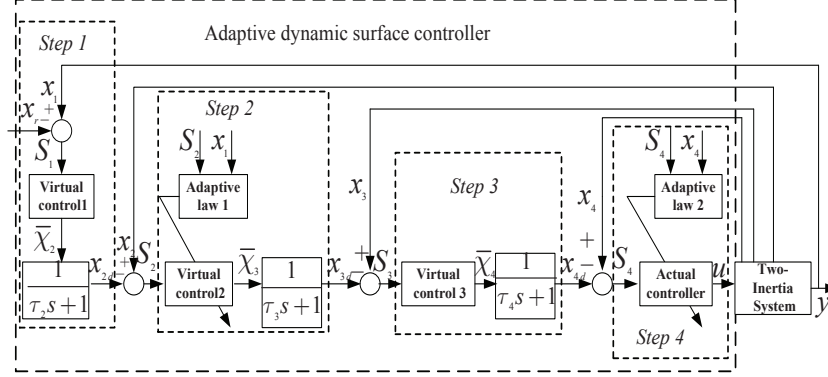


Figure 4: Block diagram of proposed control scheme

The implementation of the DSC based on FDO method is **straight forward**. The design procedures are summarized as follows.

Step 1. Define the fuzzy rules and membership, and determine the fuzzy basis function, establish the fuzzy logic system.

Step 2. Determine the control parameters $k_i (i = 1, \dots, 4)$ and the observer parameters σ_1, σ_2 .

Step 3. Choose the adaptive laws $\hat{\theta}_i (i = 1, 2)$ and the initial conditions $\hat{\theta}_i(0)$. Calculate the intermediate controller \bar{x}_i according to (22), (33) and (43).

Step 4. Select appropriate parameters $\tau_j, j = 2, 3, 4, \ell_1, \ell_2$, respectively. Calculate the actual control signal u according to (54).

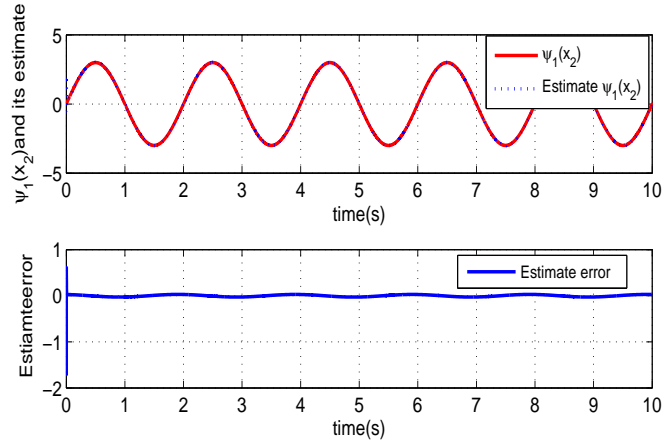
Remark 2: It should be noted that dynamic surface gains k_i , the filter time constants τ_j are required to satisfy $k_i > 0$ and $\tau_j > 0$. Thus, we can obtain sufficiently large π such that ϕ/π is arbitrarily small by increasing the control gain k_i and decreasing the filter time constant τ_j .



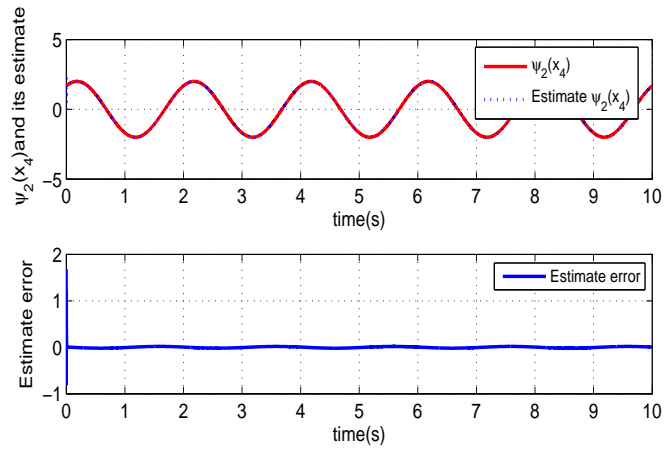
6. Simulation and Experiment Results

6.1. Simulation Results

To validate the effectiveness of the proposed control design, simulations are performed in this section. The system parameters in (1) are chosen as $J_m = 0.0226 \text{ kg/m}$, $J_l = 0.0045 \text{ kg} \cdot \text{m}^2$, $k_f = 65 \text{ N} \cdot \text{m}$, and the controller parameters



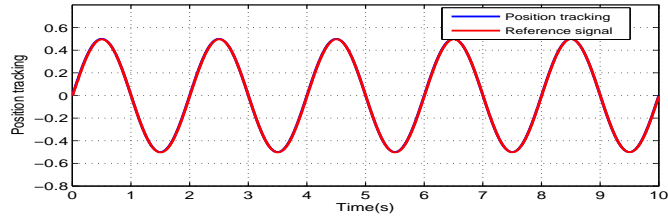
(a)



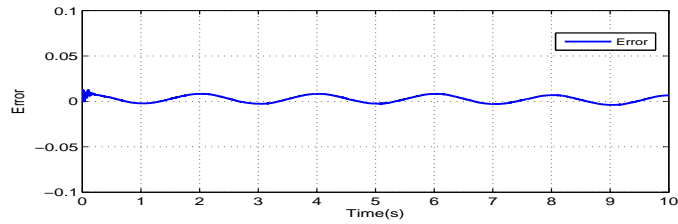
(b)

Figure 5: Actual and estimated values of disturbances: (a) Trajectory of the disturbance $\psi_1(x_2)$, (b) Trajectory of the disturbance $\psi_2(x_4)$

are $k_1 = 1$, $k_2 = 2$, $k_3 = 1$, $k_4 = 1.2$, $\tau_2 = \tau_3 = \tau_4 = 0.01$, $\ell_1 = 1$, $\ell_2 = 0.01$. In the simulation, all the initial conditions are chosen as zero. Assuming that the disturbance signals are $\psi_1 = 3\sin(\pi t)$, $\psi_2 = 2\sin(\pi t + 1)$. The actual and estimated trajectories of the lumped disturbance are depicted in Figure 5. From Figure 5, one can see that the FDO can precisely estimate the lumped disturbance.

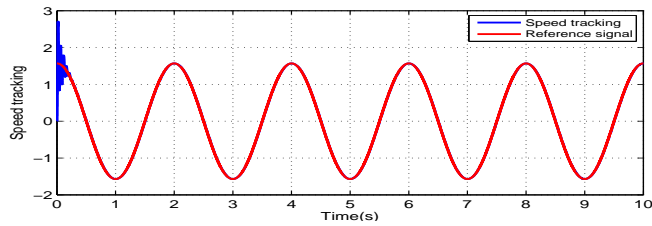


(a)

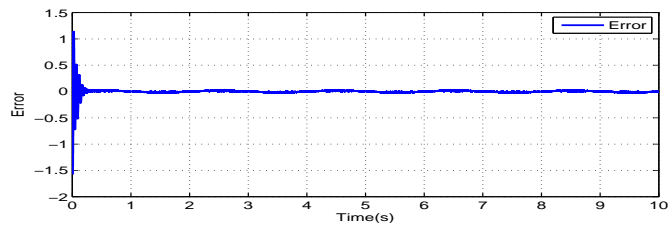


(b)

Figure 6: (a) Load position tracking of DSC with FDO, (b) tracking error



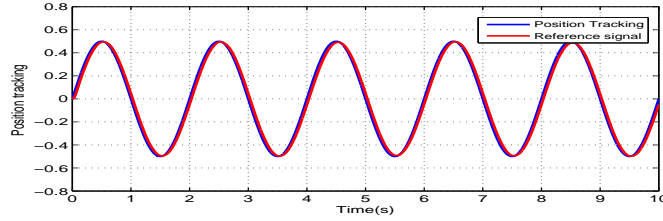
(a)



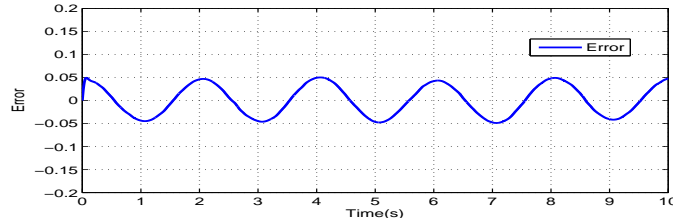
(b)

Figure 7: (a) Load speed tracking of DSC with FDO, (b) tracking error

To illustrate the tracking performance of the proposed control scheme, the reference input signal is chosen as $x_r = 0.5 \sin(\pi t)$. The simulation results are



(a)



(b)

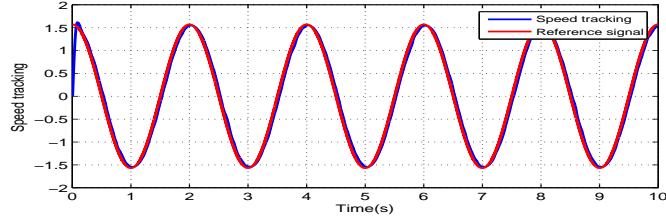
Figure 8: (a) Load position tracking of PID , (b) tracking error

shown in Figures 6-9. Figures 6-7 show the tracking performance of DSC with FDO, and Figures 8-9 shows the tracking performance of PID method. It is clear from the profiles of the position and speed tracking, that the proposed control scheme produces very good tracking performance.

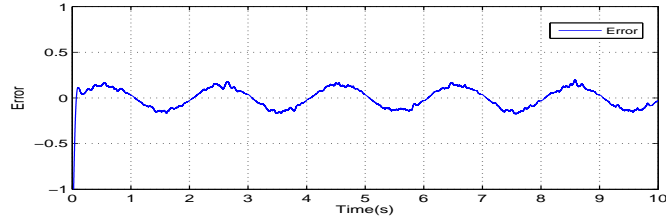
6.2. Experiment Results

A realistic two-inertia system is used as the test-rig to validate the proposed control method. The configuration of the whole experimental setup is shown in Figure 10. The experimental setup is composed of a permanent-magnet synchronous motor connected to a load machine, a PC with a 2.0GHz i5 CPU and 2G memory, and a digital signal processor (DSP, 28335). The control algorithms are written by Visual C++. The sampling time in the experiment setup is 1 ms. The positions and speeds of the drive motor and load machine are measured by sensors. Nominal parameters of the drive system are presented in Table 1.

In order to evaluate the performance of the proposed control method, the following three controllers are compared.



(a)



(b)

Figure 9: (a) Load speed tracking of PID, (b) tracking error

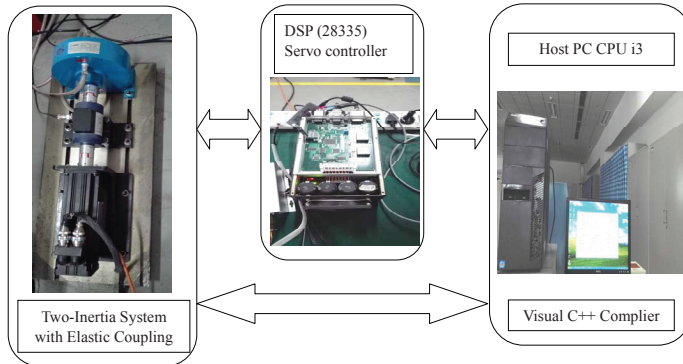


Figure 10: Block diagram of proposed control scheme

1) The proposed control scheme method: This is the proposed adaptive DSC with FDO, described in Section 4. The controller parameters are given as $k_1 = 2$, $k_2 = 1.2$, $k_3 = 1.6$, $k_4 = 3$, $\tau_2 = 0.05$, $\tau_3 = \tau_4 = 1$, $\ell_1 = 1$, $\ell_2 = 0.01$.

2) PINN [6]: This is the proportional-internal controller with neural network (NN) estimator. The control structure with state feedbacks from the torsional torque and the load-side speed which are obtained by using NN estimators.

Table 1: Parameters of the two-inertia system

Parameter	Value	Unit
Power	1.5	kW
Nominal motor voltage	230	V
Shaft length	40	cm
Nominal speed	3600	r/min
Motor inertia J_m	0.0062	$Kg \cdot m$
Load inertia J_l	0.004106	$Kg \cdot m$
Stiffness coefficient k_f	65	$N \cdot m$

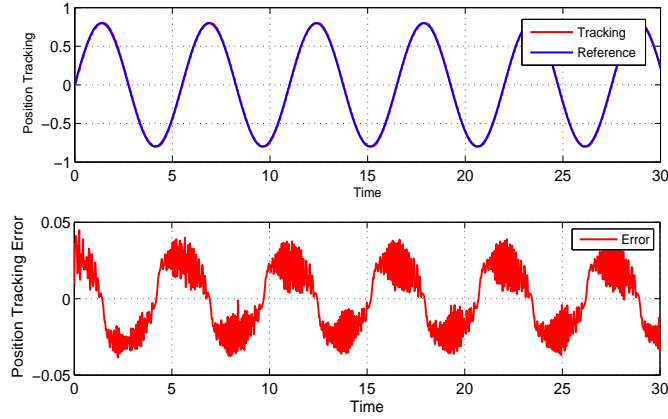


Figure 11: Position tracking and tracking error of PID method

The controller parameters and NN parameters are chosen the same as [6]. The transfer function of the PI controller is defined as follows

$$G(s) = K_P + K_I/s \quad (63)$$

3) PID: This is the proportional-internal-differential controller with speed feedback. The controller gains are tuned by $k_p = 20$, $k_i = 0.5$, and $k_d = 3$.

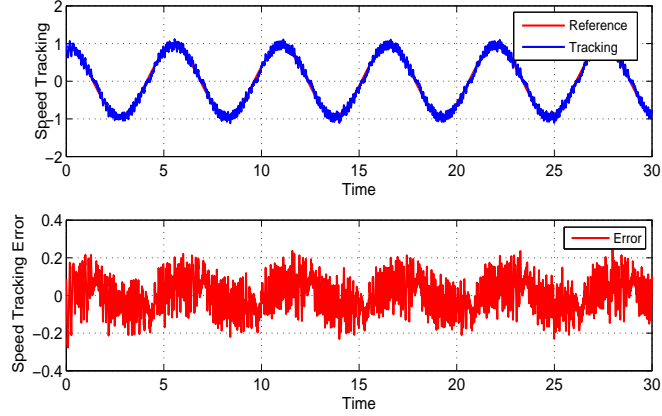


Figure 12: Speed tracking and tracking error of PID method

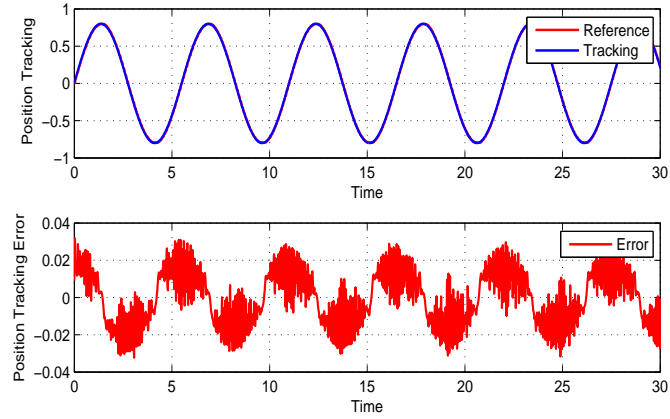


Figure 13: Position tracking and tracking error of PINN method

This controller is also similar to the PID controller proposed in [1], and the expression of the conventional PID controller is defined as

$$u_{PID} = k_p e + k_i \int_0^t e(t) dt + k_d \dot{e}. \quad (64)$$

For fair comparison, the input signal of three algorithms has the same reference signal $x_r = 0.8 \sin(2\pi t/5.5)$. The tracking control performance of the

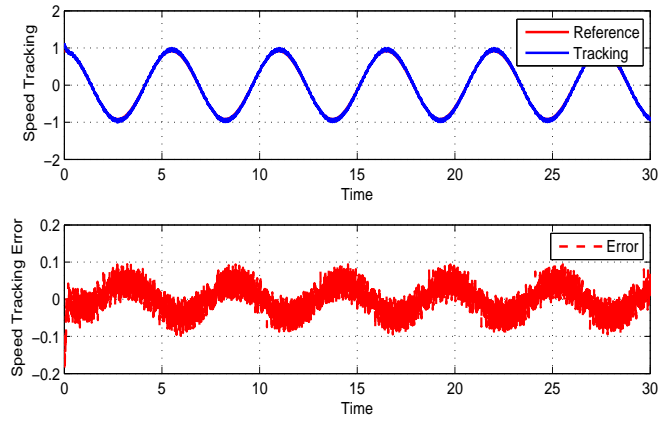


Figure 14: Speed tracking and tracking error of PINN method

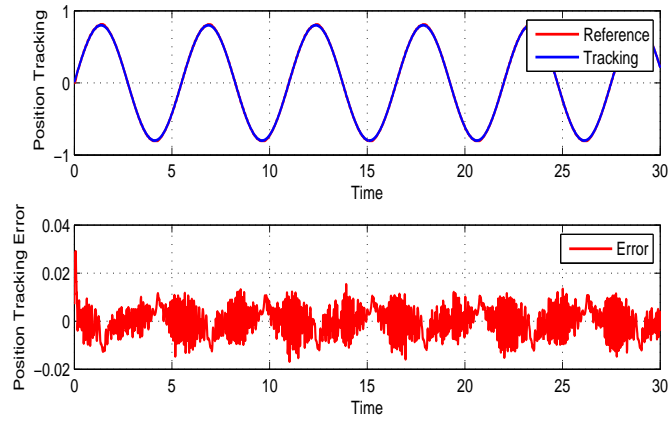


Figure 15: Position tracking and tracking error of the proposed method

three controllers are depicted in Figures 11-16, where the tracking performance of the drive system with proposed control scheme and FDO is given in Figures 15-16, and as a comparative result, the PID and PINN control methods are given in Figures 11-14. It is clearly see from Figures 11-12 that there are speed vibrations, and the output can not accurately track the reference signal. Figures 13-14 show that the tracking performance is better than PID control method, but worse than proposed control scheme which is shown in Figures 15-16. It is

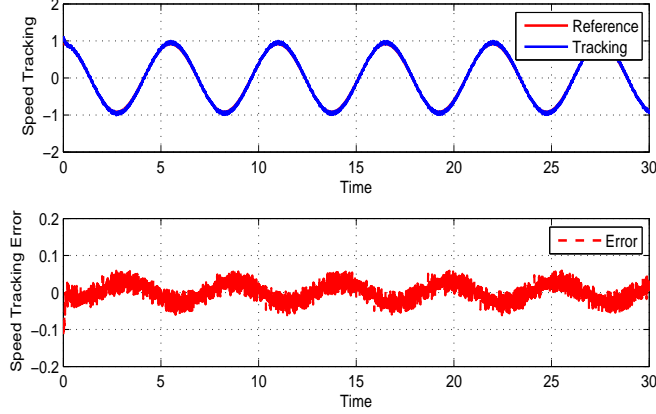


Figure 16: Speed tracking and tracking error of the proposed method

clear from the comparison of three control methods that the proposed control scheme produces a very good tracking performance, and there is no obvious vibration.

To further compare the performance of different controllers, two indices are adopted: 1) Integrated absolute error $IAE = \int |e(t)|dt$; 2) Integrated square error: $ISDE = \int (e(t) - e_0)^2 dt$ (e_0 is the mean value of the error). For fair comparison, the proposed control scheme, PID and PINN control methods for the same reference signal $x_r = 0.8\sin(2\pi t/5.5)$, and the controller parameters are fixed. The experimental results in terms of performance indices are given in Table 2. From Table 2, it is clearly shown that the proposed control scheme outperforms the other two controllers in term of IAE and ISDE, and thus provides better control, it can acquire smaller tracking error.


Remark 3: The control methods reported in [1], [6] considered the linear two inertia systems, where the  nonlinearities including uncertain and extern disturbance are ignored. These methods may be able to achieve stable control of the two-inertia system.. However, the nonlinearities can affect the performance of control system. In this paper, we proposed an alternative control, which is able to address the unknown disturbance by using dynamic surface control

Table 2: Performance comparison for difference methods.

$x_r = 0.8\sin(2\pi t/5.5)$				
	Position error		Speed error	
	IAE	ISDE	IAE	ISDE
PID	0.5949	0.0139	2.2941	0.2481
PINN	0.3699	0.0061	1.0646	0.0572
DSCFDO	0.1898	0.0018	1.0079	0.0559

scheme with FDO.

7. Conclusions

This paper proposed a vibration suppression control design scheme for two-inertia systems using the adaptive DSC based on FDO. The lumped disturbance including the uncertainty and disturbance encountered in the two-inertia system was estimated by the FDO without requiring a prior information. The vibration of the two-inertia system was effectively suppressed by the proposed DSC design procedure based on the estimation values. The semiglobally uniform ultimate boundedness of all closed-loop signals can be guaranteed and the convergence of the tracking error to an arbitrary small residual set can be achieved. Simulation and experiment results illustrated the effectiveness of proposed method.

Acknowledgement

This work is support by the National Natural Science Foundation of China (61433003, 61273150, 61203066, 61321002), the Research Fund for the Doctoral Program of Higher Education of China (20121101110029).

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